Math CS 122A HW7

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> **Question 1** *Apply the representation $f(z) = w_0 + \zeta(z)^n$ to $\cos(z)$ with $z_0 = 0$. Determine $\zeta(z)$ explicitly.*

Pf:

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Question 2 *Show by use of (36), or directly, that $|f(z) \leq 1$ for $|z| \leq 1$ implies*

$$\frac{|f'(z)|}{(1 - |f(z)|^2)} \leq \frac{1}{1 - |z|^2}$$

Pf:

Question 3 *Prove that the arc of smallest noneuclidean length that joins two given points in the unit disk is a circular arc which is orthogonal to the unit circle. (Make use of a linear transformation that carries one end point to the origin, the other to a point on the positive real axis.) The shortest noneuclidean length is called the noneuclidean distance between the end points. Derive a formula for the noneuclidean distance between z_1 and z_2. Answer:*

$$\frac{1}{2} \log \frac{1 + \left| \frac{z_1 - z_2}{1 - \bar{z}_1 z_2} \right|}{1 - \left| \frac{z_1 - z_2}{1 - \bar{z}_1 z_2} \right|}$$

Pf: