Math CS 122A HW4

Zih-Yu Hsieh

February 7, 2025

1

Question 1 *Ahlfors Pg. 96 Problem 2:*
Map the region between $|z| = 1$ *and* $|z - \frac{1}{2}| = \frac{1}{2}$ *on a half plane.*

Pf:

2

Question 2 *Ahlfors Pg. 97 Problem 5:*
Map the inside of the right-hand branch of the hyperbola $x^2 - y^2 = a^2$ *on the disk* $|w| < 1$ *so that the focus corresopnds to* $w = 0$ *and the vertex to* $w = -1$.

Pf:

3

Question 3 *Ahlfors Pg. 78 Problem 4:*
Show that any linear transformation which transforms the real axis into itself can be written with real coefficient.

Pf:

4

Question 4 *Ahlors Pg. 80 Problem 3:*
If the consecutive vertices z_1, z_2, z_3, z_4 of a quadrilateral lie on a circle, prove that

$$|z_1 - z_3| \cdot |z_2 - z_4| = |z_1 - z_2| \cdot |z_3 - z_4| + |z_2 - z_3| \cdot |_1 - z_4|$$

and interpret the result geometrically.

ϕ
Pf:

5

Question 5 *Ahlfors Pg. 83 Problem 4:*
Find the linear transformation which carries the circle $|z| = 2$ *into* $|z + 1| = 1$*, the point* -2 *into the origin, and the origin into* i*.*

Pf:

6

Question 6 *Ahlfors Pg. 84 Problem 1:*
If z_1, z_2, z_3, z_4 are points on a circle, show that z_1, z_3, z_4 and z_2, z_3, z_4 determine the same orientation if and only If $(z_1, z_2, z_3, z_4) > 0$.

Pf:

7

Question 7 *Ahlfors Pg. 88 Problem 6:*
Find all circles which are orthogonal to $|z| = 1$ *and* $|z - 1| = 4$.

Pf: