Math 111C HW4

Zih-Yu Hsieh

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Question 1 *Let F be a field and $f \in F[x]$ be an irreducible polynomial. Prove that all roots of $f(x)$ in $\overline{F}$ have the same multiplicity.*

Pf:

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Question 2 *(a) Let $\zeta_6 \in \mathbb{C}$ be a primitive 6^{th} root of unity. Find $m_{\zeta_6,\mathbb{Q}}(x)$.*

(b) Let $m, n \in \mathbb{N}$ such that $m \equiv 2 (\mod 6)$ and $n \equiv 4 (\mod 6)$. Prove that $f(x) = x^m + x^n + 1$ is not irreducible over $\mathbb{Q}$.

Pf:

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Question 3 *Prove that if F is an infinite field, then its multiplicative group F is never cyclic.*

Pf:

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Question 4 *Let K/F be a field extension and $m, n \in \mathbb{N}$. Let $\alpha, \beta \in K$ with $[F(\alpha) : F] = m$ and $[F(\beta) : F] = n$.*

(a) Show that $[F(\alpha, \beta) : F] \leq mn$.

(b) If $\gcd(m, n) = 1$, show that $[F(\alpha, \beta) : F] = mn$.

Pf:

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Question 5 *Let K be a finite field. Show that K is not algebraically closed.*

Pf: