Math 111C HW6

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Question 1 *Let E be a splitting field of $f(x) \in F[x]$ and $G = Aut(E/F)$. Prove that:*

(a) If $f(x)$ is irreducible, then G acts transitively on the set of all roots of $f(x)$, i.e. if α, β are two roots of $f(x)$ in E, there exists $\sigma \in G$ with $\sigma(\alpha) = \beta$.

(b) If $f(x)$ has no repeated roots and G acts transitively on the roots, then $f(x)$ is irreducible.

Pf:

2

Question 2 *In each part, find the degree of the extension K/F.*

(a) *Splitting field $K \subseteq \mathbb{C}$ of $f(x) = x^4 - 4$ over $F = \mathbb{Q}$.*

(b) *Splitting field $K \subseteq \mathbb{C}$ of $f(x) = x^6 - 2$ over $F = \mathbb{Q}$.*

(c) *Splitting field K of $f(x) = x^{10} - 2$ over $F = \mathbb{F}_5$.*

Pf:

3

Question 3 *Let L be the splitting field of $f(x) = x^3 + x + 1$ over $\mathbb{Q}$ contained in $\mathbb{C}$. Prove that $Aut(L/\mathbb{Q}) \cong S_3$.*

Pf:

4

Question 4 *Calculate the splitting field of* $f(x) = x^3 + x + 1 \in \mathbb{F}_2[x]$.

Pf:

5

Question 5 *Let $f(x) \in \mathbb{F}_p[x]$ be an irreducible polynomial of degree 3. Prove that $f(x)$ is irreducible over $\mathbb{F}_{p^5}$.*

Pf: