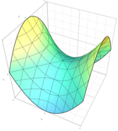

Cálculo computacional II

Unidade 2: Regra da cadeia

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Sumário

Caso 1: $z = f(x,y)$, $x = g(t)$ e $y = h(t) \Rightarrow z = f(g(t), h(t))$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dg}{dt} + \frac{\partial f}{\partial y}\frac{dh}{dt}$$

ou

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Caso 2:

$z = f(x,y)$, $x = g(s,t)$ e $y = h(s,t) \Rightarrow z = f(g(s,t),h(s,t))$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

e

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Teorema (Caso 2)

Sejam $f : D_f \subset \mathbb{R}^2 \to \mathbb{R}$ *uma diferenciável em* D_f *e as funções* $g : D_g \subset \mathbb{R}^2 \to \mathbb{R}$ *e* $h : D_h \subset \mathbb{R}^2 \to \mathbb{R}$ *são diferenciáveis tal que* $(g(s,t), h(s,t)) \in D_f$*. Então* $z(s,t) = f(g(s,t), h(s,t))$ *é uma função diferenciável em* $\mathbb{R}^2$ *tal que*

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Exemplo

Calcule $\dfrac{\partial z}{\partial s}$ *e* $\dfrac{\partial z}{\partial t}$ *para* $z = e^x + sen(y)$, $x = st^2$ *e* $y = s^2 t$

Regra da cadeia

Lista 2

Exemplo

Calcule $\dfrac{\partial z}{\partial s}$ *e* $\dfrac{\partial z}{\partial t}$ *para* $z = e^x + sen\,(y)$, $x = st^2$ *e* $y = s^2 t$

Solução: Aplicar a regra da cadeia.

$f_x = e^x$ e $f_y = \cos(y)$

$x_s = t^2$ e $x_t = 2st$

$y_s = 2st$ e $y_t = s^2$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \Rightarrow \frac{\partial z}{\partial s} = (e^x)t^2 + (\cos(y))(2st)$$

$$\frac{\partial z}{\partial s} = t^2 e^x + 2\,s\,t\,\cos(y)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \Rightarrow \frac{\partial z}{\partial t} = (e^{x})(2st) + (\cos(y))(s^{2})$$

$$\frac{\partial z}{\partial t} = 2\,s\,t\,e^{x} + s^{2}\cos(y)$$

Caso 3:

$w = f(x, y, z), x = g(s, t, r), y = h(s, t, r)$ e $z = v(s, t, r) \Rightarrow$

$w = f(g(s, t, r), h(s, t, r), v(s, t, r))$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial y}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

Exemplo

$Calcule\ \frac{\partial w}{\partial s}, \frac{\partial w}{\partial t}\ e\ \frac{\partial w}{\partial r}\ para\ z = x^2y + y^2z^3, x = rse^t,$
$y = rs^2e^{-t}\ e\ z = sr^2 sen(t)$

Exemplo

Calcule $\dfrac{\partial w}{\partial s}, \dfrac{\partial w}{\partial t}$ *e* $\dfrac{\partial w}{\partial r}$ *para* $z = x^2y + y^2z^3$, $x = rse^t$, $y = rs^2e^{-t}$ *e* $z = sr^2 sen\,(t)$

Solução: Aplicar a regra da cadeia.

$$w_x = 2xy, \quad w_y = x^2 + 2yz^3, \quad w_z = 3y^2z^2$$

$$x_s = re^t, \quad y_s = 2rse^{-t}, \quad z_s = r^2 \mathrm{sen}\,(t)$$

$$x_t = rse^t, \quad y_t = -rs^2e^{-t}, \quad z_t = sr^2\cos(t)$$

$$x_r = se^t, \quad y_r = s^2e^{-t}, \quad z_r = 2sr\mathrm{sen}\,(t)$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial y}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} \Rightarrow$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial y}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} \Rightarrow$$

$$\frac{\partial w}{\partial s} = 2xy(re^{t}) + (x^2 + 2yz^3)(2sye^{-t}) + 3y^2z^2(r^2\text{sen}\,(t))$$

Regra da cadeia

Lista 2

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial y}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} \Rightarrow$$

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Regra da cadeia

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial y}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} \Rightarrow$$

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$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t} \Rightarrow$$

$$\frac{\partial w}{\partial t} = 2xy(rse^{t}) - (x^2 + 2yz^3)(rs^2e^{-t}) + 3y^2z^2(r^2 s\cos(t))$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial y}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} \Rightarrow$$

$$\frac{\partial w}{\partial s} = 2xy(re^{t}) + (x^2 + 2yz^3)(2sye^{-t}) + 3y^2z^2(r^2\mathrm{sen}\,(t))$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t} \Rightarrow$$

$$\frac{\partial w}{\partial t} = 2xy(rse^{t}) - (x^2 + 2yz^3)(rs^2e^{-t}) + 3y^2z^2(r^2s\cos(t)$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} \Rightarrow$$

Regra da cadeia

Lista 2

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial y}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} \Rightarrow$$

$$\frac{\partial w}{\partial s} = 2xy(re^{t}) + (x^2 + 2yz^3)(2sye^{-t}) + 3y^2z^2(r^2\operatorname{sen}(t))$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t} \Rightarrow$$

$$\frac{\partial w}{\partial t} = 2xy(rse^{t}) - (x^2 + 2yz^3)(rs^2e^{-t}) + 3y^2z^2(r^2s\cos(t))$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} \Rightarrow$$

$$\frac{\partial w}{\partial r} = 2xy(se^{t}) + (x^2 + 2yz^3)(s^2e^{t}) + 3y^2z^2(2sr\operatorname{sen}(t))$$

Teorema (Caso geral)

Sejam $f : D_f \subset \mathbb{R}^n \to \mathbb{R}$ *uma diferenciável em* D_f *e as funções* $g_j : D_{g_j} \subset \mathbb{R}^m \to \mathbb{R}$ *diferenciáveis tais que*

$$x_j = g_j(t_1, t_2, \ldots, t_m)$$

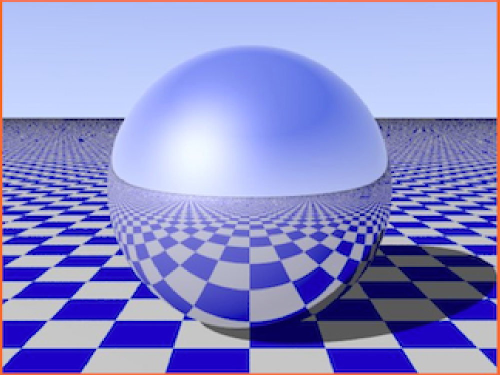
$$(x_1(g_1(\vec{t})), x_2(g_2(\vec{t})), \ldots, x_n(g_n(\vec{t})) \in D_f$$

Então $u(x_1, x_2, \ldots, x_n) = u(x_1(g_1(\vec{t})), x_2(g_2(\vec{t})), \ldots, x_n(g_n(\vec{t})))$ *é uma função diferenciável e*

$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x}{\partial t_i}$$

para $i = 1,, 2, 3, \ldots, m$

Comentários e dúvidas sobre a Lista 2



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