Hydrogen Spectrum

INTRODUCTION

When a sample of 'pure' Hydrogen gas is supplied enough energy to produce light, you should see red light being given off. This red light when passed through a prism is separated out into different colors with the most intense being of course red. The reason we see distinct bands of light instead of a continuous spectrum is because this is evidence of the quantization of energy. The color of light is attributed to its energy and when the Hydrogen electrons drop down an energy level it releases photon with the same amount of energy.

The purpose of this experiment is to observe the spectrum for Hydrogen to determine the wavelengths of the different colored lines, the 'orbit' numbers $n_1$and n_2, the Rydberg constant, R_H and ultimately the ionization energy of the hydrogen atom.

We will be adjusting the angle of the grating and will need the following equations obtained from the geometry of the set-up,

$$\theta_{in} = \frac{\theta_0 - \theta_a}{2} \qquad \theta_{out} = \theta - (\theta_a + \theta_{in})$$

$$\Delta_1 = Dcos\left(\theta_{in}\right) \quad \Delta_2 = Dcos\left(\theta_{out}\right) \qquad \Delta_{tot} = \Delta_1 - \Delta_2$$

$$\Delta_{tot} == m_d \lambda$$

Where, θ_{in} is the angle of incident beam with respect to the grating, θ_0 is the reference angle achieved during the alignment process, θ_a is the optic axis angle,θ is the angle of the spectrum line, Δ_i are the path lengths, D is the grating constant (D = 1/1200nm) and finally, m_d is the diffraction order 0,$\pm 1, \pm 2 \ldots$

$$E = \frac{hc}{\lambda}$$

Using this and the different angles we will get we can calculate $\lambda, \sigma_\lambda and E, \sigma_E$ Finally, we will plot $\frac{1}{\lambda}$ vs $(\frac{1}{n_1^2} - \frac{1}{n_2^2})$ and fit a line to determine which n_1 gives us the best fit. We will use this graph to find the value of the Rydberg constant and it's uncertainty for Hydrogen using the equation shown below.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

UNCERTAINTIES

Measurement Uncertainties $\sigma_{\theta_0}, \sigma_{\theta_A}, \sigma_\theta$: The image we observed has a width, when we measured its angle, we assumed it to be in the middle, this is of course not the case. We assumed the image was insignificantly slanted and measured the angle of the left point and the angle of the right point. After getting the average value we have a $\pm$ of $\frac{1}{4}$ the total width. ($\frac{1}{2}$ was too much, we can be more certain of a middle point than that.) Now our instrument is only so precise, we had an uncertainty of $\frac{1}{2}$ an arc minute. We will assume these two values are not correlated and add them in quadrature. Since θ_0 and θ_A are measured the same way, they have the same uncertainties. σ_θ has an uncertainty of half an arc minute as well. The way θ was measured is focusing and making the color disappear under the cross-hair of the telescope, this allows for relatively little uncertainty in the measurement of theta.

Assumptions: We are assuming a perfect vacuum, this is not the case. The hydrogen lamp is also assumed to be 100% Hydrogen, this is not the case and we in did in fact see contamination. This contamination consisted of not distinct colors but a smeared spectrum of colors at different angles, this could of easily been nitrogen or oxygen producing this effect. Background light affects how well you can see the spectrum lines and therefore your uncertainty. There is of course, minor aberrations caused by the optics in the experiment, these are unavoidable. We will assume no uncertainty with the value given for the grooves, this would lead to a very big error if it was off by even 1. We tried our best to beat the effect of parallax by viewing the angle from above and the same place.

MATH

Rewriting the equations shown in the introduction, one can obtain the following,

$$\lambda = \frac{D\left(\cos\left(\frac{\theta_0}{2} - \frac{\theta_A}{2}\right) - \cos\left(-\theta + \frac{\theta_0}{2} + \frac{\theta_A}{2}\right)\right)}{m}$$

Of course we have the uncertainty of λ using, the propagation of uncertainty rules.

$$\sigma_\lambda = \sqrt{\frac{\partial\lambda}{\partial\theta_0}^2 \sigma_{\theta_0}^2 + \frac{\partial\lambda}{\partial\theta_A}^2 \sigma_{\theta_A}^2 + \frac{\partial\lambda}{\partial\theta}^2 \sigma_\theta^2}$$

We have,

$$\frac{\partial\lambda}{\partial\theta_A} = \frac{D\left(\frac{\sin\left(\frac{\theta_0}{2} - \frac{\theta_A}{2}\right)}{2} + \frac{\sin\left(-\theta + \frac{\theta_0}{2} + \frac{\theta_A}{2}\right)}{2}\right)}{m}$$

$$\frac{\partial\lambda}{\partial\theta_0} = \frac{D\left(-\frac{\sin\left(\frac{\theta_0}{2} - \frac{\theta_A}{2}\right)}{2} + \frac{\sin\left(-\theta + \frac{\theta_0}{2} + \frac{\theta_A}{2}\right)}{2}\right)}{m}$$

$$\frac{\partial\lambda}{\partial\theta} = \frac{D\sin\left(-\theta + \frac{\theta_0}{2} + \frac{\theta_A}{2}\right)}{m}$$

We have uncertainty values of $\sigma_{\theta_0} = \sigma_{\theta_A} = \sqrt{\frac{(174-(173.5+3/60))}{4}^2 + 1/120^2}$ of course this is converted to radians by multiplying by $\frac{\pi}{180}$

RESULTS

Figure 2 shown below has the best fit and therefore is the correct n_1 value we should use. The slope is $1.09 \cdot 10^{16}\ nm$ or $1.09 \cdot 10^7\ m$ this is very close to the accepted value found online to be $1.0973731568508 \cdot 10^7$ We can calculate the percent deviation from the accepted value by,

$$\frac{1.0973731568508 - 1.09}{1.0973731568508} = 0.672\%$$

Now our % Uncertainty is

$$\frac{\sigma_\lambda}{\lambda} = \frac{2.19e13}{1.09e16} = 2\%$$

Our % Uncertainty covers our % deviation so we should be able to conclude the experiment as successful.

$$\bar{\lambda} = \frac{\sum_{i=1}^{N} \frac{\lambda_i}{\sigma^2}}{\sum_{i=1}^{N} \frac{1}{\sigma^2}} \qquad \sigma_{\bar{\lambda}} = \sqrt{\frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma^2}}}$$

Using weighted means we can calculate the average of our three orders and plot them.

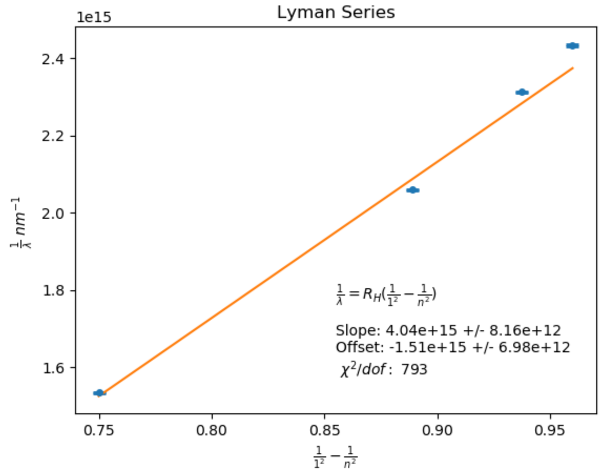

Figure 1. Lynman Series has a χ^2/dof worse than that of Figure. 2, this is because it is actually supposed to represent the wavelengths in the ultraviolet range.

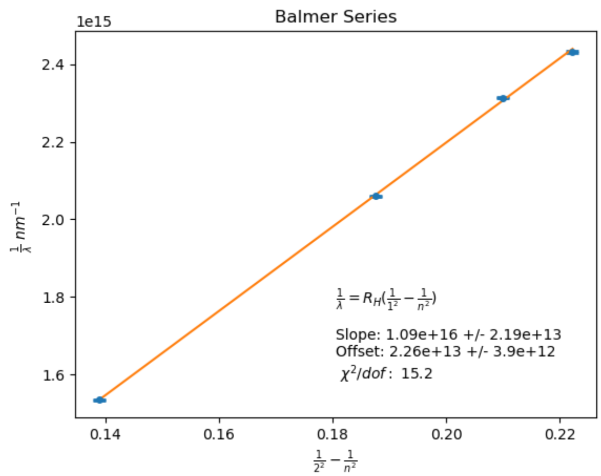

Figure 2. The Balmer Series where $n_1 = 2$ has the best χ^2/dof value and therefore is the series we should use to determine our R_H constant. The slope shown is represented in nanometers, if we wanted to represent the R_H constant in meters we have to multiply by 1e-9

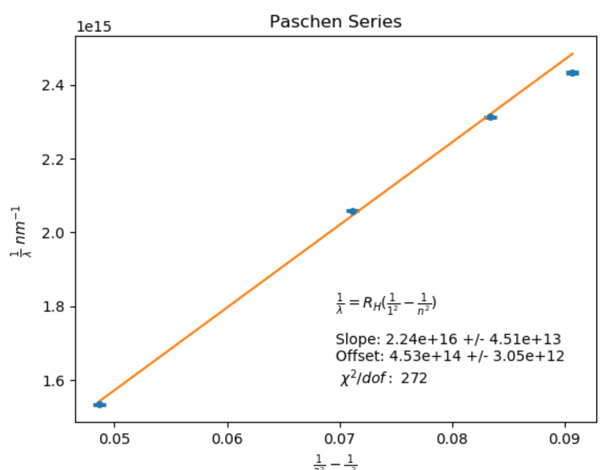

Figure 3. Paschen Series has a χ^2/dof worse than that of Figure. 2, keep in mind n_2^2 is just a series of $(n_1 + k)^2$ where k = 1,2,3.. for Fig 1,2,3

In order to plot the uncertainties of $\frac{1}{\lambda}$ we need to consider the propagation of the uncertainty, this should become: $\sigma_{\frac{1}{\lambda}} = \frac{-\sigma_\lambda}{\lambda^2}$

	λ	σ_λ	ΔE	$\sigma_{\Delta E}$
Order 1	4.12, 4.33, 4.85, 6.56 $(10^{-7}\ m)$	1.04, 1.06, 1.09, 1.17 $(10^{-9}\ m)$	4.82, 4.59, 4.10, 3.03 $(10^{-19}\ J)$	1.22, 1.12, .923, 0.541 (10$^-$
Order 2	4.11, 4.34, 4.83, 6.51 $(10^{-7}\ m)$	5.98, 5.97, 5.89, 4.93 $(10^{-10}\ m)$	4.84, 4.58, 4.11, 3.05 $(10^{-19}\ J)$	7.04, 6.31, 5.00, 2.31 (10$^-$
Order 3	N/A, 4.32, 4.86, N/A $(10^{-7}\ m)$	N/A, 3.30, 2.63, N/A $(10^{-10}\ m)$	N/A, 4.60, 4.09, N/A $(10^{-19}\ J)$	N/A, 3.52, 2.21 N/A (10$^-$

Table 1. The wavelengths, Delta E, and their uncertainties are represented in their respective order. {Dark Violet, Violet, Blue, Red}

The uncertainty in energy can be determined using,

$$\frac{dE}{d\lambda} = \frac{-hc}{\lambda^2}$$

$$\sigma_{\Delta E} = \sqrt{(\frac{dE}{d\lambda}\sigma_\lambda)^2}$$

CONCLUSION

It is important for the values of $\theta_0, \theta, \theta_A$ to be converted to radians before using them in the uncertainty equations. We acquired a value of $1.09 \cdot 10^7\ m$ or $1.09 \cdot 10^16\ nm$ This resulted in a percent deviation from the accepted value of 0.672% which is very close. The % Uncertainty we obtained was 2.0% which covered our error by 3 uncertainties. Considering this I would call the experiment a success.