Homework 5

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November 9, 2024

1. Find the closest point from $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ -2 \end{pmatrix}$ to the subspace spanned by

$$\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}.$$

Use any `MATLAB` command that you think is useful to do this computation.

`Some further explanation on my computation on MATLAB:`
Since there is no solution to the original problem of $rref([A|b])$ where A is the matrix with the three augmented vectors. We need to compute the least square solution. To do that, I compute via row operations $A^TAx = A^Tb$ where A^TA becomes a symmetric matrix and A^Tb will be a vector.

$$\hat{A} = A'A = \begin{pmatrix} 6 & 4 & -2 \\ 4 & 6 & -8 \\ -2 & -8 & 14 \end{pmatrix}$$

$$\hat{b} = A'b = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$rref([\hat{A}|\hat{b}]) = \left(\begin{array}{ccc|c} 6 & 4 & -2 & 1 \\ 4 & 6 & -8 & -1 \\ -2 & -8 & 14 & 3 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & -2 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The rref shows that there is a free variable for the third unknown variable. We can assign that variable, let's say, z, as an arbitrary value t. Thus, we get the three equations: $x = \frac{1}{2} - t$, $y = -\frac{1}{2} + 2t$, and $z = t$. Now, putting the equation together as a vector:

$$v = \begin{pmatrix} \frac{1}{2} - t \\ -\frac{1}{2} + 2t \\ t \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

Since t is an arbitrary value and z is a linear combination of the previous two vectors, we can take the basis vector and multiply it with A, giving us the closest points:

$$A * v = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ -1 & -2 & 3 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

Running MATLAB A/b yields the same solution.

4. Let $f(x) = x^2$ on the interval $[0, 2\pi]$. In this exercise, you will compute the discrete Fourier coefficients $c_0, c_1, \ldots, c_7$ of f from the sample vector

$$\mathbf{f} = \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_7 \end{pmatrix}$$

where $f_j = f(j2\pi/8)$ for $j = 0, \ldots, 7$.

Part d) Explanation of computing $p(x)$ is explained as a comment in the MATLAB code.

e) Find out how you can plot the graph of a function in `MATLAB`. Then plot the graphs of $f(x)$ and $p_1(x)$ on the interval $[0, 2\pi]$. What do you see?

Graph is rendered on Matlab code.
Analysis of the $p_1(x)$ graph:
The graph has a sinusoidal wave that has a few points that agree with $f(x) = x^2$. However, it does have a lot of "noise" and roughly follows the original $f(x)$ line. One peculiar aspect of the $p_1(x)$ graph is the sinusoidal wave alternation from being above the original x^2 to following it and ending up below it as it approached 2π. Another noteworthy aspect of the $p_1(x)$ graph is the sudden drop off at $7\pi/4$ from following the x^2 line to 0. Overall, the graph approximately follows the original signal line, but only by agreeing at specific points.

h) Plot the graph of $f(x)$ and $q_1(x)$ on the interval $[0, 2\pi]$. Now, what do you observe?

Graph is rendered on Matlab code.
Analysis of the $q_1(x)$ graph:
The graph is still a tiny bit wavy, but it follows the original x^2 graph much more

closely. Especially between the points $\pi/2$ and $3\pi/2$, the $q_1(x)$ graph hovers/on the x^2 line. The points between 0 and $\pi/2$ also seem to be a shifted sinusoidal wave before seemingly being forced to wrap on the x^2 graph afterward. There is still a sudden drop off at around $7\pi/4$ from following the x^2 line to 0. Overall, the $q_1(x)$ graph better represents the x^2 graph but is still only an approximation that is more accurate than $p_1(x)$.