Latex Template

Zih-Yu Hsieh

July 7, 2025

1

Question 1 *Consider a puck sliding across a frictionless merry-go-round at speed v on a trajectory which passes through the center. The merry-go-round rotates at angular velocity w and has radius R.*

(a) Write down the r, ϕ coordinates of the puck as functions of time, as observed by an observer standing next to the merry-go-round. Take $t = 0$ to be the time when the puck is at the edge of the merry-go-round, the spatial origin to be at the center of the merry-go-round, and $\phi = 0$ to be the initial angular location of the puck. Is the observer in an inertial frame?

(b) Write down the r', ϕ' coordinates of the puck as functions of time, as observed by an observer sitting on the edge of the merry-go-round. Take $t = 0$ to be the time when the puck is at the edge of the merry-go-round, the spatial origin to be at the center of the merry-go-round, and $\phi' = 0$ to be the initial angular location of the puck. Is the observer in an inertial frame?

Pf:
Fxxk you.

2

Question 2 *Consider an athlete putting a shot. Naturally, she would like to maximize the distance the shot travels. If the shot is put from a height h and with initial speed v_0, what launch angle maximizes the distance traveled? Assume that v_0 and h are independent of the launch angle, and ignore air resistance (but include gravity, if it wasn't obvious). You can (and should) assume that h is "small" and give an answer to leading non-trivial order, but you should specify what counts as "small".*

Pf:

In my original attempt, after doing massive calculation, I got to the part showing the following result:

$$\sin^2(\theta) \approx \frac{1}{4}\left(1 + \sqrt{1 - \frac{4gh}{v_0^2}}\right) \tag{1}$$

However, in my final solution, I didn't use Taylor Series approximation to get an approximated results for θ, which isn't the desired form of answer.

As a result, for $\sqrt{1-x}$, its first several derivatives is given as follow:

$$\frac{d}{dx}\sqrt{1-x} = -\frac{1}{2\sqrt{1-x}}, \quad \frac{d^2}{dx^2}\sqrt{1-x} = -\frac{1}{4}(1-x)^{3/2} \tag{2}$$

Which, with the center about $x = 0$, $\sqrt{1-x} \approx 1 - \frac{1}{2}x - \frac{1}{4}x^2$ when x is small; taking up to leading term, $\sqrt{1-x} \approx 1 - \frac{1}{2}x$. Using this as a result, we get:

$$\sin^2(\theta) \approx \frac{1}{4}\left(1 + 1 - \frac{1}{2} \cdot \frac{4gh}{v_0^2}\right) = \frac{1}{2} - \frac{gh}{2v_0^2} \tag{3}$$

$$\implies \sin(\theta) \approx \frac{\sqrt{2}}{2}\sqrt{1 - \frac{gh}{2v_0^2}} \approx \frac{\sqrt{2}}{2}\left(1 - \frac{1}{2} \cdot \frac{gh}{2v_0^2}\right) = \frac{\sqrt{2}}{2}\left(1 - \frac{gh}{4v_0^2}\right) \tag{4}$$

4

Question 3

Pf:

5

Question 4

Pf: