# X-ray Spectrometer

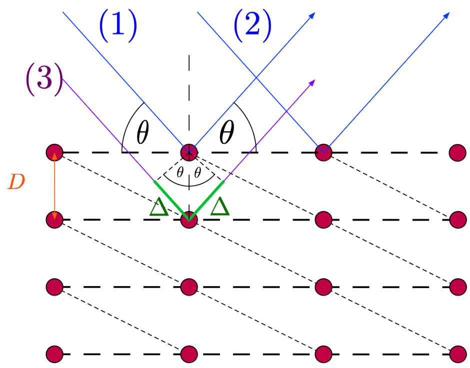
## Introduction

### Purpose

To measure the spacing between atomic planes of a crystal using Bragg's law and X-Rays.

### Supplementary

X-Rays are formed due to transitions of electrons in copper nuclei losing energy as they transition from an L or M shell to an inner K-shell. These X-Rays are directed towards a lattice structure with an angle $\theta$




$$\Delta = Dsin(\theta)$$

The path difference between the first layer and the second layer considering the same angle $\theta$ is $2\Delta$, replacing $\Delta$ we could also find that

$$2Dsin(\theta) = n\lambda$$

where $n = 1, 2, 3...$ n represents the order of diffraction. This is Bragg's condition; Due to this difference in path, there will also be a difference in the phase of the two rays and due to interference we can calculate D.

# 1 Uncertainties

## 1.1 Uncertainties Accounted for

$\sigma_R = \frac{\sqrt{N}}{T}$
Count rate uncertainty as standard

$\sigma_P = \frac{\theta_f - \theta_0}{4} = \pm 0.04165$
The peak position always lies in between three measurements (The middle measurement being the highest count we measured), quarter the possible range between two neighboring measurements and you have your peak position uncertainty. This is not the uncertainty represented on the graph, this is a somewhat coarse approximation of the uncertainty.

$\sigma_{sin(\theta)} = cos(\theta) \cdot \sigma_P$
Uncertainty represented in the graph (Calculated later and must be in radians)

## 1.2 Uncertainties that exist

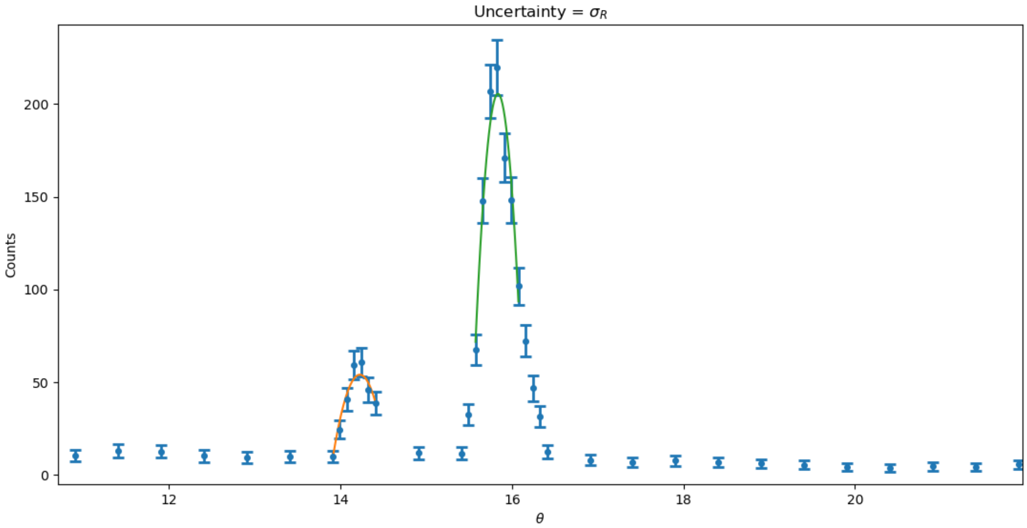
*Cosmic Radiation
*Randomness of X-Ray generation ("Braking Radiation")
*False Positives/Negatives on Geiger Muller Counter and dead time

*These are negligible due to the statistical nature of our experiment*

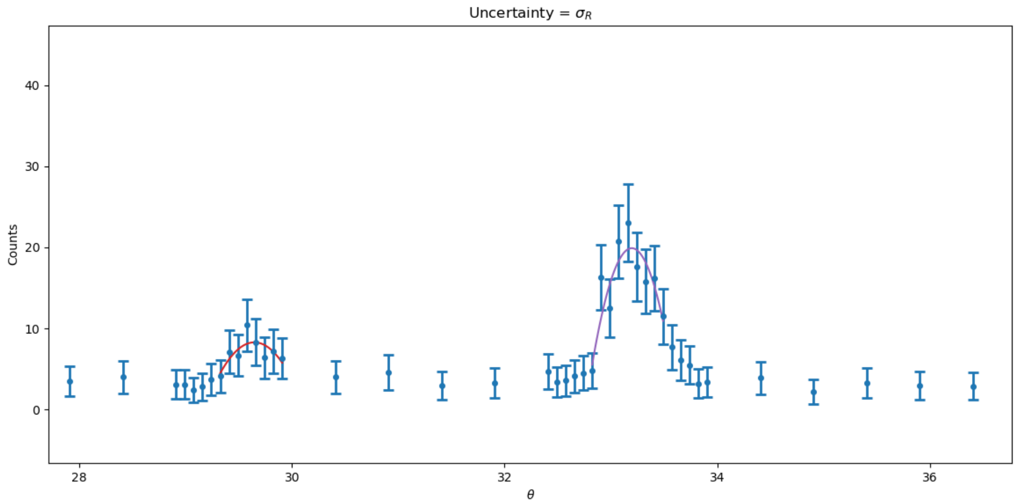
Impurities on crystal sample
Uncertainties on the exact measurements of slits used
Uncertainty in device leveling, angle measurement and parallax effects

# 2 Data

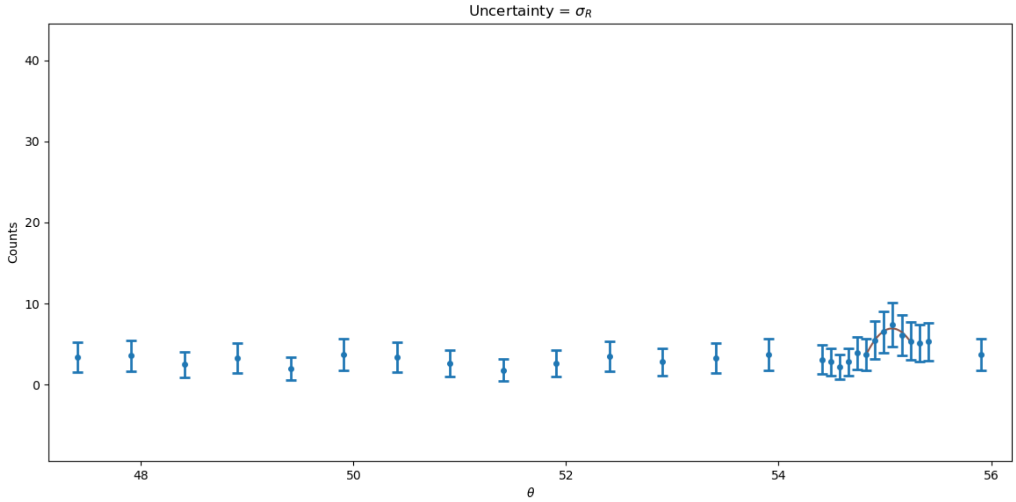
First Order Pair

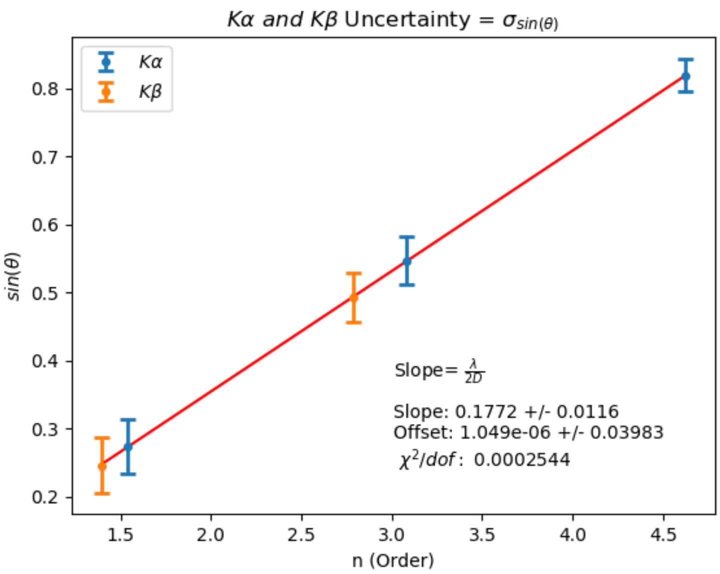



Second Order Pair




Third Order




# 3 Data Analysis

The noise drowned one of our third order pair. We can assume that the higher peaks correspond to $K\beta$ because it has a shorter wavelength. Our slope is $\frac{\lambda}{2D}$ Any offset is due to systematic bias/error such as an angle offset. I discovered that our $2\theta$ is actually 6 degrees off, $2(\theta - 3) = \theta$ through inspection of the instrument but, In an effort to minimize this I ran an algorithm to search for the smallest offset I improved it by substracting $\approx 10.93$ arc minutes off from what it reads. Which means a total offset of $2(\theta - (3 - 10.93/60)) = \theta$

In order to get D from our graphs, $D = \frac{1}{2 \cdot Slope}$
We finally get a result of $2.82\AA \pm 0.18\AA$

Comparing this to the accepted value (NIST) for NaCl crystals of 2.82Å we are on the dot. Our relative percent uncertainty is $\approx 6.55\%$

# 4 Math

Weighted Mean

$$\bar{D}_{Avg} = \frac{\sum_{i=1}^{N} \frac{D_i}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}$$

$$\sigma_{\bar{D}_{Avg}} = \sqrt{\frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}}$$

Error Propagations

$$f = sin(\theta); \ \sigma_f = cos(\theta) \cdot \sigma_\theta$$

$$f = \frac{1}{2 \cdot Slope}; \ \sigma_f = \frac{\sigma_S}{2 \ Slope^2}$$

% Relative Uncertainty

$$\frac{\sigma_D}{D} \cdot 100$$

# 5 Conclusion

The experiment was a success, we discovered an angle offset evident by analysing our data and an experimental value that supported the accepted value. With the final result being an atomic plane distance of $(2.82 \pm 0.18)\AA$ or $(.282 \pm 0.018)nm$ we differ from the accepted value (0.282nm) by 0% with a relative uncertainty of 6.55%. We also discovered in the process of graphing $K\alpha \ andK\beta$ that the larger peaks are due to $K\alpha$ rays.