Summary of lecture of survey design
141XP – Professor Esfandiari

1. Definition of the concept of validity and reliability
2. Different types of validity
3. Different measure of reliability and internal consistency
    - 3.1 Cronbach's alpha
    - 3.2 Split half reliability
    - 3.3 Test-retest reliability
    - 3.4 Parallel-forms-reliability
    - 3.5 Internal consistency reliability
    - 3.6 Inter-rater reliability

4. Using Parallel analysis to identify the dimensions of a survey
5. Using exploratory factor analysis to identify the dimensions of the survey

1. **Definition of the concept of validity and reliability**

Conceptual definition of validity
A survey is considered valid if it measures what it is designed to measure. For instance, a test of "intelligence" is supposed to measure the "ability to learn" and not an individual's measure of math and reading. That is why sometimes IQ tests that measure an individual's knowledge of math, vocabulary, and reading comprehension are considered "biased".

Conceptual definition of reliability
A survey is considered "reliable" if it has consistency. In other words, if we administer the survey at time point one and time point two, we should get similar results. Given that no learning has happened between the two intervals, and memory from the pre-survey is not affecting the post survey.

In statistical terms …

1. Definition of the concept of validity and reliability
2. Definition of different types of validity
3. How can reliability and validity be estimated
4. Definition and measures of internal consistency
5. Cronbach's alpha as a popular measure of reliability and what happens if we delete one item at a time
6. Split half reliability
7. Methods for creation of clusters using items on the scale
8. Shortening the survey while keeping a high level of validity and reliability
9. Example project from 141XP

" is often measured as "**accuracy"** and it is calculated as the difference between the population parameter and the sample estimate "**measure of bias".**

For example, the parameter for measuring the mean in the population is μ (Mue) and the sample estimate for the mean is Xbar $(\bar{X})$. The closer the sample estimate to the population parameter the higher the validity and the less the bias. That is why random sampling is so important in statistics as a sample of convenience or a sample of volunteers cannot be a good representative of the population and could lead to biased estimates of the parameter in mind.

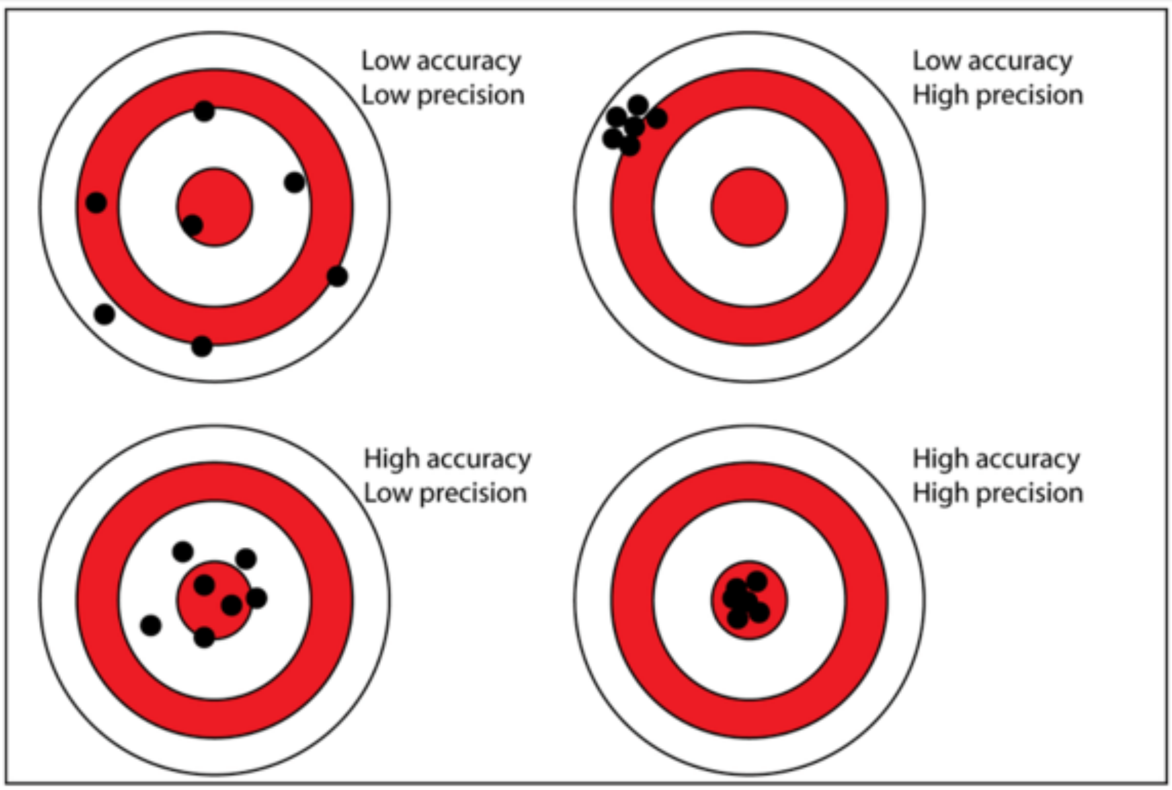
In statistical terms…

reliability is often referred to as "**precision**". The most important factor that affects reliability is the sample size. As sample size increases, the likelihood of finding similar results over repeated trials increases; thus consistency increases.

If a measure is consistent, does it mean it is also valid?

The answer to this question is "No" because you can find similar results over repeated samples that are not representative of the population. Therefore, you end up getting similar sample estimates that are "biased" estimates of the population parameter.

The figure below should clarify this concept.



Precision versus accuracy. The bullseye represents the true value, e.g., the true location of the object, while black dots represent measurements, e.g., the estimated 3D locations of the object based on the 2D images. Source: http://www.antarcticglaciers.org/glacial-geology/dating-glacial-sediments2/precision-and-accuracy-glacial-geology/. Accessed 7.4.2016.

- Low accuracy low precision is representative of biased and relatively small samples
- Low accuracy high precision is representative of biased and large samples.
- High accuracy low precision is representative of pretty representative but small samples
- High accuracy and high precision are representative of samples that are representative and acceptable as far as sample size is concerned.

Relationship of the discussion of power analysis with reliability and validity: Making sure that you create a tie between the different type of knowledge you are acquiring in this course

One of the major questions for the statistician is always: "How large of a sample do we need to ascertain reliability?"

Based on the discussion we had in power analysis, in order to reach a desirable effect size, a minimum sample size is needed to reach a desirable power. Therefore, one does not necessarily need to choose very large samples to reach draw reliable and valid conclusions. This needs to be done with a good understanding of the research question, the number of predictors, statistical methods employed, and the effect size that the client has in mind. Depending on the field and prior research sometimes a minimum effect size needed is an R-squared of 90% (if a lot of variables can be controlled; like an experiment done in a lab) and in other situations where the majority of variables that effect the outcome cannot be controlled (an example would be factors that could affect "sense of belonging" to our campus, an R-squared of 25% could be considered high.

2. **Different types of validity and examples**

content validity - assesses whether a test is representative of all aspects of the construct.

Example one: In assessment one we ascertained to have content validity by including questions that ascertained you have enough theoretical knowledge and statistical literacy of power analysis and logistic regression needed to explain the relevant results to your client.

Example two. In a study I conducted with the UCLA school of public health, one of the objectives was to measure the relationship between the knowledge of breast cancer and prevention (as measured by going for a mammogram). The survey for the knowledge of breast cancer was designed by specialists in the field.

construct validity, refers to whether a scale or test measures the construct adequately. An example is a measurement of the human intelligence (IQ test), level of emotion (self-esteem), proficiency or ability (team building).

Example: One of the objectives of psychological research is to measure or quantify constructs that relate to human emotion. Psychological research over decades of research has shown the most important constructs that related to a particular emotion.

Then an effort is made to construct instruments that measure the relevant constructs. An example is an effort that I made is designing instruments for different constructs that underlie successful team building. One of these constructs is "locus of control". It is defined as the extent to which an individual attributes their success to factors that they can control such as effort, "**internal locus of control**", and the extent to which they attribute it to factors such as chance and randomness that are not in their own control, "**external locus of control**". There are surveys in the literature whose validity and reliability is established and the researchers use them. There are also instances where the researcher has to design their own survey to measure “locus of control” within the context of interest.

Example of a predesigned survey in locus of control is “Rotter’s Scale”. A copy is provided below.

Rotter's Locus of Control Scale

For each question select the statement that you agree with the most

1. a. Children get into trouble because their patents punish them too much.

   b. The trouble with most children nowadays is that their parents are too easy with them.

2. a. Many of the unhappy things in people's lives are partly due to bad luck.

   b. People's misfortunes result from the mistakes they make.

3. a. One of the major reasons why we have wars is because people don't take enough interest in politics.

   b. There will always be wars, no matter how hard people try to prevent them.

4. a. In the long run people get the respect they deserve in this world

   b. Unfortunately, an individual's worth often passes unrecognized no matter how hard he tries

5. a. The idea that teachers are unfair to students is nonsense.

   b. Most students don't realize the extent to which their grades are influenced by accidental happenings.

6. a. Without the right breaks one cannot be an effective leader.

   b. Capable people who fail to become leaders have not taken advantage of their opportunities.

7. a. No matter how hard you try some people just don't like you.

   b. People who can't get others to like them don't understand how to get along with others.

8. a. Heredity plays the major role in determining one's personality

   b. It is one's experiences in life which determine what they're like.

9. a. I have often found that what is going to happen will happen.
   b. Trusting to fate has never turned out as well for me as making a decision to take a definite course of action.

10. a. In the case of the well prepared student there is rarely if ever such a thing as an unfair test.

b. Many times exam questions tend to be so unrelated to course work that studying in really useless.

11. a. Becoming a success is a matter of hard work, luck has little or nothing to do with it.

b. Getting a good job depends mainly on being in the right place at the right time.

12. a. The average citizen can have an influence in government decisions.

b. This world is run by the few people in power, and there is not much the little guy can do about it.

13. a. When I make plans, I am almost certain that I can make them work.

b. It is not always wise to plan too far ahead because many things turn out to- be a matter of good or bad fortune anyhow.

14. a. There are certain people who are just no good.

b. There is some good in everybody.

15. a. In my case getting what I want has little or nothing to do with luck.

b. Many times we might just as well decide what to do by flipping a coin.

16. a. Who gets to be the boss often depends on who was lucky enough to be in the right place first.

b. Getting people to do the right thing depends upon ability. Luck has little or nothing to do with it.

17. a. As far as world affairs are concerned, most of us are the victims of forces we can neither understand, nor control.

b. By taking an active part in political and social affairs the people can control world events.

18. a. Most people don't realize the extent to which their lives are controlled by accidental happenings.

b. There really is no such thing as "luck."

19. a. One should always be willing to admit mistakes.

b. It is usually best to cover up one's mistakes.

20. a. It is hard to know whether or not a person really likes you.

b. How many friends you have depends upon how nice a person you are.

21. a. In the long run the bad things that happen to us are balanced by the good ones.

b. Most misfortunes are the result of lack of ability, ignorance, laziness, or all three.

22. a. With enough effort we can wipe out political corruption.

b. It is difficult for people to have much control over the things politicians do in office.

23. a. Sometimes I can't understand how teachers arrive at the grades they give.

b. There is a direct connection between how hard 1 study and the grades I get.

24. a. A good leader expects people to decide for themselves what they should do.

b. A good leader makes it clear to everybody what their jobs are.

25. a. Many times I feel that I have little influence over the things that happen to me.

b. It is impossible for me to believe that chance or luck plays an important role in my life.

26. a. People are lonely because they don't try to be friendly.

b. There's not much use in trying too hard to please people, if they like you, they like you.

27. a. There is too much emphasis on athletics in high school.

b. Team sports are an excellent way to build character.

28. a. What happens to me is my own doing.

b. Sometimes I feel that I don't have enough control over the direction my life is taking.

29. a. Most of the time I can't understand why politicians behave the way they do.

b. In the long run the people are responsible for bad government on a national as well as on a local level.

Score one point for each of the following:
2. a, 3.b, 4.b, 5.b, 6.a, 7.a, 9.a, 10.b, 11.b, 12.b, 13.b, 15.b, 16.a, 17.a, 18.a, 20.a,

21. a, 22.b, 23.a, 25.a, 26.b, 28.b, 29.a.

A high score = External Locus of Control A

low score = Internal Locus of Control

Items in red reflect internal locus of control
Items in green represent external locus of control

Face validity is about whether a test appears to measure what it's supposed to measure.

- **Face validity** shows whether a test appears to measure what it's supposed to measure. It is a good first step for looking at the overall validity of the test.

An example from my work with 141XP

For example, one of our clients, the Center for Community Engagement at UCLA, wanted to ascertain the extent to which the different courses taught in this area across our campus help the students reach the overall goals and objectives of the center. I asked them to provide me with a list of their goals and I tried to design a relevant survey that reflected the goals of the center. I then had the directors of the Center for Community Engagement examine the survey I designed, decide the extent to which it reflected their goals, and revise it as need be. This is the first step we took in this direction. The survey has been administered to a few classes and some of the teams will be analyzing the results..

Criterion-related validity

Criterion-related validity (also called instrumental validity) is a measure of the quality of your measurement methods. The accuracy of a measure is demonstrated by comparing it with a measure that is already known to be valid.

How can you establish criterion-related validity: Suppose there is an IQ tested whose reliability and validity is established. Researcher A makes a new IQ test which is shorter and less time consuming. In order to establish criterion validity, the old measure as well as the new measure of IQ have to be administered to the same sample and if the correlation between the scores obtained on both measures is pretty high. Them it can be concluded that the new scale has criterion-related validity.

An example done in stat consulting class. Around ten years ago an agency was hired to design a survey for assessing campus climate (feeling of respect, exclusion, discrimination, friendliness in our climate etc.). This survey included over 100 questions and was administered to 5000 students and created a lot of missing data. One of the teams in the consulting class, did a study and shortened this survey to something 50-60 items using item reduction methods such as factor analysis. They showed that the correlation between the scores resulting from the shortened survey and the long survey was pretty high.

3. Different Measures of Reliability and Internal Consistency

3.1 Cronbach's alpha is a popular measure of reliability in survey design.

Cronbach's alpha is a measure of internal consistency, that is, how closely related a set of items are as a group. It is considered to be a measure of scale reliability. A "high" value for alpha does not imply that the measure is unidimensional.

Cronbach's alpha reliability coefficient normally ranges between 0 and 1. However, there is actually no lower limit to the coefficient. The closer Cronbach's alpha coefficient is to 1.0 the greater the internal consistency of the items in the scale.

Cronbach's alpha is the most common measure of internal consistency ("reliability"). It is most commonly used when you have multiple Likert questions in a survey/questionnaire that form a scale and you wish to determine if the scale is reliable.

The instrument's reliability was established using Cronbach's alpha measurement to demonstrate internal consistency. An item is considered reliable with Cronbach's alpha score greater than 0.6, acceptable between 0.6 to 0.8, with a corrected item-total correlation greater than 0.3

Cronbach's alpha is also not a measure of validity, or the extent to which a scale records the "true" value or score of the concept you're trying to measure without capturing any unintended characteristics.

More on Cronbach's alpha

Research Design and Methods
L.M. Collins, in Encyclopedia of Gerontology (Second Edition), 2007

Cronbach's alpha
Cronbach's alpha is a way of assessing reliability by comparing the amount of shared variance, or covariance, among the items making up an instrument to the amount of overall variance. The idea is that if the instrument is reliable, there should be a great deal of covariance among the items relative to the variance. Cronbach's alpha is equivalent to taking the average of all possible split-half reliabilities. Most computer packages for statistics in wide use today can compute Cronbach's alpha. Often it is helpful to examine what the Cronbach's alpha becomes after a particular item is deleted. If Cronbach's alpha goes up considerably upon deletion of an item, the item may not belong in the measure.

How to calculate Cronbach's alpha

How to Calculate Cronbach's Alpha (copied from https://statisticsbyjim.com/basics/cronbachs-alpha/

Usually, you'll have your statistical software calculate Cronbach's alpha for you. However, knowing how to calculate it yourself can help you understand it.

Below is the formula for Cronbach's alpha.

$$\alpha = \frac{N * \bar{c}}{\bar{v} + (N - 1) * \bar{c}}$$

Where

- N = number of items
- $\bar{c}$ = mean covariance between items.
- $\bar{v}$ = mean item variance.

From the above equation it is pretty clear that is average covariance is higher than average variance the value of Cronbach's alpha will be higher. That is higher correlation between the items on a survey shows higher precision/consistency and a higher Cronbach alpha.

The calculations for Cronbach's alpha involve taking the average covariance and dividing it by the average total variance. Therefore, a high alpha value requires the covariance to be high relative to the item variance. In other words, the relationships between the questions account for most of the overall variability. Additionally, the number of items is a factor. Cronbach's alpha tends to increase as you add more items.

Analysis Example Imagine a bank wants to survey customers to evaluate how satisfied they are with the timeliness of its service. You develop the following four survey questions:

- Item 1 – My telephone, email, or letter inquiry was answered in a reasonable amount of time.
- Item 2 – I am satisfied with the timeliness of the service provided.
- Item 3 – The time I waited for services was reasonable.
- Item 4 – I am satisfied with the services I received.

These questions all use a 5-point Likert scale ranging from 1 Very Dissatisfied to 5 Very Satisfied. You ask 60 customers to take the survey during the pilot study phase before distributing the survey more widely. Download the CSV dataset: Cronbachs_alpha.

For this analysis, the statistical software calculates the overall Cronbach's alpha. Then it recalculates the statistic after omitting each item because that process can provide valuable information about specific items. The statistical output is below.

$\boxed{\text{Cronbach's alpha = 0.7853}}$

Omitted Item Statistics

Omitted Variable	Adj. Total Mean	Adj. Total StDev	Item-Adj. Total Corr	Squared Multiple Corr	Cronbach's Alpha
Item 1	8.867	2.665	0.818768	0.725307	0.599499
Item 2	8.933	2.603	0.802999	0.717877	0.606279
Item 3	9.000	2.768	0.785333	0.691912	0.625996
Item 4	10.150	3.727	0.019250	0.004488	0.921674

The overall Cronbach's alpha is 0.7853. It's minimally acceptable by most standards.

Let's see if we can improve it some.

Under Omitted Item Statistics, the software recalculates Cronbach's alpha after removing an item. If omitting an item substantially increases Cronbach's alpha, consider removing that question from the instrument because it is suspect.

Removing Item 4 causes Cronbach's alpha to increase from 0.7853 to 0.921674. This result suggests that only items 1, 2, and 3 measure customer service timeliness. We should either remove item 4 or reword and retest it.

After downloading the Cronbach-alpha data, the variance covariance matrix was computed

The items along the diagonal show variances. For example ,1.777 is the variance of item one.

The off diagonal items show the covariance, For example, 1.536 is the covariance between items one and two.

Covariance between items 1 and 2 is the same as covariance between items 2 and 1.

```
> data<-data.frame(`Item 1`,`Item 2`,`Item 3`,`Item 4`)
> var(data[, 1:4])
```

	Item.1	Item.2	Item.3	Item.4
Item.1	1.77711864	1.53644068	1.329661017	0.042372881
Item.2	1.53644068	2.00310734	1.401977401	0.019774011
Item.3	1.32966102	1.40197740	1.575988701	-0.002824859
Item.4	0.04237288	0.01977401	-0.002824859	0.683615819

Calculating Cronbach alpha in R

```
>library(psych)
>alpha(Cronbach_alpha)
```

Reliability analysis

Call: alpha(x = Cronbachs_alpha)

95% confidence boundaries

	lower	alpha	upper
Feldt	0.68	0.79	0.86
Duhachek	0.71	**0.79**	0.86

Reliability if an item is dropped:

	raw_alpha	std.alpha	G6(smc)	average_r	S/N	alpha se
Item 1	0.60	0.52	0.62	0.27	1.1	0.075
Item 2	0.61	0.53	0.63	0.28	1.1	0.075
Item 3	0.63	0.55	0.65	0.29	1.2	0.069
Item 4	**0.92**	0.92	0.89	0.80	11.9	0.017

Conclusions:

- **Cronbach alpha = 0.79**
- **We are 95% confident that reliability as measured by Cronbach's alpha is between 0.68 to 0.86.**
- **If we drop item 4, reliability is the highest (0.92).**

Now we delete item 4 and recalculate reliability.

```
> newdata2<-Cronbachs_alpha[,c("Item 1","Item 2","Item 3")]
> library(psych)
> alpha(newdata2)

Reliability analysis
Call: alpha(x = newdata2)

  raw_alpha std.alpha G6(smc) average_r S/N   ase mean  sd
      0.92      0.92    0.89       0.8  12 0.017  3.4 1.2
 median_r
     0.79

    95% confidence boundaries
              lower  alpha  upper
Feldt         0.88   0.92   0.95
Duhachek      0.89   0.92   0.96

 Reliability if an item is dropped:
       raw_alpha std.alpha G6(smc) average_r S/N alpha se var.r
Item 1      0.88      0.88    0.79      0.79 7.5    0.031    NA
Item 2      0.88      0.89    0.79      0.79 7.7    0.030    NA
Item 3      0.90      0.90    0.81      0.81 8.8    0.027    NA
       med.r
Item 1  0.79
Item 2  0.79
Item 3  0.81
```

No more items need to be dropped.

Common guidelines for evaluating Cronbach's Alpha are:

- 00 to . 69 = Poor.
- 70 to . 79 = Fair.
- 80 to . 89 = Good.
- 90 to . 99 = Excellent/Strong.

However, you will notice in the literature that if a construct is measured with fewer number of questions (say five or six), then a Cronbach alpha of 0.60 may be reported as acceptable.

3.2 Split-Half Reliability

Split Half Reliability is an *internal consistency* approach to quantifying the reliability of a test, in the paradigm of classical test theory. The name comes from a simple description of the method: we split the test into two halves, calculate the score on each half for each examinee, then correlate those two columns of numbers. If the two halves measure the same thing, then the correlation is high, indicating a decent level of unidimensionality in the construct and reliability in measuring the construct.

Why do we need to estimate reliability? Well, it is one of the easiest ways to quantify the quality of the test. Some would argue, in fact, that it is a gross oversimplification. However, because it is so convenient, classical indices of reliability are incredibly popular. The most popular is coefficient alpha, which is a competitor to split half reliability.

How to Calculate Split Half Reliability

The process is simple.

1. Take the test and split it in half
2. Calculate the score of each examinee on each half
3. Correlate the scores on the two halves

The correlation is best done with the standard Pearson correlation.

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

How do we split the test into two halves? There are so many ways. Well, psychometricians generally recommend three ways:

1. First half vs last half
2. Odd-numbered items vs even-numbered items
3. Random split
4. Parallel items (It implies writing positive and negative items that covey the same message)

An alternative way of computing the reliability of a sum scale is to divide it in some random manner into two halves. If the sum scale is perfectly reliable, we would expect that the two halves are perfectly correlated (i.e., $r = 1.0$). Less than perfect reliability will lead to less than perfect correlations. We can estimate the reliability of the sum scale via the *Spearman-Brown split half* coefficient:

$$r_{spearman\ Brown} = \frac{2\, r_{XY}}{(1 + r_{XY})}$$

In this formula, $r_{spearman\ Brown}$ is the split-half reliability coefficient, and r_{xy} represents the correlation between the two halves of the scale.

3.3 Test-retest reliability conducts a test that is given twice over a given period. The scores from both tests would indicate how reliable the test is by conducting statistical measures of the results. The time interval between the two administrations should be long enough so the participants do not recall the information.

3.4 Parallel-forms-reliability. Two similar surveys are made for assessing the same construct (example two measures of depression). Both tests are administered to a group of individuals. The coefficient of correlation between the scores on the two surveys is a measure or reliability.

3.5 Internal consistency reliability looks at each item in a test that measures the same content. It measures the *consistency* of individual items of a test.

> cor(data)

	Item.1	Item.2	Item.3	Item.4
Item.1	1.00000000	0.81433945	0.794522453	0.038443572
Item.2	0.81433945	1.00000000	0.789064222	0.016898051
Item.3	0.79452245	0.78906422	1.000000000	-0.002721536
Item.4	0.03844357	0.01689805	-0.002721536	1.000000000

From the above correlation matrix, we can see that, items 1, 2, and 3 are consistent while items four is not. This result was verified by reliability analysis. When item 4 was deleted the reliability was the highest.

Additionally, Correlation of the total score with each item shows the extent to which individual items are working in the same direction as the test.

```
sum=`Item 1`+`Item 2`+`Item 3`+`Item 4`
> newdata<-data.frame(sum,`Item 1`,`Item 2`,`Item 3`,`Item 4`)
```

```
> cor(newdata)
              sum        Item.1      Item.2      Item.3
sum       1.0000000  0.91691062  0.91445841  0.894534899
Item.1    0.9169106  1.00000000  0.81433945  0.794522453
Item.2    0.9144584  0.81433945  1.00000000  0.789064222
Item.3    0.8945349  0.79452245  0.78906422  1.000000000
Item.4    0.2344051  0.03844357  0.01689805 -0.002721536
            Item.4
sum     0.234405105
Item.1  0.038443572
Item.2  0.016898051
Item.3 -0.002721536
Item.4  1.000000000
```

As indicated above item 4 not only has a low correlation with the other items, it also has a low correlation with the total score on the survey, indicating that it is not working in the same direction as the survey.

3.6 **Inter-rater reliability** measures the feedback of someone assessing the test given. **The assessment determines the validity of the test.** If multiple people score a test, the test is reliable if their scores are consistent.

4. Using Parallel analysis to identify the dimensions of the survey

Watch the following video on parallel analysis

https://www.youtube.com/watch?v=mPJ_jG4LuLk

Conducting Parallel Analysis

Parallel analysis is the first step in exploratory or in principal (EFA) component analysis (PCA). It is an objective way of finding out how many factors or components we have in our data set.

We will be comparing the eigenvalues generated from parallel analysis with the eigenvalues from the identity matrix. If the eigenvalues generated by our data, are greater than the eigenvalues that are randomly generated by the identity matrix, that tells us we have a valid factor in our data set. We will elaborate this by the analysis of the parallel analysis plot given below. Using the subset data called "tearing"; just contains the 15 questions survey, we will do the parallel analysis.

> attach(tearingq)

> View(tearingq)

> library(paran)

> library(MASS)

R command

> *paran(tearingq,cfa=TRUE)*

Output

Using eigendecomposition of correlation matrix.

Computing: 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

Results of Horn's Parallel Analysis for factor retention

450 iterations, using the mean estimate

```
--------------------------------------------------
Factor     Adjusted    Unadjusted    Estimated
           Eigenvalue  Eigenvalue    Bias
--------------------------------------------------
1          6.311381    8.409174      2.097793
--------------------------------------------------
```

Adjusted eigenvalues > 0 indicate dimensions to retain.

(1 factors retained) – only one factor was identified

R command for creating the plot of power analysis

> paran(tearingq,cfa=TRUE, graph=TRUE,color=TRUE,col=c("black","red","blue"))

Interpretation of the parallel analysis plot

1. We need to identify and define the identity matrix and then interpret the parallel analysis plot with respect to the identity matrix.
2. Identity matrix is given below

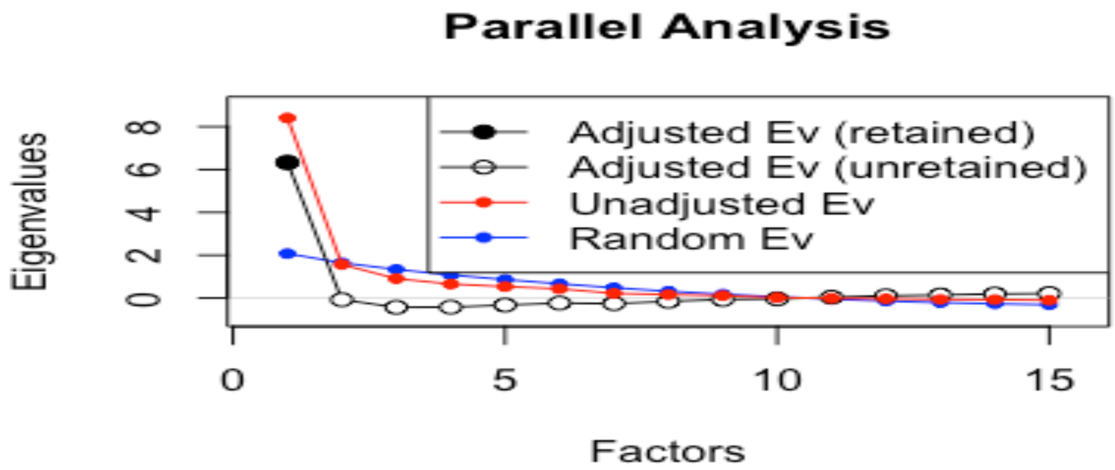
An identity matrix is always a square matrix. The order of an identity is always n, which refers to the dimensions nxn (meaning there is always the same number of rows and columns in the matrix).

Below you are given an identity matrix with 5 rows and 5 columns. As you notice all of the diagonal elements are equal to “1” and all of the “off diagonal” elements are equal to zero.
In the identity matrix nothing correlates.

$$I_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The blue line shows the random eigenvalues generated by the identity matrix
Based on the adjusted eigen-values generated by our data set, we see that there is only one factor because there is only one generated eigenvalue by our data is larger than the random eigen value generated by the identity matrix and the other 14 are not. The number of eigenvalues generated is equal to the number of questions in the survey. This tells us that our data or the tearing survey has got one valid factor.

We will now proceed with exploratory factor analysis to check this further.



Eigenvalues are a measure of the amount of variance accounted for by a factor, and so they can be useful in determining the number of factors that we need to extract. In a scree plot, we simply plot the eigenvalues for all of our factors, and then look to see where they drop off sharply.

5. **Exploratory data analysis (EDA)**

Exploratory Factor Analysis of the tearing survey data

https://www.youtube.com/watch?v=VCpVcXf_wOk

We need to install the following libraries

>library(psych)
> library(REdaS)
> library(readxl)

The first test we need to do is the Bartlett Test of Sphericity.

> bart_spher(tearingq)

Bartlett's Test of Sphericity

Call: bart_spher(x = tearingq)

X2 = 368.077
df = 105
p-value < 2.22e-16

We fail to reject the null hypothesis of sphericity. This is what we want.
> KMO(tearingq)

Kaiser-Meyer-Olkin factor adequacy
Call: KMO(r = tearingq)
Overall MSA = 0.71
MSA for each item =

q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12	q13
0.53	0.68	0.57	0.62	0.85	0.66	0.78	0.78	0.81	0.61	0.81	0.57	0.83

q14	q15
0.83	0.71

The Kaiser–Meyer–Olkin (KMO) test is a statistical measure to determine how suited data is for factor analysis. The test measures sampling adequacy for each variable in the model and the complete model. The statistic is a measure of the proportion of variance among variables that might be common variance.

In general, KMO values between 0.8 and 1indicate the sampling is adequate. KMO values less than 0.6 indicate the sampling is not adequate and that remedial action should be taken. In contrast, others set this cutoff value at 0.5.

You want the values of KMO, or factor adequacy, to be larger than 0.70 We seem to be OK there. The average is 0.71 and we should be OK.

R command for exploratory factor analysis
>fa(tearingq,nfactors = 15,rotate="oblimin")

The first thing we look at are the SS loadings which show the eigenvalues associated with each factor. It seems there are at most two underlying actors with eigen-values more than one (MR1 = 88.81) and MR 2 = 1.66. The first factor by itself explains 60% of the variance. The second factor only explains 12% of the variance.

	MR1	**MR2**	**MR3**	**MR4**	**MR5**	**MR6**	**MR7**	**MR8**
SS loadings	**8.51**	**1.66**	1.01	0.78	0.65	0.54	0.32	0.26
Proportion Var	0.57	0.11	0.07	0.05	0.04	0.04	0.02	0.02
Cumulative Var	0.57	0.68	0.75	0.80	0.84	0.88	0.90	0.91
Proportion Explained	**0.60**	**0.12**	0.07	0.05	0.05	0.04	0.02	0.02
Cumulative Proportion	0.60	0.71	0.78	0.84	0.88	0.92	0.94	0.96

	MR9	**MR10**	**MR11**	**MR12**	**MR13**	**MR14**	**MR15**
SS loadings	0.23	0.13	0.08	0.06	0.03	0.02	0.00
Proportion Var	0.02	0.01	0.01	0.00	0.00	0.00	0.00
Cumulative Var	0.93	0.94	0.94	0.95	0.95	0.95	0.95
Proportion Explained	0.02	0.01	0.01	0.00	0.00	0.00	0.00

Cumulative Proportion	0.98	0.99	0.99	1.00	1.00	1.00	1.00

Next, we look at factor loadings. The factor loadings show the relationships between each item on the test and each factor that the model extracts. These relationships are summarized within a factor loading matrix, with items in rows and loadings on each factor in columns. **The loadings are usually standardized so as to be interpreted as correlations**. The factor loading should be more than 0.5 to be reasonable. We see that questions one to fourteen load to factor one and question 15 loads to factor two. This confirms that there is a single underlying factor. There are fifteen factors because we have 15 questions. **Only question 15 has a low factor loading with factor one (MR!). The other factor loading are all higher than 0.56.**

Standardized loadings (pattern matrix) based upon correlation matrix

	MR1	**MR2**	**MR3**	**MR4**	**MR5**	**MR6**	**MR7**	**MR8**	**MR9**	**MR10**
q1	0.66	0.21	-0.67	-0.05	0.10	-0.09	0.08	0.06	-0.08	0.15
q2	0.84	0.21	-0.28	-0.01	-0.12	-0.17	0.13	-0.30	0.00	-0.04
q3	0.56	-0.01	-0.16	0.37	0.42	-0.10	0.03	0.15	0.25	-0.05
q4	0.75	0.25	0.31	-0.23	0.31	-0.29	0.03	-0.03	-0.11	-0.15
q5	0.72	-0.14	0.30	-0.07	0.39	0.20	0.00	-0.01	-0.10	0.16
q6	0.83	-0.35	0.09	0.11	-0.12	-0.15	0.19	0.19	-0.22	-0.04
q7	0.84	-0.28	0.02	-0.19	0.05	-0.04	-0.32	-0.09	0.07	0.02
q8	0.89	-0.20	-0.17	-0.23	-0.14	-0.09	-0.13	0.08	0.10	-0.09
q9	0.86	-0.14	-0.20	0.10	0.10	0.32	-0.11	-0.13	-0.15	-0.04
q10	0.74	-0.52	-0.04	0.15	-0.16	0.29	0.11	0.02	0.04	-0.11
q11	0.82	-0.16	0.19	-0.37	-0.09	0.05	0.26	0.00	0.19	0.10
q12	0.68	-0.15	0.32	0.49	-0.20	-0.27	-0.06	-0.13	0.03	0.12
q13	0.76	0.48	0.02	0.19	-0.13	0.06	-0.11	0.14	-0.06	-0.01
q14	0.75	0.50	0.13	-0.13	-0.26	0.06	-0.12	0.17	0.03	0.04
q15.	**0.48**	0.66	0.18	0.12	0.02	0.27	0.12	-0.10	0.07	-0.05

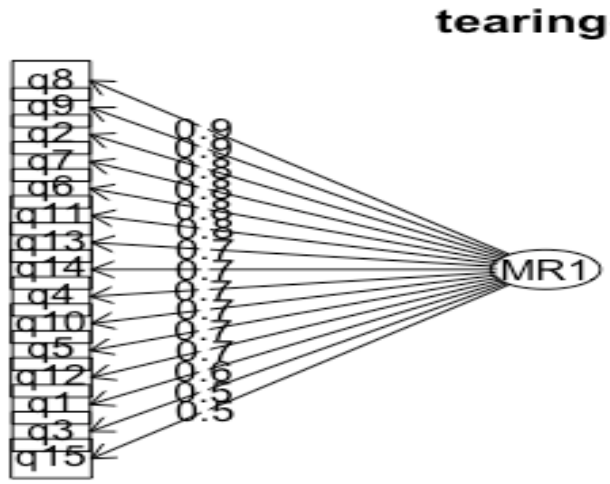
	MR11	MR12	MR13	MR14	MR15	
q1	-0.01	0.04	0.04	0.00	0	1.00
q2	-0.07	-0.02	-0.06	0.03	0	1.00
q4	-0.03	-0.03	0.06	0.01	0	1.00
q5	-0.05	-0.02	-0.08	0.02	0	0.87
q6	0.06	0.09	-0.03	0.00	0	1.00
q7	-0.04	0.14	0.02	-0.02	0	0.96
q8	0.09	-0.05	-0.08	-0.02	0	1.00
q9	0.13	-0.08	0.04	0.00	0	1.00
q10	-0.13	0.01	0.03	0.03	0	1.00
q11	0.02	-0.05	0.04	-0.04	0	0.99
q13	-0.12	-0.05	-0.01	-0.09	0	0.92
q14	0.01	-0.02	0.01	0.10	0	0.98
q15	0.08	0.12	-0.02	-0.02	0	0.84

We will not draw the diagram that shows the questions that underlie factor one

Rcodes

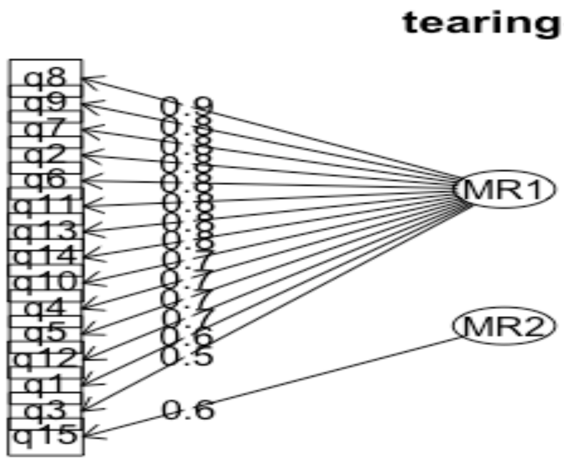
> M1<-fa(tearingq,nfactors = 1,rotate="oblimin")

> fa.diagram(M1,main="tearingq")



If we examine the factor loadings, we see that the above plot shows that the items with the higher loadings load first.

item	9	8	2	7	6	11	13	14	4	10	5	12	1	3	15
Factor loading	0.86	0.89	0.84	0.84	0.83	0.82	0.76	0.75	0.75	0.74	0.72	0.68	.066	0.56	0.48



If we try two factors, question 15 adds to factor one and question six is moved to factor two. It is possible that question two is correlated with other questions and has the lowest correlation with question 15. This is actually evident from the following table.

> cor(tearingq)

	q1	q2	q3	q4	q5	q6
q1	1.0000000	0.7794003	0.4896964	0.3972340	0.2921412	0.4481739
q2	0.7794003	1.0000000	0.4404742	0.6314624	0.4171173	0.6018140
q3	0.4896964	0.4404742	1.0000000	0.4162206	0.4439504	0.4373562
q4	0.3972340	0.6314624	0.4162206	1.0000000	0.6621423	0.5677421
q5	0.2921412	0.4171173	0.4439504	0.6621423	1.0000000	0.6003903
q6	**0.4481739**	**0.6018140**	**0.4373562**	**0.5677421**	**0.6003903**	**1.0000000**
q7	0.4788388	0.6299463	0.4178261	0.6175704	0.6662739	0.6853952
q8	0.6319127	0.7505410	0.4328307	0.5924818	0.5397765	0.7671598
q9	0.6351094	0.6986055	0.5102338	0.4887350	0.6736278	0.6813927
q10	0.3533910	0.5158120	0.4074527	0.2630970	0.5624463	0.7890931
q11	0.4048667	0.6323374	0.3018005	0.6504805	0.6616879	0.7117967
q12	0.1877846	0.5405037	0.4409365	0.4607158	0.4643524	0.7150083
q13	0.5640646	0.6855405	0.4281892	0.5963764	0.4414856	0.4947208
q14	0.4944612	0.6562041	0.2603665	0.6481232	0.4364939	0.4690224
q15	0.2976089	0.4793929	0.2679393	0.4777694	0.3353460	0.1604046

	q7	q8	q9	q10	q11	q12
q1	0.4788388	0.6319127	0.6351094	0.3533910	0.4048667	0.1877846
q2	0.6299463	0.7505410	0.6986055	0.5158120	0.6323374	0.5405037
q3	0.4178261	0.4328307	0.5102338	0.4074527	0.3018005	0.4409365

q4	0.6175704	0.5924818	0.4887350	0.2630970	0.6504805	0.4607158
q5	0.6662739	0.5397765	0.6736278	0.5624463	0.6616879	0.4643524
q6	**0.6853952**	**0.7671598**	**0.6813927**	**0.7890931**	**0.7117967**	**0.7150083**
q7	1.0000000	0.8693743	0.7510116	0.6887598	0.7223928	0.5595507
q8	0.8693743	1.0000000	0.7657939	0.7137793	0.7935056	0.5169176
q9	0.7510116	0.7657939	1.0000000	0.7779194	0.6031428	0.5149180
q10	0.6887598	0.7137793	0.7779194	1.0000000	0.6707686	0.5679827
q11	0.7223928	0.7935056	0.6031428	0.6707686	1.0000000	0.4738945
q12	0.5595507	0.5169176	0.5149180	0.5679827	0.4738945	1.0000000
q13	0.4714239	0.5534485	0.5955828	0.3743623	0.4474206	0.5348490
q14	0.5280745	0.6353411	0.5195951	0.3202325	0.6109868	0.4456337
q15	0.1738574	0.2002097	0.3691867	0.1060587	0.3218856	0.2737579

	q13	q14	q15
q1	0.5640646	0.4944612	0.2976089
q2	0.6855405	0.6562041	0.4793929
q3	0.4281892	0.2603665	0.2679393
q4	0.5963764	0.6481232	0.4777694
q5	0.4414856	0.4364939	0.3353460
q6	**0.4947208**	**0.4690224**	**0.1604046**
q7	0.4714239	0.5280745	0.1738574
q8	0.5534485	0.6353411	0.2002097
q9	0.5955828	0.5195951	0.3691867
q10	0.3743623	0.3202325	0.1060587
q11	0.4474206	0.6109868	0.3218856
q12	0.5348490	0.4456337	0.2737579
q13	1.0000000	0.8458790	0.6724498
q14	0.8458790	1.0000000	0.6742066
q15	0.6724498	0.6742066	1.0000000

5. Using exploratory factor analysis to identify the dimensions of the survey

In this part of the lecture, we will use the tearing data resulting from the survey developed by Dr. Rootman at Jules Stein Eye Institute.

The survey is given below.

Watch this video on factor analysis

https://www.youtube.com/watch?v=Ilf1XR-K3ps
Edward Boone

We first make a data frame including the 15 questions on the survey

```
> newdata<-tearing[, c("LQ 1","LQ 2","LQ 3","LQ 4","LQ 5","LQ 6","LQ 7","LQ 8","LQ 9","LQ 10","LQ 11","LQ 12","LQ 13","LQ 14","LQ 15")]
```

Then we perform principal component analysis on the data

```
> pca<-princomp((newdata))
> summary(pca)
```

Results

Importance of components:

	Comp.1	**Comp.2**	**Comp.3**	**Comp.4**
Standard deviation	4.0200226	1.8184875	1.42051248	1.3852922
Proportion of Variance	0.5565027	0.1138757	0.06948649	0.0660835
Cumulative Proportion	**0.5565027**	**0.6703783**	**0.73986480**	**0.8059483**

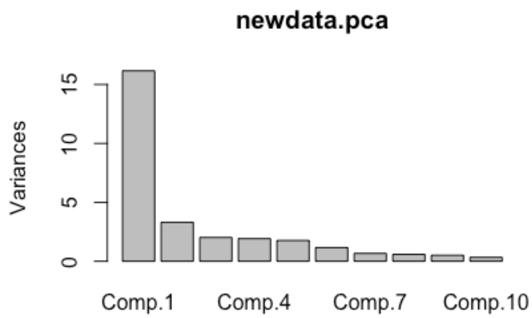
	Comp.5	**Comp.6**	**Comp.7**
Standard deviation	1.32786990	1.07689690	0.81634320
Proportion of Variance	0.06071854	0.03993544	0.02294858
Cumulative Proportion	**0.86666684**	**0.90660228**	**0.92955086**

	Comp.8	**Comp.9**	**Comp.10**
Standard deviation	0.76507918	0.7134378	0.57488673
Proportion of Variance	0.02015687	0.0175276	0.01138085
Cumulative Proportion	**0.94970773**	**0.9672353**	**0.97861618**

	Comp.11	Comp.12	Comp.13
Standard deviation	0.473902976	0.402626520	0.323822271
Proportion of Variance	0.007733732	0.005582324	0.003610968
Cumulative Proportion	**0.986349914**	**0.991932238**	**0.995543206**

	Comp.14	**Comp.15**
Standard deviation	0.306249651	0.1887708
Proportion of Variance	0.003229695	0.0012271
Cumulative Proportion	**0.998772900**	**1.0000000**

> plot(newdata.pca)



Based on the above plot, if we use six components, we can explain almost 90.7% or 91% of the variance.

Running the analysis with six components and findings the relevant factor loadings

Uniquenesses:

LQ 1	LQ 2	**LQ 3**	LQ 4	LQ 5	LQ 6	LQ 7	LQ 8	LQ 9	LQ 10
0.064	0.179	**0.549**	0.236	0.080	0.206	0.165	0.023	0.168	0.005

LQ 11	LQ 12	LQ 13	LQ 14	LQ 15
0.236	0.005	0.150	0.049	0.316

Loadings:

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
LQ 1	0.264	0.150	0.889	0.220		
LQ 2	0.400	0.255	0.628	0.351	0.267	
LQ 3	0.109	0.217	0.467	0.300	0.288	
LQ 4	0.421		0.210	0.469	0.531	0.202
LQ 5	0.224	0.371	0.108	0.219	0.808	0.139
LQ 6	0.132	0.564	0.268	0.379	0.233	0.435
LQ 7	0.108	0.430	0.281	0.619	0.346	0.239
LQ 8	0.180	0.455	0.405	0.727	0.130	0.167
LQ 9	0.250	0.594	0.475	0.251	0.322	0.157
LQ 10		0.915	0.184	0.218	0.142	0.233
LQ 11	0.272	0.478	0.133	0.570	0.326	0.111
LQ 12	0.238	0.299	0.184	0.158		0.885
LQ 13	0.771	0.184	0.340	0.185	0.109	0.243
LQ 14	0.826	0.131	0.176	0.448	0.112	
LQ 15	0.792		0.122	0.189		

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
SS loadings	2.629	2.543	2.220	2.203	1.535	1.439
Proportion Var	0.175	0.170	0.148	0.147	0.102	0.096
Cumulative Var	0.175	0.345	0.493	0.640	0.742	0.838

Test of the hypothesis that 6 factors are sufficient.

The chi square statistic is 34.73 on 30 degrees of freedom.

The p-value is 0.253

Running the analysis with five components and findings the relevant factor loadings

> newdata.fa2<-factanal(newdata, factors=5)

> newdata.fa2

Uniquenesses:

LQ 1	LQ 2	LQ 3	LQ 4	LQ 5	LQ 6	LQ 7	LQ 8	LQ 9	LQ 10
0.136	0.192	0.573	0.005	0.355	0.228	0.193	0.023	0.182	0.032

LQ 11	LQ 12	LQ 13	LQ 14	LQ 15
0.237	0.521	0.170	0.005	0.330

Loadings:

	Factor1	Factor2	Factor3	Factor4	Factor5
LQ 1	0.146	0.261	0.854		0.207
LQ 2	0.362	0.385	0.642	0.243	0.239
LQ 3	0.343	0.122	0.476	0.245	
LQ 4	0.237	0.392	0.217	0.831	0.218
LQ 5	0.586	0.233	0.125	0.481	
LQ 6	0.749	0.142	0.262	0.277	0.213
LQ 7	0.625	0.103	0.313	0.315	0.457
LQ 8	0.593	0.159	0.439	0.195	0.608
LQ 9	0.684	0.251	0.512	0.111	0.117
LQ 10	0.952		0.216		
LQ 11	0.625	0.250	0.146	0.338	0.418
LQ 12	0.582	0.283		0.214	
LQ 13	0.275	0.766	0.357	0.151	0.129
LQ 14	0.224	0.838	0.172	0.163	0.431
LQ 15		0.774	0.151	0.179	-0.104

	Factor1	Factor2	Factor3	Factor4	Factor5
SS loadings	4.206	2.593	2.310	1.514	1.195
Proportion Var	0.280	0.173	0.154	0.101	0.080
Cumulative Var	0.280	0.453	0.607	0.708	0.788

Test of the hypothesis that 5 factors are sufficient. The chi square statistic is 47.27 on 40 degrees of freedom. The p-value is 0.2

Running the analysis with five components and findings the relevant factor loadings

```
> newdata.fa3<-factanal(newdata, factors=4)
> newdata.fa3
```

Uniquenesses:

LQ 1	LQ 2	LQ 3	LQ 4	LQ 5	LQ 6	LQ 7	LQ 8	LQ 9	LQ 10
0.005	0.206	0.680	0.202	0.443	0.247	0.208	0.143	0.207	0.005

LQ 11	LQ 12	LQ 13	LQ 14	LQ 15
0.236	0.469	0.078	0.183	0.419

Loadings:

	Factor1	Factor2	Factor3	Factor4
LQ 1	0.146	0.217	0.955	0.124
LQ 2	0.371	0.401	0.626	0.321
LQ 3	0.321	0.209	0.399	0.118
LQ 4	0.197	0.446	0.191	0.724
LQ 5	0.535	0.235	0.111	0.451
LQ 6	0.737	0.176	0.274	0.323
LQ 7	0.626	0.135	0.306	0.537
LQ 8	0.616	0.184	0.464	0.477
LQ 9	0.681	0.266	0.478	0.174
LQ 10	0.974		0.209	
LQ 11	0.622	0.185	0.216	0.544
LQ 12	0.565	0.389		0.244
LQ 13	0.273	0.845	0.339	0.139
LQ 14	0.233	0.744	0.265	0.371
LQ 15		0.743	0.121	0.111

	Factor1	Factor2	Factor3	Factor4
SS loadings	4.152	2.667	2.414	2.035
Proportion Var	0.277	0.178	0.161	0.136
Cumulative Var	0.277	0.455	0.616	0.751

Test of the hypothesis that 4 factors are sufficient. The chi square statistic is 67.46 on 51 degrees of freedom. The p-value is 0.0611

Running the analysis with three components and findings the relevant factor loadings

```
> newdata.fa4<-factanal(newdata, factors=3)
> newdara.fa4
```

Uniquenesses:

```
 LQ 1  LQ 2  LQ 3  LQ 4  LQ 5  LQ 6  LQ 7  LQ 8  LQ 9 LQ 10
0.005 0.206 0.689 0.425 0.476 0.255 0.217 0.140 0.256 0.259
LQ 11 LQ 12 LQ 13 LQ 14 LQ 15
0.297 0.471 0.184 0.154 0.353
Loadings:
      Factor1 Factor2 Factor3
LQ 1  0.167   0.238   0.954
LQ 2  0.446   0.452   0.625
LQ 3  0.338   0.170   0.410
LQ 4  0.480   0.554   0.194
LQ 5  0.656   0.281   0.120
LQ 6  0.796   0.171   0.287
LQ 7  0.805   0.192   0.313
LQ 8  0.762   0.245   0.468
LQ 9  0.667   0.250   0.486
LQ 10 0.831           0.224
LQ 11 0.748   0.311   0.215
LQ 12 0.644   0.337
LQ 13 0.293   0.782   0.344
LQ 14 0.344   0.814   0.255
LQ 15         0.796   0.108
               Factor1 Factor2 Factor3
SS loadings      5.180   2.983   2.449
Proportion Var   0.345   0.199   0.163
Cumulative Var   0.345   0.544   0.707
```

Test of the hypothesis that 3 factors are sufficient. The chi square statistic is 89.95 on 63 degrees of freedom. The p-value is 0.0145. We are rejecting the null. Three factors are not sufficient.

Decision: Let us go with factors.

Next decision we need to make is which items to include under each component:

"As a rule of thumb, your variable should have a rotated factor loading of at least |0.4| (meaning ≥ +. 4 or ≤ –. 4) onto one of the factors in order to be considered important. Some researchers use much more stringent criteria such as a cut-off of |0.7|."

Let us look at the loadings with four factors and choose items with loadings more than +0.4. There are no negative loadings.

Loadings:

	Factor1	Factor2	Factor3	Factor4
LQ 1	0.146	0.217	**0.955**	0.124
LQ 2	0.371	0.401	**0.626**	0.321
LQ 3	0.321	0.209	0.399	0.118
LQ 4	0.197	**0.446**	0.191	**0.724**
LQ 5	**0.535**	0.235	0.111	0.451
LQ 6	**0.737**	0.176	0.274	0.323
LQ 7	**0.626**	0.135	0.306	**0.537**
LQ 8	**0.616**	0.184	0.464	**0.477**
LQ 9	**0.681**	0.266	0.478	0.174
LQ 10	**0.974**		0.209	
LQ 11	**0.622**	0.185	0.216	**0.544**
LQ 12	**0.565**	0.389		0.244
LQ 13	0.273	**0.845**	0.339	0.139
LQ 14	0.233	**0.744**	0.265	0.371
LQ 15		**0.743**	0.121	0.111

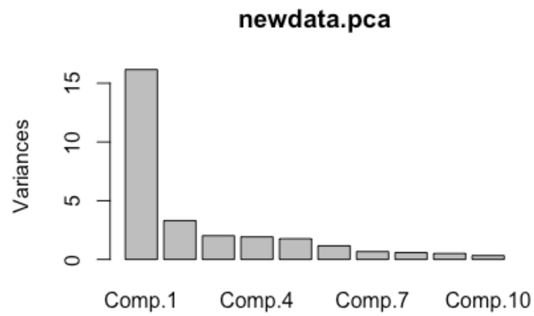
If we go with the guideline of choosing items with loadings more than +.40, we should include suggests we should include all the 15 items.

If we use the guideline of picking items with a loading of more than +0.7, we should choose items **1, 4, 6, 10, 13, 14, and 15** (see items in red).; which is seven items instead of 15.

Calculation of Cronbach alpha for a survey with seven items with loadings more than +0.7.

```
> newdata3<-tearing[, c("LQ 1","LQ 4","LQ 6","LQ 10","LQ 13","LQ 14","LQ 15")]
> library(psych)
> alpha(newdata2)
Reliability analysis
  raw_alpha
      0.86
 95% confidence boundaries
         lower alpha upper
Feldt     0.77  0.86  0.93
Duhachek  0.78  0.86  0.94
Reliability if an item is dropped:
      raw_alpha std.alpha G6(smc) average_r S/N alpha se var.r
LQ 1       0.85      0.86    0.91      0.50 5.9    0.046 0.048
LQ 4       0.84      0.84    0.89      0.47 5.3    0.050 0.046
LQ 6       0.84      0.84    0.85      0.47 5.4    0.048 0.038
LQ 10      0.87      0.87    0.88      0.52 6.5    0.040 0.027
LQ 13      0.82      0.82    0.87      0.43 4.5    0.056 0.036
LQ 14      0.82      0.82    0.87      0.44 4.7    0.054 0.035
LQ 15      0.86      0.86    0.90      0.51 6.2    0.043 0.027
      med.r
LQ 1   0.49
LQ 4   0.47
LQ 6   0.48
LQ 10  0.49
LQ 13  0.45
LQ 14  0.45
LQ 15  0.49
```

Conclusion: If we shorten the survey from 15 to 7 items, reliability drops from 0.92 to 0.86 and the 95% confidence interval for reliability drops from **(0.88 to 0.95) to (0.77 to 0.93). We have to find out if this is acceptable to the client. Reliability of 0.86 is quite acceptable and if it takes patients half the time to fill out the survey, recommending the shorter survey is acceptable. Also, visual inspection of scree plot shows that four component is quite acceptable.**



Results

Importance of components:

	Comp.1	**Comp.2**	**Comp.3**	**Comp.4**
Standard deviation	4.0200226	1.8184875	1.42051248	1.3852922
Proportion of Variance	0.5565027	0.1138757	0.06948649	0.0660835
Cumulative Proportion	**0.5565027**	**0.6703783**	**0.73986480**	**0.8059483**

With four components, we almost explain 81% of variation in the data.

Next: We will compute the coefficient of correlation between the total scores resulting from 15 item and seven item test and compare results