| Math 425 | Applied & Comput. Lin. Algebra   Fall 2024 |
| Homework | |

Homework V

For this homework, include all code and computations in a `MATLAB` file named `math425hw5.m`. You will need to submit this file along with a document containing your answers which do not involve `MATLAB`. **Do not submit a zipped (compressed) folder**.

1. Find the closest point from $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ -2 \end{pmatrix}$ to the subspace spanned by

$$\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}.$$

Use any `MATLAB` command that you think is useful to do this computation.

2. Find the least squares solution to the system $A\mathbf{x} = \mathbf{b}$ in *two* ways using `MATLAB` where

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -2 & 3 \\ 1 & 5 & -1 \\ -3 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 0 \\ 5 \\ 6 \\ 8 \end{pmatrix}.$$

However, you are not allowed to use `b \ A`.

3. The median price (in thousands of dollars) of existing homes in the Minneapolis metropolitan area from 1989 to 1999 was:

| year | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 |
| price | 86.4 | 89.8 | 92.8 | 96.0 | 99.6 | 103.1 | 106.3 | 109.5 | 113.3 | 120.0 | 129.5 |

First find an equation of the least squares line for these data using `MATLAB`. Then use the result to estimate the median price of a house in the year 2005, and the year 2010, assuming the trend continues.

4. Let $f(x) = x^2$ on the interval $[0, 2\pi]$. In this exercise you will compute the discrete Fourier coefficients $c_0, c_1, \ldots, c_7$ of f from the sample vector

$$\mathbf{f} = \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_7 \end{pmatrix}$$

where $f_j = f(j2\pi/8)$ for $j = 0, \ldots, 7$.
a) Compute the sample vector $\mathbf{f}$ in `MATLAB`.

b) Next compute the vectors $\omega_k = (\zeta_8^{0k}, \zeta_8^{1k}, \zeta_8^{2k}, \ldots, \zeta_8^{7k})^T$ for $k = 0, 1, \ldots, 7$ where $\zeta_8 = e^{i(2\pi/8)}$. In `MATLAB` the complex exponential can be computed by `exp(i*2*pi/8)`.

c) Now compute $c_k = \langle \mathbf{f}, \omega_k \rangle$ for $k = 0, \ldots, 7$. The complex inner product $\langle v, w \rangle$ is computed by `dot(v,w)` in `MATLAB`. However, do not forget to scale the result by $1/8$ since we are using the scaled inner product for the discrete Fourier transform.

d) With the discrete Fourier coefficients you have computed, set $p(x) = c_0 \cdot 1 + c_1 e^{ix} + c_2 e^{i2x} + \cdots + c_7 e^{i7x}$ where $e^{ijx} = \cos(jx) + i\sin(jx)$. Write $p(x) = p_1(x) + ip_2(x)$ where $p_1(x)$ and $p_2(x)$ are real-valued functions. Compute $p_1(x)$. This is the function that reconstructs $f(x)$.

e) Find out how you can plot the graph of a function in `MATLAB`. Then plot the graphs of $f(x)$ and $p_1(x)$ on the interval $[0, 2\pi]$. What do you see?

f) Now we will compute a modified version of a reconstruction of the signal $f(x) = x^2$ on $[0, 2\pi]$. The discrete Fourier coefficients we will compute are $c_{-4}, c_{-3}, \ldots, c_3$. For this you will need ω_k for $k = -4, -3, \ldots, 3$. And $c_k = \langle \mathbf{f}, \omega_k \rangle$ for $k = -4, -3, \ldots, 3$. Again do not forget to scale the inner product by $1/8$.

g) Let $q(x) = c_{-4} e^{i(-4)x} + c_{-3} e^{i(-3)x} + \cdots + c_3 e^{i3x}$. Write $q(x) = q_1(x) + iq_2(x)$, and compute $q_1(x)$.

h) Plot the graph of $f(x)$ and $q_1(x)$ on the interval $[0, 2\pi]$. Now what do you observe?