1. Our favorite Boeing 747 flying straight and level at 40,000ft at speed $u_0 = 774$ ft/s with $\theta_0 = 0$ ('stability axes') did not respond well to a $\delta_r = 1°$ step (Lecture 17), according to linear dynamics.

(a) Find the aileron angle δ_a required to hold the airplane into a forward slip for that rudder deflection. Clearly show/explain which set of equations you are solving to find the answers (in symbolic form), and how you determine the stability and control derivatives from the data given in Etkin & Reid.

(b) Determine the steady state bank and sideslip angles, ϕ and β, resulting from those combined aileron and rudder inputs.

(c) Find the linear lateral response of the airplane to the sudden (=step) input of those *combined* aileron and rudder angles starting from straight and level, using either the Heaviside inversion theorem as in the previous homework, or the matrix exponential method demonstrated in class. Show your brief Matlab code and your nice plots of the response.

(d) Is the linear evolution reasonable? Is the linearization valid? Explain.

2. That same Boeing 747 cruising at 40,000 feet at speed $u_0 = 774$ ft/s with $\theta_0 = 0$ ('stability axes') executes a standard 2 minute turn ($= 3°/\text{s} = 180°/\text{min}$).

(a) Find the aileron δ_a, rudder δ_r, sideslip β and bank (roll) ϕ angles required to maintain that nicely coordinated turn. Clearly show/explain which set of equations you are solving to find the answers (in symbolic form), and how you determine the stability and control derivatives from the data in Etkin & Reid.

(b) Find the linear lateral response of the airplane to the sudden (=step) input of those combined aileron and rudder angles starting from straight and level, using the Heaviside inversion theorem as in the previous homework, or the matrix exponential method demonstrated in class. Show your brief Matlab code and your nice plots of the response.

(c) Is the linear evolution reasonable? Is the linearization valid? Explain.

1. (a). I intend to use steady-state lateral force & moment equations

So the following system takes V_0 as input & gives the output δ_a, δ_r, $\sin\phi_0$ required to hold the state.

$$\begin{bmatrix} 0 & Y_{\delta r} & mg \\ L_{\delta a} & L_{\delta r} & 0 \\ N_{\delta a} & N_{\delta r} & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta a \\ \delta r \\ \sin\phi_0 \end{bmatrix} = - \begin{bmatrix} Y_v \\ L_v \\ N_v \end{bmatrix} \cdot V_0 .$$

With non-dimensional values, following conversions are used.

$$Y_{\delta r} = C_{y\delta r} \cdot \frac{1}{2} \cdot \rho \cdot u_0^2 \cdot S = 0.1157 \times 0.00237 \cdot 774^2 \cdot 5650$$
$$= 928136$$

$$Y_{\delta a} = C_{y\delta a} \cdot \frac{1}{2} \cdot \rho \cdot u_0^2 \cdot S =$$

$$L_{\delta a} = C_{l\delta a} \cdot \frac{1}{2} \cdot \rho \cdot u_0^2 \cdot S \cdot b = 23188984$$

$$L_{\delta r} = C_{l\delta r} \cdot \frac{1}{2} \cdot \rho \cdot u_0^2 \cdot S \cdot b = 11848386$$

$$N_{\delta a} = C_{n\delta a} \cdot \frac{1}{2} \cdot \rho \cdot u_0^2 \cdot Sb = 338525$$

$$N_{\delta r} = C_{n\delta r} \cdot \frac{1}{2} \cdot \rho \cdot u_0^2 \cdot Sb = -21259390.5 .$$

$Y_v = -1.103 \times 10^3$

$L_v = -6.885 \times 10^4$

$N_v = 4.790 \times 10^4$.

So the system has the following form:

$$\begin{bmatrix} 0 & 9.28\times10^5 & 6.36\times10^5 \\ 2.32\times10^7 & 1.18\times10^7 & 0 \\ 3.39\times10^5 & -2.13\times10^8 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta_a \\ \frac{\pi}{180} \\ \sin\phi_0 \end{bmatrix}$$

$$= - \begin{bmatrix} -1.103\times10^3 \\ -6.885\times10^4 \\ 4.790\times10^4 \end{bmatrix} \cdot V_0$$

We need to rearrange this system:

$$9.28\times10^5 \cdot \frac{\pi}{180} + 6.36\times10^5 \cdot \sin\phi_0 = 1.103\times10^3 \cdot V_0$$

$$2.32\times10^7 \cdot \delta_a + 1.18\times10^7 \cdot \frac{\pi}{180} = 6.885\times10^4 \cdot V_0,$$

$$3.39\times10^5 \cdot \delta_a - 2.13\times10^8 \cdot \frac{\pi}{180} = -4.4790\times10^4 \cdot V_0.$$

$$-1.103\times10^{3}\cdot V_o + 6.36\times10^{5}\cdot \sin\phi_o = \overbrace{9.28\times10^{5}\cdot\frac{\pi}{180}}^{B}.$$

$$2.32\times10^{7}\cdot S_a - 6.885\times10^{4}\cdot V_o = -1.18\times10^{7}\cdot\frac{\pi}{180}.$$

$$3.39\times10^{5}\cdot S_a + 4.479\times10^{4}\cdot V_o = 2.13\times10^{8}\cdot\frac{\pi}{180}.$$

$$\begin{bmatrix} 0 & -1.103\times10^{3} & 6.36\times10^{5} \\ 2.32\times10^{7} & -6.885\times10^{4} & 0 \\ 3.39\times10^{5} & 4.479\times10^{4} & 0 \end{bmatrix} \cdot \begin{bmatrix} S_a \\ V_o \\ \sin\phi_o \end{bmatrix}$$

$$= B$$

Solve by Matlab.

$\boxed{S_a = 0.2322}$

$V_o = 81.2420$

$\sin\phi_o = 0.1662.$

(b).

$\phi = \sin^{-1}(0.1662)$

$= 0.1669 = 9.5626^\circ$

$\beta = \tan^{-1}\left(\frac{81.24}{774}\right) = 0.1045$ rad

$= 5.99^\circ$.

(c). Refer to the attached code

(d). Most part sensical, except roll rate.

V settles to 48.97 fps, fine.

p goes to -15.15 rad/s Nonsense.

r goes to -3.5493×10^{-5} rad/s

very nice. almost

no yaw.

ϕ is 0.0533 ≈ 3.05°

fine

The linearization is valid,

just δa, δr don't lead to

steady state. when hold fixed.

2.

(a). $\dot{\psi}$ = turn rate = $3°/s$.

$$\tan\phi_0 = \frac{\dot{\psi}\cdot u_0}{g} \rightarrow \text{gives } \phi_0 = \frac{\frac{3\pi}{180}\cdot 774}{32.2}$$

$$Z_0 = \frac{-m\cdot g\cdot\cos\theta_0}{\cos\phi_0}$$

$= 0.899$ rad
$= 51.51°$.

$$X_0 = mg\cdot\sin\theta_0 = 0 \rightarrow \theta_0 = 0.$$

$$Y_0 = 0 = Y_v\cdot v_0 + Y_{\delta r}\cdot\delta r$$

$$L_0 = L_v\cdot v_0 + \cancel{L_p\cdot p_0} + L_r\cdot r_0 + L_{\delta a}\cdot\delta a + L_{\delta r}\cdot\delta r.$$
$$= 0.$$

$$N_0 = N_v\cdot v_0 + \cancel{N_p\, p_0} + N_r\cdot r_0 + N_{\delta a}\cdot\delta a + N_{\delta r}\cdot\delta r.$$

$$p_0 = -\dot{\psi}\cdot\sin\theta_0 = 0.$$

$$q_0 = \dot{\psi}\cdot\cos\theta_0\cdot\sin\phi_0 = \frac{3\pi}{180}/s\cdot 0.7827 = 0.041$$

$$r_0 = \dot{\psi}\cdot\cos\theta_0\cdot\cos\phi_0 = \frac{3\pi}{180}\cdot 0.6224 = 0.0326.$$

$L_v = -6.885\times10^4$ $\quad$ $N_r = -6.590\times10^6$

$L_r = 7.321\times10^6$ $\quad$ $N_v = 4.790\times10^4$

$L_{\delta a} = 2.32\times10^7$ $\quad$ $L_{\delta r} = 1.18\times10^7$

$N_{\delta a} = 3.39\times10^5$ $\quad$ $N_{\delta r} = -2.13\times10^8$

$$-6.885\times10^{4}\cdot v_0 + 7.321\times10^{6}\cdot r_0 + 2.32\times10^{7}\cdot\delta a$$
$$+ 7.321\times10^{6}\cdot\delta r = 0.$$

$$4.790\times10^{4}\cdot v_0 - 6.59\times10^{6}\cdot r_0 + 3.39\times10^{5}\cdot\delta a$$
$$- 2.13\times10^{8}\cdot\delta r = 0.$$

$$Y_{\delta r} = 928316 \qquad Y_v = -1.103\times10^{3}$$

$$-1.103\times10^{3}\cdot v_0 = -928316\cdot\delta r.$$

$$\delta r = \frac{1.103\times10^{3}}{928316}\cdot v_0,$$

$$v_0 = \frac{928316}{1103}\cdot\delta r.$$

$$\therefore -6.885\times10^{4}\cdot\frac{928316}{1103}\cdot\delta r + 7.321\times10^{6}\cdot 0.0326$$
$$2.32\times10^{7}\cdot\delta a + 7.321\times10^{6}\cdot\delta r = 0,$$

$$4.790\times10^{4}\cdot\frac{928316}{1103}\cdot\delta r - 6.59\times10^{6}\cdot 0.0326$$
$$+3.39\times10^{5}\cdot\delta a - 2.13\times10^{8}\,\delta r = 0.$$

$$\begin{bmatrix} 2.32\times10^{7} & -6.9\times10^{4}\frac{928316}{1103} + 7.321e6 \\ 3.39\times10^{5} & 4.79\times10^{4}\cdot\frac{928316}{1103} - 2.13e8 \end{bmatrix}\begin{bmatrix}\delta a \\ \delta r\end{bmatrix} = \begin{bmatrix} -7.321\times10^{6}\cdot 0.0326 \\ 6.59\times10^{6}\cdot 0.0326. \end{bmatrix}$$

$$\begin{cases} \delta_a = -0.0131 \\ \delta_r = -0.0013 \\ \phi_0 = 0.899 = 51.51^\circ \end{cases}$$

by matlab solving previous system.

(b). Please refer to Matlab code attached:

(c). $v_\infty \approx -36.9872$ fine

$p = 11.3650$ rad/s nonsense.

$r = -3.6981 \times 10^{-6}$ rad/s good.

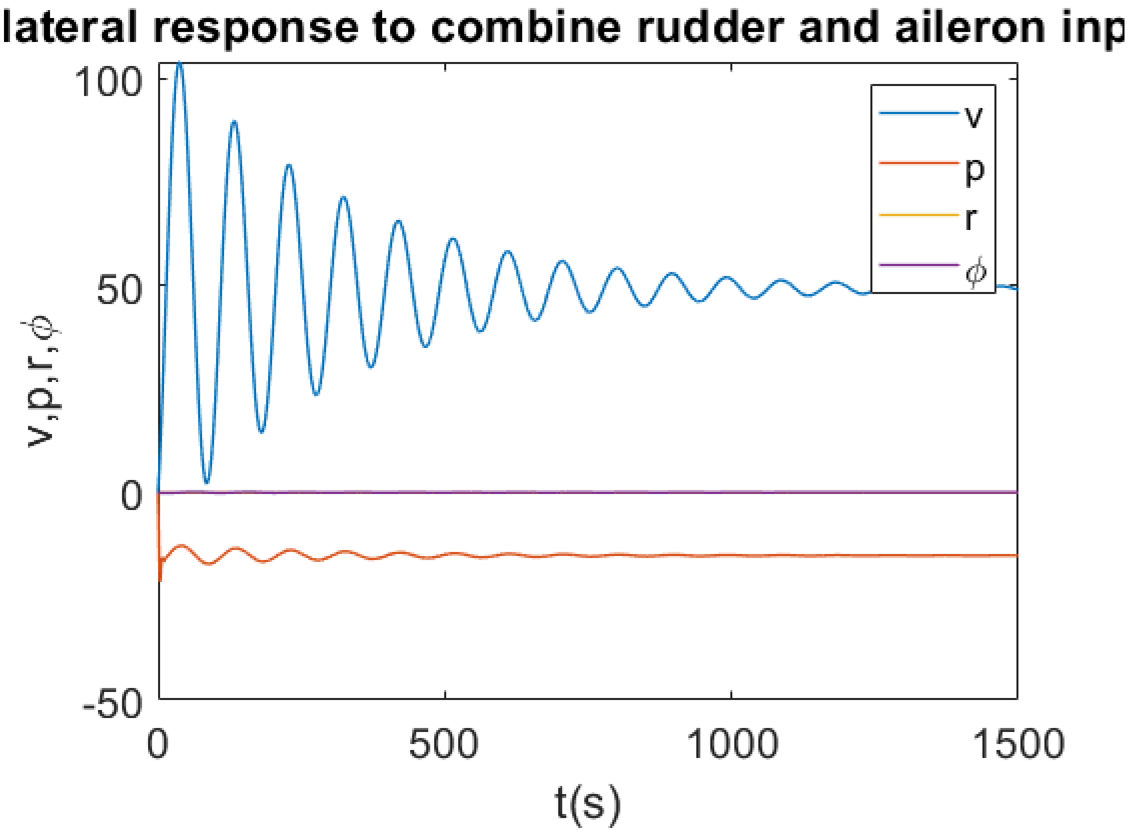
$\phi = 0.0125$ rad fine.

Still not stable, but the linearization is reasonable.

(won't go back to stable with fixed δ_a, δ_r).

```
close all; clc; clear all;
% Aircraft parameters
A = [-0.0069, 0.0140, 0, -32.3; -0.0906, -0.3151, 773.98, 0; 0.0001,
 -0.0010, -0.4285, 0; 0, 0, 1, 0]; % State matrix
B = [-0.0002, 9.66; -17.85, 0; -1.15, 0; 0, 0]; % Input matrix
u0 = 774;
% Step input for aileron and rudder
c = [pi/180;0.2322];
xinf = -A\B*c;
tf = 1500; dt = 1; nf = fix(tf/dt);
eAdt = expm(A*dt);
x = zeros(4,1); dx = x-xinf; xt = x;
disp(x);
for it = 1:nf
    dx = eAdt * dx;
    x = xinf + dx;
    xt = [xt, x];
end
figure(1),hold off
plot((0:nf)*dt, xt, 'linewidth', 1);
set(gca, 'fontsize', 16), xlabel('t(s)'), ylabel('v,p,r,\phi');
title('lateral response to combine rudder and aileron input');
legend('v','p','r','\phi');
```

```
     0
     0
     0
     0
```



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    dx = eAdt * dx;
    x = xinf + dx;
    xt = [xt, x];
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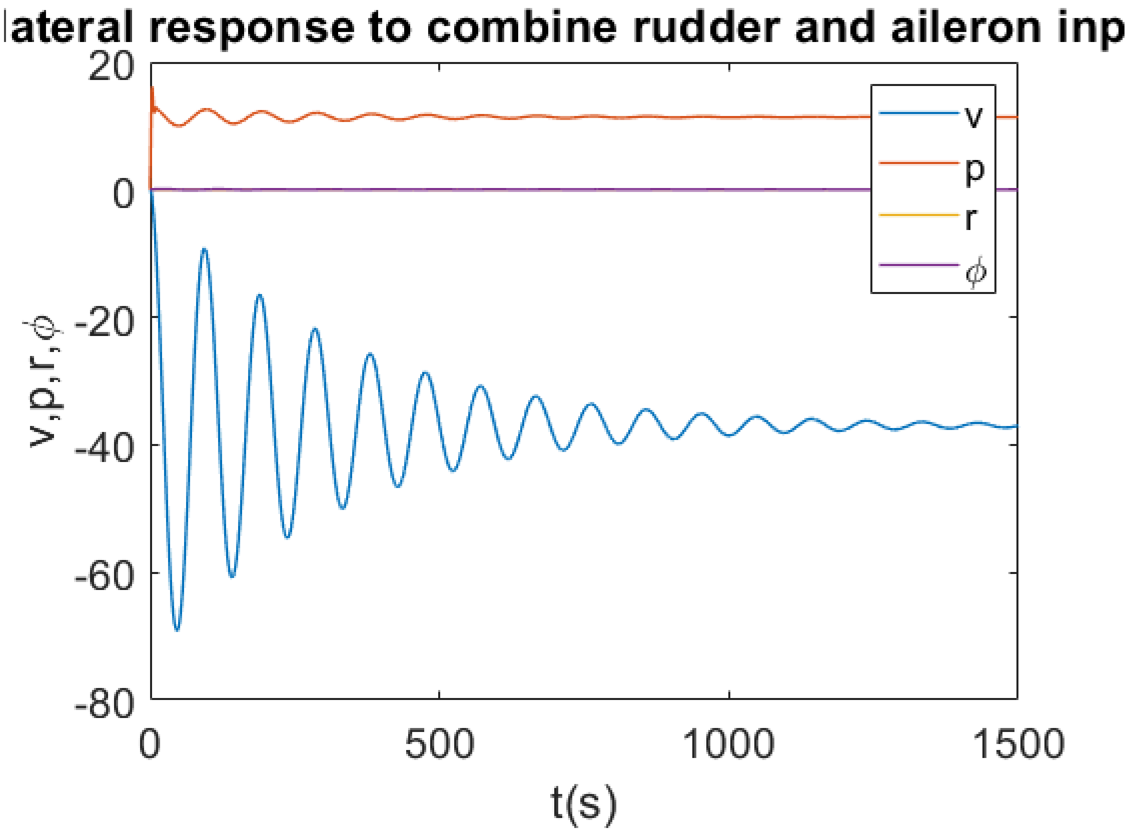
```

```
     0
     0
     0
     0
```



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