Phys 103 HW2 Pass2

Zih-Yu Hsieh

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Question 1

Pf:

(a)

In part (a), I misused the notation, where instead of $x(0) = A\cos(\phi) = 0$, I should've written $0 = x(0) = A\cos(\phi)$ (because we're given $x(0) = 0$, not $A\cos(\phi) = 0$).

(b)

When doing the calculation, I forgot to assume that $t_0 \in [0, \frac{2\pi}{w_1})$ is a local maximum of the function, but only assumed $x'(t_0) = 0$ (which is not sufficient for local maximum). The rest of the calculation follows.

Question 3

Pf:

(a)

In this section when calculating the power dissipated by the damping force, I should've use that the power dissipated $P_{\text{dissipated}} = -F_{\text{damp}}v = bv^2$ instead (or else it's phrasing that the power dissipated is negative, or the damping force is providing positive power).

Question 4

Pf:

(a)

First, to explain more about the period, to prove the minimality of T, suppose for $t_1, t_2 \in [-\frac{T}{2}, \frac{T}{2})$ it satisfies $f(t_1) = f(t_2)$, then since for index $i = 1, 2$, we have $-\frac{T}{2} \leq t_i < \frac{T}{2}$, hence $-\frac{1}{2} \leq \frac{t_i}{T} < \frac{1}{2}$, so $0 \leq \frac{1}{2} + \frac{t_i}{T} < 1$. Therefore, $\lfloor \frac{1}{2} + \frac{t_i}{T} \rfloor = 0$. Hence, we get the following:

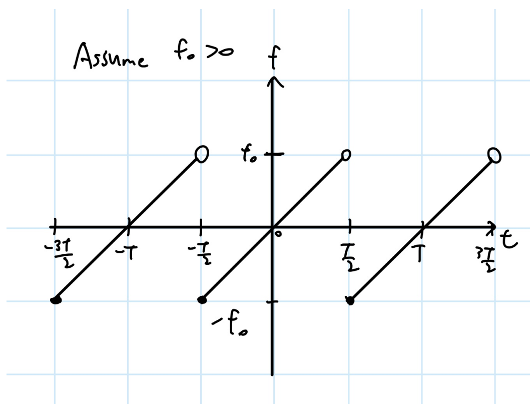
$$\frac{f_0 t_1}{T} = f_0\left(\frac{t_1}{T} + \left\lfloor \frac{1}{2} + \frac{t_1}{T} \right\rfloor\right) = f(t_1) = f(t_2) = f_0\left(\frac{t_2}{T} + \left\lfloor \frac{1}{2} + \frac{t_2}{T} \right\rfloor\right) = \frac{f_0 t_2}{T} \tag{1}$$

$$\implies t_1 = t_2 \tag{2}$$

(Note: above assumes $f_0, T \neq 0$).

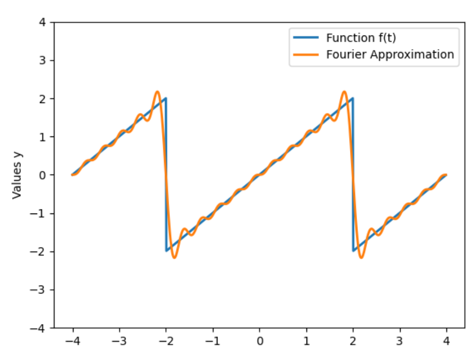
This shows that the period can't be less than T (or else there should be two distinct points in the above interval, where their function values are the same), which T is the period.

For the sketch, I think it's because the minus sign is too close, nevertheless I redrew the graph to make the minus sign more obvious:



(b)

For the graph, I forgot to add the labels for each axis. Here's the modified version: Here's the modified code:



```
import matplotlib.pyplot as plt
import numpy as np
import math

#define constant (Note: both need to be positive)
T = 4
f_0 = 4

#define original function
def f(t):
   return f_0*(t/T - math.floor(1/2+t/T))

#define fourier series approximation, with n=10
def fourier(t):
   output = 0

   for n in range(1,11):
      output += (-1)**(n+1) * f_0 / (n* math.pi) * math.sin(2*n* math.pi * t/T)

   return output

#plot function
t = np.arange(-f_0, f_0, 0.01)

Function = []
Fourier = []
for i in range(len(t)):
   Function.append(f(t[i]))
   Fourier.append(fourier(t[i]))

plt.plot(t,Function, lw=2, label='Function f(t)')
plt.plot(t,Fourier, lw=2, label = 'Fourier Approximation')
plt.ylim(-T,T)
plt.xlabel("Points t")
plt.ylabel("Values y")

plt.legend()
plt.show()
```

Question 6

Pf:

(a)

I forgot to answer the question whether it's going faster or slower eventually. This depends on the context:

- If the oscillator is undamped, then the amplitude of the system is always the same. Hence, the period of the system stays constant, which it's not slower or faster.
- If the oscillator is damped, then its maximum amplitude is constantly decreasing, so when maximum amplitude is lower, with $T = \frac{2\pi}{w_0}\left(1 + \frac{\phi_{\text{max}}^2}{16}\right)$, we get that T is decreasing. Hence, each cycle actually takes time less than 2 seconds; Since each cycle takes less time, within the same time elapsed there are more cycles happened than default, hence it records a higher time elapsed than the actual time, showing that the pendulum eventually runs faster.

(b)

For this question I should include the units for time, so I should say 2 seconds instead (and say 86400 seconds in the integrand).