Math 425 Applied & Comput. Lin. Algebra Fall 2024
Homework

Homework III

For this homework, include all code and computations in a `MATLAB` file named `math425hw3.m`. You will need to submit this file along with a document containing your answers which do not involve `MATLAB`.

1.a) Let $\mathbf{v} \in \mathbb{R}^m$ and $\mathbf{w} \in \mathbb{R}^n$. Show that the $m \times n$ matrix $\mathbf{v}\mathbf{w}^T$ has rank equal to 1.
b) Conversely, show that if A is an $m \times n$ matrix with $\text{rank}(A) = 1$, then $A = \mathbf{v}\mathbf{w}^T$ for some $\mathbf{v} \in \mathbb{R}^m$ and $\mathbf{w} \in \mathbb{R}^n$.

2. Is $\begin{pmatrix} 3 \\ 0 \\ -1 \\ -2 \end{pmatrix}$ a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -1 \\ 3 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \end{pmatrix}$? Give an answer using `MATLAB`.

3. Let

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \mathbf{v}_4 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}.$$

Use `MATLAB`, if convenient, to answer the following questions.
a) Do $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ span $\mathbb{R}^3$? Why or why not?
b) Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ linearly independent ? Why or why not?
c) Do $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ form a basis for $\mathbb{R}^3$? Why or why not? If not, is it possible to choose some subset which is a basis?
d) What is the dimension of the span of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$? Justify your answer.

4.a) Create a function called `myGS` which takes as input an $m \times n$ matrix A where $\text{rank}(A) = n \leq m$. The output is an $m \times n$ matrix B whose columns form an orthonormal basis of the vector space spanned by the columns of A. Use the Gram-Schmidt process.
b) Use `myGS` to compute an orthonormal basis for $\mathbb{R}^4$ starting with the following set of vectors:

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

c) Modify your function to `myGS2` so that it computes an orthonormal basis "on the fly" (as we have learned last week). Use `myGS2` on the input in part **b)**.