CS 182: Problem Set 4

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September 1, 2018

Introduction: Welcome to the fourth official homework for CS182! As you are hopefully already aware, this PDF comprises the written component of the second problem set. In addition to solving the problems found below, you will also need to complete the coding part of the assignment, found in the Github repo. Finally, we'd like to remind you that while you are allowed a partner for the coding part of the assignment, you are **NOT** allowed a partner for this and all future written components. All written work should be yours and yours alone. This being said, in addition to being able to ask questions at office hours, you are allowed to discuss questions with fellow classmates, provided 1) you note the people with whom you collaborated, and 2) you **DO NOT** copy any answers. Please write up the solutions to all problems independently.

Collaborators:

Problem 1 (The Coin Problem) – 3 Points: Ankit and Aidi decide to play a coin game to show how we can use HMMs for sequence analysis problems. Aidi tosses first, then they take turns based on rules described below. The game finishes when the subsequence "HTH" appears, and whoever last flips the coin wins. Each player can flip the coin for multiple turns in a row, and the rules for stopping and switching to the other partner are as follows:

1. Every time Aidi flips the coin, she also flips an extra unfair coin (P(H) = 0.3). She stops if the extra unfair coin lands heads. Otherwise, she keeps flipping the fair and extra biased coin (at the same time). The flips of the extra coin are not recorded.
2. Every time Ankit flips the coin, he only flips the fair coin until H appears (and all flips are recorded).

You're given a sequence of recorded coin fips. You'd like to infer the winner and the flips of each player.

Describe an HMM to model this game (draw a diagram with nodes rep and edges/arrows).

Hint: Make sure to draw an HMM model! **NOT** an FSM!

Solution 1:

Problem 2 (Typing Simulation) – 9 Points: For this problem, you will be playing a typing simulation. Let random variable E represent the observed key press, and X represent the hidden (intended) key press. We have a language with 4 letters (A, B, C, D), and a keyboard arranged as a circle.

A	B
C	D

At any time, the probability of hitting the intended key is 50%, and the probability of hitting the neighboring keys is 25%. For example, $P(E|X = \text{B})$:

0.25	0.5
0	0.25

We will construct a filtering model for constructing the belief state for this problem.

(a) (1 Point) Assuming a uniform prior distribution, calculate the condition probability table (CPT) of $P(X = x|E = e)$ for all x and e.

(b) (2 Points) Now let the prior distribution be:

x	P(X=x)
A	0.4
B	0.2
C	0.1
D	0.3

Calculate the CPT $P(X = x|E = e)$ for all x and e.

(c) (3 Points) Consider the following transition model for $P(X'|X)$:

	A'	B'	C'	D'
Begin	1	0	0	0
A	0.5	0.5	0	0
B	0	0.5	0.5	0
C	0.5	0	0	0.5
D	0.25	0.25	0.25	0.25

For this problem we are concerned with true (hidden) state sequences, as opposed to observations. What is the probability under this model of the sequence of letters "A B B C D"? How about "A A B A"? What is $P(X_3 = x|X_1 = \text{A}, X_2 = \text{B})$ for all x?

(d) (3 Points) Finally we consider the full filtering problem in which we compute $P(X_n|E_1, \ldots, E_n)$. Let "A B B C D" be the sequence of observed key strokes. What is the current belief state of the model? That is compute $P(X_n = x|E_1 = \text{A}, E_2 = \text{B}, E_3 = \text{B}, E_4 = \text{C}, E_5 = \text{D})$ for all x and $n = 2, 3, 4, 5$.

Hint:

$$P(X_n|E_1, ..., E_n) \propto P(E_n|X_n) \sum_{x_{n-1}} P(X_n|x_{n-1})B(x_{n-1})$$

Solution 2:

Problem 3 (Robotic Motion Planning) – 3 Points: Describe using pseudocode an RRT-based planning algorithm that uses more than two trees. Make sure to consider issues such as the maximum number of allowable trees, when to start a tree, and when to attempt connections between trees.

What are the types of problems for which this algorithm would perform better than RRT or bi-directional RRT?

Solution 3: