

Review: number systems



Z. Jerry Shi

Department of Computer Science and Engineering
University of Connecticut

CSE3666: Introduction to Computer Architecture

Outline

- Binary numbers
 - Binary to decimal
 - Decimal to binary
- Hexadecimal numbers
 - Hexadecimal to decimal
 - Decimal to hexadecimal

`0b1101110` =

`0xC0FFEE` =

<i>n</i>	<i>2ⁿ</i>
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

Decimal number

- We are familiar with decimal numbers
 - The radix is 10
- There are ten digits: 0, 1, 2, ..., 8, 9

Example: 3 6 6 6

$$\begin{aligned} 3666 &= 3 \times 10^3 + 6 \times 10^2 + 6 \times 10^1 + 6 \times 10^0 \\ &= 3000 + 600 + 60 + 6 \end{aligned}$$

In this example, each 6 carries different values.

Each time a digit moves to left by one place, the value is increased by 10.

Does the radix have to be 10?

Binary numbers

- If the radix is 2, we have binary numbers
 - We only need two digits, 0 and 1
 - Each digit is also called a **bit** (a **binary digit**)

Given a sequence of bits:

$$b_{n-1} b_{n-2} \dots b_2 b_1 b_0$$

The value is

$$b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \dots + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

b_{n-1} is the most significant bit (MSB). b_0 is the least significant bit (LSB).

Example

- Convert a binary number to a decimal number

11010101

128	64	32	16	8	4	2	1
b_7	b_6	b_5	b_4	b_3	b_2	b_1	b_0
1	1	0	1	0	1	0	1

← Weights

The answer is:

$$\begin{aligned} &1 \times 2^7 + 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^0 \\ &= 128 + 64 + 16 + 4 + 1 \\ &= 213 \end{aligned}$$

Decimal to binary

- Convert decimal number 78 to a binary number.
 - Human method. Do arithmetic in decimal.

78

128	64	32	16	8	4	2	1
b_7	b_6	b_5	b_4	b_3	b_2	b_1	b_0

← Weights

Bits we need to figure out

We start from b_0 . What is the value of b_0 ?

Decimal to binary - 2

78

128	64	32	16	8	4	2	1
b_7	b_6	b_5	b_4	b_3	b_2	b_1	b_0
							0

The right most bit is 0 because 78 is even, or when 78 is divided by 2, the remainder is 0

$$78 \% 2 = 0$$

$$78 = 39 * 2 + 0$$

How about the rest of the bits?

Decimal to binary - 3

78

128	64	32	16	8	4	2	1
b_7	b_6	b_5	b_4	b_3	b_2	b_1	b_0
						1	0

We have figured out b_0 by dividing 78 by 2.

$$78 = 39 * 2 + 0$$

To figure out b_1 , we divide the quotient 39 by 2. The remainder is 1.
So b_1 is 1

$$39 = 19 * 2 + 1$$

Decimal to binary: Complete steps

We repeat dividing the quotient by 2, until it is 0.

Number	Quotient // 2	Remainder % 2	Note
78	39	0	Remainder is 0. So $b_0 = 0$.
39	19	1	Remainder is 1. So $b_1 = 1$.
19	9	1	Remainder is 1. So $b_2 = 1$.
9	4	1	Remainder is 1. So $b_3 = 1$.
4	2	0	Remainder is 0. So $b_4 = 0$.
2	1	0	Remainder is 0. So $b_5 = 0$.
1	0	1	Remainder is 1. So $b_6 = 1$.

The answer is 0b 100 1110.

We will use 0b or a subscript of 2 for bits

Hexadecimal

- The radix is 16
 - There are 16 digits: 0-9, and a - f

Hex digits	0 – 9	a	b	c	d	e	f
Decimal value	0 – 9	10	11	12	13	14	15

- Use 0x or a subscript of 16 to indicate hexadecimal numbers
0xABCD or ABCD₁₆

Conversion between hexadecimal and decimal

The method is similar to the conversion between binary and decimal
The difference: the radix is 16

Example: hex to decimal

Convert 0x368A to a decimal number

16^4	16^3	16^2	16^1	16^0	← Weights
	3	6	8	A	

$$3 \times 16^3 + 6 \times 16^2 + 8 \times 16^1 + 10 \times 16^0 = 13962$$

↑
The right most digit A is
10 in decimal

Example: decimal to hex

Convert 846 to a hexadecimal number

We repeat dividing the numbers by 16. The remainders are hex digits.

Number	Quotient // 16	Remainder % 16	Note
846	52	14	The lowest hex digit is E (14 in decimal)
52	3	4	The next hex digit is 4
3	0	3	The highest digit is 3

So $846 = 0x34E$.

Number in other radices

- The methods are similar to what we have learned for binary and hexadecimal numbers.

Octal numbers (radix is 8)

$$72_8 = 7 \times 8 + 2 = 58$$

Ternary numbers (radix is 3)

$$121_3 = 1 \times 3^2 + 2 \times 3 + 1 = 16$$

Exercises:

Convert 58 back to an octal number and 16 back to a ternary number.

Exercise

- Convert a hexadecimal number to a decimal number
- Convert a hexadecimal number to a decimal number

Randomly pick a number.

Exercises:

[Binary Numbers \(zhijieshi.github.io\)](https://zhijieshi.github.io)

Config: select the number of bits and uncheck “bits are signed”.