First Name: _____ Last Name: _____

Student-No: _____ Section: _____

| Grade: |
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## Indefinite Integrals

1. [9 marks] Each part is worth 3 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $\int x^2\sqrt{8-x^3}\,dx$ for $x < 2$.

   Answer:



   (b) Calculate the indefinite integral $\int x\sqrt{x-1}\,dx$ for $x > 1$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int \ln(1+x^2)\,dx$.

Answer:

## Definite Integrals

2. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $\int_0^{\pi} \sin^3(x)\, dx$.

Answer:



(b) Calculate $\int_{-1}^{1} \left( x^2 e^{-x^3} + x^5 \cos(x) \right) dx$.

Answer:

(c) (A Little Harder): Calculate $\int_1^e (\ln x)^2 \, dx$.

| Answer: |
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| |

## Riemann Sum, FTC, and Volumes

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

   (a) Calculate the infinite sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i}{n^2\,(4 + i^2/n^2)}$$

   by first writing it as a definite integral. Then, **evaluate this integral**.

   Answer:



   (b) Define $F(x)$ and $g(x)$ by $F(x) = \int_1^x \ln t\, dt$ and $g(x) = x\, F(x^2)$ for $x > 1$. Calculate $g'(e)$.

   Answer:

(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $y = x^2$ and $y = 6x - 5$ about the horizontal line $y = -2$. **Do not evaluate the integral**.

| Answer: |
| --- |
| |

4. (a) [2 marks] Plot the finite area enclosed by $y^2 = 6 + x$ and $2y = x - 2$.



(b) [4 marks] Write a definite integral with specific limits of integration that determines this area. **Do not evaluate the integral**.

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5. A solid has as its base the region in the $xy$-plane between $y = 1 - x^2/16$ and the $x$-axis. The cross-sections of the solid perpendicular to the $x$-axis are isosceles right triangles (i.e. $45 - 45 - 90$ triangles) with the longest side (i.e. the hypoteneuse) in the base.

   (a) [4 marks] Write a definite integral that determines the volume of the solid.



   (b) [2 marks] **Evaluate the integral** to find the volume of the solid.

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