Math 425	Applied & Comput. Lin. Algebra	Fall 2024
Homework		

Homework II

For this homework, include all code and computations in a `MATLAB` file named `math425hw2.m`. You will need to submit this file along with a document containing your answers which do not involve `MATLAB`.

1.a) Create a function called `myPartialPivot` which takes as input an $n \times n$ matrix A. The output is an $n \times n$ matrix U which is upper triangular. This time we are not assuming that A is regular, i.e., along the way some pivots could be zero. Even if a pivot is not zero, use partial pivoting to identify a better pivot to continue with Gaussian elimination.
b) Create a function called `myRank` which takes as input an $n \times n$ matrix A and computes the rank of A using `myPartialPivot`.
c) To test your function `myRank`, generate a random 5×3 matrix P and a random 3×5 matrix Q. Let $A = PQ$. Does your function compute the rank of A to be 3?

2. Suppose an $n \times n$ matrix A is *strictly column diagonally dominant.* This means that for each $j = 1, \ldots, n$

$$|a_{jj}| > \sum_{i \neq j}^{n} |a_{ij}|.$$

a) Give an example of a 4×4 strictly column diagonally dominant matrix which is not a diagonal matrix.
b) Show that if Gaussian elimination with partial pivoting is used on a strictly column diagonally dominant matrix no row interchanges occur.
c) Modify `myPartialPivot` slightly so that it counts and prints the number of row interchanges during the application of the function. Then test this function on your example from part **a)**. The number of row interchanges should be zero.

3. Let A be an $n \times n$ symmetric matrix. Describe a strategy of *symmetric pivoting* so that after the Gaussian elimination the matrix A is reduced to a diagonal matrix D. Make sure to argue that after each symmetric pivoting the resulting matrix is still symmetric. [Hint: this will require both row and column operations]