

Math CS 122A HW8

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Question 1 Ahlfors Pg. 148 Problem 2:

Prove that the region obtained from a simply connected region by removing m points has the connectivity $m + 1$, and find a homology basis.

Pf:

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Question 2 Ahlfors Pg. 148 Problem 4:

Show that single-valued analytic branches of $\log z$, z^α and z^z can be defined in any simply connected region which does not contain the origin.

Pf:

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Question 3 Ahlfors Pg. 148 Problem 5:

Show that a single-valued analytic branch of $\sqrt{1-z^2}$ can be defined in any region such that the points ± 1 are in the same component of the complement. What are the possible values of

$$\int \frac{dz}{\sqrt{1-z^2}}$$

over a closed curve in the region?

Pf: