GroupAssignment

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Load the necessary dataset and library:

```
library(tidyverse)
library(MASS)

Market <- read_csv('Market.csv')
Portfolio <- read_csv('SampleE.csv')
Portfolio_sorted <- Portfolio[, c("Date", sort(setdiff(names(Portfolio), "Date")))]
```

Principal Component Analysis (without the Market):

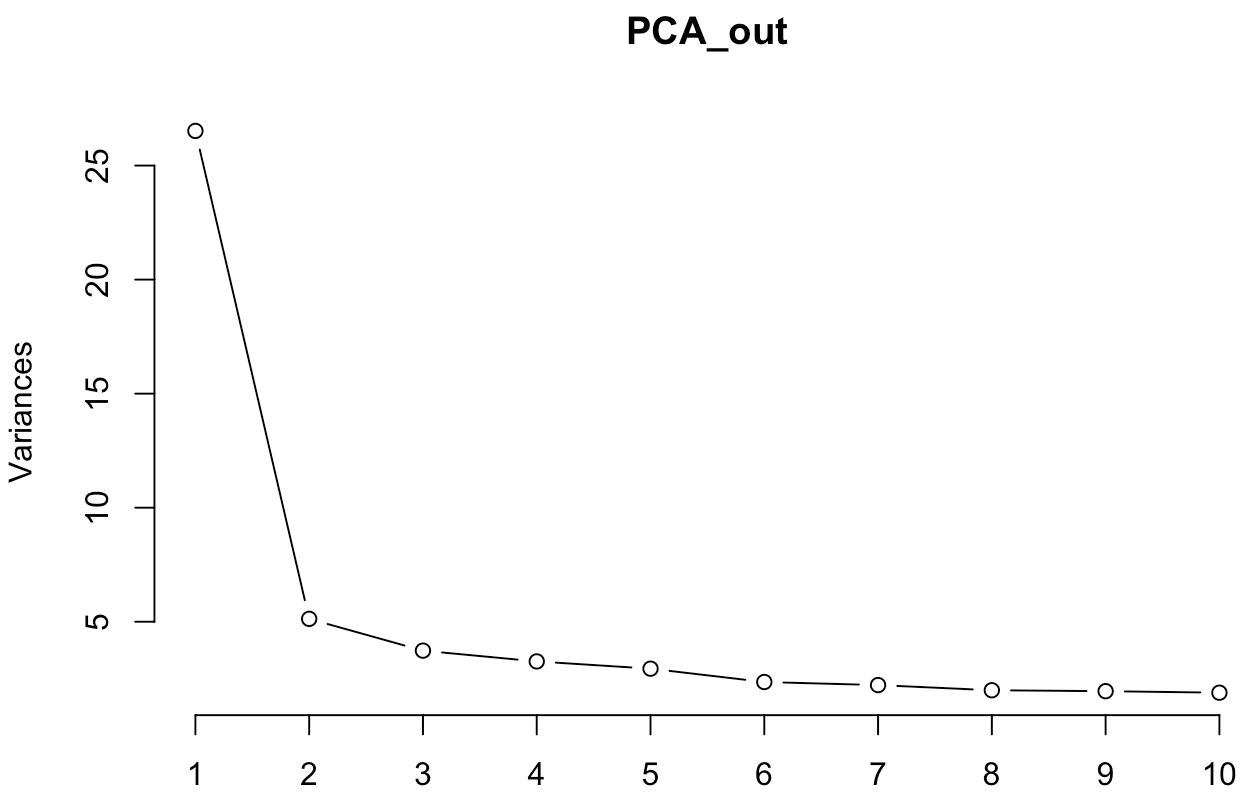
```
PCA_out <- Portfolio %>%
  column_to_rownames('Date') %>%
  prcomp(scale. = TRUE)

summary(PCA_out)
```

```
## Importance of components:
##                            PC1     PC2     PC3    PC4     PC5     PC6     PC7
## Standard deviation     5.1497 2.26425 1.93200 1.8056 1.71524 1.53553 1.49095
## Proportion of Variance 0.2652 0.05127 0.03733 0.0326 0.02942 0.02358 0.02223
## Cumulative Proportion  0.2652 0.31646 0.35379 0.3864 0.41581 0.43939 0.46162
##                             PC8     PC9    PC10    PC11   PC12    PC13    PC14
## Standard deviation     1.41269 1.39839 1.37446 1.31929 1.3078 1.29368 1.25627
## Proportion of Variance 0.01996 0.01955 0.01889 0.01741 0.0171 0.01674 0.01578
## Cumulative Proportion  0.48157 0.50113 0.52002 0.53742 0.5545 0.57126 0.58704
##                            PC15    PC16    PC17   PC18    PC19    PC20    PC21
## Standard deviation     1.19694 1.19076 1.16803 1.1489 1.13165 1.12293 1.09514
## Proportion of Variance 0.01433 0.01418 0.01364 0.0132 0.01281 0.01261 0.01199
## Cumulative Proportion  0.60137 0.61555 0.62919 0.6424 0.65520 0.66781 0.67980
##                            PC22    PC23    PC24    PC25    PC26    PC27    PC28
## Standard deviation     1.07403 1.06149 1.04677 1.03821 1.01929 1.00854 0.98603
## Proportion of Variance 0.01154 0.01127 0.01096 0.01078 0.01039 0.01017 0.00972
## Cumulative Proportion  0.69134 0.70261 0.71356 0.72434 0.73473 0.74490 0.75463
##                            PC29   PC30    PC31    PC32    PC33    PC34    PC35
## Standard deviation     0.95643 0.9381 0.92345 0.92110 0.90464 0.88825 0.87803
## Proportion of Variance 0.00915 0.0088 0.00853 0.00848 0.00818 0.00789 0.00771
## Cumulative Proportion  0.76377 0.7726 0.78110 0.78958 0.79777 0.80566 0.81337
##                            PC36    PC37    PC38    PC39    PC40    PC41   PC42
## Standard deviation     0.86521 0.85658 0.85130 0.82429 0.81444 0.79706 0.7876
## Proportion of Variance 0.00749 0.00734 0.00725 0.00679 0.00663 0.00635 0.0062
## Cumulative Proportion  0.82085 0.82819 0.83544 0.84223 0.84887 0.85522 0.8614
##                            PC43    PC44    PC45    PC46    PC47    PC48    PC49
## Standard deviation     0.77199 0.75353 0.75130 0.73696 0.72582 0.71337 0.69512
## Proportion of Variance 0.00596 0.00568 0.00564 0.00543 0.00527 0.00509 0.00483
## Cumulative Proportion  0.86738 0.87306 0.87870 0.88413 0.88940 0.89449 0.89932
##                            PC50    PC51    PC52    PC53    PC54    PC55    PC56
## Standard deviation     0.68633 0.66487 0.65488 0.65214 0.63446 0.63146 0.61270
## Proportion of Variance 0.00471 0.00442 0.00429 0.00425 0.00403 0.00399 0.00375
## Cumulative Proportion  0.90403 0.90845 0.91274 0.91700 0.92102 0.92501 0.92876
##                            PC57    PC58    PC59    PC60    PC61    PC62    PC63
## Standard deviation     0.59070 0.58971 0.56677 0.56104 0.55066 0.54252 0.53667
## Proportion of Variance 0.00349 0.00348 0.00321 0.00315 0.00303 0.00294 0.00288
## Cumulative Proportion  0.93225 0.93573 0.93894 0.94209 0.94512 0.94807 0.95095
##                            PC64   PC65    PC66   PC67    PC68    PC69    PC70
## Standard deviation     0.51731 0.5104 0.49663 0.4896 0.48213 0.47211 0.47050
## Proportion of Variance 0.00268 0.0026 0.00247 0.0024 0.00232 0.00223 0.00221
## Cumulative Proportion  0.95362 0.9562 0.95869 0.9611 0.96341 0.96564 0.96786
##                           PC71    PC72    PC73    PC74    PC75    PC76    PC77
## Standard deviation     0.4578 0.44575 0.43720 0.42894 0.42664 0.41144 0.40255
## Proportion of Variance 0.0021 0.00199 0.00191 0.00184 0.00182 0.00169 0.00162
## Cumulative Proportion  0.9699 0.97194 0.97385 0.97569 0.97751 0.97920 0.98083
##                            PC78    PC79    PC80    PC81    PC82    PC83   PC84
## Standard deviation     0.38914 0.38157 0.37175 0.36522 0.35475 0.35032 0.3323
## Proportion of Variance 0.00151 0.00146 0.00138 0.00133 0.00126 0.00123 0.0011
## Cumulative Proportion  0.98234 0.98380 0.98518 0.98651 0.98777 0.98900 0.9901
##                            PC85    PC86    PC87    PC88    PC89    PC90    PC91
## Standard deviation     0.32273 0.31815 0.29642 0.28948 0.28023 0.26635 0.25921
## Proportion of Variance 0.00104 0.00101 0.00088 0.00084 0.00079 0.00071 0.00067
```

```
## Cumulative Proportion  0.99114 0.99215 0.99303 0.99387 0.99466 0.99537 0.99604
##                           PC92    PC93    PC94    PC95    PC96    PC97    PC98
## Standard deviation     0.24012 0.23799 0.22679 0.22112 0.21145 0.19581 0.19260
## Proportion of Variance 0.00058 0.00057 0.00051 0.00049 0.00045 0.00038 0.00037
## Cumulative Proportion  0.99661 0.99718 0.99770 0.99818 0.99863 0.99901 0.99939
##                           PC99   PC100
## Standard deviation     0.18145 0.16882
## Proportion of Variance 0.00033 0.00028
## Cumulative Proportion  0.99972 1.00000
```

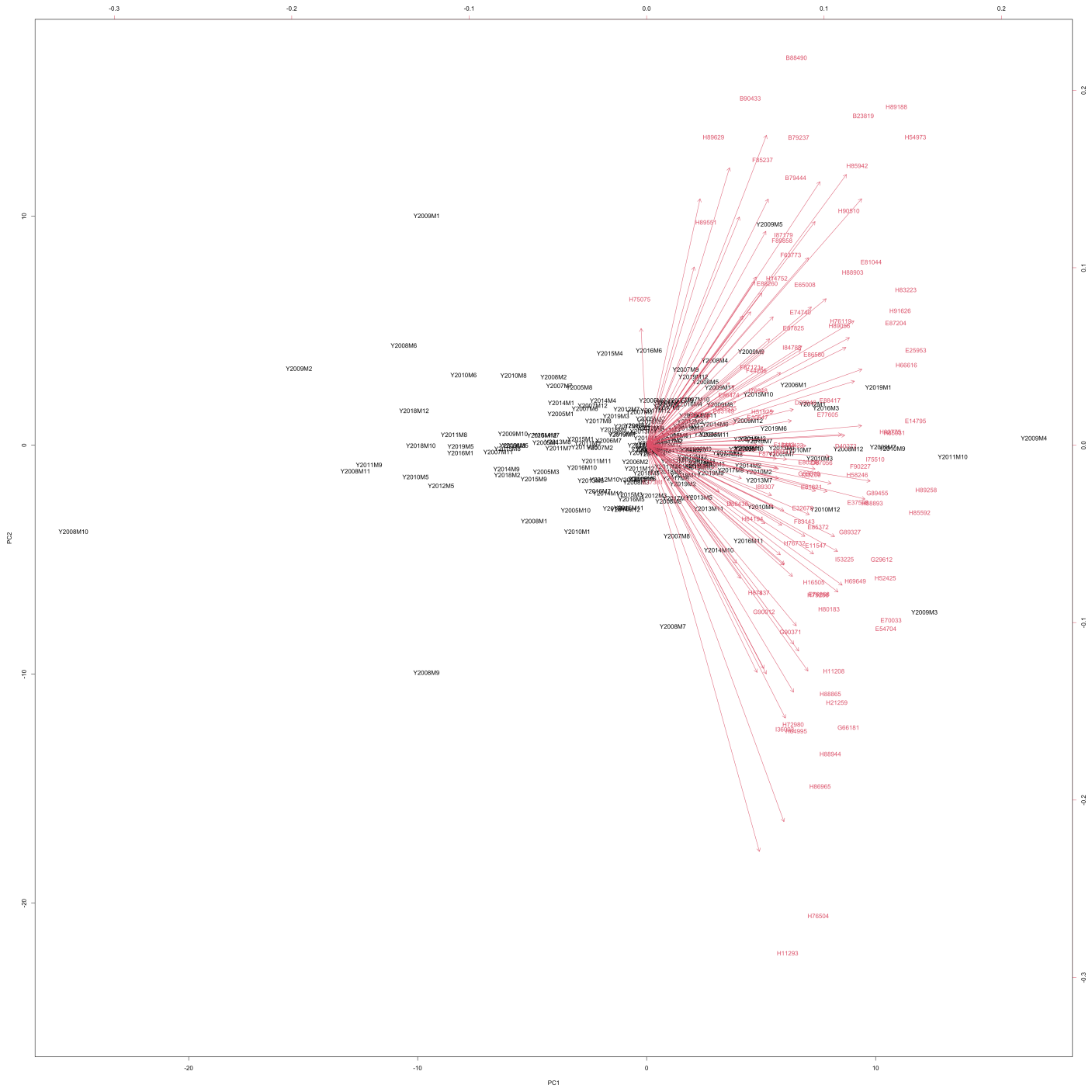
```
screeplot(PCA_out,type = 'l')
```



The elbow appears at the second PC, therefore 2 PCs should be used.

We're now visualising the biplot:

```
biplot(PCA_out, scale=0, cex=1, arrow.len=0.1, xlab = "PC1", ylab = "PC2")
```




```
cor(Market$MarketReturn, PCA_out$x)
```

```
##            PC1        PC2        PC3          PC4         PC5        PC6
## [1,] 0.9482118 0.02547051 0.08543341 -0.008306986 0.003291803 0.01619725
##              PC7        PC8         PC9        PC10        PC11       PC12
## [1,] 0.008500451 0.03742005 -0.04950894 -0.01807384 -0.07072269 0.07358135
##              PC13        PC14        PC15        PC16       PC17        PC18
## [1,] -0.009670309 0.007502947 -0.05289717 -0.04726281 0.09030535 -0.02193839
##              PC19       PC20      PC21        PC22        PC23        PC24
## [1,] -0.08877097 0.03018928 0.0321534 0.005385156 -0.01184957 -0.02830492
##              PC25        PC26        PC27       PC28        PC29       PC30
## [1,] -0.03654239 0.000227439 -0.01487146 0.01358262 -0.02771432 -0.0258521
##              PC31       PC32        PC33        PC34         PC35       PC36
## [1,] 0.007992551 0.02891946 0.005074535 -0.03166501 -0.001575187 0.04900572
##              PC37       PC38         PC39       PC40        PC41        PC42
## [1,] 0.001991793 0.02390102 -0.007551323 0.01091808 0.008262909 0.004068899
##              PC43        PC44        PC45         PC46       PC47       PC48
## [1,] -0.001218935 -0.05938677 0.002579002 -0.006325572 -0.0239496 0.02706316
##              PC49       PC50       PC51         PC52        PC53        PC54
## [1,] -0.004453102 0.01423751 0.02209727 -0.007019861 -0.01187656 0.002793666
##              PC55        PC56       PC57       PC58         PC59       PC60
## [1,] -0.03669683 0.009465507 0.05945069 0.02603025 -0.004100718 -0.0030897
##            PC61       PC62       PC63      PC64         PC65       PC66
## [1,] 0.0157114 0.01815713 0.02683059 0.0106839 -0.004162977 0.02119016
##              PC67       PC68        PC69        PC70         PC71       PC72
## [1,] -0.01316936 0.03207101 0.008681188 -0.02460998 -0.009607016 0.02149124
##              PC73        PC74       PC75        PC76        PC77        PC78
## [1,] 0.009234486 -0.01160023 0.01170874 -0.06042441 -0.04113049 -0.03580059
##              PC79       PC80       PC81       PC82        PC83         PC84
## [1,] -0.003149978 0.02357134 -0.0141897 -0.0226965 0.001627093 -0.000665063
##              PC85        PC86        PC87      PC88        PC89        PC90
## [1,] -0.006395398 0.002089858 -0.01895932 -0.026898 0.001341493 0.008124895
##              PC91        PC92        PC93         PC94        PC95        PC96
## [1,] -0.001108442 0.006359851 0.006533315 -0.001452072 0.006733771 -0.00833431
##              PC97      PC98        PC99       PC100
## [1,] -0.008416285 0.0302142 -0.01887151 0.001481463
```

```
# Load necessary library
PC_df <- as.data.frame(PCA_out$x)
# Re-attach the Date column to the PCA results
PC_df$Date <- rownames(PC_df)
PC_df$Date <- as.Date(paste0(gsub("Y", "", gsub("M", "-", PC_df$Date)), "-01"), format =
"%Y-%m-%d")
```

Now we plot the market return overtime

```
Market$Date <- as.Date(paste0(gsub("Y", "", gsub("M", "-", Market$Date)), "-01"), format
= "%Y-%m-%d")
```

```
library(patchwork)
```

```
## Warning: package 'patchwork' was built under R version 4.2.3
```

```
##
## Attaching package: 'patchwork'
```

```
## The following object is masked from 'package:MASS':
##
##     area
```

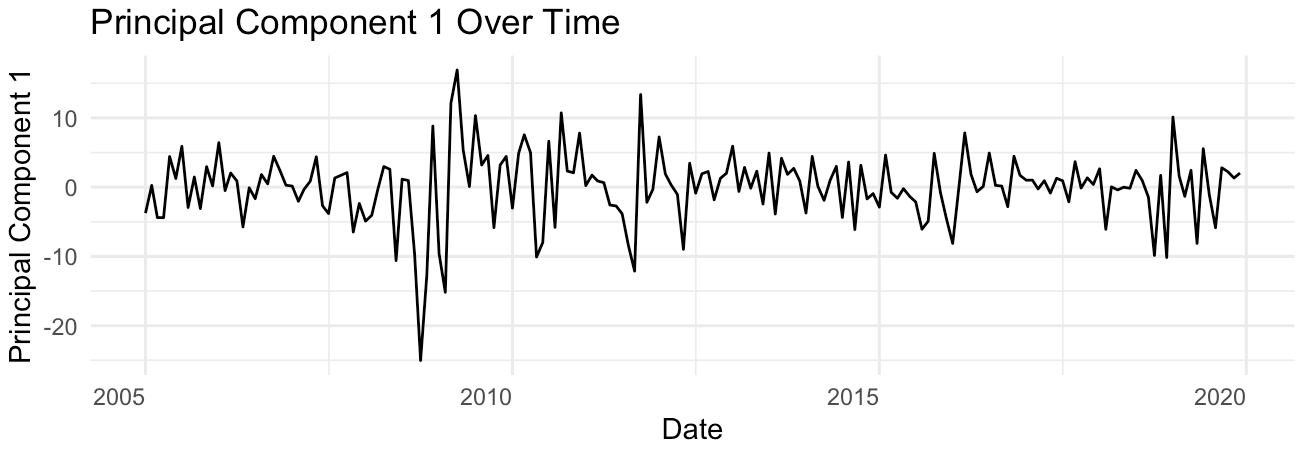
```
PC_df <- as.data.frame(PCA_out$x)
# Re-attach the Date column to the PCA results
PC_df$Date <- rownames(PC_df)
PC_df$Date <- as.Date(paste0(gsub("Y", "", gsub("M", "-", PC_df$Date)), "-01"), format =
"%Y-%m-%d")
# Format Date for Market
Market$Date <- as.Date(paste0(gsub("Y", "", gsub("M", "-", Market$Date)), "-01"), format
= "%Y-%m-%d")

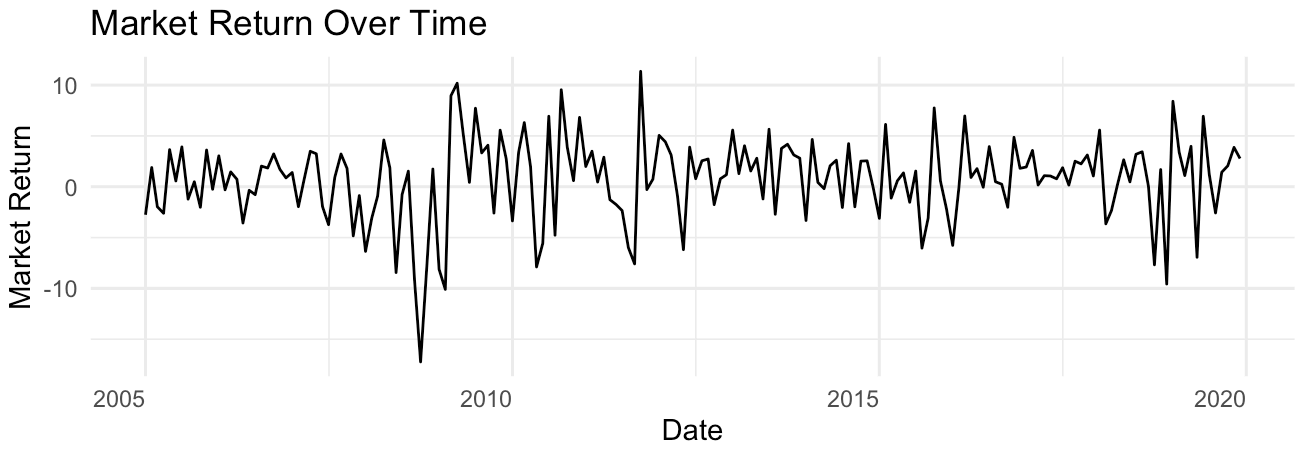
# Create individual plots
p1 <- ggplot(Market, aes(x = Date, y = MarketReturn)) +
  geom_line() +
  labs(title = "Market Return Over Time", x = "Date", y = "Market Return") +
  theme_minimal() +
  theme(axis.text.x = element_text(hjust = 1))

p2 <- ggplot(PC_df, aes(x = Date, y = PC1)) +
  geom_line() +
  labs(title = "Principal Component 1 Over Time", x = "Date", y = "Principal Component
1") +
  theme_minimal() +
  theme(axis.text.x = element_text(hjust = 1))

# Combine the two plots one on top of another
combined_plot <- p2 / p1

# Display the combined plot
combined_plot
```






```
cor(Market$MarketReturn, PC_df$PC1)
```

```
## [1] 0.9482118
```

```
FA <- Portfolio_sorted %>%
  column_to_rownames('Date')%>%
  scale() %>%
  factanal(factors = 2,rotation = 'none',scores = 'none')

loadings(FA)
```

```
##
## Loadings:
##        Factor1 Factor2
## B23819  0.647  -0.437
## B79237  0.451  -0.408
## B79444  0.441  -0.332
## B88490  0.455  -0.476
## B90433  0.315  -0.391
## D29946  0.457
## D40272  0.569
## D79122  0.157
## D86728  0.167
## D87056  0.488
## D88436  0.250
## D89935  0.225
## D90423  0.413
## E10547  0.301
## E11547  0.479   0.106
## E14795  0.786
## E25953  0.781  -0.116
## E32678  0.434   0.114
## E37381
## E37568  0.595
## E54704  0.686   0.257
## E65008  0.452  -0.163
## E70033  0.700   0.249
## E74740  0.441  -0.119
## E76868  0.488   0.152
## E77605  0.512
## E80233  0.460
## E81044  0.666  -0.220
## E81621  0.460
## E83149  0.214
## E85372  0.487
## E86474  0.226
## E86580  0.476  -0.119
## E87204  0.729  -0.128
## E87825  0.427  -0.123
## E88260  0.342  -0.161
## E88417  0.525
## F10443  0.394
## F44206  0.309
## F63773  0.415  -0.240
## F83143  0.449   0.145
## F85237  0.349  -0.318
## F87121  0.293
## F87430  0.348
## F89858  0.391  -0.219
## F90227  0.615
## G29612  0.668   0.140
## G59010  0.461
## G66181  0.557   0.370
```

```
## G89327  0.588   0.117
## G89455  0.659
## G90012  0.319   0.182
## G90371  0.399   0.212
## H11208  0.537   0.300
## H11293  0.382   0.609
## H14752  0.381  -0.210
## H16505  0.473   0.189
## H21259  0.538   0.353
## H51925  0.331
## H52425  0.677   0.233
## H54973  0.784  -0.345
## H58246  0.596
## H64194  0.286
## H64995  0.412   0.330
## H66616  0.764
## H69649  0.590   0.146
## H72980  0.414   0.362
## H75075         -0.133
## H76119  0.563  -0.135
## H76504  0.475   0.572
## H76732  0.421   0.156
## H79238  0.487   0.217
## H80183  0.522   0.257
## H82775  0.714
## H83223  0.759  -0.162
## H85592  0.788   0.151
## H85931  0.714
## H85942  0.610  -0.244
## H86965  0.495   0.427
## H87837  0.307   0.164
## H88865  0.513   0.306
## H88893  0.632   0.104
## H88903  0.607  -0.191
## H88944  0.521   0.392
## H89056  0.558
## H89188  0.744  -0.456
## H89211  0.385
## H89258  0.811   0.105
## H89551  0.165  -0.165
## H89629  0.196  -0.280
## H90510  0.584  -0.205
## H91626  0.735
## I33209  0.467
## I36003  0.373   0.328
## I53225  0.553   0.149
## I75510  0.652
## I76948  0.313
## I84788  0.413
## I87179  0.380  -0.210
## I89307  0.325
##
```

```
##                Factor1 Factor2
## SS loadings     25.878   4.524
## Proportion Var   0.259   0.045
## Cumulative Var   0.259   0.304
```

```
FA$uniquenesses
```

```
##    B23819    B79237    B79444    B88490    B90433    D29946    D40272    D79122
## 0.3912097 0.6293650 0.6955141 0.5668274 0.7481287 0.7899860 0.6763636 0.9741149
##    D86728    D87056    D88436    D89935    D90423    E10547    E11547    E14795
## 0.9707576 0.7589146 0.9346285 0.9476760 0.8290931 0.9093442 0.7592216 0.3813457
##    E25953    E32678    E37381    E37568    E54704    E65008    E70033    E74740
## 0.3762545 0.7988369 0.9992016 0.6404821 0.4639081 0.7694565 0.4473002 0.7909872
##    E76868    E77605    E80233    E81044    E81621    E83149    E85372    E86474
## 0.7386797 0.7380012 0.7874563 0.5077476 0.7856030 0.9537057 0.7547821 0.9451753
##    E86580    E87204    E87825    E88260    E88417    F10443    F44206    F63773
## 0.7588818 0.4521970 0.8023386 0.8571180 0.7204874 0.8444488 0.9029004 0.7700242
##    F83143    F85237    F87121    F87430    F89858    F90227    G29612    G59010
## 0.7772810 0.7769643 0.9114973 0.8786968 0.7988666 0.6196147 0.5339736 0.7868229
##    G66181    G89327    G89455    G90012    G90371    H11208    H11293    H14752
## 0.5530950 0.6409740 0.5630812 0.8650027 0.7961466 0.6214938 0.4835459 0.8107601
##    H16505    H21259    H51925    H52425    H54973    H58246    H64194    H64995
## 0.7404197 0.5854588 0.8891742 0.4874311 0.2656048 0.6412909 0.9110402 0.7217097
##    H66616    H69649    H72980    H75075    H76119    H76504    H76732    H79238
## 0.4113546 0.6305802 0.6975426 0.9820605 0.6642667 0.4471749 0.7981035 0.7156794
##    H80183    H82775    H83223    H85592    H85931    H85942    H86965    H87837
## 0.6614714 0.4876662 0.3970311 0.3556248 0.4898746 0.5678154 0.5729334 0.8791497
##    H88865    H88893    H88903    H88944    H89056    H89188    H89211    H89258
## 0.6426158 0.5892965 0.5949049 0.5743187 0.6820992 0.2388789 0.8509791 0.3320347
##    H89551    H89629    H90510    H91626    I33209    I36003    I53225    I75510
## 0.9457337 0.8833955 0.6169954 0.4575897 0.7796163 0.7529398 0.6724757 0.5729282
##    I76948    I84788    I87179    I89307
## 0.8977637 0.8211219 0.8112240 0.8934564
```

```
# Convert the uniquenesses to a data frame
uniquenesses_df <- data.frame(Variable = names(FA$uniquenesses), Uniqueness = FA$uniquen
esses)

# Display the uniquenesses data frame
print(uniquenesses_df)
```

```
##        Variable Uniqueness
## B23819   B23819  0.3912097
## B79237   B79237  0.6293650
## B79444   B79444  0.6955141
## B88490   B88490  0.5668274
## B90433   B90433  0.7481287
## D29946   D29946  0.7899860
## D40272   D40272  0.6763636
## D79122   D79122  0.9741149
## D86728   D86728  0.9707576
## D87056   D87056  0.7589146
## D88436   D88436  0.9346285
## D89935   D89935  0.9476760
## D90423   D90423  0.8290931
## E10547   E10547  0.9093442
## E11547   E11547  0.7592216
## E14795   E14795  0.3813457
## E25953   E25953  0.3762545
## E32678   E32678  0.7988369
## E37381   E37381  0.9992016
## E37568   E37568  0.6404821
## E54704   E54704  0.4639081
## E65008   E65008  0.7694565
## E70033   E70033  0.4473002
## E74740   E74740  0.7909872
## E76868   E76868  0.7386797
## E77605   E77605  0.7380012
## E80233   E80233  0.7874563
## E81044   E81044  0.5077476
## E81621   E81621  0.7856030
## E83149   E83149  0.9537057
## E85372   E85372  0.7547821
## E86474   E86474  0.9451753
## E86580   E86580  0.7588818
## E87204   E87204  0.4521970
## E87825   E87825  0.8023386
## E88260   E88260  0.8571180
## E88417   E88417  0.7204874
## F10443   F10443  0.8444488
## F44206   F44206  0.9029004
## F63773   F63773  0.7700242
## F83143   F83143  0.7772810
## F85237   F85237  0.7769643
## F87121   F87121  0.9114973
## F87430   F87430  0.8786968
## F89858   F89858  0.7988666
## F90227   F90227  0.6196147
## G29612   G29612  0.5339736
## G59010   G59010  0.7868229
## G66181   G66181  0.5530950
## G89327   G89327  0.6409740
## G89455   G89455  0.5630812
```

```
## G90012   G90012  0.8650027
## G90371   G90371  0.7961466
## H11208   H11208  0.6214938
## H11293   H11293  0.4835459
## H14752   H14752  0.8107601
## H16505   H16505  0.7404197
## H21259   H21259  0.5854588
## H51925   H51925  0.8891742
## H52425   H52425  0.4874311
## H54973   H54973  0.2656048
## H58246   H58246  0.6412909
## H64194   H64194  0.9110402
## H64995   H64995  0.7217097
## H66616   H66616  0.4113546
## H69649   H69649  0.6305802
## H72980   H72980  0.6975426
## H75075   H75075  0.9820605
## H76119   H76119  0.6642667
## H76504   H76504  0.4471749
## H76732   H76732  0.7981035
## H79238   H79238  0.7156794
## H80183   H80183  0.6614714
## H82775   H82775  0.4876662
## H83223   H83223  0.3970311
## H85592   H85592  0.3556248
## H85931   H85931  0.4898746
## H85942   H85942  0.5678154
## H86965   H86965  0.5729334
## H87837   H87837  0.8791497
## H88865   H88865  0.6426158
## H88893   H88893  0.5892965
## H88903   H88903  0.5949049
## H88944   H88944  0.5743187
## H89056   H89056  0.6820992
## H89188   H89188  0.2388789
## H89211   H89211  0.8509791
## H89258   H89258  0.3320347
## H89551   H89551  0.9457337
## H89629   H89629  0.8833955
## H90510   H90510  0.6169954
## H91626   H91626  0.4575897
## I33209   I33209  0.7796163
## I36003   I36003  0.7529398
## I53225   I53225  0.6724757
## I75510   I75510  0.5729282
## I76948   I76948  0.8977637
## I84788   I84788  0.8211219
## I87179   I87179  0.8112240
## I89307   I89307  0.8934564
```

```
# Perform factor analysis
FA <- Portfolio_sorted %>%
  column_to_rownames('Date') %>%
  scale() %>%
  factanal(factors = 2, rotation = 'none', scores = 'none')

cor(Portfolio_sorted$E37381, Market$MarketReturn)
```

```
## [1] 0.02591092
```

```
# Extract the loadings and convert to a data frame
loadings_df <- as.data.frame(unclass(loadings(FA)))

# Add variable names from Portfolio_sorted as a new column
loadings_df$Variable <- rownames(loadings(FA))

# Reorder columns to have 'Variable' as the first column
loadings_df <- loadings_df[, c("Variable", colnames(loadings_df)[-ncol(loadings_df)])]
loadings_df <- loadings_df[order(loadings_df$Factor1, decreasing = TRUE), ]

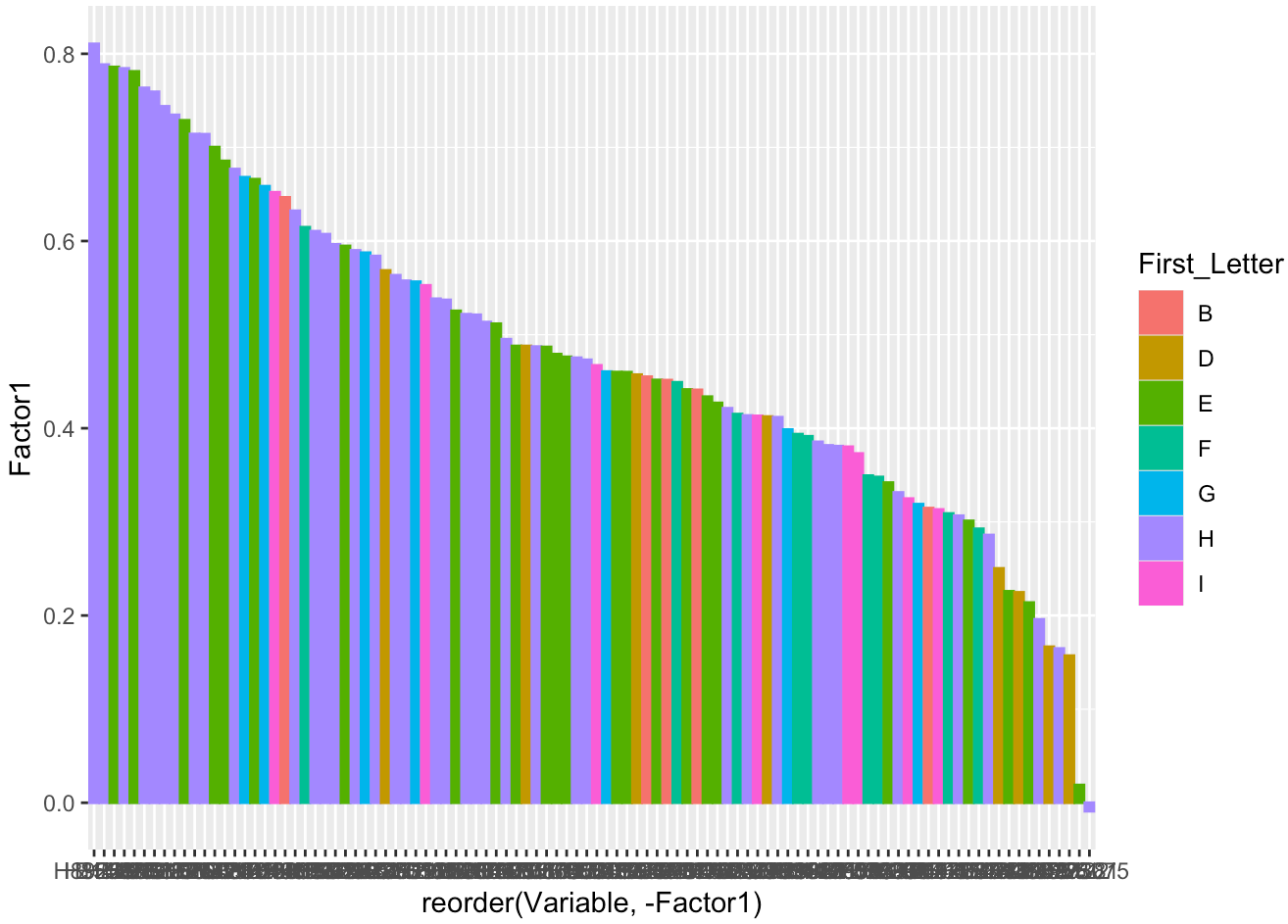

# Display the final loadings data frame
print(loadings_df)
```

```
##        Variable      Factor1      Factor2
## H89258   H89258  0.810578174  0.104508723
## H85592   H85592  0.788441244  0.150724281
## E14795   E14795  0.785888876 -0.032173213
## H54973   H54973  0.784374148 -0.345203861
## E25953   E25953  0.781186499 -0.116144268
## H66616   H66616  0.763566631 -0.074914658
## H83223   H83223  0.759370408 -0.162267610
## H89188   H89188  0.743871372 -0.455836804
## H91626   H91626  0.734714887 -0.051070212
## E87204   E87204  0.728942770 -0.128239581
## H82775   H82775  0.714271522  0.046276517
## H85931   H85931  0.714056030 -0.015628143
## E70033   E70033  0.700335371  0.249465504
## E54704   E54704  0.685575814  0.257054725
## H52425   H52425  0.676858514  0.233305677
## G29612   G29612  0.668200494  0.139723848
## E81044   E81044  0.666066775 -0.220463168
## G89455   G89455  0.658556813  0.056638501
## I75510   I75510  0.652034221  0.043712427
## B23819   B23819  0.646620877 -0.436660762
## H88893   H88893  0.632283779  0.104471639
## F90227   F90227  0.614815153  0.048846180
## H85942   H85942  0.610458484 -0.243964842
## H88903   H88903  0.607276965 -0.190539109
## H58246   H58246  0.596426466  0.054685241
## E37568   E37568  0.594763476  0.076020915
## H69649   H69649  0.590005356  0.146014094
## G89327   G89327  0.587580989  0.117397155
## H90510   H90510  0.583981630 -0.204874056
## D40272   D40272  0.568579793  0.019143844
## H76119   H76119  0.563423561 -0.135277928
## H89056   H89056  0.557541112 -0.084051053
## G66181   G66181  0.556544478  0.370341278
## I53225   I53225  0.552602017  0.148885360
## H21259   H21259  0.538316741  0.353190521
## H11208   H11208  0.537110436  0.300043684
## E88417   E88417  0.525393706 -0.059005595
## H80183   H80183  0.521798645  0.257418004
## H88944   H88944  0.521172531  0.392493560
## H88865   H88865  0.513423247  0.306252728
## E77605   E77605  0.511650856  0.014726136
## H86965   H86965  0.495073868  0.426569193
## E76868   E76868  0.487934766  0.152470503
## D87056   D87056  0.487910819  0.054958374
## H79238   H79238  0.487250408  0.216594387
## E85372   E85372  0.486872276  0.090407983
## E11547   E11547  0.479183037  0.105639364
## E86580   E86580  0.476321610 -0.119277462
## H76504   H76504  0.475336537  0.571735000
## H16505   H16505  0.473113257  0.189052268
## I33209   I33209  0.467151094  0.046358246
```

```
## G59010   G59010  0.460530837  0.032825823
## E81621   E81621  0.460066689  0.052277860
## E80233   E80233  0.459928941  0.031667962
## D29946   D29946  0.457261897 -0.030322762
## B88490   B88490  0.455041687 -0.475502579
## E65008   E65008  0.451624371 -0.163021383
## B79237   B79237  0.451426233 -0.408467208
## F83143   F83143  0.449104029  0.144960504
## E74740   E74740  0.441402778 -0.119022050
## B79444   B79444  0.441008171 -0.331662399
## E32678   E32678  0.433793027  0.113901290
## E87825   E87825  0.427084233 -0.123488926
## H76732   H76732  0.421383383  0.155918479
## F63773   F63773  0.415202486 -0.239942208
## H72980   H72980  0.413637478  0.362441488
## I84788   I84788  0.413211806 -0.090118378
## D90423   D90423  0.412514154  0.026928843
## H64995   H64995  0.411773316  0.329747671
## G90371   G90371  0.398644786  0.211951784
## F10443   F10443  0.393743802 -0.022498027
## F89858   F89858  0.391423099 -0.218880115
## H89211   H89211  0.385474723 -0.020554129
## H11293   H11293  0.381705353  0.608897205
## H14752   H14752  0.380894166 -0.210109725
## I87179   I87179  0.380258352 -0.210160735
## I36003   I36003  0.373057651  0.328462335
## F85237   F85237  0.349394683 -0.317722673
## F87430   F87430  0.348022252  0.013405330
## E88260   E88260  0.341954236 -0.161071513
## H51925   H51925  0.331426821 -0.031369437
## I89307   I89307  0.324895499  0.031529684
## G90012   G90012  0.318989080  0.182314143
## B90433   B90433  0.314736741 -0.390913844
## I76948   I76948  0.313251833 -0.064201287
## F44206   F44206  0.308997567 -0.040400079
## H87837   H87837  0.306578623  0.163866923
## E10547   E10547  0.301102797 -0.001850341
## F87121   F87121  0.292679448 -0.053459412
## H64194   H64194  0.285964526  0.084808667
## D88436   D88436  0.250334628  0.052147430
## E86474   E86474  0.225866093 -0.061892462
## D89935   D89935  0.224768122 -0.042686378
## E83149   E83149  0.213859728 -0.023834261
## H89629   H89629  0.195849008 -0.279708770
## D86728   D86728  0.166659327  0.038528109
## H89551   H89551  0.164773966 -0.164574221
## D79122   D79122  0.157130001 -0.034795125
## E37381   E37381  0.018981190  0.020771869
## H75075   H75075 -0.008997026 -0.133318094
```

```
loadings_df$First_Letter <- substr(loadings_df$Variable, 1, 1)

ggplot(data = loadings_df, aes(x=reorder(Variable, -Factor1), y = Factor1, color= First_
Letter, fill = First_Letter))+
  geom_col()
```




```
FA_v <- Portfolio %>%
  column_to_rownames('Date')%>%
  scale() %>%
  factanal(factors = 2,rotation = 'varimax',scores = 'none')

FA_p <- Portfolio %>%
  column_to_rownames('Date')%>%
  scale() %>%
  factanal(factors = 2,rotation = 'promax',scores = 'none')


loadings(FA_v)
```

```
##
## Loadings:
##        Factor1 Factor2
## I87179  0.115   0.419
## H91626  0.477   0.561
## H58246  0.456   0.389
## E14795  0.526   0.585
## D87056  0.380   0.311
## E74740  0.223   0.399
## E88417  0.325   0.417
## G89455  0.501   0.432
## H76119  0.297   0.498
## E85372  0.405   0.285
## I75510  0.487   0.436
## H88903  0.288   0.568
## H89629          0.336
## F44206  0.187   0.249
## H89258  0.641   0.507
## H89056  0.329   0.458
## H14752  0.116   0.419
## H75075 -0.102
## H89188  0.194   0.851
## H51925  0.209   0.259
## B79444          0.547
## H66616  0.480   0.599
## E86474  0.114   0.205
## G66181  0.654   0.139
## B88490          0.658
## E65008  0.199   0.437
## H86965  0.651
## G29612  0.567   0.380
## H21259  0.629   0.138
## E80233  0.344   0.307
## H72980  0.548
## H16505  0.466   0.206
## H85931  0.488   0.522
## F63773  0.118   0.465
## G90371  0.430   0.137
## E76868  0.450   0.243
## G59010  0.345   0.307
## H11208  0.590   0.175
## D89935  0.126   0.191
## B79237          0.608
## E86580  0.247   0.424
## E37568  0.470   0.372
## I36003  0.496
## F90227  0.464   0.406
## H85942  0.252   0.607
## H89211  0.255   0.290
## E81621  0.359   0.293
## E10547  0.209   0.217
## E77605  0.368   0.356
```

```
## H89551          0.233
## D88436  0.212   0.143
## I53225  0.493   0.291
## I76948  0.173   0.269
## F87430  0.253   0.240
## H64194  0.260   0.145
## F89858  0.117   0.433
## H54973  0.301   0.802
## B90433          0.498
## D29946  0.298   0.348
## H64995  0.524
## E25953  0.463   0.640
## H88893  0.516   0.379
## H80183  0.549   0.194
## I89307  0.250   0.210
## H85592  0.659   0.459
## E83149  0.132   0.170
## E81044  0.308   0.631
## D90423  0.307   0.276
## H52425  0.640   0.321
## H83223  0.414   0.657
## H11293  0.702  -0.152
## G90012  0.353   0.101
## H82775  0.532   0.479
## E87204  0.417   0.611
## H87837  0.331   0.105
## F10443  0.259   0.297
## E32678  0.385   0.231
## I84788  0.224   0.359
## E70033  0.668   0.327
## E37381
## D86728  0.144
## D40272  0.411   0.393
## H76732  0.406   0.193
## H69649  0.517   0.320
## F85237          0.472
## B23819  0.139   0.768
## E88260  0.124   0.357
## H88865  0.578   0.153
## E54704  0.663   0.311
## H88944  0.645
## F87121  0.166   0.247
## D79122          0.137
## G89327  0.494   0.338
## I33209  0.360   0.302
## H76504  0.741
## F83143  0.417   0.220
## E11547  0.410   0.269
## E87825  0.210   0.392
## H79238  0.495   0.197
## H90510  0.261   0.561
##
```

```
##                Factor1 Factor2
## SS loadings     15.692  14.711
## Proportion Var   0.157   0.147
## Cumulative Var   0.157   0.304
```

```
loadings(FA_p)
```

```
##
## Loadings:
##        Factor1 Factor2
## I87179          0.471
## H91626  0.320   0.485
## H58246  0.378   0.279
## E14795  0.370   0.492
## D87056  0.321   0.216
## E74740          0.397
## E88417  0.202   0.374
## G89455  0.412   0.312
## H76119  0.128   0.487
## E85372  0.364   0.173
## I75510  0.393   0.324
## H88903          0.578
## H89629 -0.240   0.448
## F44206  0.112   0.227
## H89258  0.550   0.342
## H89056  0.188   0.422
## H14752          0.471
## H75075 -0.168   0.154
## H89188 -0.169   0.975
## H51925  0.134   0.229
## B79444 -0.175   0.651
## H66616  0.306   0.530
## E86474          0.204
## G66181  0.743  -0.122
## B88490 -0.344   0.832
## E65008          0.456
## H86965  0.779  -0.225
## G29612  0.519   0.218
## H21259  0.712  -0.112
## E80233  0.278   0.227
## H72980  0.659  -0.195
## H16505  0.478
## H85931  0.353   0.431
## F63773          0.527
## G90371  0.467
## E76868  0.441
## G59010  0.280   0.226
## H11208  0.647
## D89935          0.181
## B79237 -0.264   0.749
## E86580  0.102   0.417
## E37568  0.403   0.252
## I36003  0.596  -0.178
## F90227  0.380   0.296
## H85942          0.644
## H89211  0.176   0.247
## E81621  0.304   0.203
## E10547  0.155   0.176
## E77605  0.285   0.277
```

```
## H89551 -0.115   0.292
## D88436  0.194
## I53225  0.470   0.141
## I76948          0.258
## F87430  0.198   0.185
## H64194  0.253
## F89858          0.488
## H54973          0.866
## B90433 -0.314   0.649
## D29946  0.201   0.300
## H64995  0.618  -0.157
## E25953  0.265   0.590
## H88893  0.457   0.240
## H80183  0.587
## I89307  0.208   0.150
## H85592  0.595   0.274
## E83149          0.152
## E81044          0.648
## D90423  0.248   0.206
## H52425  0.638   0.111
## H83223  0.197   0.632
## H11293  0.943  -0.509
## G90012  0.389
## H82775  0.429   0.357
## E87204  0.223   0.574
## H87837  0.360
## F10443  0.178   0.254
## E32678  0.365   0.114
## I84788  0.105   0.346
## E70033  0.670   0.105
## E37381
## D86728  0.134
## D40272  0.320   0.305
## H76732  0.410
## H69649  0.486   0.166
## F85237 -0.206   0.582
## B23819 -0.197   0.896
## E88260          0.390
## H88865  0.642
## E54704  0.671
## H88944  0.751  -0.169
## F87121          0.233
## D79122          0.132
## G89327  0.450   0.198
## I33209  0.300   0.214
## H76504  0.946  -0.410
## F83143  0.411
## E11547  0.379   0.150
## E87825          0.394
## H79238  0.519
## H90510          0.582
##
```

```
##                Factor1 Factor2
## SS loadings     14.736  14.059
## Proportion Var   0.147   0.141
## Cumulative Var   0.147   0.288
```

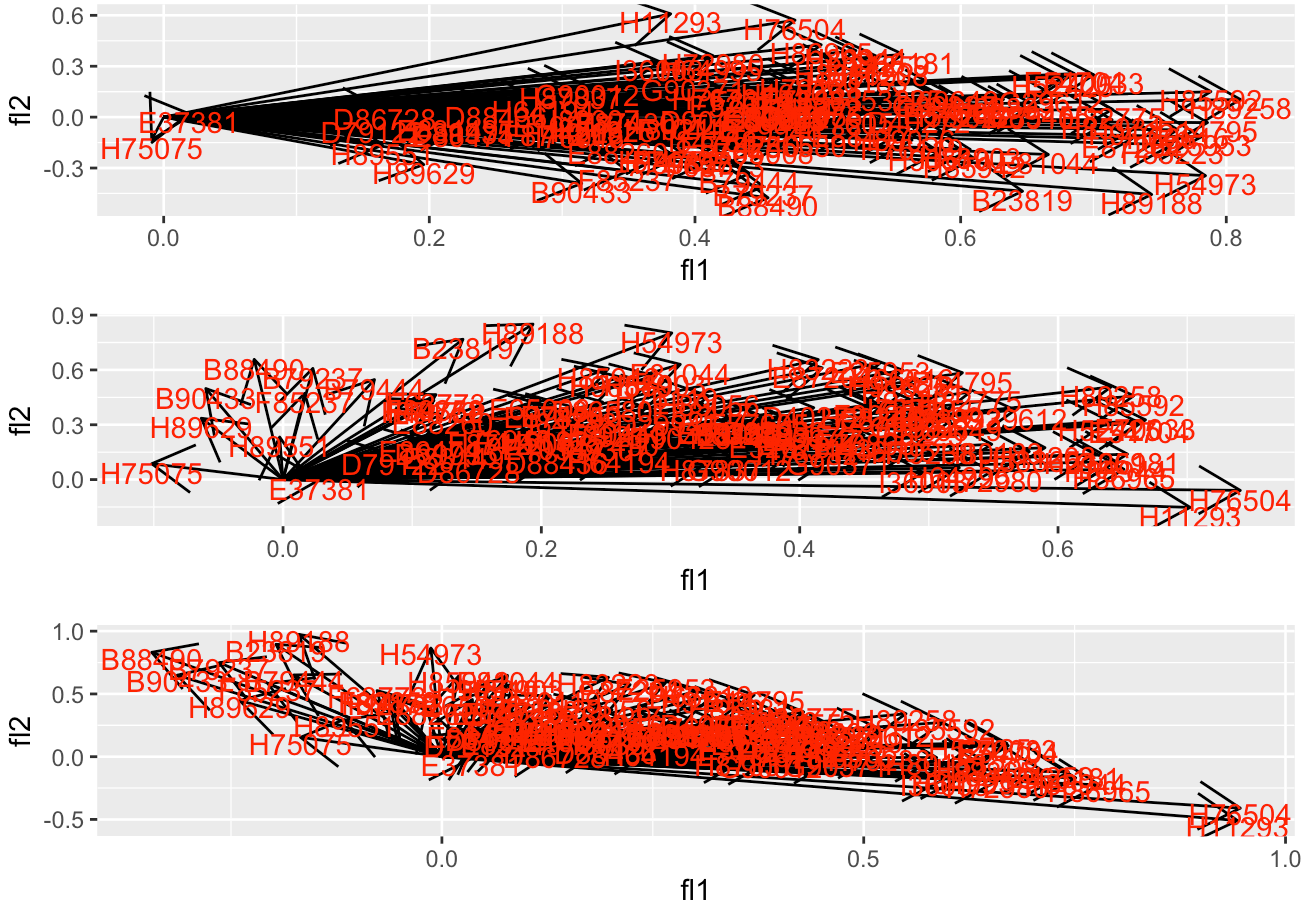
```
library(broom)
fa_df<-tidy(FA)
NoRot<-ggplot(fa_df,aes(x=fl1,y=fl2,
                 label=variable))+
  geom_segment(aes(xend=fl1,
                   yend=fl2,x=0,y=0),
               arrow = arrow())+
  geom_text(color='red',nudge_y = -0.05)
fa_df$uniqueness
```

```
##   [1] 0.3912097 0.6293650 0.6955141 0.5668274 0.7481287 0.7899860 0.6763636
##   [8] 0.9741149 0.9707576 0.7589146 0.9346285 0.9476760 0.8290931 0.9093442
##  [15] 0.7592216 0.3813457 0.3762545 0.7988369 0.9992016 0.6404821 0.4639081
##  [22] 0.7694565 0.4473002 0.7909872 0.7386797 0.7380012 0.7874563 0.5077476
##  [29] 0.7856030 0.9537057 0.7547821 0.9451753 0.7588818 0.4521970 0.8023386
##  [36] 0.8571180 0.7204874 0.8444488 0.9029004 0.7700242 0.7772810 0.7769643
##  [43] 0.9114973 0.8786968 0.7988666 0.6196147 0.5339736 0.7868229 0.5530950
##  [50] 0.6409740 0.5630812 0.8650027 0.7961466 0.6214938 0.4835459 0.8107601
##  [57] 0.7404197 0.5854588 0.8891742 0.4874311 0.2656048 0.6412909 0.9110402
##  [64] 0.7217097 0.4113546 0.6305802 0.6975426 0.9820605 0.6642667 0.4471749
##  [71] 0.7981035 0.7156794 0.6614714 0.4876662 0.3970311 0.3556248 0.4898746
##  [78] 0.5678154 0.5729334 0.8791497 0.6426158 0.5892965 0.5949049 0.5743187
##  [85] 0.6820992 0.2388789 0.8509791 0.3320347 0.9457337 0.8833955 0.6169954
##  [92] 0.4575897 0.7796163 0.7529398 0.6724757 0.5729282 0.8977637 0.8211219
##  [99] 0.8112240 0.8934564
```

```
fa_df_1<-tidy(FA_v)
VRot <-ggplot(fa_df_1,aes(x=fl1,y=fl2,
                 label=variable))+
  geom_segment(aes(xend=fl1,
                   yend=fl2,x=0,y=0),
               arrow = arrow())+
  geom_text(color='red',nudge_y = -0.05)

fa_df_2<-tidy(FA_p)
ProRot <- ggplot(fa_df_2,aes(x=fl1,y=fl2,
                 label=variable))+
  geom_segment(aes(xend=fl1,
                   yend=fl2,x=0,y=0),
               arrow = arrow())+
  geom_text(color='red',nudge_y = -0.05)

Merged <- NoRot/VRot/ProRot
Merged
```




```
FA_Bart <- Portfolio %>%
  column_to_rownames('Date')%>%
  scale() %>%
  factanal(factors = 2,rotation = 'none',scores = 'Bartlett')
FA_Bart$scores
```

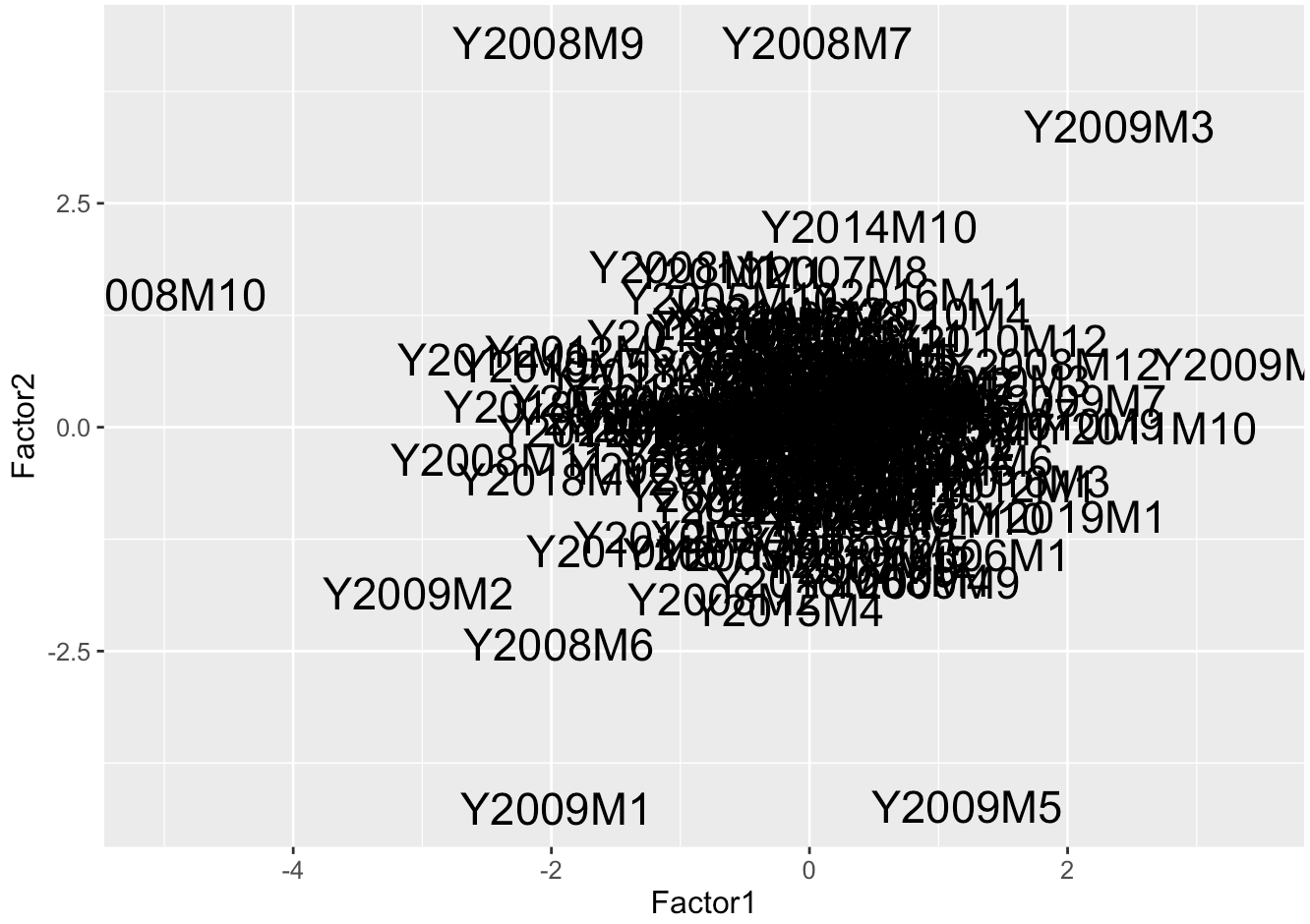
```
##               Factor1      Factor2
## Y2005M1  -0.6801668212 -0.762761250
## Y2005M2   0.2153822500 -1.289172535
## Y2005M3  -0.8632658212  0.278231028
## Y2005M4  -0.8313732905  0.189551176
## Y2005M5   0.7923617053  0.189792612
## Y2005M6   0.2894154002 -0.240716222
## Y2005M7   1.1718654897 -0.019217004
## Y2005M8  -0.4438184070 -1.419727655
## Y2005M9   0.4015128172 -1.043041766
## Y2005M10 -0.6136707312  1.438570224
## Y2005M11  0.5664681953 -0.096210196
## Y2005M12  0.0584255743 -0.593279196
## Y2006M1   1.2847927795 -1.426568053
## Y2006M2  -0.1943951130  0.668112985
## Y2006M3   0.3346218941 -0.072947546
## Y2006M4   0.2234227369 -0.963416600
## Y2006M5  -1.0720277965 -0.036736485
## Y2006M6   0.0268692013  0.156106908
## Y2006M7  -0.2532955546  0.006894858
## Y2006M8   0.2826534966  0.346464789
## Y2006M9   0.0605259042  0.269885940
## Y2006M10  0.7691575635  0.055559445
## Y2006M11  0.5009906593 -0.632086618
## Y2006M12  0.0336085247  0.007684164
## Y2007M1   0.0178859893  0.088254904
## Y2007M2  -0.3827329559 -0.217335152
## Y2007M3  -0.0069404822 -0.728535925
## Y2007M4   0.2567220175 -0.770267749
## Y2007M5   0.8609499887 -0.463964218
## Y2007M6  -0.4920501010 -0.885167368
## Y2007M7  -0.6988218190 -1.409945365
## Y2007M8   0.1833076768  1.740801247
## Y2007M9   0.4186609314 -1.570063671
## Y2007M10  0.3752129483 -0.731152261
## Y2007M11 -1.1901657553 -0.023067831
## Y2007M12 -0.3823176350 -1.210291305
## Y2008M1  -0.9590978658  1.792478590
## Y2008M2  -0.6633862830 -1.913443433
## Y2008M3  -0.0730560792  0.945737916
## Y2008M4   0.6764348427 -1.707224023
## Y2008M5   0.4983086331 -1.373806759
## Y2008M6  -1.9404524669 -2.425540927
## Y2008M7   0.0669402885  4.291692638
## Y2008M8   0.0400190417  1.234537260
## Y2008M9  -2.0176962542  4.290144209
## Y2008M10 -5.0429711641  1.486191163
## Y2008M11 -2.4042716079 -0.356711097
## Y2008M12  1.8888532889  0.704715299
## Y2009M1  -1.9599195916 -4.258348133
## Y2009M2  -3.0295929393 -1.849625724
## Y2009M3   2.4047963383  3.365085790
```

```
## Y2009M4   3.4125971776  0.704264752
## Y2009M5   1.2240064277 -4.234063424
## Y2009M6  -0.1020720556  0.285588485
## Y2009M7   2.0315125415  0.299005372
## Y2009M8   0.5736146717 -0.347864536
## Y2009M9   0.8912789229 -1.725363559
## Y2009M10 -1.0234314697 -0.455090496
## Y2009M11  0.6950141787 -1.049302730
## Y2009M12  0.7407732610 -0.292458008
## Y2010M1  -0.6132165365  1.722499048
## Y2010M2   0.8515557287  0.496224972
## Y2010M3   1.4439733339  0.524033689
## Y2010M4   0.9782665227  1.263211887
## Y2010M5  -1.9710971576  0.680854213
## Y2010M6  -1.4462997146 -1.376291721
## Y2010M7   1.3411298361  0.150198293
## Y2010M8  -1.0794404133 -1.221073360
## Y2010M9   2.0075293444  0.041087715
## Y2010M10  0.4458941014  0.383634386
## Y2010M11  0.3465706718 -0.768678389
## Y2010M12  1.4754726276  0.974245824
## Y2011M1   0.1410631074 -0.431806891
## Y2011M2   0.3513537044 -0.337999662
## Y2011M3   0.1560331774 -0.297463731
## Y2011M4   0.1682541878  0.091873659
## Y2011M5  -0.4894408769  0.155423405
## Y2011M6  -0.4763537134  0.063847431
## Y2011M7  -0.7499205502 -0.115676902
## Y2011M8  -1.6802030336 -0.035489404
## Y2011M9  -2.4475746620  0.760739875
## Y2011M10  2.6358395512 -0.002714622
## Y2011M11 -0.4264374338  0.146695684
## Y2011M12 -0.1254086178  0.774941458
## Y2012M1   1.4907693167 -0.658565359
## Y2012M2   0.4394847730 -0.435814780
## Y2012M3   0.0112964055  1.194767404
## Y2012M4  -0.2196569731 -0.003658865
## Y2012M5  -1.7678985004  0.854458105
## Y2012M6   0.5894470907  0.289093243
## Y2012M7  -0.1375160713 -0.814489426
## Y2012M8   0.3975527083  0.224372481
## Y2012M9   0.4565943739  0.020581221
## Y2012M10 -0.3102575399  0.644052628
## Y2012M11  0.1518310733  0.350947602
## Y2012M12  0.4316998410  0.169060415
## Y2013M1   1.1213649485 -0.017222480
## Y2013M2  -0.1522968663  0.565165079
## Y2013M3   0.4961388598  0.273477973
## Y2013M4  -0.0859206261  0.001266183
## Y2013M5   0.4055061869  0.763492640
## Y2013M6  -0.4973675314  0.681597421
## Y2013M7   0.8896900133  0.450396518
```

```
## Y2013M8  -0.7336036836 -0.373985185
## Y2013M9   0.7586820013  0.209407919
## Y2013M10  0.3723999553 -0.368297830
## Y2013M11  0.3775980842  1.025612998
## Y2013M12  0.1778564478 -0.007632303
## Y2014M1  -0.7156083686 -0.530477828
## Y2014M2   0.8368059302  0.307749341
## Y2014M3   0.0128737587 -0.336952637
## Y2014M4  -0.2172110347 -0.943643422
## Y2014M5   0.1654245541 -0.095003580
## Y2014M6   0.5745139362 -0.393496128
## Y2014M7  -0.7588577555 -0.199304111
## Y2014M8   0.6100554069  0.260211331
## Y2014M9  -1.1820491514  0.425221572
## Y2014M10  0.4673270802  2.245225539
## Y2014M11 -0.4220764616  1.091492355
## Y2014M12 -0.2440930745  1.280436425
## Y2015M1  -0.6222900094  0.036421093
## Y2015M2   0.8506667154 -0.195169343
## Y2015M3  -0.1695979629  0.835609601
## Y2015M4  -0.1604917576 -2.032266839
## Y2015M5  -0.0950164717  0.738114861
## Y2015M6  -0.3527674823  1.167204018
## Y2015M7  -0.4654230182  1.157384206
## Y2015M8  -1.1647487091  0.016350412
## Y2015M9  -0.9773100642  1.022231752
## Y2015M10  0.9925777262 -1.016969196
## Y2015M11 -0.1274079724  1.059390569
## Y2015M12 -0.9561373461  0.124002657
## Y2016M1  -1.5506742444  0.089519628
## Y2016M2   0.0354727920 -0.170319575
## Y2016M3   1.5930040776 -0.601556047
## Y2016M4   0.4178728891 -0.958049928
## Y2016M5  -0.1623897141  1.051142053
## Y2016M6  -0.0006293894 -1.720827067
## Y2016M7   0.9359163990  0.111399249
## Y2016M8   0.0660966020  0.110877222
## Y2016M9   0.0720927651 -0.554009949
## Y2016M10 -0.5113714507  0.132303050
## Y2016M11  0.8459919609  1.484952609
## Y2016M12  0.3004297683  0.562574663
## Y2017M1   0.2419057287 -0.825451918
## Y2017M2   0.1620198291  0.171338836
## Y2017M3  -0.0744270732  0.036304736
## Y2017M4   0.1268671878  0.513171040
## Y2017M5  -0.2195087813 -0.303106555
## Y2017M6   0.2512300920  0.671711371
## Y2017M7   0.2519551292 -0.861322773
## Y2017M8  -0.4405289923 -0.228867263
## Y2017M9   0.7095591422  0.261997663
## Y2017M10 -0.0691167036  0.212196355
## Y2017M11  0.2409035379  0.829811793
```

```
## Y2017M12  0.0912445040 -0.821659472
## Y2018M1   0.5432567508 -0.490356476
## Y2018M2  -1.2928834166  0.601821335
## Y2018M3  -0.0031315356 -0.012548356
## Y2018M4  -0.0301225294 -0.540353916
## Y2018M5  -0.0337612905  0.532736357
## Y2018M6  -0.0862010669  0.541197906
## Y2018M7   0.4683304046  0.301679608
## Y2018M8   0.0731929558  0.580890684
## Y2018M9  -0.1942983545 -0.454797408
## Y2018M10 -1.9899028619  0.234150611
## Y2018M11  0.2769682850  0.826818807
## Y2018M12 -1.8931628152 -0.581919259
## Y2019M1   2.0309702480 -0.990315057
## Y2019M2   0.2445903795  0.696986315
## Y2019M3  -0.2215900567 -0.565844500
## Y2019M4   0.4753890500  0.387499511
## Y2019M5  -1.5778663007  0.305299411
## Y2019M6   1.1504463979 -0.366780039
## Y2019M7  -0.2182501468 -0.167976927
## Y2019M8  -1.0845972531  0.105360585
## Y2019M9   0.5745053862  0.475594017
## Y2019M10  0.3926624421  0.405411788
## Y2019M11  0.2880557948 -0.467925471
## Y2019M12  0.4654142399 -1.495759283
```

```
ggplot(FA_Bart$scores,
       aes(x=Factor1,y=Factor2, label=Portfolio$Date))+
  geom_text(size=6)+theme(text= element_text(size=12))
```




```
common_variance <- 1 - FA$uniquenesses

# Step 6: Total variance explained by common factors
total_common_variance_explained <- sum(common_variance) / length(common_variance)
total_common_variance_explained
```

```
## [1] 0.3040289
```

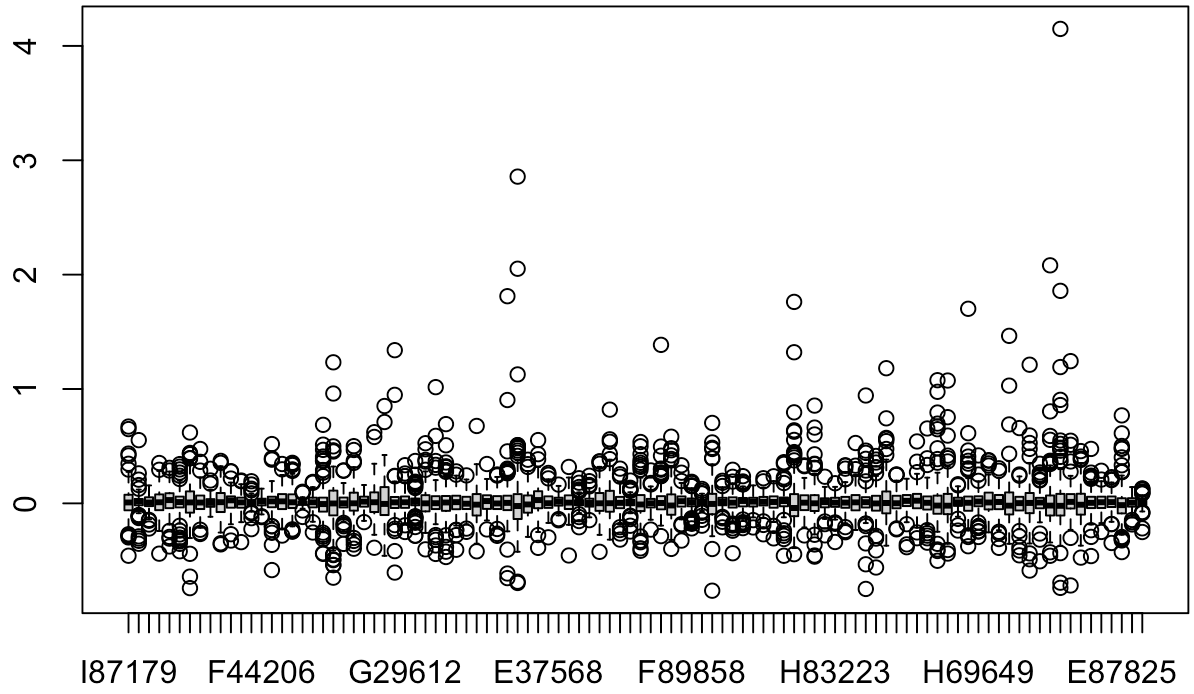
```
sum(is.na(Portfolio))
```

```
## [1] 0
```

```
Portfolio$Date <- as.Date(paste0(gsub("Y", "", gsub("M", "-", Portfolio$Date)), "-01"),
format = "%Y-%m-%d")

Portfolio <- Portfolio %>%
  column_to_rownames("Date")

boxplot <- Portfolio %>%
  boxplot()
```


0
1
2
3
4
I87179
F44206
G29612
E37568
F89858
H83223
H69649
E87825

```
boxplot
```

```
## $stats
##            [,1]       [,2]       [,3]       [,4]       [,5]      [,6]
## [1,] -0.2317930 -0.0860320 -0.1388540 -0.2414670 -0.1851500 -0.175708
## [2,] -0.0608235 -0.0169790 -0.0241135 -0.0594880 -0.0420195 -0.035685
## [3,]  0.0078195  0.0180750  0.0076160  0.0095595  0.0286040  0.011018
## [4,]  0.0752900  0.0383765  0.0532505  0.0785915  0.0886995  0.059197
## [5,]  0.2752810  0.1204810  0.1569470  0.2833640  0.2778920  0.198187
##            [,7]       [,8]       [,9]     [,10]     [,11]      [,12]      [,13]
## [1,] -0.3034290 -0.2135060 -0.1173780 -0.255277 -0.182542 -0.1643400 -0.0721840
## [2,] -0.0814995 -0.0496735 -0.0302580 -0.075892 -0.038116 -0.0384990 -0.0156315
## [3,]  0.0082050  0.0071535  0.0043025  0.010442  0.026331  0.0084715  0.0070510
## [4,]  0.1048580  0.0686140  0.0370295  0.090470  0.061407  0.0502640  0.0254890
## [5,]  0.3533830  0.2271010  0.1168830  0.324848  0.181550  0.1672680  0.0841400
##           [,14]     [,15]     [,16]      [,17]     [,18]      [,19]     [,20]
## [1,] -0.0965290 -0.161176 -0.227781 -0.2010830 -0.046637 -0.1607560 -0.252061
## [2,] -0.0181115 -0.034022 -0.050103 -0.0410085 -0.010125 -0.0352130 -0.072381
## [3,]  0.0170080  0.017474  0.017550  0.0095150  0.004714  0.0091995 -0.006260
## [4,]  0.0410120  0.058186  0.081107  0.0790370  0.016518  0.0500935  0.050198
## [5,]  0.1284440  0.194716  0.247902  0.2585690  0.053835  0.1418540  0.223557
##           [,21]     [,22]      [,23]      [,24]      [,25]      [,26]
## [1,] -0.4115230 -0.158982 -0.2857140 -0.1532240 -0.2749110 -0.4586580
## [2,] -0.1051985 -0.035874 -0.0637810 -0.0291455 -0.0759445 -0.1021700
## [3,] -0.0008550  0.000676  0.0004920  0.0150000  0.0098710 -0.0048635
## [4,]  0.1115910  0.051011  0.0935525  0.0588335  0.0963515  0.1422970
## [5,]  0.3218170  0.177758  0.3166670  0.1605430  0.3463130  0.4227990
##           [,27]      [,28]      [,29]      [,30]      [,31]      [,32]
## [1,] -0.1802170 -0.1730800 -0.0979150 -0.2583510 -0.1822410 -0.2542370
## [2,] -0.0404975 -0.0394695 -0.0169835 -0.0701985 -0.0500605 -0.0655745
## [3,]  0.0089250  0.0096935  0.0150470  0.0002855  0.0014870  0.0075655
## [4,]  0.0556570  0.0604535  0.0426800  0.0717505  0.0629065  0.0644140
## [5,]  0.1913420  0.1836610  0.1241140  0.2792640  0.2034070  0.2400000
##           [,33]      [,34]      [,35]      [,36]      [,37]     [,38]
## [1,] -0.1941980 -0.1827910 -0.3524590 -0.1495260 -0.2037410 -0.244611
## [2,] -0.0514080 -0.0496935 -0.1063080 -0.0485735 -0.0497415 -0.059055
## [3,]  0.0105745 -0.0030790 -0.0112595  0.0201150 -0.0007415  0.004178
## [4,]  0.0669445  0.0573630  0.0811645  0.0709300  0.0633375  0.067167
## [5,]  0.2423660  0.2085660  0.3426970  0.2132040  0.2307690  0.238518
##           [,39]      [,40]      [,41]      [,42]      [,43]      [,44]
## [1,] -0.4232560 -0.2935110 -0.2507180 -0.1711130 -0.1052130 -0.1876770
## [2,] -0.1368635 -0.0816850 -0.0427605 -0.0395910 -0.0200650 -0.0495950
## [3,] -0.0233890 -0.0094260  0.0285550  0.0012735  0.0150110  0.0064115
## [4,]  0.0808720  0.0694945  0.1063085  0.0612935  0.0494915  0.0702775
## [5,]  0.3758170  0.2828900  0.3181820  0.2104300  0.1524970  0.2257380
##           [,45]      [,46]      [,47]     [,48]      [,49]     [,50]     [,51]
## [1,] -0.0847930 -0.1220180 -0.2756680 -0.317453 -0.1856870 -0.086088 -0.269336
## [2,] -0.0174175 -0.0286600 -0.0657265 -0.073220 -0.0423795 -0.017001 -0.067792
## [3,]  0.0089860  0.0028285  0.0007145  0.001399  0.0044425  0.008219 -0.009721
## [4,]  0.0292300  0.0411200  0.0903090  0.109812  0.0565920  0.032000  0.072510
## [5,]  0.0959410  0.1329330  0.3191490  0.381988  0.1811180  0.097074  0.224761
##           [,52]      [,53]      [,54]      [,55]      [,56]      [,57]
## [1,] -0.1433090 -0.1945950 -0.3193280 -0.1399170 -0.1175530 -0.0670330
## [2,] -0.0361350 -0.0524310 -0.0998510 -0.0285750 -0.0255630 -0.0105525
```

```
## [3,]  0.0006640  0.0034185 -0.0186025  0.0191920  0.0101445  0.0124075
## [4,]  0.0391465  0.0631110  0.0787890  0.0609595  0.0429515  0.0278860
## [5,]  0.1479730  0.1951910  0.3273810  0.1784670  0.1324390  0.0842510
##           [,58]      [,59]      [,60]      [,61]      [,62]     [,63]
## [1,] -0.2871290 -0.0951020 -0.1707690 -0.1353890 -0.1222960 -0.128525
## [2,] -0.0948685 -0.0168585 -0.0371800 -0.0267925 -0.0277905 -0.018638
## [3,] -0.0048395  0.0119810  0.0062355  0.0212180  0.0150410  0.013913
## [4,]  0.0834360  0.0485530  0.0529635  0.0470845  0.0490575  0.056468
## [5,]  0.3417430  0.1411380  0.1752990  0.1478820  0.1569500  0.138029
##           [,64]     [,65]      [,66]      [,67]     [,68]     [,69]      [,70]
## [1,] -0.1629770 -0.121439 -0.3906810 -0.2101700 -0.271323 -0.135757 -0.1419270
## [2,] -0.0385075 -0.025682 -0.1122500 -0.0590605 -0.066713 -0.034847 -0.0307820
## [3,]  0.0122730  0.020687 -0.0406000  0.0058575  0.000746  0.011056  0.0061280
## [4,]  0.0613515  0.051528  0.0809815  0.0709050  0.071885  0.044569  0.0490215
## [5,]  0.1945950  0.159646  0.3583810  0.2225720  0.259009  0.143943  0.1440540
##          [,71]      [,72]      [,73]      [,74]      [,75]      [,76]
## [1,] -0.173957 -0.2328570 -0.1722450 -0.2372160 -0.3700790 -0.1611840
## [2,] -0.038913 -0.0611200 -0.0364305 -0.0640385 -0.0874545 -0.0432405
## [3,]  0.006582  0.0088370  0.0104255 -0.0005805  0.0029220  0.0010935
## [4,]  0.056520  0.0724355  0.0542620  0.0661190  0.1041680  0.0664205
## [5,]  0.185732  0.2477300  0.1900270  0.2398700  0.3823530  0.1988390
##           [,77]      [,78]     [,79]     [,80]      [,81]      [,82]      [,83]
## [1,] -0.1648040 -0.2133040 -0.185942 -0.248322 -0.3535010 -0.1168780 -0.2378340
## [2,] -0.0237575 -0.0429950 -0.054358 -0.091162 -0.0946200 -0.0222140 -0.0650170
## [3,]  0.0216275  0.0232215  0.004844 -0.019231 -0.0202385  0.0093555  0.0050790
## [4,]  0.0727550  0.0827430  0.060820  0.067733  0.0828515  0.0496335  0.0599255
## [5,]  0.2149080  0.2581910  0.224921  0.267946  0.3234900  0.1490910  0.2200570
##           [,84]     [,85]      [,86]      [,87]      [,88]      [,89]
## [1,] -0.1365580 -0.253287 -0.2346750 -0.3535790 -0.2248320 -0.3508310
## [2,] -0.0253415 -0.053173 -0.0510255 -0.0970875 -0.0622770 -0.0859295
## [3,]  0.0161905  0.007815  0.0121375 -0.0200135  0.0018895  0.0037355
## [4,]  0.0621840  0.094120  0.0734345  0.0958645  0.0572555  0.0907245
## [5,]  0.1923300  0.251596  0.2241150  0.3700000  0.2356000  0.3533760
##           [,90]      [,91]      [,92]      [,93]      [,94]      [,95]
## [1,] -0.1657780 -0.3421050 -0.3706730 -0.2318610 -0.3697950 -0.1853740
## [2,] -0.0431290 -0.1064390 -0.1065400 -0.0449655 -0.0987565 -0.0414095
## [3,]  0.0074655 -0.0195960 -0.0213945  0.0130035 -0.0119765  0.0046030
## [4,]  0.0484190  0.0692995  0.0829540  0.0810980  0.0842195  0.0575400
## [5,]  0.1838400  0.3312300  0.2994350  0.2404420  0.3525180  0.1917950
##           [,96]      [,97]      [,98]      [,99]     [,100]
## [1,] -0.1726650 -0.1614080 -0.2746480 -0.1222590 -0.0708880
## [2,] -0.0408075 -0.0321905 -0.0878820 -0.0291965 -0.0136560
## [3,]  0.0098585  0.0107040 -0.0156535  0.0062905  0.0047475
## [4,]  0.0586425  0.0603320  0.0449555  0.0454040  0.0249650
## [5,]  0.1932190  0.1824320  0.2431040  0.1424330  0.0700000
##
## $n
##   [1] 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180
##  [19] 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180
##  [37] 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180
##  [55] 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180
##  [73] 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180 180
```

```
##  [91] 180 180 180 180 180 180 180 180 180 180
##
## $conf
##              [,1]       [,2]         [,3]         [,4]       [,5]         [,6]
## [1,] -0.008210076 0.01155599 -0.001494868 -0.006701604 0.01320971 -0.000155897
## [2,]  0.023849076 0.02459401  0.016726868  0.025820604 0.04399829  0.022191897
##             [,7]         [,8]         [,9]        [,10]      [,11]        [,12]
## [1,] -0.01374162 -0.006776775 -0.003621696 -0.009149828 0.01461055 -0.001981785
## [2,]  0.03015162  0.021083775  0.012226696  0.030033828 0.03805145  0.018924785
##            [,13]      [,14]      [,15]       [,16]        [,17]       [,18]
## [1,] 0.002208393 0.01004525 0.00661501 0.002097891 -0.004622308 0.001576354
## [2,] 0.011893607 0.02397075 0.02833299 0.033002109  0.023652308 0.007851646
##             [,19]        [,20]       [,21]       [,22]       [,23]       [,24]
## [1,] -0.0008467263 -0.020695669 -0.02638549 -0.00955612 -0.01803658 0.004639043
## [2,]  0.0192457263  0.008175669  0.02467549  0.01090812  0.01902058 0.025360957
##            [,25]       [,26]        [,27]        [,28]       [,29]      [,30]
## [1,] -0.01041965 -0.03365346 -0.002398755 -0.002074057 0.008020654 -0.0164313
## [2,]  0.03016165  0.02392646  0.020248755  0.021461057 0.022073346  0.0170023
##           [,31]        [,32]       [,33]        [,34]       [,35]       [,36]
## [1,] -0.0118167 -0.007742758 -0.00336343 -0.015686642 -0.03333743 0.006041521
## [2,]  0.0147907  0.022873758  0.02451243  0.009528642  0.01081843 0.034188479
##            [,37]       [,38]        [,39]        [,40]     [,41]       [,42]
## [1,] -0.01405839 -0.01068669 -0.049030893 -0.027229842 0.0109997 -0.01060729
## [2,]  0.01257539  0.01904269  0.002252893  0.008377842 0.0461103  0.01315429
##            [,43]        [,44]       [,45]        [,46]       [,47]       [,48]
## [1,] 0.006819592 -0.007705434 0.003492499 -0.005389229 -0.01766122 -0.02015599
## [2,] 0.023202408  0.020528434 0.014479501  0.011046229  0.01909022  0.02295399
##             [,49]       [,50]       [,51]       [,52]       [,53]        [,54]
## [1,] -0.007213002 0.002448336 -0.02624384 -0.00820162 -0.01018845 -0.039640262
## [2,]  0.016098002 0.013989664  0.00680184  0.00952962  0.01702545  0.002435262
##            [,55]       [,56]       [,57]       [,58]       [,59]        [,60]
## [1,] 0.008647858 0.002075804 0.007880742 -0.02583775 0.004277733 -0.004380362
## [2,] 0.029736142 0.018213196 0.016934258  0.01615875 0.019684267  0.016851362
##           [,61]       [,62]       [,63]        [,64]      [,65]       [,66]
## [1,] 0.01251778 0.005990899 0.005068048 0.0005129804 0.01159427 -0.06335615
## [2,] 0.02991822 0.024091101 0.022757952 0.0240330196 0.02977973 -0.01784385
##             [,67]       [,68]       [,69]        [,70]        [,71]       [,72]
## [1,] -0.009448049 -0.01557617 0.001703476 -0.003270159 -0.004656786 -0.00689133
## [2,]  0.021163049  0.01706817 0.020408524  0.015526159  0.017820786  0.02456533
##              [,73]       [,74]       [,75]       [,76]      [,77]       [,78]
## [1,] -0.0002550153 -0.01590866 -0.01964466 -0.01182086 0.01026159 0.008413808
## [2,]  0.0211060153  0.01474766  0.02548866  0.01400786 0.03299341 0.038029192
##             [,79]         [,80]         [,81]        [,82]        [,83]
## [1,] -0.008720081 -0.0379434678 -0.0411386525 0.0008942896 -0.009635009
## [2,]  0.018408081 -0.0005185322  0.0006616525 0.0178167104  0.019793009
##           [,84]        [,85]        [,86]        [,87]       [,88]       [,89]
## [1,] 0.00588295 -0.009531144 -0.002519687 -0.042736733 -0.01218739 -0.01706838
## [2,] 0.02649805  0.025161144  0.026794687  0.002709733  0.01596639  0.02453938
##             [,90]        [,91]         [,92]        [,93]        [,94]
## [1,] -0.003315764 -0.040292064 -0.0437104972 -0.001842525 -0.033524897
## [2,]  0.018246764  0.001100064  0.0009214972  0.027849525  0.009571897
##             [,95]        [,96]         [,97]         [,98]        [,99]
```

```
## [1,] -0.007049911 -0.001853353 -0.0001920276 -3.129727e-02 -0.002494921
## [2,]  0.016255911  0.021570353  0.0216000276 -9.726189e-06  0.015075921
##            [,100]
## [1,] 0.0001992498
## [2,] 0.0092957502
##
## $out
##   [1]  0.671642  0.346154 -0.279148  0.418349 -0.285906  0.649652 -0.457986
##   [8]  0.295641 -0.269986  0.435201 -0.298046 -0.128376  0.159788 -0.116705
##  [15] -0.307571 -0.353123 -0.327228  0.207156 -0.105701 -0.288750  0.552910
##  [22]  0.262351  0.127886 -0.116686 -0.107191 -0.102173 -0.220083 -0.185047
##  [29] -0.144416 -0.152071 -0.439859  0.352860  0.293574  0.287273 -0.240765
##  [36] -0.308418 -0.376947  0.310457 -0.293433 -0.353712 -0.319820 -0.238095
##  [43]  0.241302  0.296642 -0.301429  0.320792  0.343026 -0.301196 -0.418966
##  [50] -0.332344  0.213289 -0.196128  0.269044 -0.271899  0.441103 -0.640592
##  [57] -0.741176 -0.441176  0.619433  0.433735  0.429630  0.395161  0.393939
##  [64]  0.286832 -0.257726 -0.267969  0.475336 -0.230435  0.362994  0.200000
##  [71]  0.302222  0.172337 -0.348584  0.371441 -0.358925  0.354248 -0.269643
##  [78] -0.325057  0.280838  0.278635  0.216798  0.193649 -0.339112  0.199696
##  [85] -0.077364  0.128161 -0.086437 -0.225574  0.193502  0.114065  0.087500
##  [92]  0.164802 -0.129427 -0.112424 -0.120468 -0.200268 -0.583333 -0.257333
##  [99]  0.394470 -0.362718  0.518402  0.381925 -0.232739 -0.199585  0.297489
## [106] -0.281604  0.344231  0.339869 -0.244131 -0.247407 -0.227208  0.335223
## [113]  0.346592  0.354443  0.296983 -0.238028  0.283921 -0.118700  0.092277
## [120]  0.095453  0.101505 -0.062092 -0.254935  0.187549 -0.164769  0.179618
## [127]  0.267962 -0.436159 -0.275484 -0.312733  0.370079 -0.441441  0.467742
## [134]  0.371428  0.513347 -0.292308  0.686682  0.409722  0.238095  0.291209
## [141]  0.391941 -0.256484 -0.452208 -0.538462  0.960557  0.463612  0.498458
## [148] -0.540111 -0.496753 -0.490323 -0.449036  1.233334 -0.649832  0.453252
## [155] -0.446222 -0.228025 -0.186555  0.288430 -0.354155 -0.200180  0.285059
## [162] -0.170498  0.353933 -0.359629  0.500000 -0.328255 -0.302564  0.466667
## [169]  0.372414 -0.396610 -0.164021 -0.388168  0.580699  0.622222  0.711429
## [176]  0.851171  0.240318 -0.189905 -0.417422 -0.605263  1.339394  0.948819
## [183] -0.222222 -0.243015  0.236769  0.261502 -0.191703 -0.250279  0.333728
## [190]  0.238632  0.237850 -0.129983 -0.125952 -0.134949  0.250983 -0.281943
## [197] -0.206863  0.370435 -0.152552 -0.283657  0.163418  0.134347 -0.169566
## [204]  0.189324  0.175717  0.175036  0.147718 -0.116176 -0.113099  0.525939
## [211]  0.293509  0.365112 -0.402161  0.380153  0.472952  0.311997  0.590059
## [218] -0.442017 -0.295181  1.016239 -0.368242  0.336532 -0.318885 -0.277284
## [225]  0.370370  0.275472 -0.380734 -0.468992  0.324074 -0.422727  0.692913
## [232]  0.316279  0.507772  0.357388 -0.278846  0.291005 -0.231366 -0.405657
## [239]  0.279745  0.250692 -0.277748 -0.233690  0.244648 -0.248951 -0.220779
## [246]  0.676796 -0.419204 -0.227954  0.344127 -0.267435 -0.223900  0.250899
## [253]  0.250212  0.243171 -0.283742 -0.404355  0.303694 -0.613301 -0.653465
## [260]  1.811429  0.903688  0.454639  0.263233  0.278313  0.267606  0.291866
## [267]  0.269436  0.442786  0.509091  0.461923 -0.685000  0.500385  2.051649
## [274]  1.127451  2.857143  0.490000  0.508043  0.494924 -0.696095  0.415094
## [281]  0.482143  0.342352  0.318628  0.381526 -0.277190 -0.388889  0.430322
## [288]  0.553247  0.385331  0.218506 -0.296845  0.250000  0.265711 -0.137168
## [295]  0.164083 -0.148049 -0.455776  0.318439  0.109078 -0.119883 -0.154362
## [302] -0.208835  0.180412  0.219731 -0.095588  0.142857  0.101639  0.121795
## [309] -0.146552  0.194000  0.160947  0.148639  0.241438  0.366906 -0.423640
```

```
## [316]  0.366359  0.345946  0.560000  0.386852  0.538462  0.819517  0.433000
## [323] -0.264059 -0.316722  0.254457 -0.208568 -0.233239  0.303665  0.233645
## [330] -0.195751 -0.108159 -0.246253 -0.116046  0.199352  0.109890  0.136187
## [337]  0.126728  0.116803 -0.137371  0.114900  0.294581  0.536474 -0.326716
## [344] -0.414444  0.499205  0.319194  0.378617 -0.289847 -0.393184  0.335616
## [351] -0.340000  0.354430  0.429907 -0.229560  0.178213  0.288120  0.165721
## [358]  0.294118 -0.286364  0.271111  0.496815  0.237762  0.416667  1.386029
## [365]  0.304207  0.240876  0.258487  0.244718 -0.398148  0.464286  0.375394
## [372]  0.482036  0.582766  0.419947  0.388539  0.195370 -0.325509 -0.181426
## [379]  0.268219  0.333201 -0.191943 -0.168521 -0.145250  0.151795  0.157814
## [386]  0.177332 -0.136287 -0.157515 -0.218789  0.188500 -0.178612 -0.129831
## [393]  0.151198 -0.071823 -0.195078 -0.143939 -0.081545  0.108169  0.087219
## [400] -0.077410  0.088624 -0.108952  0.126141 -0.763636  0.475524  0.703297
## [407]  0.395062 -0.399038  0.532770  0.485781  0.393946 -0.160977 -0.209720
## [414]  0.179711 -0.118081  0.197528  0.151630 -0.130693 -0.177580 -0.436055
## [421] -0.239122 -0.218551 -0.185366  0.293694  0.190557  0.231940 -0.194111
## [428]  0.184059 -0.178817 -0.216523 -0.140948  0.166793  0.166443 -0.189847
## [435]  0.191631 -0.164213  0.237914 -0.210682 -0.146547 -0.270350 -0.188509
## [442]  0.192650  0.219754 -0.157527 -0.196856  0.236756 -0.306174 -0.145232
## [449] -0.458057 -0.283396  0.360047 -0.311699 -0.261166  0.209068  0.345139
## [456] -0.235756  0.227216  0.177793  1.761628  0.426049 -0.444295  1.321428
## [463]  0.375839  0.437870  0.401274  0.448276  0.638800  0.419355  0.795918
## [470]  0.626437  0.554688 -0.280390  0.324850  0.332071 -0.359878  0.346251
## [477] -0.455859  0.854791  0.662531  0.310981  0.428874  0.341696 -0.274618
## [484]  0.602484 -0.465852  0.308023 -0.181545 -0.158657 -0.279714  0.232948
## [491] -0.337904  0.176056 -0.166723 -0.178606 -0.252542  0.212245  0.333010
## [498] -0.207727  0.328863 -0.240720  0.303441  0.211486  0.329365  0.527706
## [505]  0.299390 -0.350190 -0.748231  0.943195 -0.198538 -0.532675  0.286885
## [512]  0.461147  0.254577  0.389217  0.441480 -0.178480  0.192689 -0.176642
## [519] -0.560976  0.315108 -0.308087  0.357005  0.414270 -0.411546  0.382911
## [526]  0.263451  0.314079 -0.284091  0.743590  1.181818  0.475410  0.572222
## [533]  0.563830  0.423810  0.544248 -0.227241  0.253042  0.249127  0.248821
## [540] -0.219485 -0.182085 -0.358565 -0.383253  0.364461 -0.304809  0.541342
## [547] -0.254018  0.272601 -0.343700 -0.305147 -0.282263  0.335644  0.654966
## [554] -0.234206  0.394201 -0.260841  0.794258  0.648477  0.387268  0.672921
## [561] -0.501959 -0.402804  0.355556  0.697452 -0.416529  0.482051  1.076923
## [568]  0.975610 -0.353909  0.417391 -0.407804 -0.438903  0.366834  0.395190
## [575]  0.592592  1.074102  0.753472 -0.192093  0.167237 -0.135424 -0.237794
## [582]  0.157528 -0.279070  0.462274 -0.375000 -0.293955  0.613605  1.701518
## [589]  0.290237  0.347307  0.305874  0.409148  0.371975  0.300642 -0.200071
## [596]  0.251958 -0.293814 -0.215294 -0.245813  0.419054 -0.203553 -0.244264
## [603]  0.196607 -0.166866  0.349650  0.377391  0.373089  0.330171  0.285023
## [610] -0.262858 -0.389009  0.307046 -0.312148 -0.248500  1.464789  1.029412
## [617]  0.433915  0.690184 -0.288573 -0.412500 -0.382199 -0.241830  0.658333
## [624]  0.241667 -0.454545  0.252587  0.409863 -0.488950 -0.437372 -0.587591
## [631]  1.212389  0.528000  0.593750  0.476635  0.232342  0.270567  0.201171
## [638] -0.506453 -0.317257  0.246787  0.206920 -0.263761  0.345998  0.252097
## [645] -0.427921 -0.458333  0.555556  2.081395  0.358407  0.589577  0.804182
## [652]  0.360638  0.348958  0.372368  0.344498  1.858333 -0.693478  0.859417
## [659]  0.520000  0.486842  0.905229  0.406690  1.191781  0.460645 -0.738924
## [666] -0.435233  0.370591  0.480392  0.549669  0.370748  4.148734 -0.717882
## [673]  0.547692  0.512281  1.243620  0.366211  0.277040 -0.301898  0.457005
```

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## [680]  0.409233 -0.474499  0.390963  0.380000 -0.293238  0.476081  0.214555
## [687]  0.229339 -0.460842 -0.234646  0.268874  0.242424  0.236121  0.229350
## [694]  0.285070  0.208365 -0.250000 -0.181049 -0.181250 -0.346282  0.210697
## [701]  0.228731  0.203666 -0.324357  0.501812 -0.426025  0.476923 -0.309322
## [708] -0.293768 -0.312925  0.481013  0.769231  0.274390  0.273256  0.279816
## [715]  0.401961  0.333333  0.612500 -0.165335 -0.184476 -0.164476 -0.145032
## [722] -0.081721  0.085375 -0.218750 -0.250534  0.119488  0.127907  0.130153
## [729]  0.102953 -0.085827 -0.075599  0.088805  0.092233
##
## $group
##   [1]   1   1   1   1   1   1   1   1   1   1   1   2   2   2   2   2   2   2
##  [19]   2   2   2   2   2   2   2   2   3   3   3   3   4   4   4   5   5   5
##  [37]   5   5   5   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6
##  [55]   7   7   7   7   7   7   7   7   7   8   8   8   8   8   8   9   9   9
##  [73]  10  10  10  10  11  11  11  11  11  12  12  12  13  13  13  13  13  13
##  [91]  13  13  13  14  14  15  15  15  15  15  15  15  15  15  15  16  16  16  17
## [109]  17  17  17  17  17  17  17  17  17  18  18  18  18  18  19  19  19  19
## [127]  20  20  20  20  20  20  20  20  20  20  20  20  20  20  20  20  21  21
## [145]  21  21  21  21  21  21  21  21  21  21  21  22  22  22  22  22  22  22
## [163]  23  23  23  23  23  23  23  23  24  25  25  25  26  26  27  27  27  27
## [181]  27  27  27  27  27  28  28  28  28  28  28  29  29  29  29  29  29  29
## [199]  29  29  29  29  29  29  29  29  29  29  29  30  30  30  30  30  30  31
## [217]  31  31  31  31  31  31  31  31  31  32  32  32  32  32  32  32  32  32
## [235]  32  32  33  33  33  33  33  33  34  34  34  35  35  36  36  37  37  37
## [253]  37  37  37  38  38  38  38  38  38  38  38  38  38  38  38  38  39  39  39
## [271]  39  39  39  39  39  39  39  39  39  39  39  40  40  40  41  41  41  41
## [289]  41  42  42  42  42  43  43  43  44  44  45  45  45  45  45  45  45  45
## [307]  45  45  46  46  46  46  46  47  47  47  47  48  48  48  48  48  49  49
## [325]  49  49  49  49  49  50  50  50  50  50  50  50  50  50  50  50  51  51
## [343]  51  51  51  51  51  51  51  51  51  51  51  52  52  52  52  53  53  53
## [361]  53  53  53  53  53  53  53  53  54  54  54  54  54  54  54  55  55  55
## [379]  55  55  55  56  56  56  56  56  56  56  56  56  56  56  56  56  57  57  57
## [397]  57  57  57  57  57  57  57  58  58  58  58  58  58  58  58  58  59  59  59
## [415]  59  59  59  59  60  60  60  60  60  60  60  60  60  60  61  61  61  61  61
## [433]  61  61  61  61  61  62  62  63  63  63  63  63  64  64  64  65  65  65
## [451]  65  65  65  65  65  65  65  65  66  66  66  66  66  66  66  66  66  66
## [469]  66  66  66  67  67  67  68  68  68  68  68  68  68  68  68  68  68  68
## [487]  69  69  69  69  70  70  70  70  71  71  71  71  71  71  71  71  72  72
## [505]  72  73  73  73  73  73  73  73  73  73  73  73  73  73  73  74  74  74  74
## [523]  74  74  74  74  74  74  75  75  75  75  75  75  75  76  76  76  76  76
## [541]  77  77  77  78  78  78  78  78  79  79  79  79  79  79  79  79  80  80
## [559]  80  80  80  80  80  80  80  80  80  80  80  81  81  81  81  81  81  81
## [577]  81  82  82  82  82  82  83  83  83  83  83  83  83  83  83  83  83  83
## [595]  84  84  84  84  84  84  84  84  84  84  85  85  85  85  86  86  86  86
## [613]  86  86  87  87  87  87  88  88  88  88  88  88  88  88  89  89  89  89
## [631]  89  89  89  89  90  90  90  90  90  90  90  90  90  90  91  91  91  91
## [649]  91  91  91  91  91  91  91  92  92  92  92  92  92  92  92  92  92  92
## [667]  92  92  92  92  92  93  93  93  93  93  93  93  94  94  94  94  94  95
## [685]  95  95  95  95  95  95  95  95  95  96  96  96  97  97  97  97  97  97
## [703]  98  98  98  98  98  98  98  98  98  98  98  98  98  98  98  99  99  99
## [721]  99 100 100 100 100 100 100 100 100 100 100 100 100
##
```

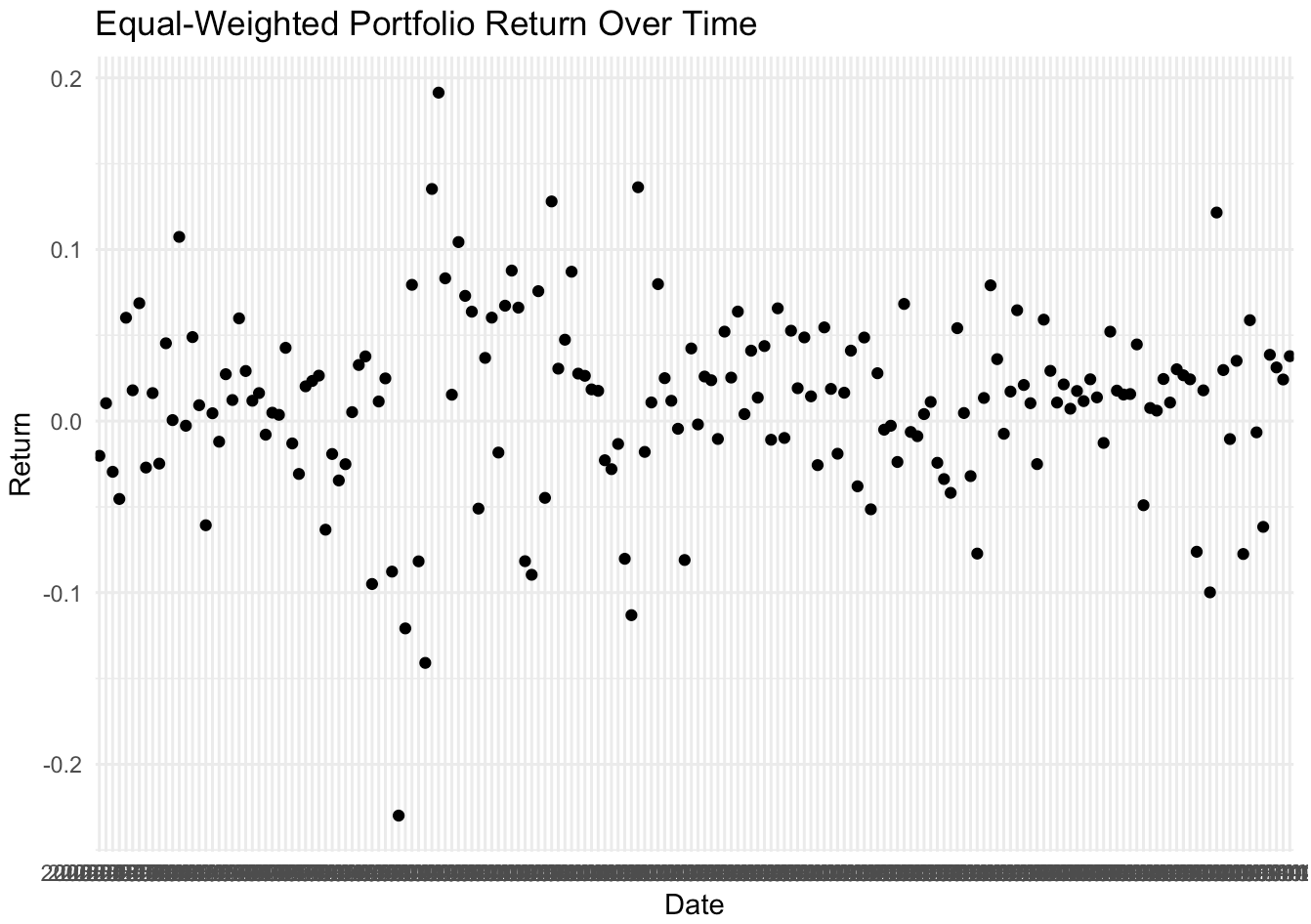
```
## $names
##   [1] "I87179" "H91626" "H58246" "E14795" "D87056" "E74740" "E88417" "G89455"
##   [9] "H76119" "E85372" "I75510" "H88903" "H89629" "F44206" "H89258" "H89056"
##  [17] "H14752" "H75075" "H89188" "H51925" "B79444" "H66616" "E86474" "G66181"
##  [25] "B88490" "E65008" "H86965" "G29612" "H21259" "E80233" "H72980" "H16505"
##  [33] "H85931" "F63773" "G90371" "E76868" "G59010" "H11208" "D89935" "B79237"
##  [41] "E86580" "E37568" "I36003" "F90227" "H85942" "H89211" "E81621" "E10547"
##  [49] "E77605" "H89551" "D88436" "I53225" "I76948" "F87430" "H64194" "F89858"
##  [57] "H54973" "B90433" "D29946" "H64995" "E25953" "H88893" "H80183" "I89307"
##  [65] "H85592" "E83149" "E81044" "D90423" "H52425" "H83223" "H11293" "G90012"
##  [73] "H82775" "E87204" "H87837" "F10443" "E32678" "I84788" "E70033" "E37381"
##  [81] "D86728" "D40272" "H76732" "H69649" "F85237" "B23819" "E88260" "H88865"
##  [89] "E54704" "H88944" "F87121" "D79122" "G89327" "I33209" "H76504" "F83143"
##  [97] "E11547" "E87825" "H79238" "H90510"
```

```
# Step 1: Calculate the mean return across all stocks for each time period

Portfolio$Portfolio_Return <- rowMeans(Portfolio[ , sapply(Portfolio, is.numeric)], na.r
m = TRUE)

# Step 3: Create a dataframe for ggplot
portfolio_data <- Portfolio %>%
  rownames_to_column("Date")

# Step 4: Plot the time series with ggplot
ggplot(portfolio_data, aes(x = Date, y = Portfolio_Return)) +
  geom_point() +
  labs(title = "Equal-Weighted Portfolio Return Over Time", x = "Date", y = "Return") +
  theme_minimal()
```




```
PortfolioTr <-
  Portfolio %>%
  t()
PortfolioTr <- as.data.frame(PortfolioTr)

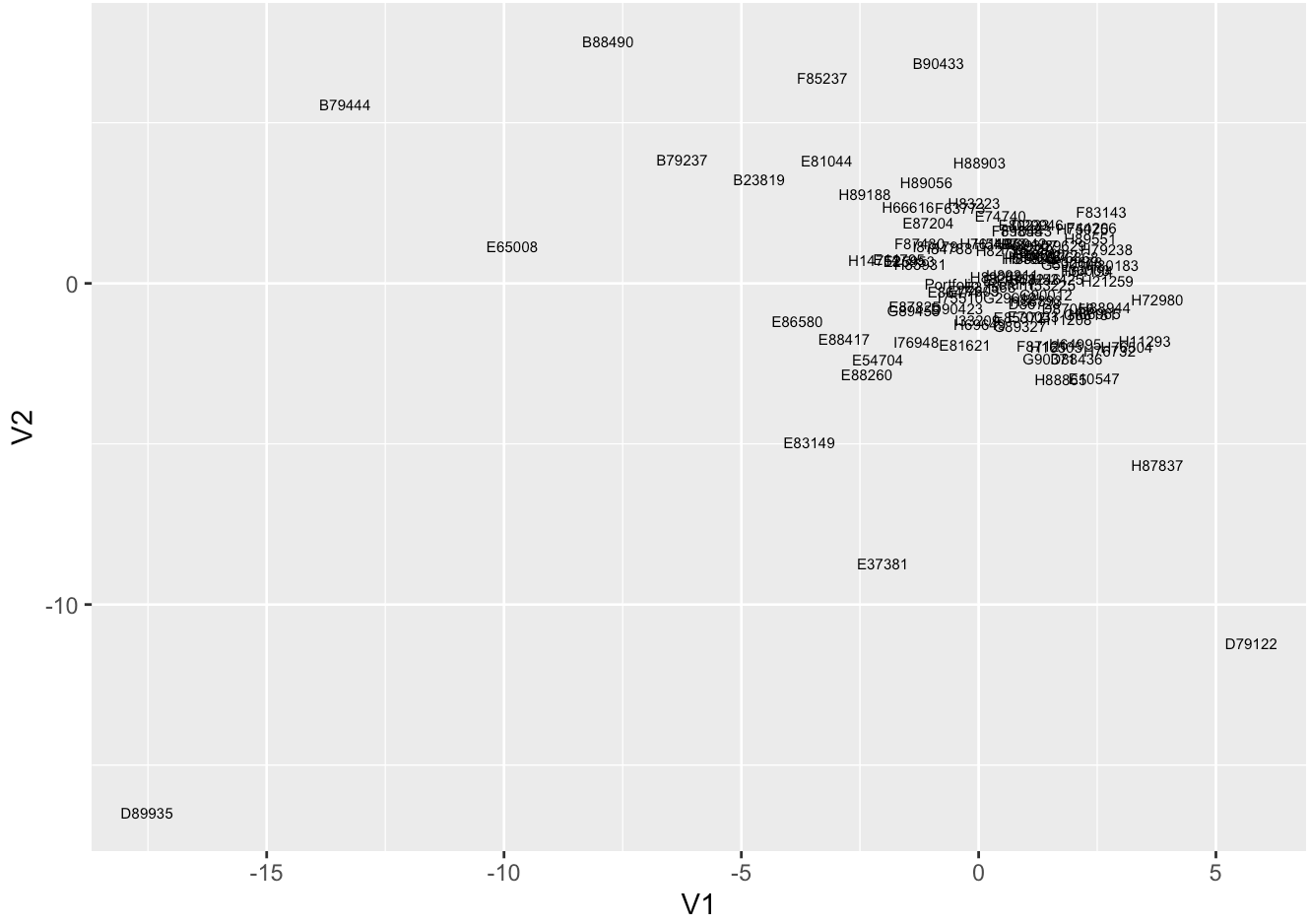
PortfolioTr %>%
  scale() %>%
  dist() -> dd

rownames(PortfolioTr)->attributes(dd)$Labels

cmds<-cmdscale(dd,eig = T)

cmds$points %>%
  as.data.frame()%>%
  rownames_to_column(var = 'Stocks')->df

ggplot(df,aes(x=V1,y=V2,label=`Stocks`))+
  geom_text(size=2)
```

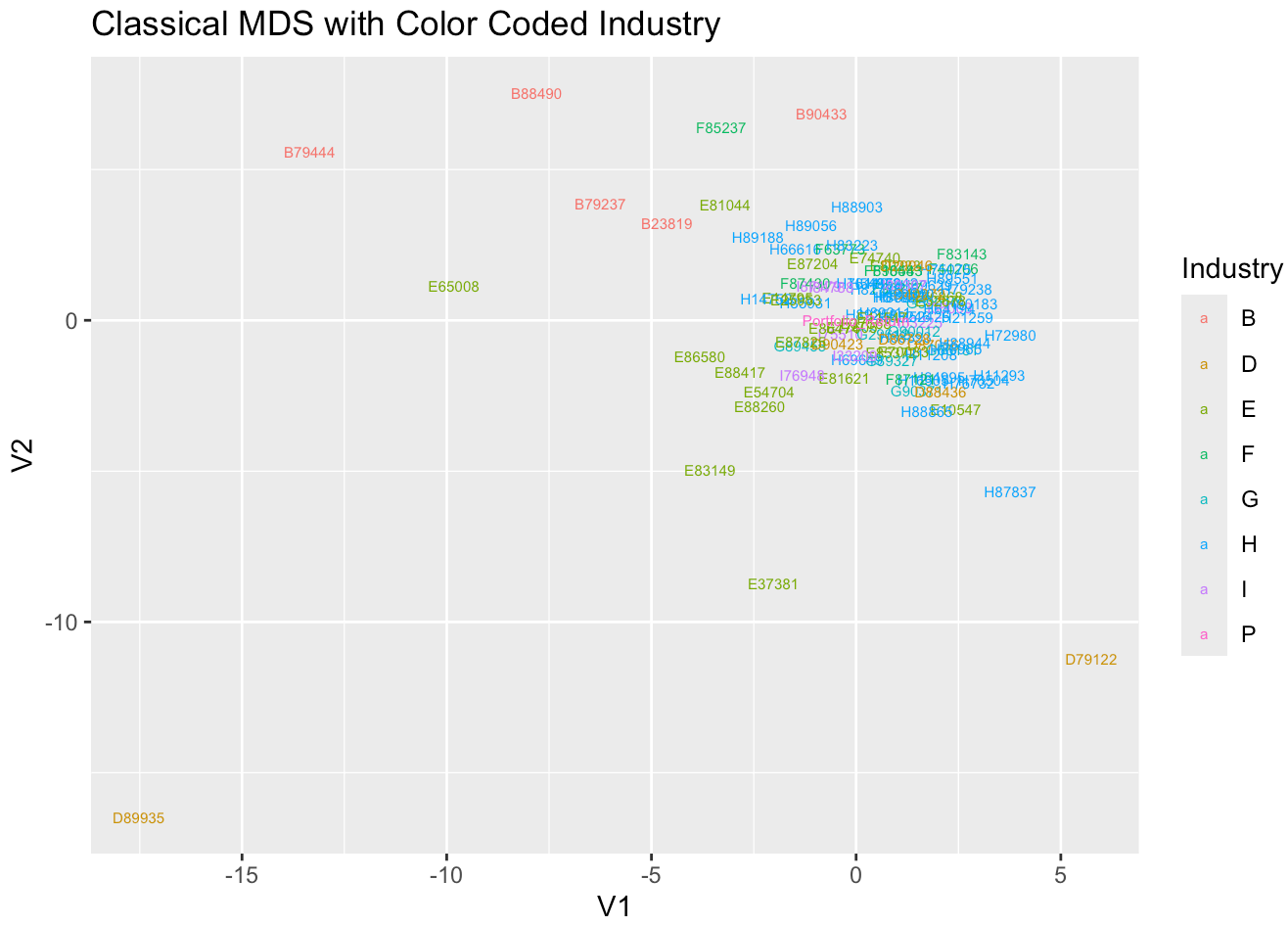



```
min(cmds$eig)
```

```
## [1] -4.039399e-13
```

```
PortfolioTr$First_Letter <- substr(rownames(PortfolioTr), 1, 1)

df<-add_column(df,Industry=PortfolioTr$First_Letter)
ggplot(df,aes(x=V1,y=V2,col=Industry,label=`Stocks`))+
  geom_text(size=2) +
  labs(title = "Classical MDS with Color Coded Industry")
```

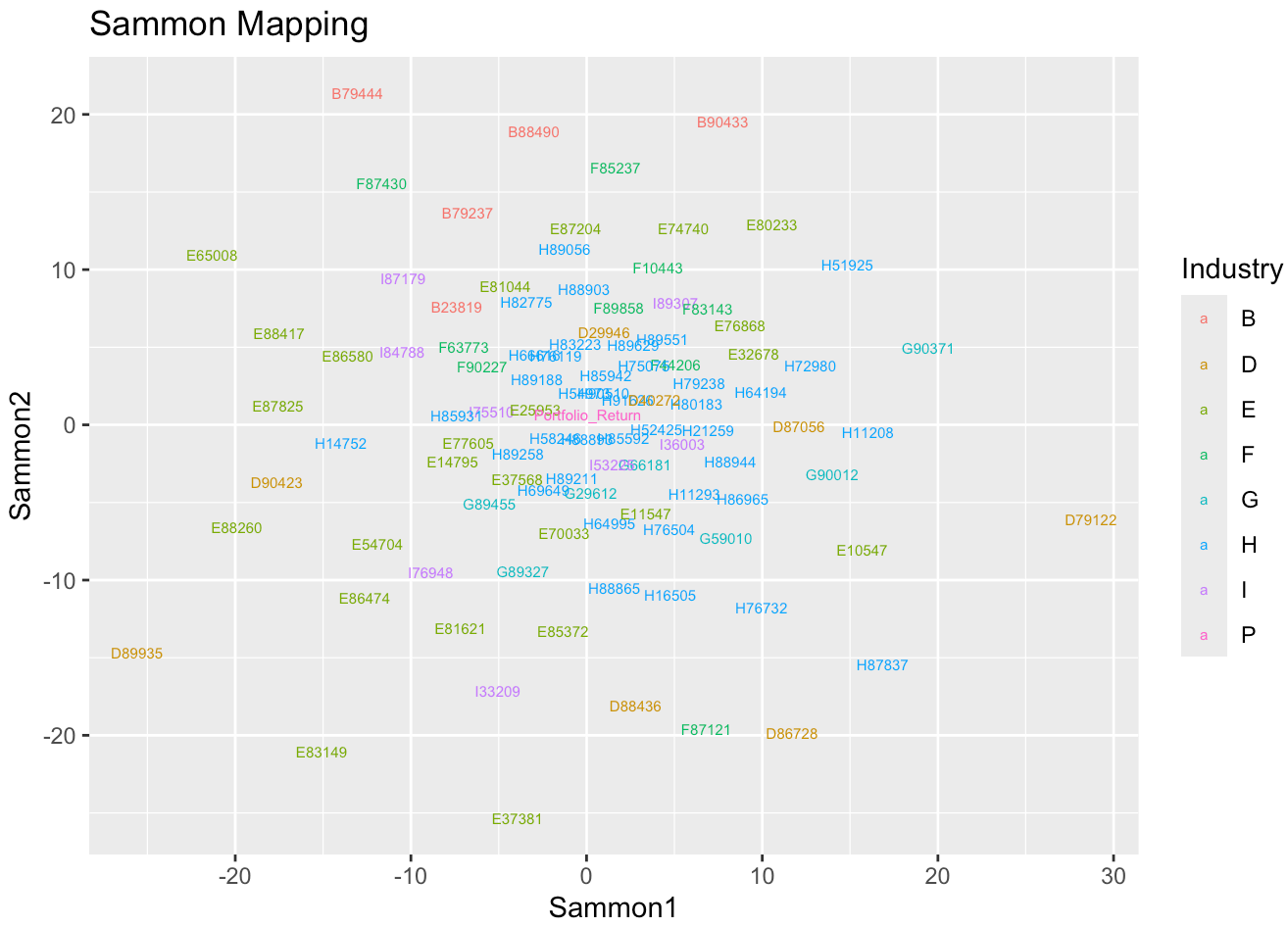



```
smds<-sammon(dd)
```

```
## Initial stress        : 0.58628
## stress after  10 iters: 0.36500, magic = 0.004
## stress after  20 iters: 0.21950, magic = 0.043
## stress after  30 iters: 0.16766, magic = 0.020
## stress after  40 iters: 0.12276, magic = 0.226
## stress after  50 iters: 0.10874, magic = 0.021
## stress after  60 iters: 0.10339, magic = 0.241
## stress after  70 iters: 0.09687, magic = 0.500
## stress after  80 iters: 0.09573, magic = 0.500
## stress after  90 iters: 0.09520, magic = 0.500
## stress after 100 iters: 0.09487, magic = 0.500
```

```
df<-add_column(df,Sammon1=smds$points[,1],
  Sammon2=smds$points[,2])

ggplot(df,aes(x=Sammon1,y=Sammon2,col=Industry,label=`Stocks`))+
  geom_text(size=2) +
  labs(title = "Sammon Mapping")
```


Sammon Mapping
Sammon1
Sammon2
Industry
B
D
E
F
G
H
I
P
Portfolio_Return