① → (a) → $f(x) = x^2 + 4$

$\lambda x.(x^2 + \lambda y.(y))\ (x)(4)$

(b) → $f(a, b) = a + b$

↳ $\lambda a(a + \lambda b(b))$

(c) → $f(x) = x^{-2}$

↳ $\lambda x(x^{-2})$

(d) → $f(x) = x * x^{-1}$

↳ $\lambda x($

② → (A) → $xy = = yx$

↳ não são α-equivalentes pois elas só possuem variáveis independentes

(b) → $\lambda x.x(\lambda y.xy) = = \lambda y.y(\lambda x.yx)$

$\lambda y(x(\lambda y(xy))\{y/x\})$

$\lambda y(y(\lambda y(yy)))$

$\lambda y(y(\lambda x(yy)\{x/y\}))$

$\lambda y(y(\lambda x(yx)))$

não α-equivalentes, e combinadores

$\{y/x\}$ e $\{y/y\}$

(c) → $((\lambda x.x(\lambda xy.xyzy)x)xy) = = ((\lambda y.y(\lambda z.yzwz)y)yx)$

$((\lambda y.x(\lambda y.xyzy)x\{y/x\})xy)$

$((\lambda y.y(\lambda w.yyzy)y)xy$

$((\lambda y.y(\lambda z.yyzy)\{z/y\}y)xy$

③ → $(x(\lambda y.xy))[x \to yz]$

↳ $(yz(\lambda y.yzy))$

4 → a → $(\lambda x.(\lambda y.y*y-(\lambda z.z+x)4)3)2$

↳ $(\lambda y.y*y-(\lambda z.z+2)4)3$

$(3*3-(\lambda z.z+2))4$

$9-(4+2)=3$

b → $(\lambda x.x+(\lambda y.y*y)b)a$

↳ $(a+(\lambda y.y*y))b$

$(a+(b^2))=a+b^2$

c → $(\lambda x.(\lambda y.x+((\lambda x.8)7))6)5$

$(\lambda y.5+((\lambda x.8)7))6$

$(5+((\lambda x.8)7)))$

$5+(8)=13$

d → $((\lambda x.(\lambda y.x+y))3)7$

$((\lambda y.7+y))3$

$(7+3)=10$

5 → a → $(\lambda x.+x\ 1)2$

↳ $(+2\ 1)$

b → $(\lambda x.xx)(\lambda x.xx)$

↳ $(xx)[x\to(\lambda x.xx)]$

$(\lambda x.xx)(\lambda x.xx)$ # loop infinito

c → $(\lambda x.x(xy))(\lambda u.u)$

↳ $(x.(xy))[x\to(\lambda u.u)]$

$(\lambda u.u.(\lambda u.u).y)$

$u.y[u\to\lambda u.u]$

$(\lambda u.u)y=y$

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(d) → $(\lambda y.(\lambda x.y*y+x))\,y$
$(\lambda x.y^2+x)$

(e) → $(\lambda x.((\lambda y.(yx))(\lambda i.i)))(\lambda p.\lambda q.p)$ $x \to (\lambda p.\lambda q.p)$
$((\lambda y.(y.(\lambda p.\lambda q.p)))(\lambda i.i))$ $y \to (\lambda i.i)$
$(\lambda i.(i.(\lambda p.\lambda q.p)))$

(f) → $(\lambda x.x)((\lambda y.(\lambda x.xy))x)$ $y \to x$
$(\lambda x.x)(\lambda x.(x.x))$ $x \to (\lambda x.(xx))$
$\lambda x.(xx)$

(g) → $(\lambda x.xx)(\lambda y.y)$ $x \to (\lambda y\,y)$
$(\lambda y\,y)(\lambda y\,y)$ $y \to (\lambda y\,y)$
$\lambda y\,y$ ++

(c) → and true true
$((\lambda a.(\lambda b.(a\ b\ False)))(\lambda x.\lambda y.x))(\lambda x.\lambda y.x)$ $a \to (\lambda x.\lambda y.x)$
$(\lambda b.(\lambda x.\lambda y.x\ b\ False)(\lambda x.\lambda y.x))$ $b \to (\lambda x\lambda y\,x)$
$((\lambda x.\lambda y.x)(\lambda x.\lambda y.x)\ False)$
$(\lambda x.\lambda y.x)(\lambda x.\lambda y.x)(\lambda x.\lambda y.y)$ $x \to (\lambda x\lambda y\,x)$
$(\lambda y\,\lambda x\lambda y\,x)(\lambda x\lambda y\,y)$ $y \to (\lambda x\lambda y\,y)$
$(\lambda x\,\lambda y\,x) = true$

(b) → and true false
$((\lambda a(\lambda b(a\ b\ false)))(\lambda x\lambda y\,x))(\lambda x\lambda y\,y)$ $a \to (\lambda x\lambda y\,x)$
$(\lambda b((\lambda x\lambda y\,x)\ b\ false))(\lambda x\lambda y\,y)$ $b \to (\lambda x\lambda y\,y)$
$(\lambda x\lambda y\,x)(\lambda x\lambda y\,y)(\lambda x\lambda y.y)$ true false false $x \to (\lambda x\lambda y\,y)$
$\lambda y(\lambda x\lambda y\,y)(\lambda x\lambda y\,y)$ $y \to (\lambda x\lambda y\,y)$
$\lambda x\lambda y\,y = false$

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c → and false false F F

$(\lambda a.(\lambda b.(a\ b\ false)))\ (\lambda x\ \lambda y\ y)\ (\lambda x\ \lambda y\ y)$ a → F

$(\lambda b\ (false\ b\ false))\ false$ b → false

$(false)(false)(false)$

$(\lambda x\ \lambda y\ y)\ f\ f$ x → f

$(\lambda y\ y)\ f$ y → f

f

d → and false true

$(\lambda a.(\lambda b.(a\ b\ false)))\ false\ true$ a → false

$(\lambda b\ (false\ b\ false))\ true$ b → true

$false\ true\ false)$

$(\lambda x\ \lambda y\ y)\ true\ false$ x → true

$(\lambda y\ y)\ false$ y → false

false

e → not true

$(\lambda a.\ a\ false\ true)\ true$ a → true

$(true\ false\ true)$

$(\lambda x\ \lambda y\ x)\ false\ true$ x → false

$\lambda y\ false\ true$ y → true

false

f → not false

$(\lambda a.\ a\ false\ true)\ false$ a → false

$(false\ false)\ true$

$(\lambda x\ \lambda y\ y)\ (false)\ true$ x → false

$(\lambda y\ y)\ true$ y → true

true

g → or true true

$(\lambda a.(\lambda b\ (a\ true\ b)))\ true\ true$ a → true

$(\lambda b\ (true\ true\ b))\ true$ b → true

tilibra $(true\ true\ true)$

$(\lambda x\, \lambda y\, x)$ true true — $x \to$ true

λy true (true) — $y \to$ true

true

(h) $\to$ or true false

$(\lambda a\, (\lambda b\, (a$ true $b)))$ true false — $a \to$ true

$(\lambda b\, ($true true $b))$ false — $b \to$ false

$(\lambda x\, \lambda y\, x)$ true false — $x \to$ true

$(\lambda y$ true$)$ false — $y \to$ false

true

(i) $\to$ or false true

$(\lambda a\, .\, (\lambda b\, (a$ true $b)))$ false true — $a \to$ false

$\lambda b\, ($false true $b)$ true — $b \to$ true

$(\lambda x\, \lambda y\, y)$ true true

true

(j) $\to$ or false false

$(\lambda a\, (\lambda b\, (a$ true $b))$ false false — $a \to$ false

$(\lambda b\, ($false true $b)$ false — $b \to$ false

false true false

false

(K) $\to$ if true a b

$(\lambda e\, (\lambda a\, (\lambda b\, (e\, a\, b)))$ true a b — $e \to$ true

$(\lambda a\, (\lambda b\, ($true $a\, b)))\, a\, b$ — $a \to a$

$(\lambda b\, ($true $a\, b))\, b$ — $b \to b$

(true a b)

a

(l) $\to$ if false a b

$(\lambda e\, (\lambda a\, (\lambda b\, (e\, a\, b)))$ false a b — $e \to$ false

$(\lambda a\, (\lambda b\, ($false $a\, b)))\, a\, b$ — $a \to a$; $b \to b$

false a b

b

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7) → função sucessor → $\lambda n.\lambda f.\lambda x.\ f\ (n\ f\ x)$

função predecessor → $\lambda n.\lambda f.\lambda x.\ n\ (\lambda g.\lambda h.h\ (g\ f))\ (\lambda u.x)\ (\lambda u.u)$

0 → $\lambda f\lambda x.x$

1 → $\lambda f\lambda x.\ f\ x$

2 → $\lambda f\lambda x\ f\ (f\ x)$

3 → $\lambda f\lambda x\ f\ (f\ (f\ x))$

função soma → $\lambda m.\lambda n.\lambda f.\lambda x.\ m\ f\ (n\ f\ x)$

Sucessor de 0

$\lambda n.\lambda f.\lambda x.\ f\ (n\ f\ x)\ 0$ $\quad n \to \lambda f\lambda x.x$

$\lambda f.\lambda x.\ f\ ((\lambda f.\lambda x.x)\ f\ x)$ $\quad f \to f$ e $x \to x$

$\lambda f.\lambda x.\ f\ x = 1$ ||

Sucessor de 1

$\lambda n.\lambda f.\lambda x.\ (f\ (n\ f\ x))\ (\lambda f\lambda x.f\ x)$ $\quad n \to \lambda f\lambda x\ f\ x$

$\lambda f.\lambda x\ (f\ (\lambda f\lambda x\ f\ x)\ f\ x))$ $\quad f \to f \quad x \to x$

$\lambda f.\lambda x\ (f\ (f\ x)) = 2$ ||

soma 0+1

$\lambda m.\lambda n.\lambda f.\lambda x.\ m\ f\ (n\ f\ x)\ (\lambda f\lambda x.x)\ (\lambda f.\lambda x.f\ x)$ $\quad m \to 0$

$\lambda n.\lambda f.\lambda x.\ (\lambda f\lambda x.x)\ f\ (n\ f\ x)\ (\lambda f.\lambda x.f\ x)$ $\quad n \to 1$

$\lambda f.\lambda x.\ ((\lambda f\lambda x.x)\ f\ (\lambda f.\lambda x.f\ x)\ f\ x)$ $\quad f \to f \quad x \to x$

$\lambda f.\lambda x.\ (x\ f\lambda x.x)\ f\ (f\ x)$ $\quad 0 \to f \quad x \to x$

$\lambda f.\lambda x.\ (x\ f) \to 1$ ||

soma 1+2

$(\lambda m\lambda n.\lambda f\lambda x.m\ f\ (n\ f\ x))\ (\lambda f.\lambda x.f\ (f\ x))\ (\lambda f.\lambda x.f\ x)$ $\quad m \to 2$

$(\lambda n.\lambda f.\lambda x.\ (\lambda f.\lambda x.f\ (f\ x))\ f\ (n\ f\ x))\ (\lambda f.\lambda x.f\ x)$ $\quad n \to 1$

$\lambda f.\lambda x.\ (\lambda f.\lambda x.f\ (f\ x))\ f\ ((\lambda f.\lambda x.f\ x)\ f\ x)$ $\quad f \to f\ ;\ x \to x$

$\lambda f.\lambda x.\ (\lambda f.\lambda x.f\ (f\ x))\ f\ (f\ x)$ $\quad 1 \to f\ ;\ x \to x$

$\lambda f.\lambda x.\ f\ (f\ (f\ x)) = 3$ ||

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Já foi mostrado os números 0, 1, 2 a partir daí é possível utilizar a formula

$$n = \lambda f . \lambda x . f^{on} x$$

Onde $f^{on} = \underbrace{f \circ f \ldots f}_{n \text{ vezes}}$.

função subtração $\Rightarrow \lambda m . \lambda n . (n \text{ pred}) m$

função pred $\Rightarrow \lambda n . \lambda f . \lambda x . n (\lambda g . \lambda h . h (g f)) (\lambda u . x) (\lambda u . u)$

$1 - 2 \Rightarrow \lambda m . \lambda n (n \text{ pred}) m (1) (2)$ $m \to 1$ $n \to 2$

$(1 \text{ pred}) 2$

$(\lambda f \lambda x . f x \quad \lambda n . \lambda f . \lambda x . n (\lambda g . \lambda h . h (g f)) (\lambda u x) (\lambda u . u)) (\lambda f . \lambda x . f (f x))$ $h \to f$ $g \to g$

$(\lambda f . \lambda x . f x \quad \lambda n . \lambda f . \lambda x . n (f)) (\lambda u . x) (\lambda u . u) (\lambda f . \lambda x . f (f x))$ $f \to f$

$(\lambda x f x \quad \lambda n . \lambda f . \lambda x . n) (\lambda u . x) (\lambda u . u) (\lambda f . \lambda x . f (f x))$ $x \to (\lambda u x)$

$(f (\lambda u . x) \lambda n . \lambda f . \lambda x . n) (\lambda u . u) (\lambda f . \lambda x . f (f x))$

(8) → Combinador Y utiliza do ponto fixo e cálculo lambda para implementar funções recursivas. Ponto fixo é o ponto da função onde o valor retornado é igual ao parâmetro. Portanto, quando a função recebe o ponto fixo ela retorna o próprio ponto fixo

$$Y f = f (Y f)$$