534hw1

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Problem 1

```r
det <- function(R){
  eigenvalues <- eigen(R, only.values = TRUE)$values
  det <- prod(abs(eigenvalues))
  return(det)
}

logdet <- function(R){
  return(log(det(R)))
}
```

We verify the function we wrote by computing the logarithm of the determinant of a 2×2 matrix:

$$M = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

The determinant of M is:

$$\det(M) = 3 \times 4 - 1 \times 2 = 10$$

Taking the logarithm:

$$\log(\det(M)) = \log 10 \approx 2.302585$$

```r
M <- matrix(c(3,1,2,4), nrow = 2, byrow = TRUE)
cat("The logarithm of determinant M:", logdet(M), "\n")
```

```
## The logarithm of determinant M: 2.302585
```

This result aligns with our value of $\log(\det(M))$.

Problem 2

The marginal likelihood of [1|A] is given by:

$$p(D_1, D_A \mid [1|A]) = \frac{\Gamma\left(\frac{n+|A|+2}{2}\right)}{\Gamma\left(\frac{|A|+2}{2}\right)} \cdot (\det M_A)^{-1/2} \cdot \left(1 + D_1^T D_1 - D_1^T D_A M_A^{-1} D_A^T D_1\right)^{-(n+|A|+2)/2}$$

Taking the logarithm we have:

$$\log p(D_1, D_A \mid [1|A]) = \log\Gamma\left(\frac{n+|A|+2}{2}\right) - \log\Gamma\left(\frac{|A|+2}{2}\right) - \frac{1}{2}\log(\det M_A) - \frac{n+|A|+2}{2}\log(1 + D_1^T D_1 - D_1^T D_A M_A^{-1} D_A^T D_1$$

```
logmarglik <- function(data, A){
  n = nrow(data)
  a = length(A)

  D_1 = data[, 1]
  D_A = data[, A]
  I_A <- diag(a)
  M_A = I_A + t(D_A) %*% D_A

  first_term <- lgamma((n + a + 2) / 2) - lgamma((a + 2) / 2)
  second_term <- (-1 / 2) * logdet(M_A)
  third_term <- -(n + a + 2) / 2 * log((1 + t(D_1) %*% D_1 - t(D_1) %*% D_A %*% solve(M_A) %*% t(D_A) %

  ans <- first_term + second_term + third_term
  return (ans)
}

mypath <- "C:/Users/ncwbr/Desktop/erdata.txt"
data <- as.matrix(read.table(mypath, header = FALSE))
cat("The dimension of the data:", dim(data), "\n")
```

```
## The dimension of the data: 158 51
```

```
cat("Result:", logmarglik(data, c(2,5,10)), "\n")
```

```
## Result: -59.97893
```