Young's Double Slit Experiment

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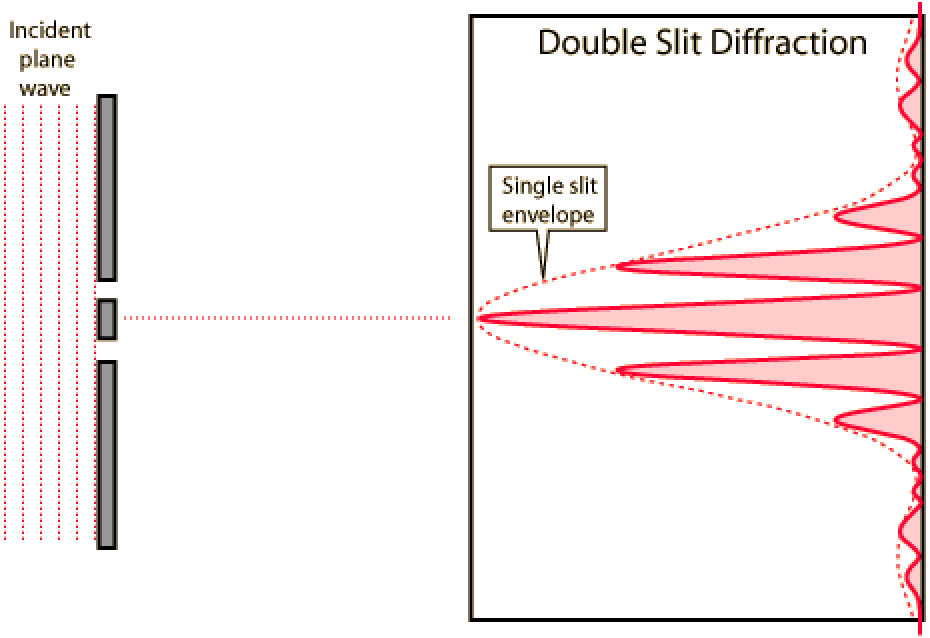
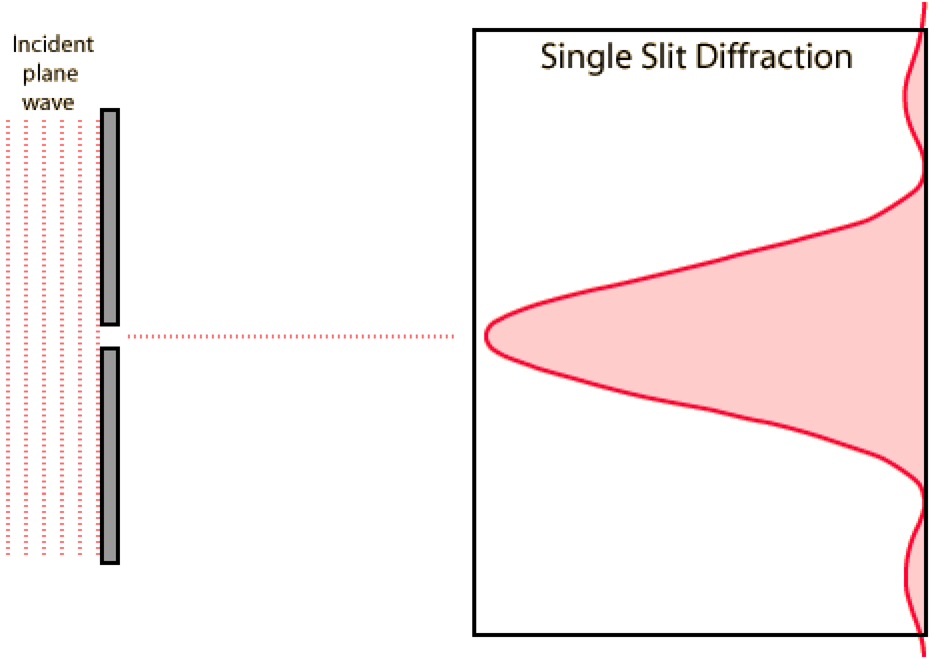
Particle Wave Duality of Photons



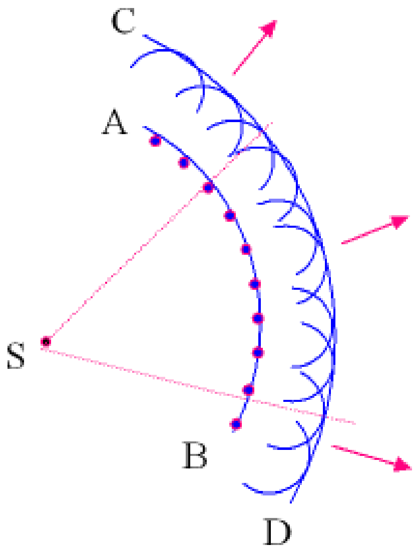
- During this time there were discussions for light being a particle or a wave
- Evidence for the wave like nature of light came from interference in Young's double slit experiment

Wave Properties



- Interference in the double slit is obvious; it’s not in the single slit.
- We saw interference in single slits, this led to the development

Huygens' Principle



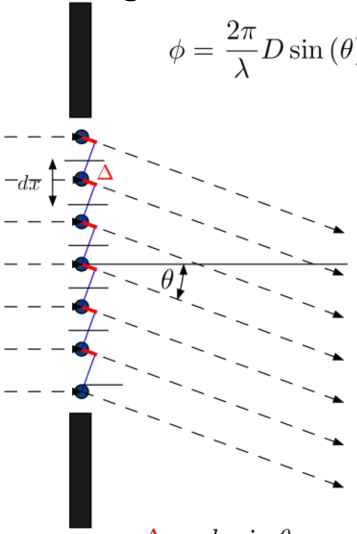
Huygens' Principle:
Each wavefront is the envelope of the wavelets. Each point on a wavefront acts as an independent source to generate wavelets for the next wavefront. AB and CD are two wavefronts.

- This works the same way for planar wavefronts
- You could think of this instead by imagining infinitesimal small slits

Fraunhofer diffraction

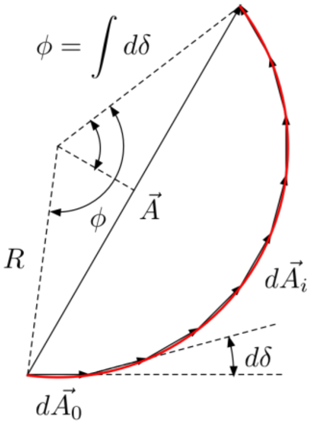
For single slit,

$$\phi = \frac{2\pi}{\lambda} D \sin(\theta)$$



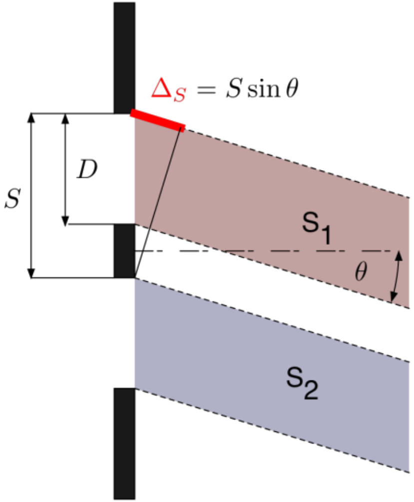
$$\Delta = dx \sin\theta$$

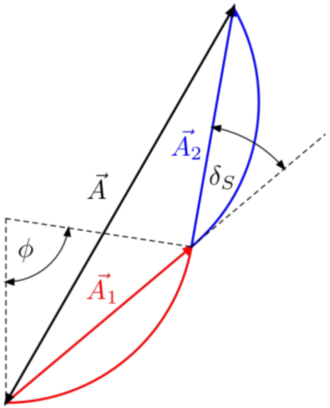
$$\phi = \int d\delta$$



Under the assumption that the source is very far away… (This way the rays are paraxial)

A Theoretical Model for far-field diffraction





$$\vec{A} = \sum_i^N d\vec{A}_i$$

$$|d\vec{A}_i| = \frac{dx}{D}|\vec{A}_0|$$

As N ⇒ ∞

$$|\vec{A}| = 2R\sin(\phi/2)$$

$$\phi = \tfrac{2\pi}{\lambda} D \sin(\theta)$$

$$R = |\vec{A}_0|/\phi$$

Finally,

$$|\vec{A}| = |\vec{A}_0| \frac{\sin(\phi/2)}{\phi/2.}$$

$$I(\theta) = I_0 \left(\frac{\sin(\phi/2)}{\phi/2.}\right)^2$$

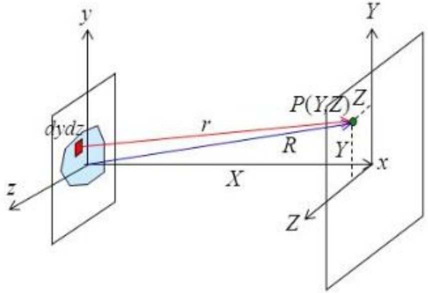
For double slit, add a phase shift,

$$|\vec{A}| = |\vec{A}_1 + \vec{A}_2| = 2|\vec{A}_0| \frac{\sin(\phi/2)}{\phi/2} \cos(\psi/2)$$

$$I(\theta) = |\vec{A}|^2$$

A quick introduction to Fourier optics

We can represent a plane wave propagating through a slit using Huygens' principle as spherical waves propagating from the source of infinitesimal slits (dy away)



Where b-a is the total slit width

$$E(x, y, z) = \int_a^b E_0 \frac{e^{jkr}}{r} dz$$

Using some geometry you can find in "Introduction to Fourier optics" by Joseph W. Goodman

$$r = \sqrt{(Z - z)^2 + D^2}$$

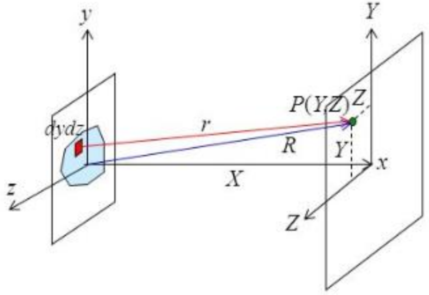
Let's assume a paraxial approximation (small angle approximation) (far field) and other approximations to get the Fresnel integral

$$e^{-jkD} \frac{1}{D} \int_a^b e^{-\frac{1}{2}jk\frac{(Z-z)^2}{d}} dz$$

A continuation of Fourier optics

Making the Fraunhofer assumptions, (b-a) << Z and z << Z (Basically, we are at the far-field) we get,

$$\frac{e^{-jkd}e^{-\frac{jkZ^2}{d}}}{d}\int_a^b e^{-jk_z z}dz$$



Now, this is exactly like a Fourier transform,

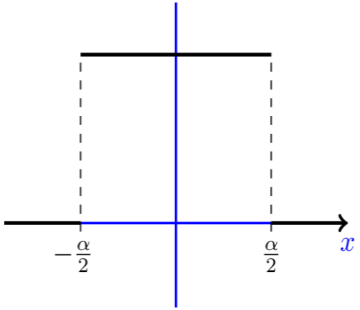
$$E(x,y,z) = E_0\int_a^b e^{-jk_z z}dz$$

To get the expected intensity function of a slit we could use an Aperture function and replace the function to be,

$$E(x,y,z) = E_0\int_{-\infty}^{\infty} g(x)e^{-jk_z z}dz = E_0\mathcal{F}(g(x))$$

Example of Fourier Optics

Let's say we have an aperture function defining a single slit of width α, it should look something like this,



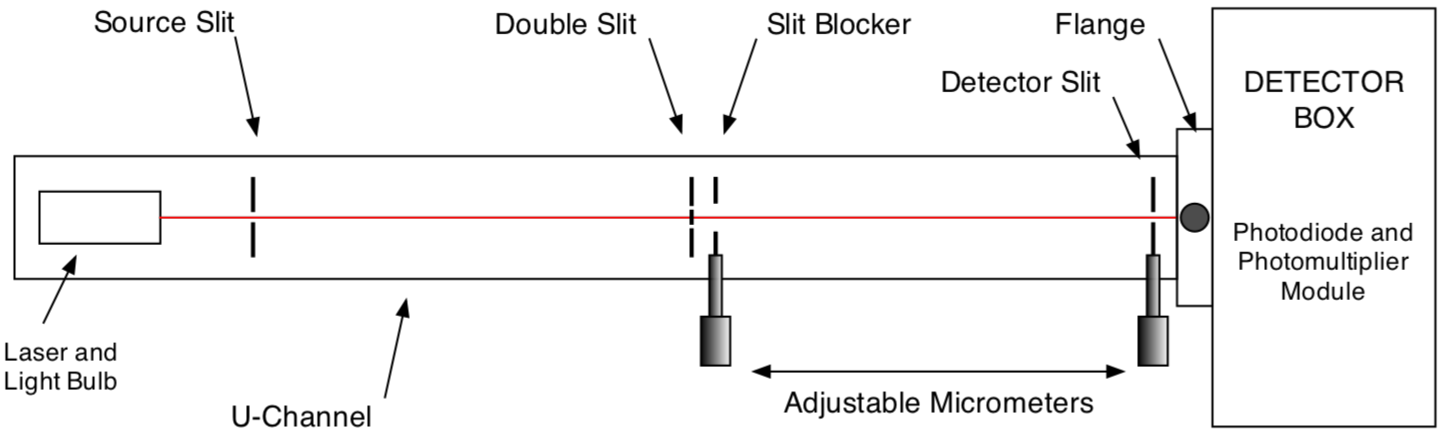
You will see that the Fourier transform of this function is

$$sinc\left(\frac{k_x\alpha}{2}\right)$$

Which means we get the following relationship when squaring the electric field,

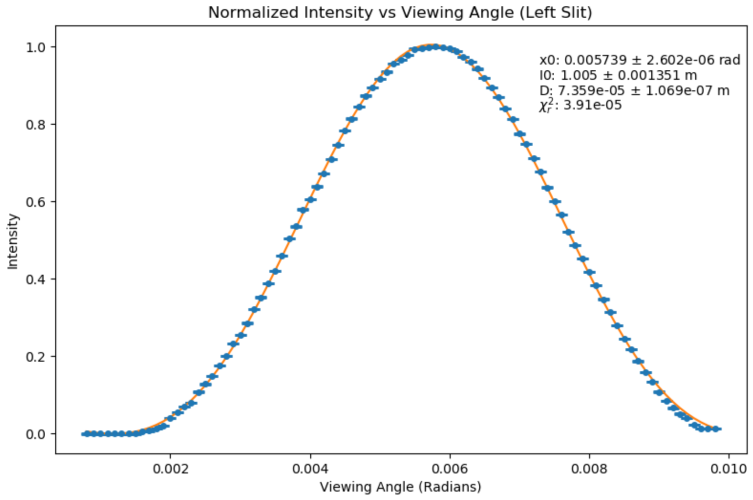
$$I = I_0 sinc^2\left(\frac{k_x\alpha}{2}\right) = I_0\left(\frac{\sin\left(\frac{k_x\alpha}{2}\right)}{\frac{k_x\alpha}{2}}\right)^2$$

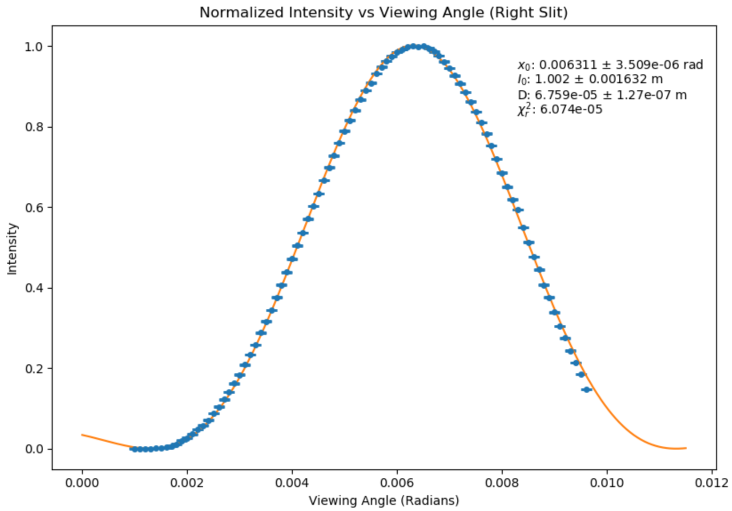
Experimental Set-up



Specifications
Photomultiplier tube: Hamamatsu R 212
Preamplifier-Discriminator: Amptek A-111
Interference Filter: 546 nm, 10 nm FWHM
All Slit Widths: 0.09 mm
Double Slit Separations: 0.35, 0.40, 0.45 mm
Laser (Class II): 670 ± 20 nm<1.0 mW
Universal Power Supply
Input: 110 - 230 VAC, 50 - 60 Hz
Output: 15 V DC
Optical Light Path: 1 meter
Overall Instrument Length: 52 inches

Single Slit Plots

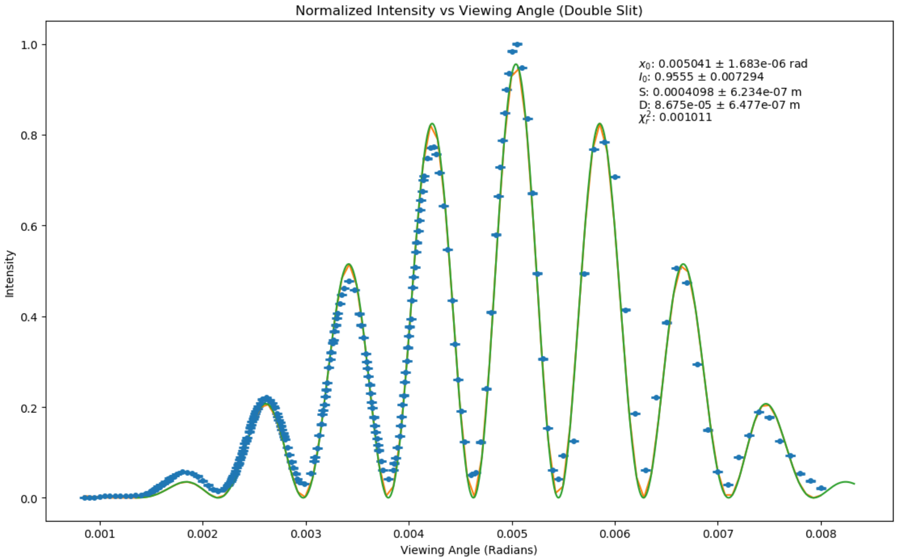




$V_{max} = 0.802 \pm 0.0005$ V

$V_{max} = 0.611 \pm 0.0005$ V

Double Slit Plot



- Background noise was subtracted from both single and double slit.

Data Fit

$$\theta \approx \frac{x - x_0}{L}$$

$$\phi = \frac{2\pi}{\lambda} D sin(\theta)$$

Single Slit,

$$I(\theta) = I_0 sinc\left(\frac{\phi}{2}\right)^2$$

Double Slit,

$$\psi = \frac{2\pi}{\lambda} S sin(\theta)$$

$$I(\theta) = I_0 sinc^2\left(\frac{\phi}{2}\right)\cos^2\left(\frac{\psi}{2}\right)$$

Parameters for Single Slit:

D: Slit Width

I_0: Max intensity

x_0: Location of Maximum (rad)

Parameters for Double Slit:

D

I_0

x_0

S: Slit Separation

Results

The weighted mean of the slit widths resulted in:

$$7.13 \cdot 10^{-5} \pm 8 \cdot 10^{-8}\ m$$

The slit separation gotten was:

$$0.00041 \pm 6 \cdot 10^{-7} \mathrm{m}$$

Uncertainty Propagation

For the voltage we recorded an instrumental uncertainty of about $\pm 0.0005\ V$ for the double slit and about $\pm\ 0.005\ V$ for the single slit.

This can be propagated through the normalization function by applying the same ratios to the uncertainty

Uncertainty can be further analyzed in the x-direction

$$\theta = \frac{x - x_0}{L}$$

$$\frac{\partial\theta}{\partial x_0} = \frac{x}{L}$$

$$\frac{\partial\theta}{\partial x} = -\frac{x_0}{L}$$

$$\frac{\partial\theta}{\partial L} = \frac{-(x - x_0)}{L^2}$$

$$\sigma_\theta = \sqrt{\left(\frac{\partial\theta}{\partial x_0}\sigma_{x_0}\right)^2 + \left(\frac{\partial\theta}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial\theta}{\partial L}\sigma_L\right)^2}$$

References

"Introduction to Fourier Optics" Joseph W. Goodman, 2ed, Pg. 73

http://wanda.fiu.edu/boeglinw/courses/Modern_lab_manual3/interference.html

https://www.teachspin.com/two-slit

http://www.pstcc.edu/departments/natural_behavioral_sciences/Web%20Physics/Chapter038.htm

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/slits.html