HW 3

Problem 1

Consider a subset of the MNIST data set. The original data set is a widely used database of handwritten digits (0-9) used to train and test image classifiers. It consists of 70,000 images, 28x28 pixels each. The images are grayscale and have been pre-processed to ensure consistency

For this homework the digits **3, 4, 5 and 9** have been selected and split into training, validation, and test sets

Part (a):

Train and test the performance of a single layer MLP for the following two classification tasks using a binary cross-entropy loss function.

- Train the single-layer MLP to classify digits 3 and 5

Packages:

```
library(torch)
library(MASS)
library(ggplot2)
library(dplyr)
```

```
Attaching package: 'dplyr'


The following object is masked from 'package:MASS':

    select
```

```
The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

```r
library(tidyr)
library(readr)
library(ROCR)
library(viridis)
```

```
Loading required package: viridisLite
```

```r
library(glmnet)
```

```
Loading required package: Matrix


Attaching package: 'Matrix'

The following objects are masked from 'package:tidyr':

    expand, pack, unpack

Loaded glmnet 4.1-8
```

```r
viridis_colors <- viridis(100)
```

Load Data:

```r
train_main <- read_csv("mnist_train.csv")
```

```
Rows: 11200 Columns: 785
-- Column specification ----------------------------------------------------
Delimiter: ","
dbl (785): label, V2, V3, V4, V5, V6, V7, V8, V9, V10, V11, V12, V13, V14, V...

i Use `spec()` to retrieve the full column specification for this data.
i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
val_main <- read_csv("mnist_val.csv")
```

```
Rows: 2400 Columns: 785
-- Column specification ------------------------------------------------
Delimiter: ","
dbl (785): label, V2, V3, V4, V5, V6, V7, V8, V9, V10, V11, V12, V13, V14, V...

i Use `spec()` to retrieve the full column specification for this data.
i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
test_main <- read_csv("mnist_test.csv")
```

```
Rows: 2400 Columns: 785
-- Column specification ------------------------------------------------
Delimiter: ","
dbl (785): label, V2, V3, V4, V5, V6, V7, V8, V9, V10, V11, V12, V13, V14, V...

i Use `spec()` to retrieve the full column specification for this data.
i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

Partition into Train/Test/Validation tensors:

```
# Separate into 3/5
train_35 <- train_main %>%
  filter(label == 3 | label == 5) %>%
  mutate(label = ifelse(label == 3, 0, 1))

val_35 <- val_main %>%
  filter(label == 3 | label == 5) %>%
  mutate(label = ifelse(label == 3, 0, 1))

test_35 <- test_main %>%
  filter(label == 3 | label == 5) %>%
  mutate(label = ifelse(label == 3, 0, 1))

# train_35 %>%
#   group_by(label) %>%
#   summarize(count = n())

# Training Data
```

```r
x_train_35 <- train_35 %>%
  select(-label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

y_train_35 <- train_35 %>%
  pull(label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

# Validation Data
x_val_35 <- val_35 %>%
  select(-label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

y_val_35 <- val_35 %>%
  pull(label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

# Test Data
x_test_35 <- test_35 %>%
  select(-label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

y_test_35 <- test_35 %>%
  pull(label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())
```

MLP Code:

```r
# MLP model definition: MLP with one hidden layer
mlp_1_layer = function(input_dim, hidden1) {
  nn_module(
    initialize = function() {
      self$fc1 = nn_linear(input_dim, hidden1)
      self$fc2 = nn_linear(hidden1, 1)
    },
    forward = function(x) {
```

```
    x %>%
        self$fc1() %>% nnf_relu() %>%
        self$fc2() %>% torch_sigmoid()
    }
  )
}
```

Train and Output the model:

```
# Parameters
seed_parameter <- 123
n <- 10000
p <- 784
corr_factor <- 0.5
flip_factor <- 0.15
epochs <- 50
batch_size <- 16
h1 <- 32
base_lr <- 0.1
decay_rate <- 0.7

# Set seed
set.seed(seed_parameter)
torch_manual_seed(seed_parameter)

# Print model config
cat("Trying MLP with 1 hidden layer of size", h1, "\n")
```

```
Trying MLP with 1 hidden layer of size 32
```

```
# Initialize model
model = mlp_1_layer(p, h1)()
optimizer = optim_sgd(model$parameters, lr = base_lr)
loss_fn = nn_bce_loss()

train_loss_history = c()
val_loss_history = c()

for (epoch in 1:epochs) {
  lr = base_lr * decay_rate^(epoch - 1)
  optimizer$param_groups[[1]]$lr = lr
```

```r
  idx = sample(nrow(x_train_35))
  x_shuffled = x_train_35[idx, ]
  y_shuffled = y_train_35[idx, ]

  for (i in seq(1, nrow(x_shuffled), by = batch_size)) {
    end_idx = min(i + batch_size - 1, nrow(x_shuffled))
    x_batch = x_shuffled[i:end_idx, ]
    y_batch = y_shuffled[i:end_idx, , drop = FALSE]

    optimizer$zero_grad()
    y_pred = model(x_batch)
    loss = loss_fn(y_pred, y_batch)
    loss$backward()
    optimizer$step()
  }

  with_no_grad({
    train_pred <- model(x_train_35)
    val_pred <- model(x_val_35)
    train_loss <- loss_fn(train_pred, y_train_35)$item()
    val_loss <- loss_fn(val_pred, y_val_35)$item()
  })

  train_loss_history = c(train_loss_history, train_loss)
  val_loss_history = c(val_loss_history, val_loss)
}

# Store results in keyed list
key = paste0("Batch_", batch_size, "_Hidden_", h1)
results = list(train = train_loss_history, val = val_loss_history)
```

```r
# Model Output

# Plotting training and validation loss
loss_df = data.frame(
  Epoch = 1:epochs,
  Training = train_loss_history,
  Validation = val_loss_history
)

loss_long = pivot_longer(loss_df, cols = c("Training", "Validation"),
```

```
                       names_to = "LossType", values_to = "Loss")

# Separate plots for Training and Validation loss
ggplot(loss_long %>% filter(LossType == "Training"), aes(x = Epoch, y = Loss)) +
  geom_line(color = "pink") +
  theme_minimal() +
  ggtitle("Training Loss For 1 Layer MLP")
```

Training Loss For 1 Layer MLP
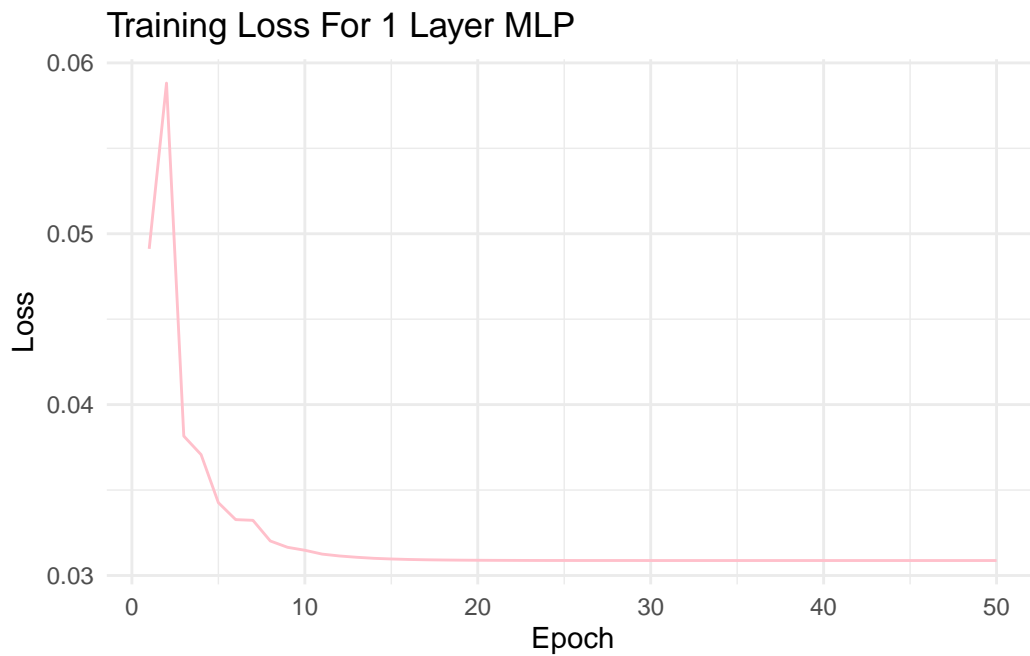

```
# Plot Validation Loss
ggplot(loss_long %>% filter(LossType == "Validation"), aes(x = Epoch, y = Loss)) +
  geom_line(color = "green", linetype = "dashed") +
  theme_minimal() +
  ggtitle("Validation Loss For 1 Layer MLP")
```

Validation Loss For 1 Layer MLP
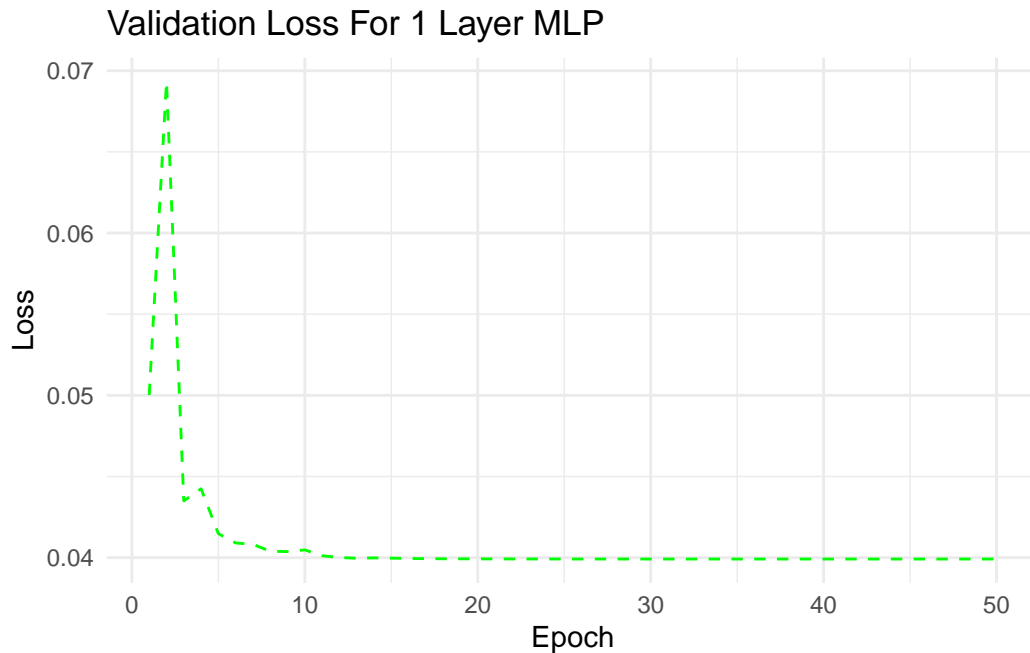
- Select the MLP that performs the best based on the validation set for the mini-batch size s = 64, 128, 256 and the dimension of the hidden layer 64, 128, 256. Fix the number of epochs to 30.

```
# Parameters
seed_parameter = 123
n = 10000
p = 784
corr_factor = 0.5
flip_factor = 0.15
epochs = 30
batch_sizes = c(64, 128, 256)
hidden_layers = c(64, 128, 256)  # two-layer configurations
base_lr = 0.1
decay_rate = 0.7

# Training loop
results = list()
best_val_loss = Inf
best_model = NULL
best_config = NULL

for (batch_size in batch_sizes) {
```

```r
for (h1 in hidden_layers) {

  # Print model config
  cat("Trying MLP with 1 hidden layer of size", h1, "Batch Size", batch_size, "\n")

  # Initialize model
  model = mlp_1_layer(p, h1)()
  optimizer = optim_sgd(model$parameters, lr = base_lr)
  loss_fn = nn_bce_loss()

  train_loss_history = c()
  val_loss_history = c()

  for (epoch in 1:epochs) {
      lr = base_lr * decay_rate^(epoch - 1)
      optimizer$param_groups[[1]]$lr = lr

      idx = sample(nrow(x_train_35))
      x_shuffled = x_train_35[idx, ]
      y_shuffled = y_train_35[idx, ]

      for (i in seq(1, nrow(x_shuffled), by = batch_size)) {
        end_idx = min(i + batch_size - 1, nrow(x_shuffled))
        x_batch = x_shuffled[i:end_idx, ]
        y_batch = y_shuffled[i:end_idx, , drop = FALSE]

        optimizer$zero_grad()
        y_pred = model(x_batch)
        loss = loss_fn(y_pred, y_batch)
        loss$backward()
        optimizer$step()
      }

      with_no_grad({
        train_pred <- model(x_train_35)
        val_pred <- model(x_val_35)
        train_loss <- loss_fn(train_pred, y_train_35)$item()
        val_loss <- loss_fn(val_pred, y_val_35)$item()
      })

      train_loss_history = c(train_loss_history, train_loss)
      val_loss_history = c(val_loss_history, val_loss)
```

```
    }

    # Store results in keyed list
    key = paste0("Batch_", batch_size, "_Hidden_", h1)
    results[[key]] = list(train = train_loss_history, val = val_loss_history)

    # Update best model if needed
    if (min(val_loss_history) < best_val_loss) {
      best_val_loss = min(val_loss_history)
      best_model = model
      best_config = key
    }
  }
}
```

```
Trying MLP with 1 hidden layer of size 64 Batch Size 64
Trying MLP with 1 hidden layer of size 128 Batch Size 64
Trying MLP with 1 hidden layer of size 256 Batch Size 64
Trying MLP with 1 hidden layer of size 64 Batch Size 128
Trying MLP with 1 hidden layer of size 128 Batch Size 128
Trying MLP with 1 hidden layer of size 256 Batch Size 128
Trying MLP with 1 hidden layer of size 64 Batch Size 256
Trying MLP with 1 hidden layer of size 128 Batch Size 256
Trying MLP with 1 hidden layer of size 256 Batch Size 256
```

```
cat("Best model config:", best_config, "\n")
```

```
Best model config: Batch_64_Hidden_64
```

```
cat("Best val loss:", best_val_loss, "\n")
```

```
Best val loss: 0.05090108
```

```
# Plotting training and validation loss
loss_df = data.frame()
for (key in names(results)) {
  df = data.frame(
    Epoch = 1:epochs,
    Training = results[[key]]$train,
    Validation = results[[key]]$val,
```

```
    Model = key
  )
  loss_df = rbind(loss_df, df)
}
loss_long = pivot_longer(loss_df, cols = c("Training", "Validation"),
                         names_to = "LossType", values_to = "Loss")

# Separate plots for Training and Validation loss
ggplot(loss_long %>% filter(LossType == "Training"),
       aes(x = Epoch, y = Loss, color = Model)) +
  geom_line() + theme_minimal() + ggtitle("Training Loss 3/5")
```
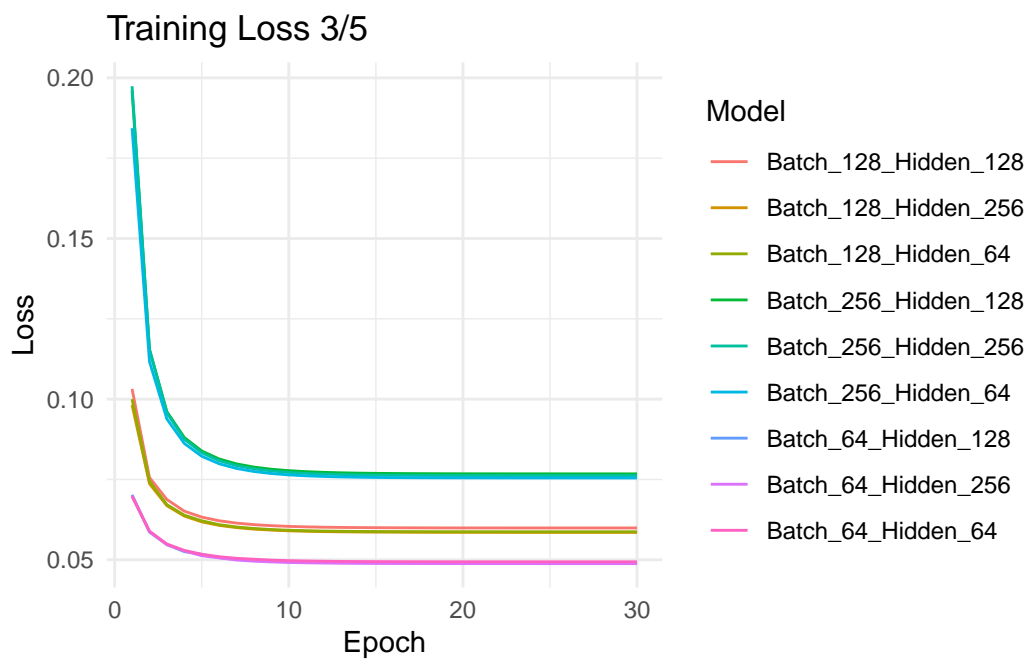

Training Loss 3/5

```
ggplot(loss_long %>% filter(LossType == "Validation"),
       aes(x = Epoch, y = Loss, color = Model)) +
  geom_line(linetype = "dashed") + theme_minimal() + ggtitle("Validation Loss 3/5")
```

Validation Loss 3/5

Model

- – – Batch_128_Hidden_128
- – – Batch_128_Hidden_256
- – – Batch_128_Hidden_64
- – – Batch_256_Hidden_128
- – – Batch_256_Hidden_256
- – – Batch_256_Hidden_64
- – – Batch_64_Hidden_128
- – – Batch_64_Hidden_256
- – – Batch_64_Hidden_64
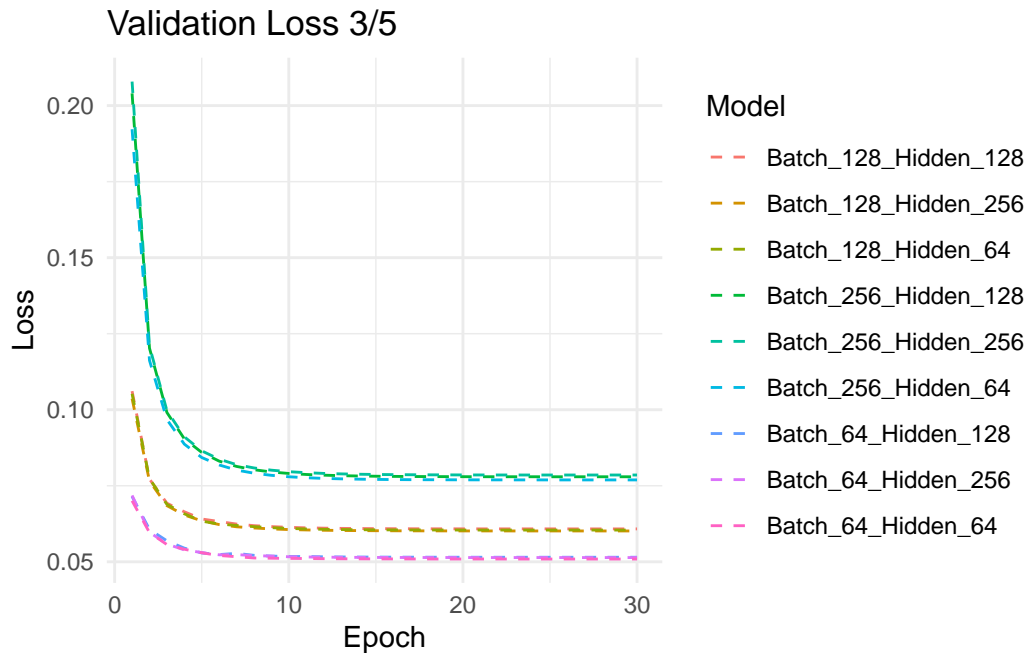
- Report the performance of the best performing MLP in the test data set by plotting the ROC curve and calculating the AUC

The best model config was Batch_64_Hidden_64, meaning batch size = 64 and with 64 hidden layer size. It had the lowest training lost and validation loss at 0.051, closest to the Batch 64 and Hidden 256 model.

Plotting the ROC curve and calculating the AUC:

```
with_no_grad({
  pred_probs_tensor <- best_model(x_test_35)  # Predict probabilities
})

pred_probs <- as.numeric(pred_probs_tensor)
true_labels <- as.numeric(y_test_35)


# Calc ROC
pred <- prediction(pred_probs, true_labels)
perf_m <- performance(pred, "tpr", "fpr")

# Calc AUC
auc <- performance(pred, "auc")
```

```
auc_value <- auc@y.values[[1]]
cat("AUC =", auc_value, "\n")
```

```
AUC = 0.9980987
```

```
# Plot the Curve
plot(perf_m, colorize = TRUE, colorkey.label = "Cutoff",
     colorize.palette = viridis_colors,
     main = "ROC Curve 3 and 5 Prediction, 64 Batch Size, 64 Hidden Layer Size")
abline(a = 0, b = 1, lty = 2, col = "black")
```

OC Curve 3 and 5 Prediction, 64 Batch Size, 64 Hidden Laye
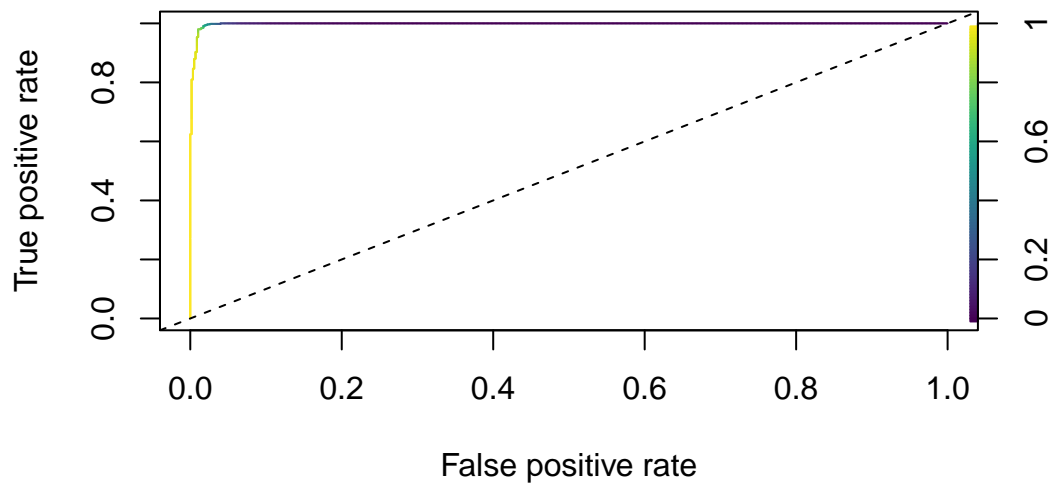
- Repeat the exercise to classify the digits 4 and 9

Partition into Train/Test/Validation tensors:

```
# Separate into 4/9
train_49 <- train_main %>%
  filter(label == 4 | label == 9) %>%
  mutate(label = ifelse(label == 4, 0, 1))

val_49 <- val_main %>%
  filter(label == 4 | label == 9) %>%
```

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```r
  mutate(label = ifelse(label == 4, 0, 1))

test_49 <- test_main %>%
  filter(label == 4 | label == 9) %>%
  mutate(label = ifelse(label == 4, 0, 1))

# Training Data
x_train_49 <- train_49 %>%
  select(-label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

y_train_49 <- train_49 %>%
  pull(label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

# Validation Data
x_val_49 <- val_49 %>%
  select(-label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

y_val_49 <- val_49 %>%
  pull(label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

# Test Data
x_test_49 <- test_49 %>%
  select(-label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())

y_test_49 <- test_49 %>%
  pull(label) %>%
  as.matrix() %>%
  torch_tensor(dtype = torch_float())
```

```r
# Parameters
seed_parameter = 123
n = 10000
```

```r
p = 784
corr_factor = 0.5
flip_factor = 0.15
epochs = 30
batch_sizes = c(64, 128, 256)
hidden_layers = c(64, 128, 256)  # two-layer configurations
base_lr = 0.1
decay_rate = 0.7

# Training loop
results = list()
best_val_loss = Inf
best_model = NULL
best_config = NULL

for (batch_size in batch_sizes) {
  for (h1 in hidden_layers) {

    # Print model config
    cat("Trying MLP with 1 hidden layer of size", h1, "Batch Size", batch_size, "\n")

    # Initialize model
    model = mlp_1_layer(p, h1)()
    optimizer = optim_sgd(model$parameters, lr = base_lr)
    loss_fn = nn_bce_loss()

    train_loss_history = c()
    val_loss_history = c()

    for (epoch in 1:epochs) {
        lr = base_lr * decay_rate^(epoch - 1)
        optimizer$param_groups[[1]]$lr = lr

        idx = sample(nrow(x_train_49))
        x_shuffled = x_train_49[idx, ]
        y_shuffled = y_train_49[idx, ]

        for (i in seq(1, nrow(x_shuffled), by = batch_size)) {
          end_idx = min(i + batch_size - 1, nrow(x_shuffled))
          x_batch = x_shuffled[i:end_idx, ]
          y_batch = y_shuffled[i:end_idx, , drop = FALSE]
```

```
          optimizer$zero_grad()
          y_pred = model(x_batch)
          loss = loss_fn(y_pred, y_batch)
          loss$backward()
          optimizer$step()
        }

        with_no_grad({
          train_pred <- model(x_train_49)
          val_pred <- model(x_val_49)
          train_loss <- loss_fn(train_pred, y_train_49)$item()
          val_loss <- loss_fn(val_pred, y_val_49)$item()
        })

        train_loss_history = c(train_loss_history, train_loss)
        val_loss_history = c(val_loss_history, val_loss)
      }

    # Store results in keyed list
    key = paste0("Batch_", batch_size, "_Hidden_", h1)
    results[[key]] = list(train = train_loss_history, val = val_loss_history)

    # Update best model if needed
    if (min(val_loss_history) < best_val_loss) {
      best_val_loss = min(val_loss_history)
      best_model = model
      best_config = key
    }
  }
}
```

```
Trying MLP with 1 hidden layer of size 64 Batch Size 64
Trying MLP with 1 hidden layer of size 128 Batch Size 64
Trying MLP with 1 hidden layer of size 256 Batch Size 64
Trying MLP with 1 hidden layer of size 64 Batch Size 128
Trying MLP with 1 hidden layer of size 128 Batch Size 128
Trying MLP with 1 hidden layer of size 256 Batch Size 128
Trying MLP with 1 hidden layer of size 64 Batch Size 256
Trying MLP with 1 hidden layer of size 128 Batch Size 256
Trying MLP with 1 hidden layer of size 256 Batch Size 256
```

```
cat("Best model config:", best_config, "\n")
```

```
Best model config: Batch_64_Hidden_256
```

```
cat("Best val loss:", best_val_loss, "\n")
```

```
Best val loss: 0.1159538
```

The best model config was Batch_64_Hidden_256, meaning batch size $= 64$ and with a hidden layer size 256. It had the lowest training loss with 0.118

Plotting training and validation loss

```
loss_df = data.frame()
for (key in names(results)) {
  df = data.frame(
    Epoch = 1:epochs,
    Training = results[[key]]$train,
    Validation = results[[key]]$val,
    Model = key
  )
  loss_df = rbind(loss_df, df)
}
loss_long = pivot_longer(loss_df, cols = c("Training", "Validation"),
                         names_to = "LossType", values_to = "Loss")

# Separate plots for Training and Validation loss
ggplot(loss_long %>% filter(LossType == "Training"),
       aes(x = Epoch, y = Loss, color = Model)) +
  geom_line() + theme_minimal() + ggtitle("Training Loss 4/9")
```

Training Loss 4/9
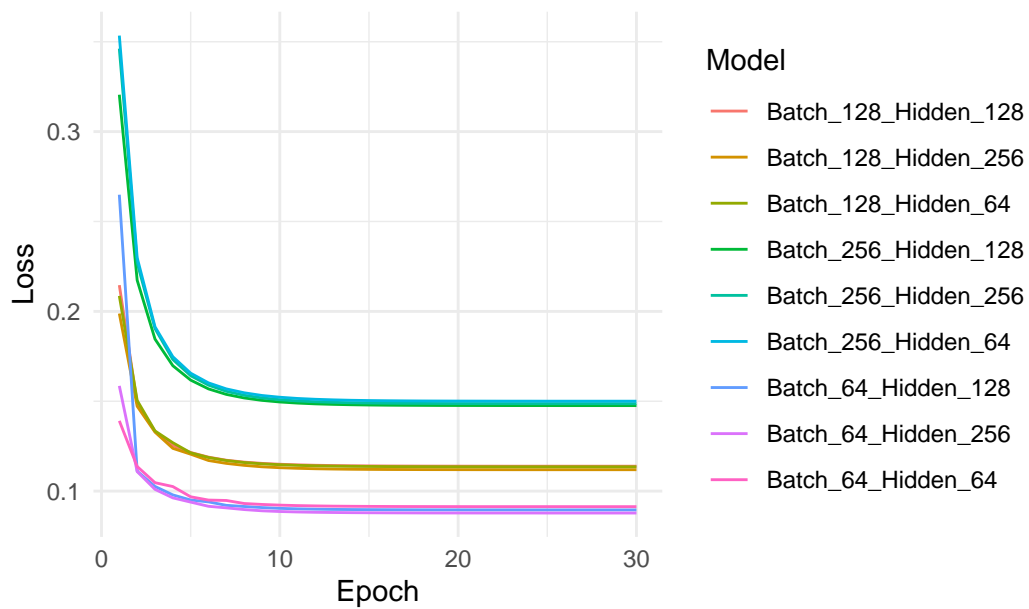

```
ggplot(loss_long %>% filter(LossType == "Validation"),
       aes(x = Epoch, y = Loss, color = Model)) +
  geom_line(linetype = "dashed") + theme_minimal() + ggtitle("Validation Loss 4/9")
```

Validation Loss 4/9
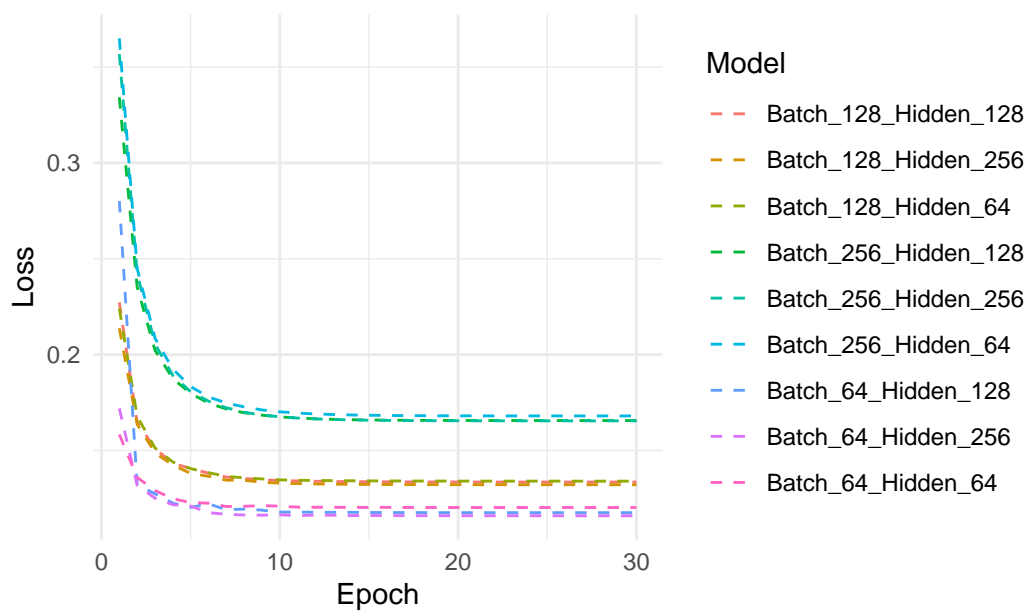
Plotting the ROC curve and calculating the AUC:

```
with_no_grad({
  pred_probs_tensor <- best_model(x_test_49)  # Predict probabilities
})


pred_probs <- as.numeric(pred_probs_tensor)
true_labels <- as.numeric(y_test_49)



# Calc ROC
pred <- prediction(pred_probs, true_labels)
perf_m <- performance(pred, "tpr", "fpr")

# Calc AUC
auc <- performance(pred, "auc")
auc_value <- auc@y.values[[1]]
cat("AUC =", auc_value, "\n")
```

```
AUC = 0.9900191
```

```
# Plot the Curve
plot(perf_m, colorize = TRUE, colorkey.label = "Cutoff",
     colorize.palette = viridis_colors,
     main = "ROC Curve 4/9 Prediction, 64 Batch Size, 256 Hidden Layer Size")
abline(a = 0, b = 1, lty = 2, col = "black")
```

ROC Curve 4/9 Prediction, 64 Batch Size, 256 Hidden Layer
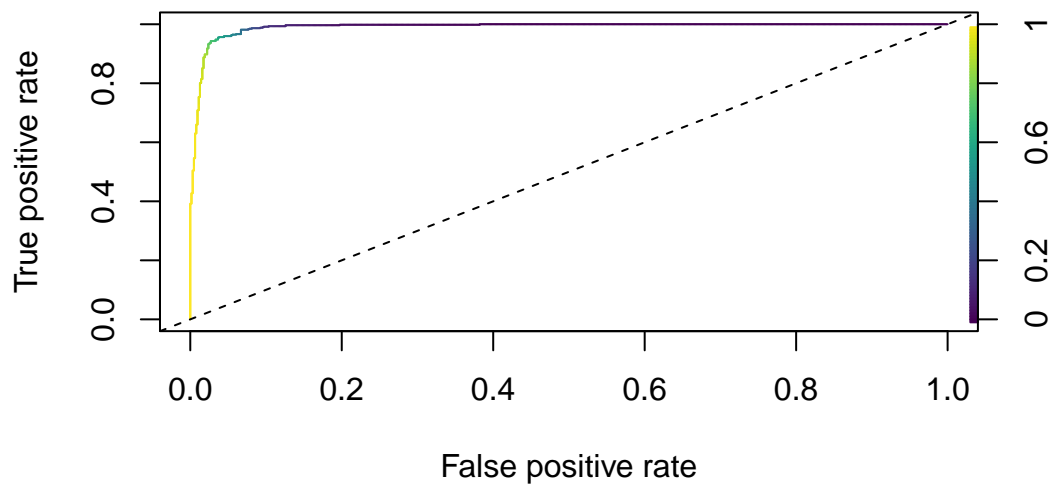
- Comment on the results

I can see that the performance is much better on the test data for the 4 and 9 predictions. I can see this because of the ROC curve being much sharper in the 4-9 data set, and the AUC from 3/5 to 4/9 went from 0.7274219 to 0.9898937, with the same model. This is likely because 3 looks closer to a 5 than 4 to a 9, which could explain why the 4 to 9 classifier was much better than the 2 to 5 classifier

Part (b):

Compare the performance of the best single-layer MLP you have trained for the two classification tasks to that of logistic regression. Specifically, train logistic regression on the train data set and test its performance on the test data set. Plot its ROC and calculate its AUC.

Logistic for 3 and 5 Prediction:

```
predictors <- paste0("V", 2:785)
rhs <- paste(predictors, collapse = " + ")
model_eq <- paste("label ~", rhs)
X <- as.matrix(train_35[, 2:785])
y <- as.matrix(train_35$label)

l1 <- glm(data=train_35, formula = formula(model_eq), family = binomial())
```

```
Warning: glm.fit: algorithm did not converge

Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```
summary(l1)
```

```
Call:
glm(formula = formula(model_eq), family = binomial(), data = train_35)

Coefficients: (171 not defined because of singularities)
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  5.270e+00  3.549e+04   0.000    1.000
V2                 NA         NA      NA      NA
V3                 NA         NA      NA      NA
V4                 NA         NA      NA      NA
V5                 NA         NA      NA      NA
V6                 NA         NA      NA      NA
V7                 NA         NA      NA      NA
V8                 NA         NA      NA      NA
V9                 NA         NA      NA      NA
V10                NA         NA      NA      NA
V11                NA         NA      NA      NA
V12                NA         NA      NA      NA
V13                NA         NA      NA      NA
V14                NA         NA      NA      NA
V15                NA         NA      NA      NA
V16                NA         NA      NA      NA
V17                NA         NA      NA      NA
V18                NA         NA      NA      NA
V19                NA         NA      NA      NA
V20                NA         NA      NA      NA
V21                NA         NA      NA      NA
V22                NA         NA      NA      NA
V23                NA         NA      NA      NA
V24                NA         NA      NA      NA
V25                NA         NA      NA      NA
V26                NA         NA      NA      NA
V27                NA         NA      NA      NA
V28                NA         NA      NA      NA
V29                NA         NA      NA      NA
V30                NA         NA      NA      NA
```

V31	NA	NA	NA	NA
V32	NA	NA	NA	NA
V33	NA	NA	NA	NA
V34	NA	NA	NA	NA
V35	NA	NA	NA	NA
V36	NA	NA	NA	NA
V37	NA	NA	NA	NA
V38	7.505e+01	1.487e+06	0.000	1.000
V39	NA	NA	NA	NA
V40	NA	NA	NA	NA
V41	NA	NA	NA	NA
V42	NA	NA	NA	NA
V43	NA	NA	NA	NA
V44	NA	NA	NA	NA
V45	NA	NA	NA	NA
V46	NA	NA	NA	NA
V47	NA	NA	NA	NA
V48	3.541e+04	2.919e+08	0.000	1.000
V49	NA	NA	NA	NA
V50	8.945e+02	2.981e+07	0.000	1.000
V51	NA	NA	NA	NA
V52	NA	NA	NA	NA
V53	NA	NA	NA	NA
V54	NA	NA	NA	NA
V55	NA	NA	NA	NA
V56	NA	NA	NA	NA
V57	NA	NA	NA	NA
V58	NA	NA	NA	NA
V59	NA	NA	NA	NA
V60	NA	NA	NA	NA
V61	9.298e+02	1.008e+07	0.000	1.000
V62	NA	NA	NA	NA
V63	5.781e+01	1.939e+06	0.000	1.000
V64	1.701e+05	2.586e+09	0.000	1.000
V65	−1.844e+04	2.794e+08	0.000	1.000
V66	1.678e+04	2.536e+08	0.000	1.000
V67	−5.869e+01	2.775e+06	0.000	1.000
V68	−2.029e+04	3.053e+08	0.000	1.000
V69	2.462e+04	3.690e+08	0.000	1.000
V70	−2.002e+04	2.957e+08	0.000	1.000
V71	NA	NA	NA	NA
V72	3.038e+03	2.991e+07	0.000	1.000
V73	2.026e+02	2.744e+06	0.000	1.000

V74	−1.002e+05	8.311e+08	0.000	1.000
V75	6.868e+04	5.706e+08	0.000	1.000
V76	−3.870e+04	3.211e+08	0.000	1.000
V77	1.197e+02	1.061e+06	0.000	1.000
V78	1.050e+02	2.316e+06	0.000	1.000
V79	−1.021e+02	2.854e+06	0.000	1.000
V80	1.396e+02	3.510e+06	0.000	1.000
V81	−6.371e+00	6.630e+06	0.000	1.000
V82	NA	NA	NA	NA
V83	NA	NA	NA	NA
V84	NA	NA	NA	NA
V85	NA	NA	NA	NA
V86	NA	NA	NA	NA
V87	NA	NA	NA	NA
V88	NA	NA	NA	NA
V89	NA	NA	NA	NA
V90	−2.734e+02	1.618e+06	0.000	1.000
V91	−1.071e+02	1.747e+06	0.000	1.000
V92	−6.493e+00	1.675e+06	0.000	1.000
V93	−2.123e−01	7.800e+05	0.000	1.000
V94	3.064e+01	5.893e+05	0.000	1.000
V95	−3.236e+01	1.404e+06	0.000	1.000
V96	1.181e+02	8.516e+05	0.000	1.000
V97	−7.466e+01	5.357e+05	0.000	1.000
V98	8.481e+01	4.786e+05	0.000	1.000
V99	3.138e+00	5.864e+05	0.000	1.000
V100	−1.056e+02	6.026e+05	0.000	1.000
V101	−3.771e+01	6.759e+05	0.000	1.000
V102	7.863e+01	6.253e+05	0.000	1.000
V103	−5.875e+01	3.961e+05	0.000	1.000
V104	1.512e+01	3.476e+05	0.000	1.000
V105	−1.125e+01	3.846e+05	0.000	1.000
V106	−4.919e+01	3.575e+05	0.000	1.000
V107	6.607e+01	3.524e+05	0.000	1.000
V108	−5.304e+01	4.397e+05	0.000	1.000
V109	−1.007e+02	9.952e+05	0.000	1.000
V110	8.526e+01	4.391e+06	0.000	1.000
V111	NA	NA	NA	NA
V112	NA	NA	NA	NA
V113	NA	NA	NA	NA
V114	NA	NA	NA	NA
V115	NA	NA	NA	NA
V116	NA	NA	NA	NA

V117	−5.905e+02	1.100e+07	0.000	1.000
V118	−2.727e+00	5.736e+05	0.000	1.000
V119	8.967e+01	4.717e+05	0.000	1.000
V120	−4.784e+01	4.241e+05	0.000	1.000
V121	6.939e+00	2.958e+05	0.000	1.000
V122	1.131e+01	3.024e+05	0.000	1.000
V123	−5.444e+01	2.582e+05	0.000	1.000
V124	4.927e+01	2.331e+05	0.000	1.000
V125	−2.750e+01	3.593e+05	0.000	1.000
V126	−7.194e+01	3.229e+05	0.000	1.000
V127	1.233e+02	2.898e+05	0.000	1.000
V128	−5.243e+01	2.925e+05	0.000	1.000
V129	2.360e+01	2.759e+05	0.000	1.000
V130	−1.403e+01	1.729e+05	0.000	1.000
V131	3.316e+00	1.398e+05	0.000	1.000
V132	−1.253e+01	1.170e+05	0.000	1.000
V133	−1.510e+01	1.531e+05	0.000	1.000
V134	3.571e+01	1.846e+05	0.000	1.000
V135	−3.609e+01	1.883e+05	0.000	1.000
V136	4.952e+01	2.804e+05	0.000	1.000
V137	6.483e+01	4.576e+05	0.000	1.000
V138	−3.905e+01	2.989e+06	0.000	1.000
V139	5.288e+02	1.974e+06	0.000	1.000
V140	NA	NA	NA	NA
V141	NA	NA	NA	NA
V142	NA	NA	NA	NA
V143	7.662e+02	1.141e+08	0.000	1.000
V144	−3.054e+01	1.155e+06	0.000	1.000
V145	−4.526e+01	9.407e+05	0.000	1.000
V146	2.800e+01	4.974e+05	0.000	1.000
V147	−4.417e+01	2.730e+05	0.000	1.000
V148	1.616e+01	2.131e+05	0.000	1.000
V149	−3.026e+00	1.730e+05	0.000	1.000
V150	−1.444e+01	1.819e+05	0.000	1.000
V151	2.748e+01	1.874e+05	0.000	1.000
V152	−7.453e+01	2.118e+05	0.000	1.000
V153	9.105e+01	2.354e+05	0.000	1.000
V154	−3.691e+01	1.801e+05	0.000	1.000
V155	−6.413e+00	1.674e+05	0.000	1.000
V156	2.246e+01	1.782e+05	0.000	1.000
V157	−2.648e+01	1.331e+05	0.000	1.000
V158	−4.720e+00	1.003e+05	0.000	1.000
V159	1.229e+01	9.974e+04	0.000	1.000

V160	−1.317e+01	1.081e+05	0.000	1.000
V161	2.889e+01	1.113e+05	0.000	1.000
V162	−5.258e+01	1.411e+05	0.000	1.000
V163	4.009e+01	1.621e+05	0.000	1.000
V164	−4.286e+01	2.094e+05	0.000	1.000
V165	−4.134e+01	4.671e+05	0.000	1.000
V166	5.082e+01	1.860e+06	0.000	1.000
V167	NA	NA	NA	NA
V168	NA	NA	NA	NA
V169	NA	NA	NA	NA
V170	NA	NA	NA	NA
V171	NA	NA	NA	NA
V172	−1.016e+02	1.015e+06	0.000	1.000
V173	8.374e+01	7.382e+05	0.000	1.000
V174	−2.561e+01	2.878e+05	0.000	1.000
V175	1.634e+01	2.029e+05	0.000	1.000
V176	1.013e+01	1.706e+05	0.000	1.000
V177	2.412e+00	1.449e+05	0.000	1.000
V178	−8.628e+00	1.624e+05	0.000	1.000
V179	−1.157e−01	1.217e+05	0.000	1.000
V180	−2.283e+00	1.443e+05	0.000	1.000
V181	1.453e+01	1.526e+05	0.000	1.000
V182	−1.403e+01	1.288e+05	0.000	1.000
V183	1.625e+01	1.065e+05	0.000	1.000
V184	−6.031e−01	9.662e+04	0.000	1.000
V185	1.187e+00	9.331e+04	0.000	1.000
V186	1.395e+01	1.109e+05	0.000	1.000
V187	−2.992e+01	9.102e+04	0.000	1.000
V188	8.829e+00	9.464e+04	0.000	1.000
V189	−1.296e+01	9.456e+04	0.000	1.000
V190	6.703e+00	9.136e+04	0.000	1.000
V191	8.340e+00	1.687e+05	0.000	1.000
V192	7.883e+00	2.142e+05	0.000	1.000
V193	8.733e+00	2.479e+05	0.000	1.000
V194	1.453e+01	6.954e+05	0.000	1.000
V195	−1.430e+02	1.150e+06	0.000	1.000
V196	NA	NA	NA	NA
V197	NA	NA	NA	NA
V198	NA	NA	NA	NA
V199	−3.313e+02	2.857e+07	0.000	1.000
V200	1.338e+02	9.006e+05	0.000	1.000
V201	−2.692e+01	7.942e+05	0.000	1.000
V202	−1.265e+01	1.863e+05	0.000	1.000

V203	3.056e+00	1.529e+05	0.000	1.000
V204	−2.341e+01	1.290e+05	0.000	1.000
V205	6.115e+00	1.073e+05	0.000	1.000
V206	2.102e+01	1.256e+05	0.000	1.000
V207	−9.849e+00	1.319e+05	0.000	1.000
V208	−2.229e+01	1.314e+05	0.000	1.000
V209	3.274e+01	1.532e+05	0.000	1.000
V210	−3.798e+00	1.051e+05	0.000	1.000
V211	−5.826e+00	9.271e+04	0.000	1.000
V212	6.667e+00	9.621e+04	0.000	1.000
V213	−4.477e+00	1.012e+05	0.000	1.000
V214	2.350e+00	8.680e+04	0.000	1.000
V215	−4.749e+00	8.260e+04	0.000	1.000
V216	−1.923e+00	9.220e+04	0.000	1.000
V217	9.068e+00	1.059e+05	0.000	1.000
V218	2.362e+01	9.277e+04	0.000	1.000
V219	−4.377e+01	1.385e+05	0.000	1.000
V220	4.594e+00	1.616e+05	0.000	1.000
V221	1.009e+00	1.835e+05	0.000	1.000
V222	−1.266e+01	2.446e+05	0.000	1.000
V223	1.488e+02	1.433e+06	0.000	1.000
V224	NA	NA	NA	NA
V225	NA	NA	NA	NA
V226	NA	NA	NA	NA
V227	−1.075e+02	2.222e+06	0.000	1.000
V228	−4.812e+01	1.021e+06	0.000	1.000
V229	2.515e+01	5.141e+05	0.000	1.000
V230	1.213e+01	1.769e+05	0.000	1.000
V231	−1.052e+01	1.354e+05	0.000	1.000
V232	8.014e+00	1.318e+05	0.000	1.000
V233	−1.495e+01	1.063e+05	0.000	1.000
V234	−1.160e+01	1.105e+05	0.000	1.000
V235	1.372e+01	9.368e+04	0.000	1.000
V236	−2.933e+01	1.039e+05	0.000	1.000
V237	1.061e+01	9.781e+04	0.000	1.000
V238	1.337e+01	1.058e+05	0.000	1.000
V239	−8.130e+00	7.498e+04	0.000	1.000
V240	1.796e+01	7.150e+04	0.000	1.000
V241	−4.384e+00	7.732e+04	0.000	1.000
V242	2.415e+01	6.965e+04	0.000	1.000
V243	−4.547e+00	6.745e+04	0.000	1.000
V244	−6.419e+00	8.319e+04	0.000	1.000
V245	−1.476e+01	9.317e+04	0.000	1.000

V246	-9.572e+00	8.912e+04	0.000	1.000
V247	1.773e+01	1.228e+05	0.000	1.000
V248	-9.131e+00	1.095e+05	0.000	1.000
V249	1.494e+01	1.110e+05	0.000	1.000
V250	-9.129e+00	1.540e+05	0.000	1.000
V251	-6.954e+01	5.246e+05	0.000	1.000
V252	NA	NA	NA	NA
V253	NA	NA	NA	NA
V254	NA	NA	NA	NA
V255	3.712e+02	2.973e+06	0.000	1.000
V256	-6.647e+00	4.647e+05	0.000	1.000
V257	-7.728e+00	4.520e+05	0.000	1.000
V258	6.411e+00	1.367e+05	0.000	1.000
V259	2.190e+01	1.171e+05	0.000	1.000
V260	-7.895e+00	1.007e+05	0.000	1.000
V261	7.154e+00	8.478e+04	0.000	1.000
V262	6.366e+00	9.228e+04	0.000	1.000
V263	-1.323e+01	1.003e+05	0.000	1.000
V264	1.801e+01	1.016e+05	0.000	1.000
V265	-2.464e+00	8.006e+04	0.000	1.000
V266	1.504e+01	7.928e+04	0.000	1.000
V267	-5.469e+00	7.544e+04	0.000	1.000
V268	-7.442e-01	6.320e+04	0.000	1.000
V269	4.952e+00	6.570e+04	0.000	1.000
V270	-1.956e+01	6.230e+04	0.000	1.000
V271	1.156e+01	6.138e+04	0.000	1.000
V272	2.020e+01	7.859e+04	0.000	1.000
V273	-1.132e+01	7.814e+04	0.000	1.000
V274	-1.749e+00	8.576e+04	0.000	1.000
V275	-9.936e+00	1.091e+05	0.000	1.000
V276	9.454e+00	1.044e+05	0.000	1.000
V277	-1.543e+01	1.257e+05	0.000	1.000
V278	2.608e+00	1.481e+05	0.000	1.000
V279	8.536e+01	3.771e+05	0.000	1.000
V280	-6.296e+02	6.811e+06	0.000	1.000
V281	NA	NA	NA	NA
V282	NA	NA	NA	NA
V283	NA	NA	NA	NA
V284	-4.842e+01	6.777e+05	0.000	1.000
V285	-1.722e+01	4.056e+05	0.000	1.000
V286	-2.130e+01	9.702e+04	0.000	1.000
V287	-1.384e+01	7.142e+04	0.000	1.000
V288	-4.948e+00	8.411e+04	0.000	1.000

V289	1.661e+01	7.645e+04	0.000	1.000
V290	−1.890e+01	7.193e+04	0.000	1.000
V291	−2.900e+00	7.140e+04	0.000	1.000
V292	6.125e−02	6.556e+04	0.000	1.000
V293	1.214e+01	6.755e+04	0.000	1.000
V294	−2.767e+00	6.800e+04	0.000	1.000
V295	1.543e+01	7.027e+04	0.000	1.000
V296	1.050e+00	6.334e+04	0.000	1.000
V297	1.089e+01	6.632e+04	0.000	1.000
V298	1.212e+01	7.268e+04	0.000	1.000
V299	−8.965e+00	6.400e+04	0.000	1.000
V300	−2.405e+01	8.062e+04	0.000	1.000
V301	1.011e+01	8.398e+04	0.000	1.000
V302	−4.151e+00	8.481e+04	0.000	1.000
V303	1.077e−01	8.885e+04	0.000	1.000
V304	−1.967e+01	1.062e+05	0.000	1.000
V305	1.448e+01	9.546e+04	0.000	1.000
V306	8.066e+00	1.144e+05	0.000	1.000
V307	−2.722e+01	2.329e+05	0.000	1.000
V308	4.897e+01	1.102e+06	0.000	1.000
V309	NA	NA	NA	NA
V310	NA	NA	NA	NA
V311	NA	NA	NA	NA
V312	−1.902e+01	5.278e+05	0.000	1.000
V313	6.351e+00	2.773e+05	0.000	1.000
V314	9.435e+00	1.255e+05	0.000	1.000
V315	2.032e+00	7.834e+04	0.000	1.000
V316	−1.857e+01	6.561e+04	0.000	1.000
V317	1.002e+01	7.676e+04	0.000	1.000
V318	6.839e+00	6.142e+04	0.000	1.000
V319	−1.508e+01	7.335e+04	0.000	1.000
V320	8.780e+00	9.400e+04	0.000	1.000
V321	−3.995e−01	7.806e+04	0.000	1.000
V322	−2.977e+00	7.619e+04	0.000	1.000
V323	1.145e+00	7.682e+04	0.000	1.000
V324	3.152e+00	7.108e+04	0.000	1.000
V325	−1.969e+01	7.323e+04	0.000	1.000
V326	1.198e+00	6.783e+04	0.000	1.000
V327	−9.216e−01	6.917e+04	0.000	1.000
V328	1.993e+01	9.357e+04	0.000	1.000
V329	−1.183e+01	8.071e+04	0.000	1.000
V330	−9.116e+00	8.218e+04	0.000	1.000
V331	−9.395e−01	8.272e+04	0.000	1.000

V332	2.835e+00	9.195e+04	0.000	1.000
V333	−1.118e+01	9.830e+04	0.000	1.000
V334	−3.705e+00	1.022e+05	0.000	1.000
V335	1.398e+01	1.892e+05	0.000	1.000
V336	−4.096e+01	4.326e+05	0.000	1.000
V337	NA	NA	NA	NA
V338	NA	NA	NA	NA
V339	NA	NA	NA	NA
V340	5.104e+01	2.912e+05	0.000	1.000
V341	4.439e+00	1.599e+05	0.000	1.000
V342	4.480e+00	1.317e+05	0.000	1.000
V343	1.486e+01	1.086e+05	0.000	1.000
V344	2.342e−01	7.281e+04	0.000	1.000
V345	−9.499e+00	7.101e+04	0.000	1.000
V346	−3.721e+00	8.116e+04	0.000	1.000
V347	8.793e+00	8.433e+04	0.000	1.000
V348	−1.110e+01	7.990e+04	0.000	1.000
V349	3.493e+01	1.021e+05	0.000	1.000
V350	1.520e+01	7.657e+04	0.000	1.000
V351	−7.945e−01	8.658e+04	0.000	1.000
V352	1.103e+01	7.344e+04	0.000	1.000
V353	2.213e+01	8.311e+04	0.000	1.000
V354	1.063e+01	6.648e+04	0.000	1.000
V355	−1.042e+01	6.151e+04	0.000	1.000
V356	2.993e+00	7.742e+04	0.000	1.000
V357	−4.411e+00	8.234e+04	0.000	1.000
V358	1.827e+01	8.829e+04	0.000	1.000
V359	−2.308e+00	8.757e+04	0.000	1.000
V360	4.406e−01	8.761e+04	0.000	1.000
V361	2.665e+00	7.956e+04	0.000	1.000
V362	−3.803e−01	9.025e+04	0.000	1.000
V363	1.680e+01	2.331e+05	0.000	1.000
V364	−2.971e+01	8.220e+05	0.000	1.000
V365	NA	NA	NA	NA
V366	NA	NA	NA	NA
V367	NA	NA	NA	NA
V368	−2.972e+01	3.067e+05	0.000	1.000
V369	−1.726e+01	1.552e+05	0.000	1.000
V370	−1.067e+01	8.663e+04	0.000	1.000
V371	−3.068e+01	8.705e+04	0.000	1.000
V372	−3.267e+00	8.490e+04	0.000	1.000
V373	1.697e+00	6.707e+04	0.000	1.000
V374	3.395e+00	7.428e+04	0.000	1.000

V375	-4.328e+00	8.349e+04	0.000	1.000
V376	1.269e+01	9.559e+04	0.000	1.000
V377	-1.504e+01	9.869e+04	0.000	1.000
V378	-8.324e+00	8.380e+04	0.000	1.000
V379	-2.159e+01	6.507e+04	0.000	1.000
V380	-3.588e+00	8.552e+04	0.000	1.000
V381	-3.122e+01	8.214e+04	0.000	1.000
V382	-7.223e+00	6.976e+04	0.000	1.000
V383	1.122e+01	6.588e+04	0.000	1.000
V384	-2.889e+00	7.113e+04	0.000	1.000
V385	4.564e+00	7.847e+04	0.000	1.000
V386	-1.541e+01	7.908e+04	0.000	1.000
V387	-1.491e+00	7.336e+04	0.000	1.000
V388	2.028e-01	8.386e+04	0.000	1.000
V389	2.861e-01	6.845e+04	0.000	1.000
V390	3.763e+00	6.044e+04	0.000	1.000
V391	6.479e+00	1.867e+05	0.000	1.000
V392	-1.595e+01	1.120e+06	0.000	1.000
V393	NA	NA	NA	NA
V394	NA	NA	NA	NA
V395	-4.574e+00	1.060e+07	0.000	1.000
V396	5.112e-01	1.977e+05	0.000	1.000
V397	-1.928e+01	1.581e+05	0.000	1.000
V398	-5.771e+00	9.512e+04	0.000	1.000
V399	1.722e+01	7.050e+04	0.000	1.000
V400	3.038e-02	7.159e+04	0.000	1.000
V401	-1.353e+00	7.156e+04	0.000	1.000
V402	-8.956e+00	7.622e+04	0.000	1.000
V403	-1.511e+01	8.315e+04	0.000	1.000
V404	-2.434e+00	9.474e+04	0.000	1.000
V405	2.026e+01	9.142e+04	0.000	1.000
V406	-2.896e+00	7.555e+04	0.000	1.000
V407	3.568e+00	6.883e+04	0.000	1.000
V408	1.112e+01	7.047e+04	0.000	1.000
V409	5.615e+00	7.387e+04	0.000	1.000
V410	4.643e+00	5.314e+04	0.000	1.000
V411	-4.153e+00	7.308e+04	0.000	1.000
V412	-3.249e+00	6.686e+04	0.000	1.000
V413	3.707e+00	6.839e+04	0.000	1.000
V414	1.082e+01	7.601e+04	0.000	1.000
V415	-2.283e+00	8.313e+04	0.000	1.000
V416	-1.372e+01	7.909e+04	0.000	1.000
V417	-3.709e+00	7.869e+04	0.000	1.000

V418	1.434e+01	7.135e+04	0.000	1.000
V419	−2.151e+00	1.656e+05	0.000	1.000
V420	−9.111e+01	1.871e+06	0.000	1.000
V421	NA	NA	NA	NA
V422	NA	NA	NA	NA
V423	−8.625e+01	5.929e+06	0.000	1.000
V424	3.642e+00	2.183e+05	0.000	1.000
V425	4.750e+01	1.381e+05	0.000	1.000
V426	−3.531e+00	8.777e+04	0.000	1.000
V427	−7.361e+00	7.160e+04	0.000	1.000
V428	−1.462e+01	6.942e+04	0.000	1.000
V429	−5.751e+00	7.524e+04	0.000	1.000
V430	2.021e+00	6.993e+04	0.000	1.000
V431	−1.043e+01	9.093e+04	0.000	1.000
V432	7.381e−01	8.587e+04	0.000	1.000
V433	2.335e+00	8.120e+04	0.000	1.000
V434	4.548e+00	7.212e+04	0.000	1.000
V435	1.079e+01	6.589e+04	0.000	1.000
V436	−2.535e+01	6.062e+04	0.000	1.000
V437	9.156e−01	6.111e+04	0.000	1.000
V438	7.729e+00	6.410e+04	0.000	1.000
V439	−1.750e+01	5.883e+04	0.000	1.000
V440	1.942e+01	6.503e+04	0.000	1.000
V441	−8.876e+00	7.782e+04	0.000	1.000
V442	−6.458e+00	8.793e+04	0.000	1.000
V443	1.824e+01	8.154e+04	0.000	1.000
V444	2.123e+00	8.449e+04	0.000	1.000
V445	2.118e−01	8.766e+04	0.000	1.000
V446	−4.253e+00	8.630e+04	0.000	1.000
V447	−8.204e+00	1.574e+05	0.000	1.000
V448	−4.568e+00	1.070e+06	0.000	1.000
V449	NA	NA	NA	NA
V450	NA	NA	NA	NA
V451	4.629e+01	8.212e+05	0.000	1.000
V452	1.041e+01	3.363e+05	0.000	1.000
V453	−6.992e+01	1.597e+05	0.000	1.000
V454	2.839e+00	7.506e+04	0.000	1.000
V455	−1.199e+01	6.671e+04	0.000	1.000
V456	9.710e+00	6.972e+04	0.000	1.000
V457	−7.818e+00	7.321e+04	0.000	1.000
V458	−5.195e+00	8.340e+04	0.000	1.000
V459	1.750e+01	7.714e+04	0.000	1.000
V460	−4.062e+01	9.572e+04	0.000	1.000

V461	4.261e+01	8.663e+04	0.000	1.000
V462	-5.333e+00	7.150e+04	0.000	1.000
V463	2.013e+00	6.591e+04	0.000	1.000
V464	1.781e+00	6.201e+04	0.000	1.000
V465	2.211e-01	5.765e+04	0.000	1.000
V466	-1.992e+00	6.546e+04	0.000	1.000
V467	1.046e+01	6.417e+04	0.000	1.000
V468	-1.119e+01	6.735e+04	0.000	1.000
V469	8.471e+00	7.896e+04	0.000	1.000
V470	-1.820e+00	8.439e+04	0.000	1.000
V471	-8.696e+00	8.135e+04	0.000	1.000
V472	9.962e+00	9.370e+04	0.000	1.000
V473	1.104e+01	1.042e+05	0.000	1.000
V474	-7.578e+00	9.102e+04	0.000	1.000
V475	1.077e+01	1.601e+05	0.000	1.000
V476	1.370e+02	3.264e+06	0.000	1.000
V477	NA	NA	NA	NA
V478	NA	NA	NA	NA
V479	1.006e+01	8.574e+05	0.000	1.000
V480	-7.502e+01	6.301e+05	0.000	1.000
V481	5.059e+01	1.496e+05	0.000	1.000
V482	-2.759e+00	8.665e+04	0.000	1.000
V483	-3.238e+00	6.099e+04	0.000	1.000
V484	-4.667e+00	6.711e+04	0.000	1.000
V485	6.665e+00	7.891e+04	0.000	1.000
V486	-9.616e+00	7.662e+04	0.000	1.000
V487	-3.938e+00	7.392e+04	0.000	1.000
V488	-8.726e+00	8.422e+04	0.000	1.000
V489	-3.733e+00	8.262e+04	0.000	1.000
V490	2.044e+01	7.970e+04	0.000	1.000
V491	-6.222e+00	6.626e+04	0.000	1.000
V492	1.244e+01	7.421e+04	0.000	1.000
V493	3.694e-01	5.264e+04	0.000	1.000
V494	-6.040e+00	7.114e+04	0.000	1.000
V495	-7.999e-01	7.002e+04	0.000	1.000
V496	-1.190e+00	7.549e+04	0.000	1.000
V497	9.859e+00	8.952e+04	0.000	1.000
V498	-2.093e+00	8.303e+04	0.000	1.000
V499	9.226e+00	8.132e+04	0.000	1.000
V500	-7.421e+00	9.262e+04	0.000	1.000
V501	-2.274e+00	9.620e+04	0.000	1.000
V502	-2.129e+01	9.060e+04	0.000	1.000
V503	1.995e+00	1.793e+05	0.000	1.000

V504	-2.678e+02	3.018e+06	0.000	1.000
V505	NA	NA	NA	NA
V506	NA	NA	NA	NA
V507	-4.573e+01	4.613e+06	0.000	1.000
V508	3.568e+01	1.432e+06	0.000	1.000
V509	-2.193e+01	1.226e+05	0.000	1.000
V510	-8.893e+00	9.269e+04	0.000	1.000
V511	-1.421e-01	6.355e+04	0.000	1.000
V512	3.765e+00	7.412e+04	0.000	1.000
V513	-1.675e+01	7.769e+04	0.000	1.000
V514	2.206e+01	8.149e+04	0.000	1.000
V515	-7.574e+00	8.283e+04	0.000	1.000
V516	-5.897e+00	8.924e+04	0.000	1.000
V517	1.833e+01	8.621e+04	0.000	1.000
V518	-1.921e+01	7.520e+04	0.000	1.000
V519	1.044e+01	8.300e+04	0.000	1.000
V520	-3.640e-01	6.264e+04	0.000	1.000
V521	3.912e-02	6.830e+04	0.000	1.000
V522	1.088e+01	6.852e+04	0.000	1.000
V523	2.875e-01	7.815e+04	0.000	1.000
V524	-7.580e+00	8.412e+04	0.000	1.000
V525	6.498e+00	8.891e+04	0.000	1.000
V526	-4.692e+00	9.228e+04	0.000	1.000
V527	1.179e+01	9.588e+04	0.000	1.000
V528	-3.843e+00	1.004e+05	0.000	1.000
V529	1.587e+00	8.494e+04	0.000	1.000
V530	3.273e+01	1.051e+05	0.000	1.000
V531	2.625e-01	2.524e+05	0.000	1.000
V532	1.668e+02	1.595e+06	0.000	1.000
V533	NA	NA	NA	NA
V534	NA	NA	NA	NA
V535	NA	NA	NA	NA
V536	4.676e+01	2.139e+06	0.000	1.000
V537	-5.671e+00	1.421e+05	0.000	1.000
V538	9.743e-01	9.956e+04	0.000	1.000
V539	-7.019e+00	8.046e+04	0.000	1.000
V540	2.410e+00	7.910e+04	0.000	1.000
V541	3.732e+00	7.987e+04	0.000	1.000
V542	-1.678e+01	7.285e+04	0.000	1.000
V543	-3.115e+00	7.767e+04	0.000	1.000
V544	-2.114e+00	9.131e+04	0.000	1.000
V545	2.119e-01	9.318e+04	0.000	1.000
V546	5.835e+00	9.558e+04	0.000	1.000

V547	−1.176e+01	8.148e+04	0.000	1.000
V548	1.000e+01	7.283e+04	0.000	1.000
V549	7.476e+00	8.393e+04	0.000	1.000
V550	4.261e−01	8.818e+04	0.000	1.000
V551	−5.340e+00	9.186e+04	0.000	1.000
V552	2.728e+01	9.639e+04	0.000	1.000
V553	−1.818e+01	1.042e+05	0.000	1.000
V554	4.945e−01	1.004e+05	0.000	1.000
V555	3.021e+00	9.066e+04	0.000	1.000
V556	1.311e+00	8.328e+04	0.000	1.000
V557	−2.258e+01	7.811e+04	0.000	1.000
V558	8.739e−01	1.287e+05	0.000	1.000
V559	−1.195e+01	3.055e+05	0.000	1.000
V560	−2.096e+02	1.181e+06	0.000	1.000
V561	NA	NA	NA	NA
V562	NA	NA	NA	NA
V563	NA	NA	NA	NA
V564	6.102e+01	2.346e+06	0.000	1.000
V565	−2.725e+01	2.633e+05	0.000	1.000
V566	6.776e+00	1.160e+05	0.000	1.000
V567	4.099e+00	9.919e+04	0.000	1.000
V568	−5.590e+00	9.397e+04	0.000	1.000
V569	2.003e+01	1.004e+05	0.000	1.000
V570	−1.326e+01	1.060e+05	0.000	1.000
V571	−1.051e+00	1.255e+05	0.000	1.000
V572	−1.369e+00	1.210e+05	0.000	1.000
V573	−9.760e+00	1.057e+05	0.000	1.000
V574	7.835e+00	1.093e+05	0.000	1.000
V575	1.080e+01	9.929e+04	0.000	1.000
V576	−9.127e+00	8.391e+04	0.000	1.000
V577	2.701e+00	8.041e+04	0.000	1.000
V578	1.444e+01	8.383e+04	0.000	1.000
V579	−1.262e+01	8.431e+04	0.000	1.000
V580	−1.484e+01	9.827e+04	0.000	1.000
V581	1.659e+01	1.290e+05	0.000	1.000
V582	1.061e+01	1.026e+05	0.000	1.000
V583	−6.607e+00	9.806e+04	0.000	1.000
V584	−1.600e+01	1.103e+05	0.000	1.000
V585	3.016e+01	1.062e+05	0.000	1.000
V586	1.924e+01	1.525e+05	0.000	1.000
V587	−5.631e+01	1.142e+06	0.000	1.000
V588	6.847e+02	1.010e+07	0.000	1.000
V589	NA	NA	NA	NA

V590	NA	NA	NA	NA
V591	NA	NA	NA	NA
V592	1.199e+02	4.947e+06	0.000	1.000
V593	1.159e+01	2.702e+05	0.000	1.000
V594	1.786e+00	1.696e+05	0.000	1.000
V595	2.867e+00	1.149e+05	0.000	1.000
V596	−1.523e+01	1.211e+05	0.000	1.000
V597	2.333e+01	1.167e+05	0.000	1.000
V598	−3.676e+00	1.130e+05	0.000	1.000
V599	1.027e+01	1.287e+05	0.000	1.000
V600	−2.105e+01	1.408e+05	0.000	1.000
V601	1.582e+01	1.479e+05	0.000	1.000
V602	−1.209e+01	1.323e+05	0.000	1.000
V603	1.151e+01	1.177e+05	0.000	1.000
V604	5.250e+00	9.734e+04	0.000	1.000
V605	−6.963e+00	1.000e+05	0.000	1.000
V606	−7.519e+00	1.082e+05	0.000	1.000
V607	1.617e+01	1.004e+05	0.000	1.000
V608	−3.548e+00	1.170e+05	0.000	1.000
V609	4.007e+00	1.217e+05	0.000	1.000
V610	3.184e+00	8.760e+04	0.000	1.000
V611	2.188e+00	9.787e+04	0.000	1.000
V612	1.498e+01	1.347e+05	0.000	1.000
V613	−9.912e+00	1.457e+05	0.000	1.000
V614	−3.537e+01	1.582e+05	0.000	1.000
V615	2.576e+01	3.115e+06	0.000	1.000
V616	NA	NA	NA	NA
V617	NA	NA	NA	NA
V618	NA	NA	NA	NA
V619	NA	NA	NA	NA
V620	NA	NA	NA	NA
V621	−3.691e+01	3.906e+05	0.000	1.000
V622	2.356e+01	2.343e+05	0.000	1.000
V623	1.141e+01	1.273e+05	0.000	1.000
V624	−1.062e+01	1.212e+05	0.000	1.000
V625	−1.989e+01	1.283e+05	0.000	1.000
V626	1.354e+01	1.666e+05	0.000	1.000
V627	−1.635e+01	1.600e+05	0.000	1.000
V628	8.546e+00	1.537e+05	0.000	1.000
V629	−3.362e+01	1.634e+05	0.000	1.000
V630	1.920e+01	1.730e+05	0.000	1.000
V631	7.301e+00	1.402e+05	0.000	1.000
V632	−2.031e+01	1.189e+05	0.000	1.000

V633	3.418e+01	1.167e+05	0.000	1.000
V634	−2.453e+01	1.154e+05	0.000	1.000
V635	1.450e+00	1.043e+05	0.000	1.000
V636	7.574e−01	1.111e+05	0.000	1.000
V637	−6.494e−01	1.354e+05	0.000	1.000
V638	−2.463e+01	1.217e+05	0.000	1.000
V639	1.751e+01	1.522e+05	0.000	1.000
V640	−1.376e+01	1.743e+05	0.000	1.000
V641	1.228e+01	1.542e+05	0.000	1.000
V642	−5.023e+01	1.908e+05	0.000	1.000
V643	8.486e+01	1.167e+06	0.000	1.000
V644	2.153e+02	3.681e+06	0.000	1.000
V645	NA	NA	NA	NA
V646	NA	NA	NA	NA
V647	NA	NA	NA	NA
V648	−2.025e+02	1.911e+06	0.000	1.000
V649	6.105e+01	1.192e+06	0.000	1.000
V650	6.981e+00	2.343e+05	0.000	1.000
V651	−1.437e+01	2.070e+05	0.000	1.000
V652	1.541e+01	1.954e+05	0.000	1.000
V653	2.115e+01	1.834e+05	0.000	1.000
V654	1.956e+00	1.839e+05	0.000	1.000
V655	−2.621e−01	1.638e+05	0.000	1.000
V656	−2.305e+01	2.622e+05	0.000	1.000
V657	−4.805e+01	2.989e+05	0.000	1.000
V658	7.099e+01	3.094e+05	0.000	1.000
V659	−3.499e+01	2.080e+05	0.000	1.000
V660	4.197e+01	1.511e+05	0.000	1.000
V661	−2.466e+01	1.124e+05	0.000	1.000
V662	5.618e+00	1.135e+05	0.000	1.000
V663	−1.779e+01	1.304e+05	0.000	1.000
V664	1.452e+01	1.170e+05	0.000	1.000
V665	−3.238e+01	1.435e+05	0.000	1.000
V666	2.459e+01	1.717e+05	0.000	1.000
V667	−2.234e+01	2.066e+05	0.000	1.000
V668	7.048e+00	2.017e+05	0.000	1.000
V669	−3.708e+01	2.047e+05	0.000	1.000
V670	1.450e+02	4.869e+05	0.000	1.000
V671	−1.124e+02	7.722e+05	0.000	1.000
V672	NA	NA	NA	NA
V673	NA	NA	NA	NA
V674	NA	NA	NA	NA
V675	NA	NA	NA	NA

V676	3.013e+02	1.396e+06	0.000	1.000
V677	−1.469e+02	1.068e+06	0.000	1.000
V678	6.014e+01	5.274e+05	0.000	1.000
V679	9.541e-01	2.508e+05	0.000	1.000
V680	3.546e+01	3.382e+05	0.000	1.000
V681	−1.944e+01	3.575e+05	0.000	1.000
V682	5.857e+01	5.091e+05	0.000	1.000
V683	−1.343e+01	8.059e+05	0.000	1.000
V684	1.085e+02	1.062e+06	0.000	1.000
V685	4.110e+01	7.496e+05	0.000	1.000
V686	−6.977e+00	3.350e+05	0.000	1.000
V687	−3.966e+01	2.043e+05	0.000	1.000
V688	−2.197e+01	1.739e+05	0.000	1.000
V689	−9.332e+00	1.093e+05	0.000	1.000
V690	1.650e+01	1.092e+05	0.000	1.000
V691	−1.586e+01	1.181e+05	0.000	1.000
V692	−1.047e+01	1.184e+05	0.000	1.000
V693	−4.704e+00	1.639e+05	0.000	1.000
V694	−3.457e+01	1.829e+05	0.000	1.000
V695	−1.042e+01	2.199e+05	0.000	1.000
V696	5.157e+01	3.295e+05	0.000	1.000
V697	−8.517e+01	4.435e+05	0.000	1.000
V698	−8.303e+01	1.030e+06	0.000	1.000
V699	6.834e+02	4.498e+06	0.000	1.000
V700	NA	NA	NA	NA
V701	NA	NA	NA	NA
V702	NA	NA	NA	NA
V703	NA	NA	NA	NA
V704	NA	NA	NA	NA
V705	NA	NA	NA	NA
V706	−4.489e+01	6.295e+05	0.000	1.000
V707	1.744e+01	3.948e+05	0.000	1.000
V708	−6.624e+01	4.170e+05	0.000	1.000
V709	−2.489e+01	4.591e+05	0.000	1.000
V710	3.624e+01	7.896e+05	0.000	1.000
V711	−1.449e+02	1.555e+06	0.000	1.000
V712	−1.699e+01	1.470e+06	0.000	1.000
V713	−2.188e+02	1.271e+06	0.000	1.000
V714	−4.447e+01	2.858e+05	0.000	1.000
V715	2.178e+01	2.569e+05	0.000	1.000
V716	4.374e+01	1.633e+05	0.000	1.000
V717	−8.682e+01	1.145e+05	−0.001	0.999
V718	2.415e+01	1.680e+05	0.000	1.000

V719	−1.079e+01	1.597e+05	0.000	1.000
V720	−1.493e+00	1.559e+05	0.000	1.000
V721	−1.564e+01	2.966e+05	0.000	1.000
V722	1.517e+01	2.831e+05	0.000	1.000
V723	−8.005e+00	2.306e+05	0.000	1.000
V724	−5.585e+01	7.819e+05	0.000	1.000
V725	2.752e+02	1.167e+06	0.000	1.000
V726	−2.560e+00	1.033e+06	0.000	1.000
V727	NA	NA	NA	NA
V728	NA	NA	NA	NA
V729	NA	NA	NA	NA
V730	NA	NA	NA	NA
V731	NA	NA	NA	NA
V732	NA	NA	NA	NA
V733	NA	NA	NA	NA
V734	−7.993e+01	8.551e+05	0.000	1.000
V735	−1.154e+02	1.726e+06	0.000	1.000
V736	1.380e+01	1.212e+06	0.000	1.000
V737	−1.031e+01	1.017e+06	0.000	1.000
V738	−4.733e+01	9.048e+06	0.000	1.000
V739	3.785e+01	1.416e+07	0.000	1.000
V740	5.778e+02	2.281e+07	0.000	1.000
V741	1.286e+02	2.560e+06	0.000	1.000
V742	−1.105e+02	2.226e+05	0.000	1.000
V743	5.090e+01	2.516e+05	0.000	1.000
V744	−2.112e+02	3.675e+05	−0.001	1.000
V745	1.370e+02	2.638e+05	0.001	1.000
V746	−8.505e+01	2.717e+05	0.000	1.000
V747	−8.716e+00	3.522e+05	0.000	1.000
V748	2.301e−01	2.809e+05	0.000	1.000
V749	−3.339e+01	3.943e+05	0.000	1.000
V750	4.773e+01	7.752e+05	0.000	1.000
V751	8.187e+01	1.512e+06	0.000	1.000
V752	2.615e−01	9.383e+06	0.000	1.000
V753	−5.990e+02	9.774e+06	0.000	1.000
V754	NA	NA	NA	NA
V755	NA	NA	NA	NA
V756	NA	NA	NA	NA
V757	NA	NA	NA	NA
V758	NA	NA	NA	NA
V759	NA	NA	NA	NA
V760	NA	NA	NA	NA
V761	NA	NA	NA	NA

```
V762               NA        NA       NA       NA
V763               NA        NA       NA       NA
V764               NA        NA       NA       NA
V765               NA        NA       NA       NA
V766               NA        NA       NA       NA
V767               NA        NA       NA       NA
V768               NA        NA       NA       NA
V769               NA        NA       NA       NA
V770        -4.891e+02  1.998e+06    0.000    1.000
V771               NA        NA       NA       NA
V772         8.712e+01  2.241e+06    0.000    1.000
V773        -4.075e+01  4.191e+05    0.000    1.000
V774         1.364e+01  4.500e+05    0.000    1.000
V775        -1.544e+02  8.601e+05    0.000    1.000
V776         3.086e+02  1.840e+06    0.000    1.000
V777               NA        NA       NA       NA
V778               NA        NA       NA       NA
V779               NA        NA       NA       NA
V780               NA        NA       NA       NA
V781               NA        NA       NA       NA
V782               NA        NA       NA       NA
V783               NA        NA       NA       NA
V784               NA        NA       NA       NA
V785               NA        NA       NA       NA
```

(Dispersion parameter for binomial family taken to be 1)

```
    Null deviance: 7.7923e+03  on 5620  degrees of freedom
Residual deviance: 2.4419e-07  on 5007  degrees of freedom
AIC: 1228
```

Number of Fisher Scoring iterations: 25

Plotting the ROC curve and calculating the AUC:

```
X <- as.data.frame(test_35[, 2:785])
y <- test_35$label
pred <- as.numeric(predict(l1, X))
```

```
Warning in predict.lm(object, newdata, se.fit, scale = 1, type = if (type == :
prediction from rank-deficient fit; attr(*, "non-estim") has doubtful cases
```

```
probs <- as.numeric(1 / (1 + exp(-pred)))
true_labels <- as.numeric(y)

# Calc ROC
pred <- prediction(probs, true_labels)
perf_m <- performance(pred, "tpr", "fpr")

# Calc AUC
auc <- performance(pred, "auc")
auc_value <- auc@y.values[[1]]
cat("AUC =", auc_value, "\n")
```

```
AUC = 0.9806825
```

```
# Plot the Curve
plot(perf_m, colorize = TRUE, colorkey.label = "Cutoff",
     colorize.palette = viridis_colors,
     main = "ROC Curve 3 and 5 Prediction, 64 Batch Size, 64 Hidden Layer Size")
abline(a = 0, b = 1, lty = 2, col = "black")
```

OC Curve 3 and 5 Prediction, 64 Batch Size, 64 Hidden Laye
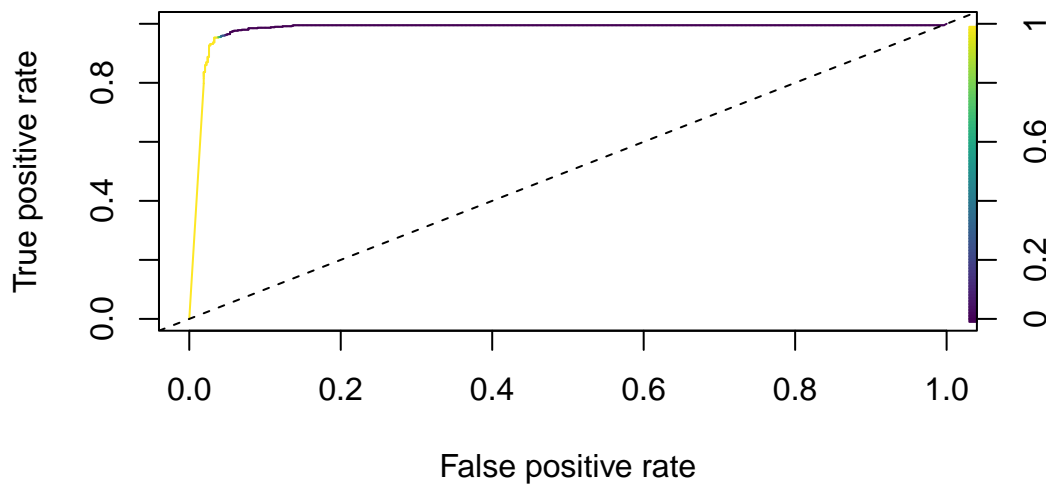
The performance of the Logistic Regression model was about the same than that of the MLP. The MLP AUC 0.9980987 was while the logistic regression AUC was 0.9806825. This indicates

that the Logistic Regression model was classifying a little worse for the digits 3/5. Overall, both models are fairly good, because AUC is high(close to 1).

Logistic for 4 and 9 Prediction:

```r
predictors <- paste0("V", 2:785)
rhs <- paste(predictors, collapse = " + ")
model_eq <- paste("label ~", rhs)
X <- as.matrix(train_49[, 2:785])
y <- as.matrix(train_49$label)


l2 <- glm(data = train_49, formula = formula(model_eq), family = binomial())
```

```
Warning: glm.fit: algorithm did not converge


Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```r
summary(l2)
```

```
Call:
glm(formula = formula(model_eq), family = binomial(), data = train_49)

Coefficients: (196 not defined because of singularities)
            Estimate Std. Error   z value Pr(>|z|)
(Intercept) -3.589e+14  5.061e+06  -70915381    <2e-16 ***
V2                NA         NA         NA        NA
V3                NA         NA         NA        NA
V4                NA         NA         NA        NA
V5                NA         NA         NA        NA
V6                NA         NA         NA        NA
V7                NA         NA         NA        NA
V8                NA         NA         NA        NA
V9                NA         NA         NA        NA
V10               NA         NA         NA        NA
V11               NA         NA         NA        NA
V12               NA         NA         NA        NA
V13               NA         NA         NA        NA
V14               NA         NA         NA        NA
V15               NA         NA         NA        NA
V16               NA         NA         NA        NA
```

V17	NA	NA	NA	NA
V18	NA	NA	NA	NA
V19	NA	NA	NA	NA
V20	NA	NA	NA	NA
V21	NA	NA	NA	NA
V22	NA	NA	NA	NA
V23	NA	NA	NA	NA
V24	NA	NA	NA	NA
V25	NA	NA	NA	NA
V26	NA	NA	NA	NA
V27	NA	NA	NA	NA
V28	NA	NA	NA	NA
V29	NA	NA	NA	NA
V30	NA	NA	NA	NA
V31	NA	NA	NA	NA
V32	NA	NA	NA	NA
V33	NA	NA	NA	NA
V34	NA	NA	NA	NA
V35	NA	NA	NA	NA
V36	NA	NA	NA	NA
V37	7.519e+14	1.167e+08	6443440	<2e-16 ***
V38	1.339e+16	4.910e+08	27265416	<2e-16 ***
V39	-1.107e+16	4.413e+08	-25080037	<2e-16 ***
V40	NA	NA	NA	NA
V41	NA	NA	NA	NA
V42	NA	NA	NA	NA
V43	NA	NA	NA	NA
V44	NA	NA	NA	NA
V45	NA	NA	NA	NA
V46	NA	NA	NA	NA
V47	-1.576e+16	3.013e+08	-52303955	<2e-16 ***
V48	9.041e+15	2.351e+08	38455553	<2e-16 ***
V49	NA	NA	NA	NA
V50	-3.637e+15	1.429e+08	-25458978	<2e-16 ***
V51	NA	NA	NA	NA
V52	NA	NA	NA	NA
V53	NA	NA	NA	NA
V54	NA	NA	NA	NA
V55	NA	NA	NA	NA
V56	NA	NA	NA	NA
V57	NA	NA	NA	NA
V58	NA	NA	NA	NA
V59	NA	NA	NA	NA

```
V60             NA        NA        NA         NA
V61             NA        NA        NA         NA
V62             NA        NA        NA         NA
V63             NA        NA        NA         NA
V64     -1.414e+16  2.957e+08  -47812296   <2e-16 ***
V65      1.540e+15  2.184e+08    7051661   <2e-16 ***
V66     -2.501e+15  8.958e+07  -27918330   <2e-16 ***
V67      5.904e+14  1.232e+08    4792253   <2e-16 ***
V68     -9.322e+15  2.623e+08  -35544577   <2e-16 ***
V69     -7.118e+15  2.574e+08  -27648666   <2e-16 ***
V70             NA        NA        NA         NA
V71     -4.999e+17  1.175e+10  -42556703   <2e-16 ***
V72      1.563e+17  3.487e+09   44819177   <2e-16 ***
V73     -3.895e+15  1.799e+08  -21653777   <2e-16 ***
V74     -1.034e+15  1.086e+08   -9518726   <2e-16 ***
V75     -1.749e+15  7.765e+07  -22527046   <2e-16 ***
V76     -4.724e+14  6.369e+07   -7416109   <2e-16 ***
V77      1.960e+15  4.887e+07   40102798   <2e-16 ***
V78     -1.838e+15  4.023e+07  -45679879   <2e-16 ***
V79     -4.543e+14  4.095e+07  -11091588   <2e-16 ***
V80     -5.750e+13  4.736e+07   -1214003   <2e-16 ***
V81     -1.375e+15  7.135e+07  -19272658   <2e-16 ***
V82      4.060e+15  2.485e+08   16341899   <2e-16 ***
V83      8.610e+15  1.270e+09    6779145   <2e-16 ***
V84             NA        NA        NA         NA
V85             NA        NA        NA         NA
V86             NA        NA        NA         NA
V87             NA        NA        NA         NA
V88             NA        NA        NA         NA
V89             NA        NA        NA         NA
V90     -2.691e+16  4.146e+08  -64894345   <2e-16 ***
V91     -1.062e+15  6.483e+07  -16377846   <2e-16 ***
V92     -2.748e+14  4.285e+07   -6411534   <2e-16 ***
V93     -6.653e+14  4.345e+07  -15310526   <2e-16 ***
V94      4.311e+14  3.812e+07   11307789   <2e-16 ***
V95      1.126e+13  4.228e+07     266357   <2e-16 ***
V96     -1.760e+15  4.582e+07  -38419228   <2e-16 ***
V97     -1.624e+14  5.726e+07   -2835963   <2e-16 ***
V98     -4.023e+15  8.699e+07  -46248302   <2e-16 ***
V99      3.231e+15  1.340e+08   24120569   <2e-16 ***
V100    -1.150e+16  1.347e+08  -85331460   <2e-16 ***
V101     1.783e+15  8.221e+07   21682278   <2e-16 ***
V102    -7.439e+13  5.542e+07   -1342280   <2e-16 ***
```

V103	1.435e+15	3.793e+07	37817978	<2e-16 ***
V104	-2.659e+15	3.109e+07	-85524897	<2e-16 ***
V105	-6.297e+13	2.563e+07	-2456624	<2e-16 ***
V106	-3.784e+13	2.262e+07	-1673020	<2e-16 ***
V107	1.539e+15	2.245e+07	68572405	<2e-16 ***
V108	-3.875e+14	2.694e+07	-14385570	<2e-16 ***
V109	2.803e+15	3.704e+07	75666122	<2e-16 ***
V110	-5.645e+15	5.975e+07	-94484380	<2e-16 ***
V111	-6.369e+15	2.752e+08	-23144490	<2e-16 ***
V112	NA	NA	NA	NA
V113	NA	NA	NA	NA
V114	NA	NA	NA	NA
V115	NA	NA	NA	NA
V116	NA	NA	NA	NA
V117	NA	NA	NA	NA
V118	3.026e+15	7.146e+07	42349390	<2e-16 ***
V119	-1.723e+15	3.893e+07	-44265206	<2e-16 ***
V120	-1.518e+14	2.782e+07	-5457129	<2e-16 ***
V121	3.890e+14	2.308e+07	16858729	<2e-16 ***
V122	-5.173e+14	2.397e+07	-21583726	<2e-16 ***
V123	-6.266e+14	2.584e+07	-24248563	<2e-16 ***
V124	2.211e+15	3.393e+07	65165435	<2e-16 ***
V125	-3.214e+15	3.828e+07	-83967009	<2e-16 ***
V126	4.283e+15	5.002e+07	85630780	<2e-16 ***
V127	-6.697e+14	6.914e+07	-9686743	<2e-16 ***
V128	2.001e+15	7.842e+07	25519736	<2e-16 ***
V129	1.092e+15	5.082e+07	21480438	<2e-16 ***
V130	-2.134e+15	3.890e+07	-54851441	<2e-16 ***
V131	3.131e+14	2.894e+07	10815687	<2e-16 ***
V132	1.866e+15	2.126e+07	87777981	<2e-16 ***
V133	-2.487e+14	1.828e+07	-13604683	<2e-16 ***
V134	-4.021e+13	1.663e+07	-2418576	<2e-16 ***
V135	-1.182e+15	1.673e+07	-70645937	<2e-16 ***
V136	-5.016e+14	1.797e+07	-27906775	<2e-16 ***
V137	-1.581e+15	2.312e+07	-68357447	<2e-16 ***
V138	2.558e+15	3.540e+07	72276648	<2e-16 ***
V139	4.611e+15	9.187e+07	50186124	<2e-16 ***
V140	-5.522e+16	2.477e+10	-2228849	<2e-16 ***
V141	NA	NA	NA	NA
V142	NA	NA	NA	NA
V143	NA	NA	NA	NA
V144	NA	NA	NA	NA
V145	-1.402e+16	1.030e+09	-13608429	<2e-16 ***

```
V146    -2.348e+15  4.787e+07   -49042568   <2e-16 ***
V147     9.108e+14  2.896e+07    31447288   <2e-16 ***
V148    -3.307e+14  2.004e+07   -16502401   <2e-16 ***
V149    -8.025e+13  1.821e+07    -4408063   <2e-16 ***
V150    -2.173e+14  1.656e+07   -13122729   <2e-16 ***
V151     4.914e+13  1.840e+07     2670485   <2e-16 ***
V152    -7.298e+14  2.177e+07   -33527903   <2e-16 ***
V153     2.050e+15  2.540e+07    80730046   <2e-16 ***
V154    -1.045e+15  3.076e+07   -33955664   <2e-16 ***
V155     1.786e+13  3.384e+07      527844   <2e-16 ***
V156     1.249e+15  4.039e+07    30915158   <2e-16 ***
V157    -2.853e+15  3.908e+07   -72996701   <2e-16 ***
V158     1.398e+15  3.121e+07    44792304   <2e-16 ***
V159    -6.832e+14  2.343e+07   -29161824   <2e-16 ***
V160    -6.338e+14  1.734e+07   -36561653   <2e-16 ***
V161    -4.854e+14  1.384e+07   -35063204   <2e-16 ***
V162    -7.985e+13  1.272e+07    -6275166   <2e-16 ***
V163     7.382e+14  1.226e+07    60196634   <2e-16 ***
V164    -3.673e+14  1.307e+07   -28092801   <2e-16 ***
V165     2.421e+14  1.579e+07    15339990   <2e-16 ***
V166     7.788e+14  2.451e+07    31776107   <2e-16 ***
V167    -3.691e+15  8.210e+07   -44957973   <2e-16 ***
V168            NA         NA          NA       NA
V169            NA         NA          NA       NA
V170            NA         NA          NA       NA
V171            NA         NA          NA       NA
V172            NA         NA          NA       NA
V173     5.681e+15  3.372e+08    16845518   <2e-16 ***
V174     1.697e+15  3.397e+07    49952610   <2e-16 ***
V175    -2.622e+14  2.002e+07   -13097517   <2e-16 ***
V176    -3.365e+14  1.530e+07   -21992961   <2e-16 ***
V177     5.606e+14  1.399e+07    40083297   <2e-16 ***
V178    -4.287e+14  1.342e+07   -31942791   <2e-16 ***
V179     3.111e+13  1.430e+07     2175490   <2e-16 ***
V180     1.474e+14  1.620e+07     9101726   <2e-16 ***
V181    -5.744e+14  1.927e+07   -29813952   <2e-16 ***
V182     1.080e+15  2.228e+07    48455387   <2e-16 ***
V183    -3.054e+14  2.350e+07   -12997261   <2e-16 ***
V184    -6.700e+14  2.562e+07   -26148336   <2e-16 ***
V185     4.240e+13  2.710e+07     1564245   <2e-16 ***
V186     1.511e+14  2.317e+07     6520316   <2e-16 ***
V187     1.029e+15  1.753e+07    58702140   <2e-16 ***
V188    -2.025e+14  1.379e+07   -14680094   <2e-16 ***
```

```
V189     -2.545e+14  1.139e+07  -22336134  <2e-16 ***
V190     -5.137e+14  1.022e+07  -50250290  <2e-16 ***
V191     -1.226e+14  1.016e+07  -12065219  <2e-16 ***
V192      4.205e+14  1.030e+07   40842347  <2e-16 ***
V193      2.987e+14  1.161e+07   25732889  <2e-16 ***
V194     -2.198e+15  1.834e+07 -119865103  <2e-16 ***
V195      9.761e+14  5.417e+07   18018825  <2e-16 ***
V196      6.910e+15  3.042e+09    2271870  <2e-16 ***
V197            NA         NA         NA      NA
V198            NA         NA         NA      NA
V199            NA         NA         NA      NA
V200            NA         NA         NA      NA
V201      6.547e+14  9.705e+07    6746200  <2e-16 ***
V202     -1.237e+15  2.514e+07  -49211720  <2e-16 ***
V203      7.467e+14  1.572e+07   47504077  <2e-16 ***
V204     -5.959e+14  1.172e+07  -50861613  <2e-16 ***
V205     -2.418e+14  1.098e+07  -22016895  <2e-16 ***
V206      7.732e+14  1.084e+07   71293419  <2e-16 ***
V207     -3.759e+14  1.161e+07  -32384167  <2e-16 ***
V208      1.872e+13  1.245e+07    1502952  <2e-16 ***
V209      1.637e+14  1.400e+07   11688974  <2e-16 ***
V210     -9.719e+13  1.526e+07   -6368191  <2e-16 ***
V211      9.634e+13  1.551e+07    6209742  <2e-16 ***
V212      7.955e+14  1.605e+07   49550849  <2e-16 ***
V213     -8.517e+14  1.643e+07  -51845592  <2e-16 ***
V214      2.597e+12  1.654e+07     157070  <2e-16 ***
V215     -5.342e+14  1.370e+07  -38982582  <2e-16 ***
V216      8.218e+14  1.158e+07   70942799  <2e-16 ***
V217     -3.237e+14  9.772e+06  -33122705  <2e-16 ***
V218      3.132e+14  8.902e+06   35187513  <2e-16 ***
V219      2.686e+14  8.659e+06   31017652  <2e-16 ***
V220     -1.055e+14  8.888e+06  -11875241  <2e-16 ***
V221     -7.270e+14  9.672e+06  -75166977  <2e-16 ***
V222      4.197e+14  1.401e+07   29950533  <2e-16 ***
V223     -7.519e+12  4.358e+07    -172535  <2e-16 ***
V224     -7.599e+15  1.314e+08  -57839636  <2e-16 ***
V225            NA         NA         NA      NA
V226            NA         NA         NA      NA
V227            NA         NA         NA      NA
V228            NA         NA         NA      NA
V229     -5.257e+14  8.036e+07   -6542305  <2e-16 ***
V230     -9.627e+14  2.075e+07  -46392848  <2e-16 ***
V231      1.130e+14  1.256e+07    8996130  <2e-16 ***
```

```
V232     2.045e+14  9.975e+06   20504953   <2e-16 ***
V233    -4.210e+14  9.031e+06  -46613506   <2e-16 ***
V234     4.521e+13  9.032e+06    5005037   <2e-16 ***
V235    -1.098e+14  9.559e+06  -11488595   <2e-16 ***
V236     5.048e+14  1.047e+07   48231174   <2e-16 ***
V237     9.241e+13  1.106e+07    8357817   <2e-16 ***
V238     4.744e+14  1.099e+07   43175449   <2e-16 ***
V239    -1.875e+14  1.069e+07  -17541917   <2e-16 ***
V240    -1.550e+14  1.097e+07  -14128649   <2e-16 ***
V241    -1.370e+14  1.170e+07  -11703598   <2e-16 ***
V242     1.052e+15  1.215e+07   86645556   <2e-16 ***
V243     2.837e+14  1.124e+07   25235923   <2e-16 ***
V244    -8.295e+14  9.690e+06  -85607640   <2e-16 ***
V245     1.765e+14  8.794e+06   20071162   <2e-16 ***
V246    -4.890e+13  7.838e+06   -6239220   <2e-16 ***
V247    -1.160e+14  7.715e+06  -15030851   <2e-16 ***
V248    -3.042e+14  7.864e+06  -38683510   <2e-16 ***
V249     3.067e+14  8.374e+06   36624023   <2e-16 ***
V250    -2.990e+14  1.170e+07  -25557003   <2e-16 ***
V251    -2.224e+14  2.983e+07   -7456292   <2e-16 ***
V252     5.016e+15  1.481e+08   33878038   <2e-16 ***
V253            NA         NA         NA        NA
V254            NA         NA         NA        NA
V255            NA         NA         NA        NA
V256            NA         NA         NA        NA
V257    -1.697e+14  5.565e+07   -3048693   <2e-16 ***
V258    -2.178e+14  1.740e+07  -12514735   <2e-16 ***
V259    -2.158e+14  1.080e+07  -19986579   <2e-16 ***
V260     1.764e+14  8.883e+06   19857805   <2e-16 ***
V261    -2.176e+14  7.967e+06  -27309378   <2e-16 ***
V262     3.766e+14  7.831e+06   48095930   <2e-16 ***
V263    -3.150e+14  8.264e+06  -38115975   <2e-16 ***
V264     1.266e+14  8.818e+06   14354741   <2e-16 ***
V265     2.214e+14  8.833e+06   25067958   <2e-16 ***
V266    -1.579e+14  8.285e+06  -19059025   <2e-16 ***
V267     1.973e+14  8.029e+06   24577460   <2e-16 ***
V268    -2.652e+13  8.286e+06   -3200506   <2e-16 ***
V269     1.623e+14  8.839e+06   18355609   <2e-16 ***
V270    -5.300e+14  9.403e+06  -56359327   <2e-16 ***
V271     9.173e+14  9.557e+06   95988524   <2e-16 ***
V272     5.211e+14  9.139e+06   57020335   <2e-16 ***
V273    -1.009e+14  8.145e+06  -12385582   <2e-16 ***
V274    -3.070e+14  7.544e+06  -40695355   <2e-16 ***
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V275         5.058e+14  7.332e+06   68991922   <2e-16 ***
V276        -3.134e+13  7.347e+06   -4266238   <2e-16 ***
V277        -3.337e+14  7.922e+06  -42119456   <2e-16 ***
V278         3.222e+14  1.063e+07   30300315   <2e-16 ***
V279        -1.688e+15  2.648e+07  -63733787   <2e-16 ***
V280         2.932e+14  2.539e+08    1154881   <2e-16 ***
V281                NA         NA         NA       NA
V282                NA         NA         NA       NA
V283                NA         NA         NA       NA
V284        -1.149e+16  1.804e+08  -63675646   <2e-16 ***
V285        -2.245e+14  4.246e+07   -5287565   <2e-16 ***
V286         7.619e+14  1.515e+07   50291226   <2e-16 ***
V287        -5.547e+13  9.799e+06   -5660717   <2e-16 ***
V288        -3.477e+14  8.160e+06  -42612787   <2e-16 ***
V289         2.826e+14  7.253e+06   38961318   <2e-16 ***
V290         4.645e+13  7.431e+06    6250752   <2e-16 ***
V291         1.098e+12  7.706e+06     142540   <2e-16 ***
V292         1.677e+14  7.850e+06   21368514   <2e-16 ***
V293         3.712e+14  7.636e+06   48607367   <2e-16 ***
V294        -2.605e+13  7.162e+06   -3637885   <2e-16 ***
V295         1.801e+14  7.099e+06   25365380   <2e-16 ***
V296        -1.030e+14  7.055e+06  -14600993   <2e-16 ***
V297        -1.856e+13  7.567e+06   -2452292   <2e-16 ***
V298         2.857e+14  8.127e+06   35151181   <2e-16 ***
V299        -1.352e+14  8.884e+06  -15222079   <2e-16 ***
V300         9.527e+13  8.773e+06   10859630   <2e-16 ***
V301         4.489e+14  8.016e+06   55996154   <2e-16 ***
V302         5.785e+13  7.336e+06    7886316   <2e-16 ***
V303        -4.449e+14  7.139e+06  -62319497   <2e-16 ***
V304         1.035e+14  7.334e+06   14112869   <2e-16 ***
V305        -1.583e+14  7.891e+06  -20059486   <2e-16 ***
V306        -1.618e+14  9.917e+06  -16311274   <2e-16 ***
V307         2.755e+14  2.317e+07   11890310   <2e-16 ***
V308        -4.657e+15  3.204e+08  -14532239   <2e-16 ***
V309                NA         NA         NA       NA
V310                NA         NA         NA       NA
V311                NA         NA         NA       NA
V312                NA         NA         NA       NA
V313         2.068e+14  4.060e+07    5093461   <2e-16 ***
V314        -7.840e+14  1.437e+07  -54560548   <2e-16 ***
V315         2.938e+14  9.209e+06   31908310   <2e-16 ***
V316         1.877e+14  8.045e+06   23330570   <2e-16 ***
V317        -6.857e+13  7.331e+06   -9353317   <2e-16 ***
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V318	-1.321e+13	7.321e+06	-1804421	<2e-16 ***
V319	1.163e+14	7.428e+06	15660595	<2e-16 ***
V320	1.513e+14	7.476e+06	20238538	<2e-16 ***
V321	1.112e+13	6.946e+06	1601139	<2e-16 ***
V322	2.034e+13	6.551e+06	3104093	<2e-16 ***
V323	5.863e+13	6.494e+06	9027458	<2e-16 ***
V324	2.691e+14	6.472e+06	41584802	<2e-16 ***
V325	-3.401e+14	6.965e+06	-48830762	<2e-16 ***
V326	8.749e+13	7.705e+06	11355473	<2e-16 ***
V327	3.250e+14	8.492e+06	38266957	<2e-16 ***
V328	-2.601e+14	8.571e+06	-30344524	<2e-16 ***
V329	-1.088e+12	7.958e+06	-136761	<2e-16 ***
V330	4.534e+14	7.325e+06	61906551	<2e-16 ***
V331	-1.788e+14	7.059e+06	-25329665	<2e-16 ***
V332	-2.536e+14	7.586e+06	-33430943	<2e-16 ***
V333	1.132e+14	7.975e+06	14190287	<2e-16 ***
V334	-9.009e+13	9.298e+06	-9688935	<2e-16 ***
V335	2.019e+14	2.277e+07	8867477	<2e-16 ***
V336	3.363e+15	1.716e+08	19600027	<2e-16 ***
V337	NA	NA	NA	NA
V338	NA	NA	NA	NA
V339	NA	NA	NA	NA
V340	NA	NA	NA	NA
V341	-6.181e+14	3.797e+07	-16275623	<2e-16 ***
V342	5.222e+14	1.353e+07	38585565	<2e-16 ***
V343	-3.189e+14	8.809e+06	-36197272	<2e-16 ***
V344	-1.247e+14	8.080e+06	-15434507	<2e-16 ***
V345	2.474e+14	7.676e+06	32230666	<2e-16 ***
V346	2.342e+14	7.329e+06	31948090	<2e-16 ***
V347	-1.033e+14	7.203e+06	-14344936	<2e-16 ***
V348	-2.005e+14	7.019e+06	-28566930	<2e-16 ***
V349	3.382e+14	6.651e+06	50849600	<2e-16 ***
V350	-5.606e+13	6.570e+06	-8532863	<2e-16 ***
V351	3.954e+13	6.543e+06	6043352	<2e-16 ***
V352	1.376e+14	6.771e+06	20326227	<2e-16 ***
V353	-7.739e+13	7.260e+06	-10659804	<2e-16 ***
V354	2.375e+14	7.945e+06	29890173	<2e-16 ***
V355	1.305e+14	8.503e+06	15350101	<2e-16 ***
V356	4.756e+14	8.388e+06	56705689	<2e-16 ***
V357	5.005e+13	8.115e+06	6167890	<2e-16 ***
V358	1.311e+14	7.499e+06	17478453	<2e-16 ***
V359	-2.734e+14	7.200e+06	-37973862	<2e-16 ***
V360	3.434e+14	7.991e+06	42976484	<2e-16 ***

V361	-1.408e+14	8.155e+06	-17261827	<2e-16	***
V362	2.081e+14	9.251e+06	22490171	<2e-16	***
V363	-4.682e+14	2.116e+07	-22126210	<2e-16	***
V364	-2.878e+15	1.125e+08	-25579044	<2e-16	***
V365	NA	NA	NA	NA	
V366	NA	NA	NA	NA	
V367	NA	NA	NA	NA	
V368	NA	NA	NA	NA	
V369	-6.302e+13	3.773e+07	-1670042	<2e-16	***
V370	-7.869e+13	1.291e+07	-6095271	<2e-16	***
V371	3.402e+14	8.428e+06	40369585	<2e-16	***
V372	1.091e+14	8.299e+06	13140443	<2e-16	***
V373	-4.348e+14	7.666e+06	-56723163	<2e-16	***
V374	1.272e+14	7.157e+06	17774012	<2e-16	***
V375	2.982e+14	7.098e+06	42006210	<2e-16	***
V376	3.922e+13	7.048e+06	5564433	<2e-16	***
V377	2.171e+14	6.703e+06	32388701	<2e-16	***
V378	9.605e+13	6.681e+06	14377337	<2e-16	***
V379	2.248e+14	7.147e+06	31456091	<2e-16	***
V380	8.018e+13	7.436e+06	10783538	<2e-16	***
V381	2.475e+14	7.777e+06	31830717	<2e-16	***
V382	9.741e+13	8.194e+06	11887258	<2e-16	***
V383	2.726e+13	8.335e+06	3270831	<2e-16	***
V384	-1.790e+14	8.418e+06	-21261939	<2e-16	***
V385	2.913e+14	8.302e+06	35090947	<2e-16	***
V386	-3.205e+13	7.680e+06	-4173040	<2e-16	***
V387	1.761e+14	7.336e+06	23997572	<2e-16	***
V388	-2.986e+13	7.922e+06	-3769490	<2e-16	***
V389	1.238e+14	8.413e+06	14710093	<2e-16	***
V390	6.833e+13	9.655e+06	7077344	<2e-16	***
V391	-1.137e+14	2.172e+07	-5235498	<2e-16	***
V392	3.655e+15	1.373e+08	26613449	<2e-16	***
V393	NA	NA	NA	NA	
V394	NA	NA	NA	NA	
V395	NA	NA	NA	NA	
V396	-1.522e+15	6.542e+08	-2326439	<2e-16	***
V397	-5.998e+14	3.881e+07	-15455037	<2e-16	***
V398	-2.351e+13	1.175e+07	-2000962	<2e-16	***
V399	-3.466e+14	8.401e+06	-41253620	<2e-16	***
V400	-1.001e+13	8.265e+06	-1210563	<2e-16	***
V401	-9.242e+13	7.494e+06	-12332666	<2e-16	***
V402	-1.211e+14	7.014e+06	-17263819	<2e-16	***
V403	5.090e+13	6.997e+06	7274707	<2e-16	***

V404	-2.884e+14	7.044e+06	-40946003	<2e-16	***
V405	1.678e+14	6.935e+06	24202339	<2e-16	***
V406	-6.165e+13	6.932e+06	-8893748	<2e-16	***
V407	-5.126e+14	7.474e+06	-68583473	<2e-16	***
V408	1.289e+14	7.466e+06	17266896	<2e-16	***
V409	-5.290e+13	7.854e+06	-6735730	<2e-16	***
V410	1.087e+14	7.702e+06	14107776	<2e-16	***
V411	-2.356e+13	7.865e+06	-2995167	<2e-16	***
V412	1.101e+14	8.279e+06	13299380	<2e-16	***
V413	-4.249e+13	7.887e+06	-5386716	<2e-16	***
V414	-3.257e+14	7.508e+06	-43379350	<2e-16	***
V415	3.178e+14	7.326e+06	43386238	<2e-16	***
V416	1.156e+14	7.770e+06	14882332	<2e-16	***
V417	5.424e+13	8.410e+06	6449655	<2e-16	***
V418	-3.622e+14	1.042e+07	-34772305	<2e-16	***
V419	-3.916e+14	2.499e+07	-15668971	<2e-16	***
V420	-7.991e+15	2.952e+08	-27068200	<2e-16	***
V421	NA	NA	NA	NA	
V422	NA	NA	NA	NA	
V423	NA	NA	NA	NA	
V424	4.282e+15	3.743e+08	11439536	<2e-16	***
V425	1.063e+14	3.481e+07	3052914	<2e-16	***
V426	-7.417e+13	1.122e+07	-6610235	<2e-16	***
V427	7.340e+13	8.210e+06	8940045	<2e-16	***
V428	-1.369e+14	7.858e+06	-17419468	<2e-16	***
V429	3.632e+13	7.225e+06	5027401	<2e-16	***
V430	-1.283e+14	6.807e+06	-18842654	<2e-16	***
V431	-1.637e+14	6.932e+06	-23614458	<2e-16	***
V432	1.785e+14	7.103e+06	25126350	<2e-16	***
V433	-1.842e+14	7.219e+06	-25518804	<2e-16	***
V434	6.927e+13	7.218e+06	9596010	<2e-16	***
V435	5.167e+14	7.421e+06	69624244	<2e-16	***
V436	-1.239e+14	7.177e+06	-17264598	<2e-16	***
V437	1.280e+14	7.024e+06	18216110	<2e-16	***
V438	-1.954e+14	7.085e+06	-27577206	<2e-16	***
V439	3.330e+14	7.647e+06	43555200	<2e-16	***
V440	-5.279e+14	7.851e+06	-67237546	<2e-16	***
V441	9.052e+13	7.539e+06	12006747	<2e-16	***
V442	6.348e+13	7.231e+06	8779509	<2e-16	***
V443	-3.567e+14	7.386e+06	-48290573	<2e-16	***
V444	3.263e+12	7.764e+06	420265	<2e-16	***
V445	-8.446e+13	8.565e+06	-9860904	<2e-16	***
V446	3.939e+14	1.118e+07	35220497	<2e-16	***

V447	3.214e+13	2.797e+07	1148914	<2e-16	***
V448	1.892e+15	4.835e+08	3913533	<2e-16	***
V449	NA	NA	NA	NA	
V450	NA	NA	NA	NA	
V451	NA	NA	NA	NA	
V452	NA	NA	NA	NA	
V453	-6.432e+14	3.520e+07	-18272306	<2e-16	***
V454	1.863e+13	1.147e+07	1623966	<2e-16	***
V455	1.710e+13	7.769e+06	2201035	<2e-16	***
V456	-1.335e+14	7.564e+06	-17655510	<2e-16	***
V457	-3.006e+13	6.860e+06	-4381825	<2e-16	***
V458	5.269e+13	6.679e+06	7889666	<2e-16	***
V459	-1.214e+14	6.777e+06	-17919916	<2e-16	***
V460	-1.451e+14	7.190e+06	-20184599	<2e-16	***
V461	-2.031e+14	7.350e+06	-27630870	<2e-16	***
V462	9.874e+13	7.068e+06	13970096	<2e-16	***
V463	-2.321e+14	7.058e+06	-32889601	<2e-16	***
V464	3.368e+14	6.636e+06	50750233	<2e-16	***
V465	-2.117e+14	6.551e+06	-32312013	<2e-16	***
V466	3.656e+14	6.946e+06	52636208	<2e-16	***
V467	1.497e+14	7.465e+06	20055446	<2e-16	***
V468	-2.278e+14	7.674e+06	-29687107	<2e-16	***
V469	-2.464e+14	7.501e+06	-32845439	<2e-16	***
V470	2.194e+14	7.307e+06	30021305	<2e-16	***
V471	-1.559e+14	7.490e+06	-20815876	<2e-16	***
V472	-3.476e+14	8.038e+06	-43242538	<2e-16	***
V473	4.781e+14	9.119e+06	52433266	<2e-16	***
V474	-8.952e+12	1.270e+07	-704637	<2e-16	***
V475	1.459e+13	3.170e+07	460435	<2e-16	***
V476	1.005e+16	4.657e+08	21589841	<2e-16	***
V477	NA	NA	NA	NA	
V478	NA	NA	NA	NA	
V479	NA	NA	NA	NA	
V480	NA	NA	NA	NA	
V481	1.161e+15	3.525e+07	32944121	<2e-16	***
V482	1.739e+14	1.204e+07	14452771	<2e-16	***
V483	2.244e+14	7.580e+06	29604593	<2e-16	***
V484	9.948e+13	7.246e+06	13729199	<2e-16	***
V485	9.862e+13	6.771e+06	14566186	<2e-16	***
V486	-2.231e+14	6.468e+06	-34497559	<2e-16	***
V487	-2.653e+13	6.685e+06	-3968269	<2e-16	***
V488	3.554e+13	7.019e+06	5064254	<2e-16	***
V489	-6.695e+13	7.200e+06	-9298950	<2e-16	***

V490	3.297e+13	7.138e+06	4619314	<2e-16	***
V491	4.400e+13	6.657e+06	6609237	<2e-16	***
V492	-1.941e+13	6.579e+06	-2949779	<2e-16	***
V493	2.555e+14	6.729e+06	37970480	<2e-16	***
V494	-1.147e+14	7.164e+06	-16007938	<2e-16	***
V495	-9.512e+13	7.778e+06	-12229444	<2e-16	***
V496	5.619e+13	8.067e+06	6965334	<2e-16	***
V497	-4.125e+14	7.990e+06	-51632135	<2e-16	***
V498	-7.310e+13	7.853e+06	-9309020	<2e-16	***
V499	3.063e+14	7.819e+06	39170560	<2e-16	***
V500	-1.034e+14	8.420e+06	-12276766	<2e-16	***
V501	-5.105e+14	1.020e+07	-50067994	<2e-16	***
V502	-5.370e+14	1.490e+07	-36053167	<2e-16	***
V503	3.834e+14	3.732e+07	10273043	<2e-16	***
V504	-8.553e+15	1.857e+08	-46053388	<2e-16	***
V505	NA	NA	NA	NA	
V506	NA	NA	NA	NA	
V507	NA	NA	NA	NA	
V508	NA	NA	NA	NA	
V509	-1.578e+15	4.304e+07	-36674464	<2e-16	***
V510	-1.056e+14	1.312e+07	-8047417	<2e-16	***
V511	7.163e+12	7.849e+06	912599	<2e-16	***
V512	-3.330e+14	7.277e+06	-45758552	<2e-16	***
V513	-1.377e+14	6.912e+06	-19923563	<2e-16	***
V514	2.984e+14	6.680e+06	44671444	<2e-16	***
V515	4.815e+13	6.943e+06	6935049	<2e-16	***
V516	7.886e+13	7.308e+06	10790843	<2e-16	***
V517	-3.435e+14	7.358e+06	-46679515	<2e-16	***
V518	-1.195e+14	7.407e+06	-16139800	<2e-16	***
V519	3.954e+13	6.955e+06	5684259	<2e-16	***
V520	-6.139e+14	7.182e+06	-85481983	<2e-16	***
V521	3.521e+14	7.654e+06	46000632	<2e-16	***
V522	-5.943e+14	8.373e+06	-70979482	<2e-16	***
V523	-3.098e+13	9.038e+06	-3428082	<2e-16	***
V524	-8.964e+13	8.932e+06	-10035981	<2e-16	***
V525	2.192e+14	8.589e+06	25519289	<2e-16	***
V526	-3.635e+14	8.574e+06	-42391706	<2e-16	***
V527	-1.469e+13	8.700e+06	-1688138	<2e-16	***
V528	1.581e+14	9.811e+06	16116449	<2e-16	***
V529	1.613e+14	1.167e+07	13828362	<2e-16	***
V530	-9.042e+14	1.812e+07	-49912007	<2e-16	***
V531	-1.429e+15	4.727e+07	-30235008	<2e-16	***
V532	-5.886e+15	3.126e+08	-18826816	<2e-16	***

```
V533            NA         NA         NA        NA
V534            NA         NA         NA        NA
V535            NA         NA         NA        NA
V536            NA         NA         NA        NA
V537      2.053e+14  5.948e+07    3451135   <2e-16 ***
V538      1.355e+14  1.465e+07    9248704   <2e-16 ***
V539     -2.657e+14  8.373e+06  -31740130   <2e-16 ***
V540      2.919e+14  7.466e+06   39090861   <2e-16 ***
V541      1.177e+14  7.249e+06   16238471   <2e-16 ***
V542     -4.065e+13  7.315e+06   -5557529   <2e-16 ***
V543     -4.134e+14  7.480e+06  -55269035   <2e-16 ***
V544      3.878e+14  7.693e+06   50416429   <2e-16 ***
V545      2.738e+14  7.980e+06   34311493   <2e-16 ***
V546     -4.170e+14  8.147e+06  -51190456   <2e-16 ***
V547      4.559e+14  8.096e+06   56307602   <2e-16 ***
V548      2.869e+14  8.715e+06   32924663   <2e-16 ***
V549      6.867e+13  8.989e+06    7638684   <2e-16 ***
V550     -5.006e+14  9.802e+06  -51075453   <2e-16 ***
V551      5.950e+14  1.027e+07   57910663   <2e-16 ***
V552     -5.695e+14  1.007e+07  -56575591   <2e-16 ***
V553      8.319e+13  9.890e+06    8411725   <2e-16 ***
V554     -3.156e+14  9.842e+06  -32070056   <2e-16 ***
V555     -1.875e+14  1.045e+07  -17946549   <2e-16 ***
V556      2.535e+14  1.188e+07   21340380   <2e-16 ***
V557      9.772e+14  1.512e+07   64636386   <2e-16 ***
V558      7.739e+14  2.359e+07   32813579   <2e-16 ***
V559     -4.133e+14  6.243e+07   -6620231   <2e-16 ***
V560      1.587e+15  2.143e+08    7408584   <2e-16 ***
V561            NA         NA         NA        NA
V562            NA         NA         NA        NA
V563            NA         NA         NA        NA
V564            NA         NA         NA        NA
V565      2.316e+14  7.518e+07    3080631   <2e-16 ***
V566     -7.260e+14  1.747e+07  -41550094   <2e-16 ***
V567      4.345e+14  9.440e+06   46021681   <2e-16 ***
V568     -2.281e+14  8.229e+06  -27714988   <2e-16 ***
V569     -1.323e+14  7.978e+06  -16584233   <2e-16 ***
V570      2.750e+14  8.042e+06   34189829   <2e-16 ***
V571      9.774e+13  8.239e+06   11863237   <2e-16 ***
V572      1.009e+14  8.558e+06   11787244   <2e-16 ***
V573     -5.848e+14  9.087e+06  -64354604   <2e-16 ***
V574      6.867e+14  9.732e+06   70558968   <2e-16 ***
V575     -7.528e+14  1.025e+07  -73445291   <2e-16 ***
```

V576	−3.292e+14	1.093e+07	−30113199	<2e-16	***
V577	−1.977e+13	1.136e+07	−1739949	<2e-16	***
V578	−8.356e+14	1.157e+07	−72226606	<2e-16	***
V579	2.984e+14	1.211e+07	24642844	<2e-16	***
V580	−3.287e+14	1.220e+07	−26951928	<2e-16	***
V581	8.816e+14	1.224e+07	72029843	<2e-16	***
V582	−9.838e+14	1.234e+07	−79721897	<2e-16	***
V583	4.491e+14	1.330e+07	33762904	<2e-16	***
V584	4.132e+14	1.569e+07	26332643	<2e-16	***
V585	−5.545e+14	2.107e+07	−26317070	<2e-16	***
V586	6.296e+14	3.830e+07	16440322	<2e-16	***
V587	2.093e+15	1.207e+08	17347178	<2e-16	***
V588	3.137e+16	9.256e+08	33896887	<2e-16	***
V589	NA	NA	NA	NA	
V590	NA	NA	NA	NA	
V591	NA	NA	NA	NA	
V592	NA	NA	NA	NA	
V593	−2.539e+15	1.140e+08	−22262605	<2e-16	***
V594	3.444e+14	2.077e+07	16583111	<2e-16	***
V595	−2.189e+13	1.174e+07	−1864644	<2e-16	***
V596	1.860e+14	1.006e+07	18492209	<2e-16	***
V597	5.044e+14	9.359e+06	53891687	<2e-16	***
V598	3.709e+13	9.273e+06	3999491	<2e-16	***
V599	−4.772e+14	9.854e+06	−48425660	<2e-16	***
V600	3.171e+14	1.032e+07	30724651	<2e-16	***
V601	4.754e+14	1.160e+07	40973673	<2e-16	***
V602	3.077e+14	1.258e+07	24454933	<2e-16	***
V603	3.443e+14	1.412e+07	24388303	<2e-16	***
V604	3.063e+13	1.449e+07	2114225	<2e-16	***
V605	9.718e+14	1.534e+07	63368985	<2e-16	***
V606	−5.766e+14	1.536e+07	−37543632	<2e-16	***
V607	1.210e+15	1.516e+07	79824044	<2e-16	***
V608	−1.597e+15	1.522e+07	−104942824	<2e-16	***
V609	7.934e+14	1.550e+07	51203275	<2e-16	***
V610	−2.417e+14	1.602e+07	−15087439	<2e-16	***
V611	−1.220e+15	1.826e+07	−66810110	<2e-16	***
V612	−6.786e+14	2.237e+07	−30341983	<2e-16	***
V613	1.402e+15	3.262e+07	42997869	<2e-16	***
V614	3.620e+14	5.484e+07	6601736	<2e-16	***
V615	−1.033e+16	2.633e+08	−39224271	<2e-16	***
V616	NA	NA	NA	NA	
V617	NA	NA	NA	NA	
V618	NA	NA	NA	NA	

```
V619          NA         NA         NA        NA
V620          NA         NA         NA        NA
V621   6.068e+15  1.554e+08   39051842   <2e-16 ***
V622   5.577e+14  2.760e+07   20206454   <2e-16 ***
V623  -2.665e+14  1.565e+07  -17028978   <2e-16 ***
V624  -4.061e+13  1.241e+07   -3271767   <2e-16 ***
V625  -7.277e+14  1.155e+07  -63017447   <2e-16 ***
V626   2.850e+13  1.112e+07    2562425   <2e-16 ***
V627   2.985e+14  1.159e+07   25759318   <2e-16 ***
V628  -3.358e+13  1.265e+07   -2654246   <2e-16 ***
V629  -1.282e+15  1.484e+07  -86438230   <2e-16 ***
V630   1.094e+15  1.933e+07   56573630   <2e-16 ***
V631  -5.108e+14  2.127e+07  -24016486   <2e-16 ***
V632   9.051e+14  2.175e+07   41614185   <2e-16 ***
V633   5.689e+14  2.187e+07   26020590   <2e-16 ***
V634  -1.769e+15  2.173e+07  -81441155   <2e-16 ***
V635   6.669e+14  2.158e+07   30904947   <2e-16 ***
V636  -9.233e+14  2.139e+07  -43153083   <2e-16 ***
V637   7.563e+14  2.112e+07   35807975   <2e-16 ***
V638   8.763e+14  2.147e+07   40822677   <2e-16 ***
V639  -1.324e+15  2.590e+07  -51110869   <2e-16 ***
V640   9.903e+14  3.457e+07   28646906   <2e-16 ***
V641  -7.290e+14  5.896e+07  -12364943   <2e-16 ***
V642  -1.036e+15  1.060e+08   -9772856   <2e-16 ***
V643   1.129e+16  2.767e+09    4079750   <2e-16 ***
V644          NA         NA         NA        NA
V645          NA         NA         NA        NA
V646          NA         NA         NA        NA
V647          NA         NA         NA        NA
V648          NA         NA         NA        NA
V649  -6.616e+15  1.333e+08  -49630828   <2e-16 ***
V650   5.966e+14  4.076e+07   14635755   <2e-16 ***
V651  -3.868e+14  2.133e+07  -18137013   <2e-16 ***
V652   1.488e+15  1.657e+07   89782440   <2e-16 ***
V653  -2.782e+14  1.440e+07  -19311098   <2e-16 ***
V654   1.336e+15  1.427e+07   93609040   <2e-16 ***
V655  -1.744e+15  1.472e+07 -118487147   <2e-16 ***
V656   1.410e+15  1.677e+07   84084047   <2e-16 ***
V657   2.222e+14  2.117e+07   10497851   <2e-16 ***
V658  -1.774e+14  2.751e+07   -6448849   <2e-16 ***
V659  -3.198e+14  2.918e+07  -10960586   <2e-16 ***
V660   1.405e+15  3.577e+07   39286486   <2e-16 ***
V661  -1.151e+14  3.445e+07   -3342680   <2e-16 ***
```

V662	-2.806e+15	3.488e+07	-80444680	<2e-16	***
V663	2.385e+15	3.507e+07	67998509	<2e-16	***
V664	-1.107e+15	3.514e+07	-31512324	<2e-16	***
V665	8.798e+13	3.350e+07	2626009	<2e-16	***
V666	-2.118e+14	3.928e+07	-5392890	<2e-16	***
V667	-3.179e+14	4.863e+07	-6537435	<2e-16	***
V668	-3.414e+15	8.223e+07	-41511649	<2e-16	***
V669	1.016e+16	1.611e+08	63087219	<2e-16	***
V670	-4.475e+15	4.241e+08	-10553035	<2e-16	***
V671	-3.166e+15	9.034e+08	-3504716	<2e-16	***
V672	NA	NA	NA	NA	
V673	NA	NA	NA	NA	
V674	NA	NA	NA	NA	
V675	NA	NA	NA	NA	
V676	NA	NA	NA	NA	
V677	NA	NA	NA	NA	
V678	-4.159e+14	5.867e+07	-7088882	<2e-16	***
V679	2.703e+15	3.287e+07	82229310	<2e-16	***
V680	-1.993e+15	2.283e+07	-87308120	<2e-16	***
V681	-2.486e+13	1.916e+07	-1297272	<2e-16	***
V682	1.487e+15	1.800e+07	82605742	<2e-16	***
V683	-1.289e+14	1.921e+07	-6713184	<2e-16	***
V684	-9.399e+14	2.169e+07	-43328084	<2e-16	***
V685	2.738e+15	2.747e+07	99670223	<2e-16	***
V686	4.879e+14	4.152e+07	11752014	<2e-16	***
V687	-6.097e+15	7.202e+07	-84655611	<2e-16	***
V688	3.369e+15	8.170e+07	41234223	<2e-16	***
V689	-5.264e+14	7.067e+07	-7448807	<2e-16	***
V690	-8.668e+14	6.651e+07	-13032262	<2e-16	***
V691	1.245e+15	7.436e+07	16737652	<2e-16	***
V692	5.215e+14	8.605e+07	6060384	<2e-16	***
V693	1.005e+15	8.826e+07	11387306	<2e-16	***
V694	2.245e+15	7.770e+07	28891501	<2e-16	***
V695	-8.507e+14	1.178e+08	-7219446	<2e-16	***
V696	-6.407e+15	3.758e+08	-17048035	<2e-16	***
V697	7.117e+15	1.067e+09	6668899	<2e-16	***
V698	-3.478e+16	3.206e+09	-10846747	<2e-16	***
V699	NA	NA	NA	NA	
V700	NA	NA	NA	NA	
V701	NA	NA	NA	NA	
V702	NA	NA	NA	NA	
V703	NA	NA	NA	NA	
V704	NA	NA	NA	NA	

V705	NA	NA	NA	NA	
V706	1.833e+15	7.781e+07	23558946	<2e-16	***
V707	-1.449e+15	4.795e+07	-30218053	<2e-16	***
V708	2.620e+14	3.449e+07	7596252	<2e-16	***
V709	6.099e+13	2.777e+07	2196275	<2e-16	***
V710	-1.464e+14	2.447e+07	-5982970	<2e-16	***
V711	7.383e+14	2.611e+07	28279229	<2e-16	***
V712	-9.439e+14	3.177e+07	-29713102	<2e-16	***
V713	-2.153e+15	4.039e+07	-53306081	<2e-16	***
V714	-1.432e+15	5.967e+07	-24003466	<2e-16	***
V715	7.410e+15	1.009e+08	73439037	<2e-16	***
V716	-7.062e+15	1.177e+08	-60000915	<2e-16	***
V717	5.733e+15	4.113e+08	13940531	<2e-16	***
V718	-8.181e+15	5.290e+08	-15463974	<2e-16	***
V719	1.584e+16	3.797e+08	41720438	<2e-16	***
V720	-3.283e+16	7.743e+08	-42392213	<2e-16	***
V721	6.297e+16	1.570e+09	40119832	<2e-16	***
V722	-5.411e+16	1.248e+09	-43356700	<2e-16	***
V723	1.787e+17	3.611e+09	49495073	<2e-16	***
V724	-1.010e+17	3.285e+09	-30745393	<2e-16	***
V725	1.857e+16	2.217e+09	8375198	<2e-16	***
V726	NA	NA	NA	NA	
V727	NA	NA	NA	NA	
V728	NA	NA	NA	NA	
V729	NA	NA	NA	NA	
V730	NA	NA	NA	NA	
V731	NA	NA	NA	NA	
V732	NA	NA	NA	NA	
V733	NA	NA	NA	NA	
V734	-2.520e+15	9.538e+07	-26425714	<2e-16	***
V735	3.853e+15	8.334e+07	46236724	<2e-16	***
V736	2.428e+14	4.842e+07	5013366	<2e-16	***
V737	-5.499e+14	4.805e+07	-11444465	<2e-16	***
V738	-3.130e+14	6.436e+07	-4862925	<2e-16	***
V739	-2.976e+15	6.411e+07	-46411962	<2e-16	***
V740	2.510e+15	5.387e+07	46602045	<2e-16	***
V741	9.800e+14	6.198e+07	15810870	<2e-16	***
V742	-8.801e+14	1.312e+08	-6709949	<2e-16	***
V743	2.943e+17	4.822e+09	61043742	<2e-16	***
V744	-2.824e+17	4.551e+09	-62050938	<2e-16	***
V745	1.350e+15	1.505e+08	8967691	<2e-16	***
V746	NA	NA	NA	NA	
V747	NA	NA	NA	NA	

```
V748              NA         NA         NA         NA
V749              NA         NA         NA         NA
V750       3.610e+15  2.261e+08   15962219   <2e-16 ***
V751              NA         NA         NA         NA
V752       7.658e+16  3.511e+09   21814864   <2e-16 ***
V753              NA         NA         NA         NA
V754              NA         NA         NA         NA
V755              NA         NA         NA         NA
V756              NA         NA         NA         NA
V757              NA         NA         NA         NA
V758              NA         NA         NA         NA
V759              NA         NA         NA         NA
V760              NA         NA         NA         NA
V761              NA         NA         NA         NA
V762       2.118e+16  4.663e+08   45420875   <2e-16 ***
V763       3.353e+15  4.754e+08    7052820   <2e-16 ***
V764       1.708e+14  2.688e+08     635518   <2e-16 ***
V765      -5.072e+15  3.158e+08  -16059036   <2e-16 ***
V766       3.602e+15  1.104e+08   32633143   <2e-16 ***
V767      -1.179e+15  3.083e+08   -3823400   <2e-16 ***
V768      -1.387e+15  2.136e+08   -6493191   <2e-16 ***
V769      -1.893e+16  5.690e+08  -33263494   <2e-16 ***
V770              NA         NA         NA         NA
V771              NA         NA         NA         NA
V772              NA         NA         NA         NA
V773              NA         NA         NA         NA
V774              NA         NA         NA         NA
V775              NA         NA         NA         NA
V776              NA         NA         NA         NA
V777              NA         NA         NA         NA
V778              NA         NA         NA         NA
V779              NA         NA         NA         NA
V780              NA         NA         NA         NA
V781              NA         NA         NA         NA
V782              NA         NA         NA         NA
V783              NA         NA         NA         NA
V784              NA         NA         NA         NA
V785              NA         NA         NA         NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)
```

```
    Null deviance: 7734.1  on 5578  degrees of freedom
Residual deviance: 6055.3  on 4990  degrees of freedom
AIC: 7233.3

Number of Fisher Scoring iterations: 25
```

Plotting the ROC curve and calculating the AUC:

```
X <- as.data.frame(test_49[, 2:785])
y <- test_49$label
pred <- as.numeric(predict(l2, X))
```

```
Warning in predict.lm(object, newdata, se.fit, scale = 1, type = if (type == :
prediction from rank-deficient fit; attr(*, "non-estim") has doubtful cases
```

```
probs <- as.numeric(1 / (1 + exp(-pred)))
true_labels <- as.numeric(y)

# Calc ROC
pred <- prediction(probs, true_labels)
perf_m <- performance(pred, "tpr", "fpr")

# Calc AUC
auc <- performance(pred, "auc")
auc_value <- auc@y.values[[1]]
cat("AUC =", auc_value, "\n")
```

```
AUC = 0.9322635
```

```
# Plot the Curve
plot(perf_m, colorize = TRUE, colorkey.label = "Cutoff",
     colorize.palette = viridis_colors,
     main = "ROC Curve 3 and 5 Prediction, 64 Batch Size, 64 Hidden Layer Size")
abline(a = 0, b = 1, lty = 2, col = "black")
```

OC Curve 3 and 5 Prediction, 64 Batch Size, 64 Hidden Laye
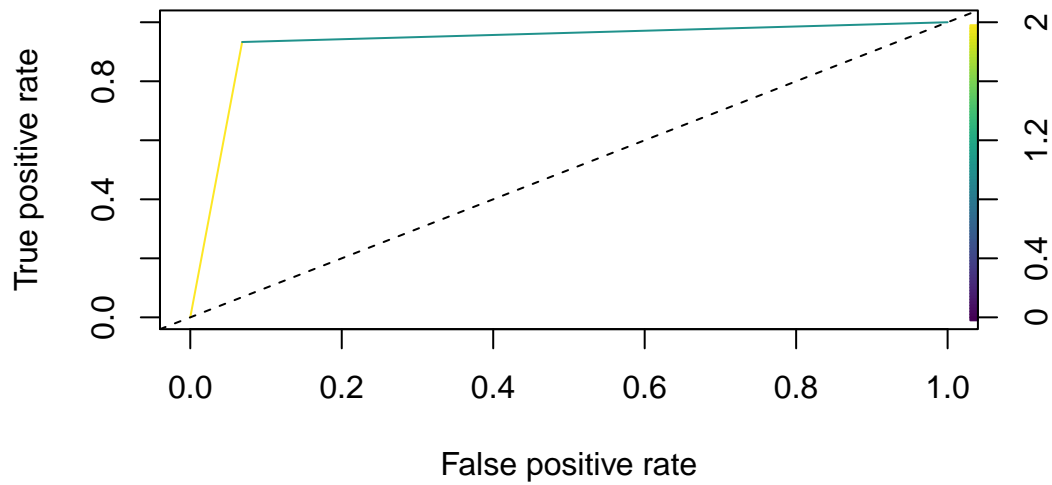
The performance of the Logistic Regression model was worse than that of the MLP. The MLP AUC was around 0.99 while the logistic regression AUC was 0.9322635. This indicates that the MLP model was classifying better for the digits 4/9. Overall, both models are fairly good, because AUC is high.