Homework 4

Thursday, April 13, 2023 11:29 AM

4.8.1 $p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$ 4.2

$(1+e^{\beta_0+\beta_1 x})\, p(x) = e^{\beta_0 + \beta_1 x}$

$p(x) + p(x)e^{\beta_0 + \beta_1 x} = e^{\beta_0 + \beta_1 x}$

$p(x) = e^{\beta_0 + \beta_1 x} - p(x)e^{\beta_0 + \beta_1 x}$

$p(x) = e^{\beta_0 + \beta_1 x}(1 - p(x))$

$e^{\beta_0 + \beta_1 x} = \frac{p(x)}{1 - p(x)}$ 4.3

HENCE EQ 4.2 = EQ 4.3

4.8.2 $p_k(x) = \frac{\pi_k \left(\frac{1}{\sqrt{2\pi}\sigma}\right) e^{-\frac{1}{2\sigma^2}(x-\mu_k)^2}}{\sum_{l=1}^{K} \pi_l \left(\frac{1}{\sqrt{2\pi}\sigma}\right) e^{-\frac{1}{2\sigma^2}(x-\mu_l)^2}}$

$= \frac{\pi_k e^{-\frac{1}{2\sigma^2}(x-\mu_k)^2}}{\sum_{l=1}^{K} \pi_l e^{-\frac{1}{2\sigma^2}(x-\mu_l)^2}}$

$\log p_k(x) = \log(\pi_k) - \frac{1}{2\sigma^2}(x-\mu_k)^2 - \log \sum_{l=1}^{K} \pi_l e^{-\frac{1}{2\sigma^2}(x-\mu_l)^2}$

$= \log(\pi_k) - \frac{1}{2\sigma^2}(x^2 - x\mu_k + \mu_k^2)$

$\delta_k(x) = \frac{x\mu_k}{2\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$

b. $X_1 \sim Pois(\lambda_1)$ $X_2 \sim Pois(\lambda_2)$

$\delta_k(x) = \frac{x\lambda_k}{2\lambda_k^2} - \frac{\lambda_k^2}{2\lambda_k^2} + \log(\pi_k)$

$\hat{\delta}_k(x) = \frac{x}{2\hat{\lambda}_k} - \frac{1}{2} + \log(\hat{\pi}_k)$ $\hat{\pi}_k = \frac{1}{n}$

$= \frac{x}{2\hat{\lambda}_k} - \frac{1}{2} + \log\left(\frac{1}{n}\right)$