STAT534HW4

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Problem 1

```r
inverseLogit <- function(x) {
  return (exp(x) / (1 + exp(x)))
}

inverseLogit2 <- function(x) {
  return (exp(x) / (1 + exp(x)) ^ 2)
}

getPi <- function(x, beta) {
  x0 <- cbind(rep(1, length(x)), x)
  return (inverseLogit(x0 %*% beta))
}

getPi2 <- function(x, beta) {
  x0 <- cbind(rep(1, length(x)), x)
  return (inverseLogit2(x0 %*% beta))
}

logisticLoglik <- function(y, x, beta) {
  Pi <- getPi(x, beta)
  return (sum(y * log(Pi)) + sum((1 - y) * log(1 - Pi)))
}

logisticLoglik_star <- function(y, x, beta) {
  Loglik <- logisticLoglik(y, x, beta)
  return (-log(2 * pi) - (1 / 2) * (beta[1]^2 + beta[2]^2) + Loglik)
}

getGradient <- function(y, x, beta) {
  gradient <- matrix(0, 2, 1)
  Pi = getPi(x, beta)

  gradient[1, 1] <- sum(y - Pi) - beta[1]
  gradient[2, 1] <- sum((y - Pi) * x) - beta[2]

  return (gradient)
}
```

```r
getHessian <- function(y, x, beta){
  hessian <- matrix(0, 2, 2)
  Pi2 <- getPi2(x, beta)

  hessian[1, 1] <- sum(Pi2) + 1
  hessian[1, 2] <- sum(Pi2 * x)
  hessian[2, 1] <- hessian[1, 2]
  hessian[2, 2] <- sum(Pi2 * x^2) + 1
  return (-hessian)
}

getcoefNR <- function(response, explanatory, beta) {
  beta <- matrix(0, 2, 1)
  y <- data[, response]
  x <- data[, explanatory]
  currentLoglik <- logisticLoglik_star(y, x, beta)

  while(1)
  {
    # Compute update: new beta = old beta - solve(H) * gradient
    H <- getHessian(y, x, beta)
    G <- getGradient(y, x, beta)
    newBeta <- beta - drop(solve(H) %*% G)
    newLoglik <- logisticLoglik_star(y, x, newBeta)

    if (newLoglik < currentLoglik)
    {
      cat("COGING ERROR!!\n")
      break
    }

    # Update beta and log-posterior
    beta <- newBeta

    # Stop if improvement is very small
    if (newLoglik - currentLoglik < 1e-4){
      break
    }
    currentLoglik <- newLoglik
  }
  return (beta)
}

# Laplace approximation of marginal likelihood p(D)
getLaplaceApprox <- function(response, explanatory, data, betaMode) {
  y <- data[, response]
  x <- data[, explanatory]
  beta <- betaMode
  # Compute penalized log-posterior at mode
  Loglik_star <- logisticLoglik_star(y, x, beta)
  H <- getHessian(y, x, beta)
  return (log(2 * pi) + Loglik_star - 0.5 * log(det(-H)))
}
```

Problem 2

```
getPosteriorMeans <- function(response, explanatory, data, betaMode, niter = 10000) {
  y <- data[, response]
  x <- data[, explanatory]

  # Initialize chain
  beta <- betaMode
  beta_chain <- matrix(0, nrow = niter, ncol = 2)

  # Sampling loop from 1 to niter
  for (i in seq_len(niter)) {
    H <- getHessian(y, x, beta)
    beta_temp <- mvrnorm(1, mu = beta, Sigma = -solve(H))
    diff <- logisticLoglik_star(y, x, beta_temp) - logisticLoglik_star(y, x, beta)

    # Metropolis acceptance
    if (diff >= 0 || log(runif(1)) <= diff) {
      beta <- beta_temp
    }
    beta_chain[i, ] <- beta
  }
  # Compute sample means
  beta_mean <- apply(beta_chain, 2, mean)
  return(beta_mean)
}
```

Problem 3

```
bayesLogistic <- function(apredictor, response, data, niter){
  # Find the MAP estimate of beta via Newton-Raphson
  betaMode <- getcoefNR(response, apredictor, data)
  log_marglik <- getLaplaceApprox(response, apredictor, data, betaMode)
  beta_mean <- getPosteriorMeans(response, apredictor, data, betaMode, niter)

  list(apredictor = apredictor,
       logmarglik = log_marglik,
       beta0bayes = beta_mean[1],
       beta1bayes = beta_mean[2],
       beta0mle = betaMode[1],
       beta1mle = betaMode[2])
}

main <- function(datafile, NumberOfIterations, clusterSize)
{
  data = read.table(datafile, header=FALSE);

  response = ncol(data);
  lastPredictor = ncol(data)-1;

  cl <- makeCluster(clusterSize, type = "SOCK")
```

```r
  clusterExport(cl, "data")
  clusterExport(cl, c("bayesLogistic",
                     "getcoefNR", "getLaplaceApprox", "getPosteriorMeans",
                     "inverseLogit", "inverseLogit2",
                     "getPi", "getPi2",
                     "logisticLoglik", "logisticLoglik_star",
                     "getGradient", "getHessian"))
  clusterEvalQ(cl, library(MASS))

  results = clusterApply(cl, 1: lastPredictor, bayesLogistic,
                        response, data,NumberOfIterations);

  for(i in 1: lastPredictor)
  {
    cat('Regression of Y on explanatory variable ',results[[i]]$apredictor,
        ' has log marginal likelihood ',results[[i]]$logmarglik,
        '\n',
        ' with beta0 = ',results[[i]]$beta0bayes,' (',results[[i]]$beta0mle,')',
        ' and beta1 = ',results[[i]]$beta1bayes,' (',results[[i]]$beta1mle,')',
        '\n');
  }

  stopCluster(cl);
}

library(MASS)
library(parallel)
library(snow)
```

```
##
## Attaching package: 'snow'

## The following objects are masked from 'package:parallel':
##
##     closeNode, clusterApply, clusterApplyLB, clusterCall, clusterEvalQ,
##     clusterExport, clusterMap, clusterSplit, makeCluster, parApply,
##     parCapply, parLapply, parRapply, parSapply, recvData, recvOneData,
##     sendData, splitIndices, stopCluster
```

```r
path <- "C:/Users/ncwbr/Desktop/534binarydata.txt"
data <- as.matrix(read.table(path, header = FALSE))
set.seed(2427348)
response <- 61
main(path, 10000, 10)
```

```
## Regression of Y on explanatory variable  1  has log marginal likelihood  -84.17025
##   with beta0 =  -0.7561469  ( -0.7549792 )  and beta1 =  0.9862237  ( 0.9842621 )
## Regression of Y on explanatory variable  2  has log marginal likelihood  -91.13804
##   with beta0 =  -0.7724316  ( -0.7785309 )  and beta1 =  -0.635131  ( -0.6468523 )
## Regression of Y on explanatory variable  3  has log marginal likelihood  -89.27079
##   with beta0 =  -0.7767911  ( -0.7837034 )  and beta1 =  0.7313956  ( 0.7288721 )
## Regression of Y on explanatory variable  4  has log marginal likelihood  -94.74949
```

```
##   with beta0 =  -0.7311459  ( -0.7332059 )  and beta1 =  0.3684432  ( 0.3689551 )
## Regression of Y on explanatory variable  5  has log marginal likelihood  -92.78677
##   with beta0 =  -0.7397618  ( -0.7550132 )  and beta1 =  0.5213229  ( 0.5229461 )
## Regression of Y on explanatory variable  6  has log marginal likelihood  -94.44806
##   with beta0 =  -0.7312794  ( -0.7327855 )  and beta1 =  0.3903294  ( 0.3891122 )
## Regression of Y on explanatory variable  7  has log marginal likelihood  -95.40641
##   with beta0 =  -0.7330991  ( -0.7273442 )  and beta1 =  0.3168921  ( 0.3108188 )
## Regression of Y on explanatory variable  8  has log marginal likelihood  -88.02388
##   with beta0 =  -0.7779883  ( -0.7780915 )  and beta1 =  0.7687005  ( 0.7713092 )
## Regression of Y on explanatory variable  9  has log marginal likelihood  -95.46512
##   with beta0 =  -0.7146493  ( -0.7135338 )  and beta1 =  0.311696  ( 0.2985043 )
## Regression of Y on explanatory variable  10  has log marginal likelihood  -94.84316
##   with beta0 =  -0.7208622  ( -0.7318991 )  and beta1 =  0.3697513  ( 0.3669237 )
## Regression of Y on explanatory variable  11  has log marginal likelihood  -91.98299
##   with beta0 =  -0.7512826  ( -0.7623398 )  and beta1 =  0.5564607  ( 0.5647126 )
## Regression of Y on explanatory variable  12  has log marginal likelihood  -96.1455
##   with beta0 =  -0.7292766  ( -0.7209605 )  and beta1 =  -0.2243654  ( -0.2290757 )
## Regression of Y on explanatory variable  13  has log marginal likelihood  -92.13999
##   with beta0 =  -0.7581035  ( -0.7556561 )  and beta1 =  0.557884  ( 0.549674 )
## Regression of Y on explanatory variable  14  has log marginal likelihood  -95.39082
##   with beta0 =  -0.7223671  ( -0.7272474 )  and beta1 =  0.3209494  ( 0.3115849 )
## Regression of Y on explanatory variable  15  has log marginal likelihood  -91.25791
##   with beta0 =  -0.7780403  ( -0.7844429 )  and beta1 =  0.6605895  ( 0.672372 )
## Regression of Y on explanatory variable  16  has log marginal likelihood  -93.49182
##   with beta0 =  -0.7444724  ( -0.7484474 )  and beta1 =  0.4871921  ( 0.4788875 )
## Regression of Y on explanatory variable  17  has log marginal likelihood  -91.31417
##   with beta0 =  -0.7657243  ( -0.7706116 )  and beta1 =  -0.6377286  ( -0.6234695 )
## Regression of Y on explanatory variable  18  has log marginal likelihood  -89.08946
##   with beta0 =  -0.7695707  ( -0.7797449 )  and beta1 =  0.7090495  ( 0.7143947 )
## Regression of Y on explanatory variable  19  has log marginal likelihood  -90.42463
##   with beta0 =  -0.7363493  ( -0.7464287 )  and beta1 =  0.7096877  ( 0.720492 )
## Regression of Y on explanatory variable  20  has log marginal likelihood  -94.86778
##   with beta0 =  -0.7385129  ( -0.7317445 )  and beta1 =  0.3560969  ( 0.3590619 )
## Regression of Y on explanatory variable  21  has log marginal likelihood  -85.74706
##   with beta0 =  -0.816563  ( -0.8156821 )  and beta1 =  0.8669246  ( 0.880807 )
## Regression of Y on explanatory variable  22  has log marginal likelihood  -84.03714
##   with beta0 =  -0.8234537  ( -0.828694 )  and beta1 =  0.9536007  ( 0.9552211 )
## Regression of Y on explanatory variable  23  has log marginal likelihood  -79.48414
##   with beta0 =  -0.8899663  ( -0.8866648 )  and beta1 =  1.202566  ( 1.211364 )
## Regression of Y on explanatory variable  24  has log marginal likelihood  -94.15746
##   with beta0 =  -0.7330072  ( -0.7385748 )  and beta1 =  -0.4186187  ( -0.4194562 )
## Regression of Y on explanatory variable  25  has log marginal likelihood  -96.58153
##   with beta0 =  -0.7185706  ( -0.7163197 )  and beta1 =  0.1678669  ( 0.1628502 )
## Regression of Y on explanatory variable  26  has log marginal likelihood  -91.73426
##   with beta0 =  -0.7343433  ( -0.7415221 )  and beta1 =  0.5618195  ( 0.5674934 )
## Regression of Y on explanatory variable  27  has log marginal likelihood  -90.40866
##   with beta0 =  -0.7679609  ( -0.7779678 )  and beta1 =  -0.645742  ( -0.6583559 )
## Regression of Y on explanatory variable  28  has log marginal likelihood  -95.21228
##   with beta0 =  -0.7289214  ( -0.7292691 )  and beta1 =  0.3274924  ( 0.329432 )
## Regression of Y on explanatory variable  29  has log marginal likelihood  -91.99002
##   with beta0 =  -0.7592473  ( -0.7606315 )  and beta1 =  0.5617578  ( 0.5644083 )
## Regression of Y on explanatory variable  30  has log marginal likelihood  -90.80862
##   with beta0 =  -0.7931392  ( -0.7833993 )  and beta1 =  0.6706027  ( 0.669029 )
## Regression of Y on explanatory variable  31  has log marginal likelihood  -90.88962
```

```
##   with beta0 =  -0.7721337  ( -0.7738447 )  and beta1 =  0.6363826  ( 0.6330321 )
## Regression of Y on explanatory variable  32  has log marginal likelihood  -90.48715
##   with beta0 =  -0.7926233  ( -0.7975917 )  and beta1 =  -0.7293692  ( -0.7260067 )
## Regression of Y on explanatory variable  33  has log marginal likelihood  -95.73493
##   with beta0 =  -0.7249628  ( -0.7246871 )  and beta1 =  0.2736161  ( 0.2782866 )
## Regression of Y on explanatory variable  34  has log marginal likelihood  -89.09465
##   with beta0 =  -0.7919612  ( -0.8035621 )  and beta1 =  -0.7521065  ( -0.7591494 )
## Regression of Y on explanatory variable  35  has log marginal likelihood  -96.68771
##   with beta0 =  -0.7150878  ( -0.7153149 )  and beta1 =  0.1350395  ( 0.1427644 )
## Regression of Y on explanatory variable  36  has log marginal likelihood  -96.73543
##   with beta0 =  -0.713951  ( -0.7149075 )  and beta1 =  0.1313825  ( 0.1318086 )
## Regression of Y on explanatory variable  37  has log marginal likelihood  -83.42107
##   with beta0 =  -0.8385768  ( -0.8374759 )  and beta1 =  0.9973077  ( 0.9953003 )
## Regression of Y on explanatory variable  38  has log marginal likelihood  -93.49103
##   with beta0 =  -0.7400759  ( -0.7509762 )  and beta1 =  -0.4660063  ( -0.4788535 )
## Regression of Y on explanatory variable  39  has log marginal likelihood  -87.4932
##   with beta0 =  -0.8016122  ( -0.8049518 )  and beta1 =  0.8105628  ( 0.8285465 )
## Regression of Y on explanatory variable  40  has log marginal likelihood  -91.01029
##   with beta0 =  -0.7760627  ( -0.7687116 )  and beta1 =  0.6382831  ( 0.6320124 )
## Regression of Y on explanatory variable  41  has log marginal likelihood  -91.2524
##   with beta0 =  -0.7639219  ( -0.7622365 )  and beta1 =  -0.6022009  ( -0.6010363 )
## Regression of Y on explanatory variable  42  has log marginal likelihood  -85.78488
##   with beta0 =  -0.8217979  ( -0.8275532 )  and beta1 =  -0.9178428  ( -0.9140379 )
## Regression of Y on explanatory variable  43  has log marginal likelihood  -94.16321
##   with beta0 =  -0.7425195  ( -0.7391006 )  and beta1 =  -0.4169062  ( -0.4184341 )
## Regression of Y on explanatory variable  44  has log marginal likelihood  -95.9192
##   with beta0 =  -0.7299893  ( -0.7226125 )  and beta1 =  0.2590645  ( 0.2565493 )
## Regression of Y on explanatory variable  45  has log marginal likelihood  -95.12644
##   with beta0 =  -0.7272501  ( -0.7314595 )  and beta1 =  -0.3407335  ( -0.3415704 )
## Regression of Y on explanatory variable  46  has log marginal likelihood  -86.60919
##   with beta0 =  -0.8184944  ( -0.8196287 )  and beta1 =  0.8992378  ( 0.8998076 )
## Regression of Y on explanatory variable  47  has log marginal likelihood  -91.62257
##   with beta0 =  -0.7629485  ( -0.7626076 )  and beta1 =  0.5716904  ( 0.5824422 )
## Regression of Y on explanatory variable  48  has log marginal likelihood  -91.46699
##   with beta0 =  -0.7702703  ( -0.7625558 )  and beta1 =  0.5928079  ( 0.5937575 )
## Regression of Y on explanatory variable  49  has log marginal likelihood  -90.41233
##   with beta0 =  -0.7658003  ( -0.76442 )  and beta1 =  0.6301173  ( 0.6375997 )
## Regression of Y on explanatory variable  50  has log marginal likelihood  -92.4211
##   with beta0 =  -0.7502928  ( -0.7521175 )  and beta1 =  -0.5388768  ( -0.5359225 )
## Regression of Y on explanatory variable  51  has log marginal likelihood  -95.32329
##   with beta0 =  -0.7258747  ( -0.7274459 )  and beta1 =  -0.3257307  ( -0.3187631 )
## Regression of Y on explanatory variable  52  has log marginal likelihood  -93.79355
##   with beta0 =  -0.7473147  ( -0.7427776 )  and beta1 =  0.4398933  ( 0.4475682 )
## Regression of Y on explanatory variable  53  has log marginal likelihood  -93.90333
##   with beta0 =  -0.7402172  ( -0.7467168 )  and beta1 =  -0.4503396  ( -0.4491826 )
## Regression of Y on explanatory variable  54  has log marginal likelihood  -91.25855
##   with beta0 =  -0.7778235  ( -0.7743789 )  and beta1 =  0.6289296  ( 0.6345326 )
## Regression of Y on explanatory variable  55  has log marginal likelihood  -90.68376
##   with beta0 =  -0.773015  ( -0.7741314 )  and beta1 =  0.6490587  ( 0.6443876 )
## Regression of Y on explanatory variable  56  has log marginal likelihood  -90.96067
##   with beta0 =  -0.75884  ( -0.7561731 )  and beta1 =  0.5967838  ( 0.6058147 )
## Regression of Y on explanatory variable  57  has log marginal likelihood  -95.2976
##   with beta0 =  -0.7213369  ( -0.7280938 )  and beta1 =  0.3210921  ( 0.3208882 )
## Regression of Y on explanatory variable  58  has log marginal likelihood  -92.44812
```

```
##   with beta0 =  -0.7429134  ( -0.7473734 )  and beta1 =  0.5347898  ( 0.5307593 )
## Regression of Y on explanatory variable  59  has log marginal likelihood  -97.02482
##   with beta0 =  -0.7106043  ( -0.7121179 )  and beta1 =  0.01292713  ( 0.01142458 )
## Regression of Y on explanatory variable  60  has log marginal likelihood  -93.98176
##   with beta0 =  -0.7345481  ( -0.738898 )  and beta1 =  -0.4355138  ( -0.4319359 )
```