EM Lab Report

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Introduction/Theory

The purpose of the experiment is to calculate the approximate value of the fundamental constant ratio $\frac{e}{m_e}$. This value can be found using a variation of the experiment conducted by J.J. Thompson which first discovered the electron. A glass container is filled with a gas sample which becomes excited by electrons. When the excited gas sample drops down to its stable energy level, it releases a photon. This means one can see the path of electrons in this gas sample. Inside the glass container, a heated filament (cathode) releases electrons by "thermionic emission" then are accelerated to a cylindrical anode. This can be seen in Fig. 2[1]

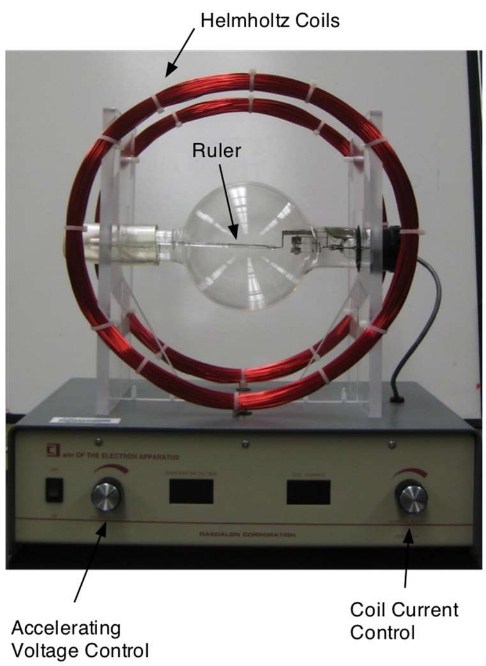


Fig. 1

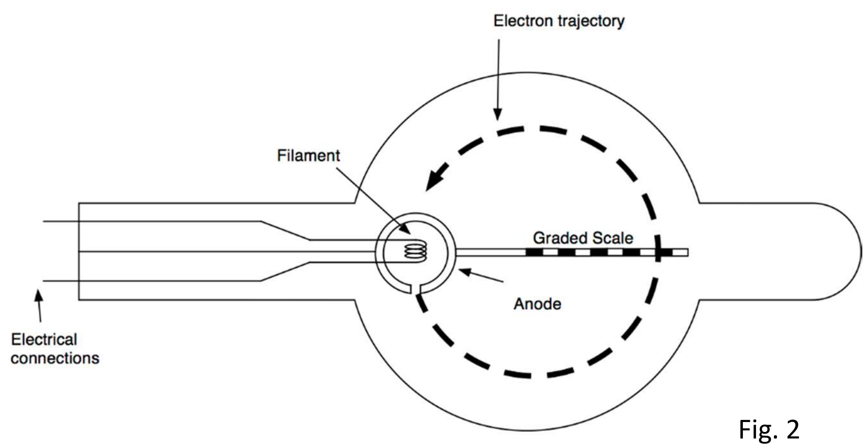


Fig. 2

When there is a voltage difference between the cathode and anode electrons travel at a velocity proportional to the difference. If no current is introduced in the Helmholtz Coils (Fig.1) then the electrons travel in a straight line out of the anode gap. Otherwise, the electrons travel in the electron trajectory shown in Fig.2 because of the magnetic field produced by the Helmholtz Coils. Varying the charge will cause a

[1] "The e/m ratio", Werner U. Boeglin. Accessed: September 5th 2019. http://wanda.fiu.edu/boeglinw/courses/Modern_lab_manual3/em_ratio.html

change in the magnetic field and therefore in the radius of the electron trajectory. We can relate the strength of the magnetic field, the radius of the electron path and the classical equations for centripetal force on a particle to each other to solve for the desired ratio.

$$E_0 = E \qquad U_e = eV \qquad K_e = \frac{1}{2} m_e v^2 \tag{1}$$

The potential energy of the electrons can be calculated by the potential difference between the cathode and the anode. These electrons convert their potential energy to kinetic energy. Equation (2) comes from (1)

$$v_e = \sqrt{\frac{2eV}{m_e}} \tag{2}$$

$$F = ev \times B = evB = \frac{m_e v_e^2}{r} \tag{3}$$

The force introduced to the particle is described by two equations, a classical particle in centripetal motion and by a particle experiencing a magnetic field. The cross product is perpendicular and simplifies.

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{u_0 n I}{R} \tag{4}$$

The magnetic field due to the Helmholtz coil is shown in 4 This can be derived by the equation of a single coil and the single coil equation can be derived by the Biot-Savart Law.[2]

$$\frac{e}{m_e} = \frac{2V}{B^2 r^2} \tag{5}$$

Equation (5) is just a combination of (2) and (3) Using equations 4 and 5, we can obtain all the constants needed and use them to approximate the desired ratio.

[2] "Introduction to Electrodynamics" David J. Griffiths 4th edition, *Pearson*

Procedures

1. Record coil diameter
2. Set voltage to 150
3. Adjust current until the electron path produces a circle and reaches a tick on the ruler (Fig.1)
4. Record current and tick mark
5. Increase current until you reach the next tick mark
6. Record current and tick mark
7. Repeat 5 and 6 until you have 10 measurements
8. Repeat 3-7 for voltages 150, 200, 300, and 400
9. Find sources of uncertainty, such as ruler, equipment and incompetence

Errors and Uncertainties

The ruler inside the apparatus had tick marks at every 5 millimeters. This should result in an uncertainty of ±2.5 millimeters. Since this was used to measure the diameter of the electron path and we need the radius “r” we can either rewrite the equations that use it or divide the uncertainty and diameter by 2.

The radius of the coils also had to be recorded. The wooden ruler used to measure that had tick marks every millimeter, so we achieved a measurement of 30.5 cm in diameter from one mid-point of the coils to the other with an uncertainty of half a millimeter. This was the measurement vertically, we did not measure it horizontally. In hindsight, this would have been a good idea because this can lead to more uncertainty.

Diameter is converted to radius This is represented by R

The machine itself could increase voltage by an interval of 2V, this means an uncertainty of ±1V. It could also increase current by 0.02A, this means an uncertainty of ±0.01A.

The number of turns in the coil was given. However, in the future it would be good practice to check the name of the apparatus and check the manual to verify any information given.

This results in the following values:

$\sigma_r = 0.00125$m $\sigma_R = 0.00025$m $\sigma_V = 1$V $\sigma_I = 0.01$A

The uncertainty values were calculated using the following equations.[1]

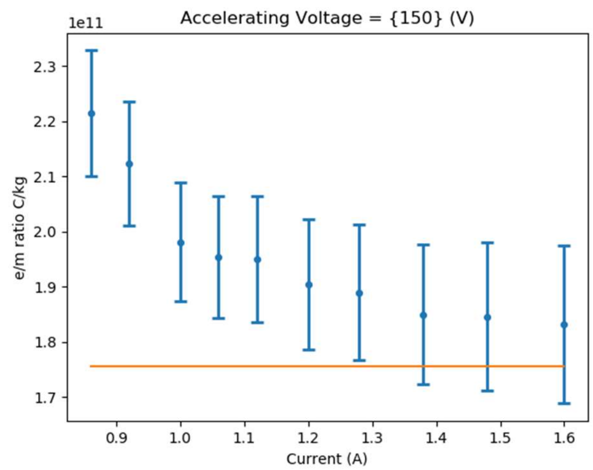
$$\sigma_B = \left(\frac{4}{5}\right)^{3/2} \mu_0 \sqrt{\left(\frac{I}{R}\right)^2 \sigma_N^2 + \left(\frac{IN}{R^2}\right)^2 \sigma_R^2 + \left(\frac{N}{R}\right)^2 \sigma_I^2}$$

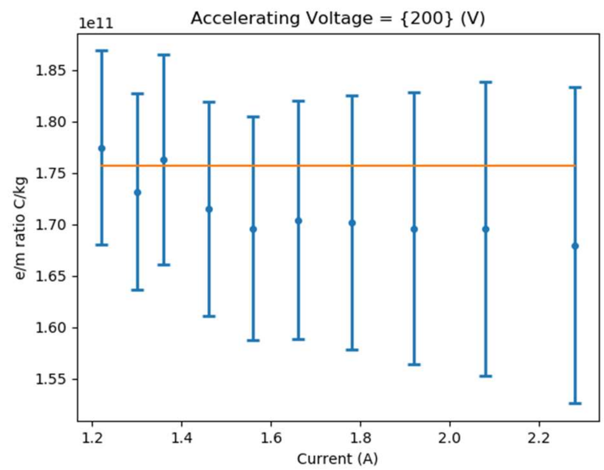
$$\sigma_{R_{em}} = \sqrt{\left(\frac{2}{B^2 r^2}\right)^2 \sigma_V^2 + \left(\frac{4V}{B^3 r^2}\right)^2 \sigma_B^2 + \left(\frac{4V}{B^2 r^3}\right)^2 \sigma_r^2}$$

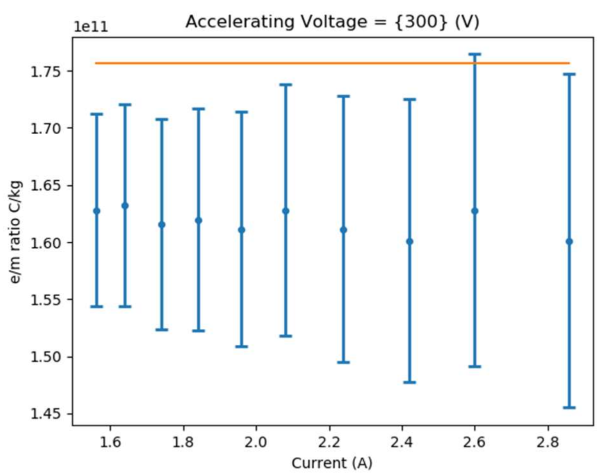
Where σ_i are the previous values. Otherwise, $\sigma_i = 0$

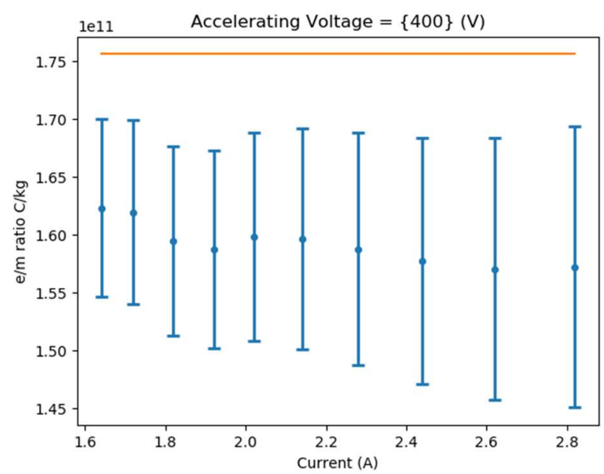
There are things which also lead to experimental errors due to assumptions. The magnetic field calculated is for the middle of the two coils. The equations for the coils assume an infinitely thin wire which is obviously not the case. The effect of parallax might have affected the perception of the ruler measurements. Energy is also lost due to friction and the equations of motion for the particle are in a vacuum, which is not the case in our experiment. Many assumptions and human errors lead to deviation from the expected value.

Results









The orange line in the graphs represents the accepted value of the EM ratio. One can see for the first graph (150V) there was some "apparent" dependence on current. This could be due to the low voltage. The second graph (200V) showed the best results. This graph showed constant overlap of the uncertainties with the actual value. This is exactly what you should expect and results in an experiment that supports the "theoretical" value. Graph 3 and 4 (300V and 400V respectively) underapproximated the value.

Accepted value of EM Ratio: 1.756e11	Average Weighted Value of EM Ratio	Weighted Uncertainty	Percent Error	$\frac{Accepted - Experimental}{Uncertainty}$ "Uncertainty Deviation"
150V	1.97e11	3.76e9	-12.0%	-5.59
200V	1.72e11	3.57e9	1.90%	0.931
300V	1.62e11	3.32e9	7.78%	4.12
400V	1.60e11	2.91e9	9.06%	5.47
Avg Weighted	1.70e11	1.67e9	3.02%	3.17

The last column represents how many uncertainties away from the accepted value we are. (This can be seen in the graphs; the second graph has more overlap with the accepted value and therefore has a lower "uncertainty deviation" in the chart.)

The weighted mean was calculated by using the formula

$$\overline{R_{em}} = \frac{\sum_{i=1}^{N} \frac{R_{em_i}}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}$$

$$\sigma_{\overline{R_{em}}} = \sqrt{\frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}}$$

Where $\overline{R_{em}}$ is the weighted mean/average and $\sigma_{\overline{R_{em}}}$ is its uncertainty.

To obtain the "Avg Weighted" in the final row, I used the weighted mean formula with the previous results. I used three significant digits because even our most inaccurate measurement was three significant digits.

Conclusion

Despite all the uncertainties, the final average value was only ~3% off from the accepted value. We found 1.70e11 with an uncertainty of ± 1.67e9 (C/Kg). Our final result was only 3.17 uncertainties away. The EM ratio calculated could be used in conjunction with another experiment to determine the value of either the charge constant e or the electron mass constant. For example, doing another experiment to calculate the charge of the electron, I could solve for the mass without having to experiment for it. The most significant source of uncertainty was the tick marks in the apparatus ruler. The biggest source of error is potentially the approximation of the electromagnetic field. I believe this is why in the first experiment (150V) you could potentially see dependence on the current. The magnetic field farther from the center is weaker. With low potential difference and low current, it is possible this effect is more apparent. The aforementioned is nothing more than speculation however, more experimentation is needed to determine why we saw the results we did in the first experiment. We could improve our results by trying to be more precise in our measurements, running more trials and improving our equipment's accuracy.

Code

```
import numpy as np
import LT.box as B

def wmean(x, sig):
    w = 1. / sig ** 2
    # weighted mean
    wm = np.sum(x * w) / np.sum(w)
    sig_wm = np.sqrt(1. / np.sum(w))
    return wm, sig_wm

def showResult(x):
    dfile = x
    #Get data from file and make an array
    #I = current D = diameter of electron beam dD = deviation in measurement of
diameter
    f = B.get_file(dfile)
    I = B.get_data(f,'I')
    D = B.get_data(f,'D')
    dD = B.get_data(f,'dD')
    V = f.par.get_value('V')
    #Physical Constant
    munot = np.pi*4e-7
    #Measured radius of coils
    R = (30.5/2) * 0.01
    #Published value of the em Ratio
    Rem_Pub = 1.6e-19/9.11e-31
    #Calculating the BField magnitude
    bField = (munot * 132 * I * ((4/5)**(3/2)) ) / R
    #D*0.1/2 converts the cm to meters and the diameter to the radius.
    r = (D*.01)/2
    Rem = (2*V) / ((bField**2)*(r**2))

    #Error Analysis
    #defining sigmas obtained
    sigmaN = 0.0
    sigmaR = 0.00025
    sigmaI = 0.01
    sigmaV = 1.00
    sigmar = dD/2

    #Defining the partial Derivatives
    partialBN = (I/R)
    partialBR = (I*132/(R**2))
    partialBI = (132/R)
    #Defining sigmaB by the error equation
    sigmaB = (4/5)**(3/2) * munot * np.sqrt( (partialBN**2 * sigmaN**2) +
(partialBR**2 * sigmaR**2) + (partialBI**2 * sigmaI**2))
    #Defining partial derivatives
    partialRemV = 2/((bField**2)*(r**2))
    partialRemB = -4*V/((bField**3)*(r**2)
```

```
    partialRemr = -4*V/((bField**2)*(r**3))
    #defining sigmaRem by the error equation
    sigmaRem = np.sqrt( (partialRemV**2 * sigmaV**2) + (partialRemB**2 * sigmaB**2) +
(partialRemr**2 * sigmar**2) )

    #Printing out stats
    print("############ Results for V" + str(V) + " #################")
    print("Actual Value of Rem: " + np.format_float_scientific(Rem_Pub))
    ARem,ErRem = wmean(Rem,sigmaRem)
    print("Average Value for Rem (Weighted): " + np.format_float_scientific(ARem))
    print("Weighted Uncertainty: " + np.format_float_scientific(ErRem))
    print("Percent Error: " + str((Rem_Pub - ARem)/(Rem_Pub)*100) + "%")
    print("Actual - Experimental/Uncertainty: " + str((Rem_Pub - ARem)/(ErRem)))

    #Making the plot
    B.pl.figure('EM Experiment')
    B.plot_exp(I,Rem,sigmaRem)
    B.plot_line(I,np.ones_like(I)*Rem_Pub)
    B.pl.xlabel('Current (A)')
    B.pl.ylabel('e/m ratio C/kg')
    B.pl.title('Accelerating Voltage = {%.0f} (V)' %(V))
    #Uncomment to show graphs
    #B.pl.show()
    return ARem, ErRem

list = ['V150.data','V200.data','V300.data','V400.data']
aa = []
nn = []
for k in list:
    n,a = showResult(k)
    aa.append(a)
    nn.append(n)
print("#################################################")
overallAV, overallAD = wmean(np.asarray(nn),np.asarray(aa))
print("Average weighted value for all trials: " +
str(np.format_float_scientific(overallAV)))
print("Average weighted uncertainty for all trials: " +
str(np.format_float_scientific(overallAD)))
print("Error percentage for all trails is: " + str((( (1.6e-19/9.11e-31) -
overallAV)/(1.6e-19/9.11e-31))*100) + "%")
print("Actual - Experimental/Uncertainty: " + str(((1.6e-19/9.11e-31) -
overallAV)/(overallAD)))
##Uncertainty is constant 0.0025
#     V150              V200           V300             V400
#  I       D        I       D       I       D       I       D
#0.86    11.0     1.22    10.0    1.56    10.0    1.64    11.0
#0.92    10.5     1.30    9.50    1.64    9.50    1.72    10.5
#1.00    10.0     1.36    9.00    1.74    9.00    1.82    10.0
#1.06    9.50     1.46    8.50    1.84    8.50    1.92    9.50
#1.12    9.00     1.56    8.00    1.96    8.00    2.02    9.00
#1.20    8.50     1.66    7.50    2.08    7.50    2.14    8.50
#1.28    8.00     1.78    7.00    2.24    7.00    2.28    8.00
#1.38    7.50     1.92    6.50    2.42    6.50    2.44    7.50
#1.48    7.00     2.08    6.00    2.60    6.00    2.62    7.00
#1.60    6.50     2.28    5.50    2.86    5.50    2.82    6.50
```