# Reduced-complexity Superbaud Timing Recovery for PAM-based Multi-h CPM Receivers

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***Abstract*—Continuous phase modulation (CPM) is a widely used modulation scheme in communication systems. However, difficulties arise with the design of CPM receivers, due to the nonlinear nature of CPM. One popular solution to this problem is to linearize CPM with pulse amplitude modulation (PAM) representation. In this paper, a reduced-complexity superbaud timing recovery method is proposed for PAM-based multi-h CPM receivers. The proposed method is non-data-aided, feedforward, and suitable for M-ary, multi-h CPM in general. The novelties of the proposed method are twofold. On one hand, the proposed method is reduced-complexity for PAM-based receivers, because it shares the same match filters with the PAM-based detectors. On the other hand, it is shown that the proposed method is able to outperform the existing method with some modulation schemes. Therefore, the proposed method provides an important synchronization component for PAM-based multi-h CPM receivers.**

## I. Introduction

Continuous phase modulation (CPM) is a joint bandwidth and power efficient modulation scheme with a constant envelope. It is particularly appealing in radio channels with nonlinear amplifiers as CPM allows us to exploit the full amplifier power without any backoff. An important generalization of CPM is the so-called multi-h modulation, which differs from the ordinary single-h format in that it uses a set of modulation indices in a cyclic manner [1]. This results in delayed merging of neighboring phase trellis paths and, ultimately, in improved error performance [2]. Optimal detection of multi-h CPM does not only require symbol timing information, but also requires the knowledge of the beginning of each index cycle, which is referred to as superbaud timing.

With CPM, one important problem is that difficulties arise with the design of CPM receivers due to the nonlinear nature of CPM. One popular method of dealing with the challenges has been the pulse amplitude modulation (PAM) representation [3]-[5]. The PAM representation has been applied to the design of reduced-complexity detectors [6]-[9], synchronizers [9]-[12], and equalizers [13]-[15].

In [10] and [11], PAM-based reduced-complexity methods have been proposed on data-aided and non-data-aided symbol timing recovery for CPM signals, respectively. Here, we further extend the method presented in [11] into the superbaud timing recovery for M-ary, multi-h CPM signals, and propose a non-data-aided reduced-complexity superbaud timing recovery method for PAM-based CPM receivers. Several superbaud synchronization schemes have been proposed in previous literatures. In [16], the received signal is fed to a suitable nonlinearity to derive the superbaud timing information. However, this method fails to work when the signal-to-noise rate (SNR) is beyond 8dB. In [17], superbaud information is derived by two decision-directed loops. But the limitation of this method is that the tracking loops are prone to false loops when operating with multi-level partial response pulse. In [18], a non-data-aided superbaud timing method is proposed. This method works well at low SNR and is suitable for M-ary, multi-h CPM in general. However, this method needs additional filters to obtain sufficient statistics for synchronization, which increases the complexity of the receivers.

Due to the fact that they are both non-data-aided, feedforward, and suitable for M-ary, multi-h CPM in general, the method presented in [18] is employed as a comparison to investigate the performance of the proposed method. Compared with [18], the merits of the proposed method are twofold. On one hand, unlike [18] which needs additional filters, the proposed method shares the match filter (MF) bank with PAM-based reduced-complexity detectors [6]-[9]. In other words, the proposed method is a reduced-complexity method for PAM-based CPM receivers. On the other hand, numerical results show that the proposed method is able to outperform [18] with some modulation schemes. Moreover, it is shown that the proposed method works independent of phase synchronization and can be applied to both PAM-based coherent and noncoherent receivers.

Notation description: $E(\bullet)$ is the expectation operator. The estimated and hypothesized value of $x$ is referred to $\hat{x}$ and $\tilde{x}$, respectively. $(\bullet)^*$ denotes the complex conjugate, and $\otimes$ denotes the convolution operation. $\mathrm{Re}(x)$ denotes the real part of $x$, $\mathrm{int}(x)$ denotes the integer part of $x$, and $|x|$ denotes the absolute value of $x$.

## II. System Model

### A. Signal model

The complex envelope of M-ary, multi-h CPM signal is described as

$$s(t;\alpha) = \sqrt{\frac{2E_s}{T_s}} \exp\{j\varphi(t;\alpha)\} \tag{1}$$

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where $E_s$ is the symbol energy, $T_s$ is the symbol period and $\alpha = \{\alpha_i\}$ belong to an M-ary alphabet $\{\pm 1, \pm 3, \cdots, \pm(M-1)\}$. The phase of the signal is given by

$$\varphi(t;\alpha) = 2\pi \sum_i \alpha_i h_{\underline{i}} q(t - iT_s) \tag{2}$$

where $\{h_{\underline{i}}\}_{i=0}^{N_h-1}$ is a set of $N_h$ modulation indices, and $\underline{i} = i \bmod N_h$. The phase pulse $q(t)$ is the time-integral of a frequency pulse with a duration $LT_s$ and an area of 1/2, where $L$ is the memory length of CPM. $L = 1$ indicates that the signal is full response and $L > 1$ means that the signal is partial response. In numerical examples discussed later, we consider two shapes of frequency pulse, i.e., length-$LT_s$ rectangular (LREC) and length-$LT_s$ raised-cosine (LRC).

Using the PAM-based model for M-ary, multi-h CPM, we describe the right hand of Equation (1) as a superposition of PAM waveforms

$$s(t;\alpha) \cong \sqrt{\frac{2E_s}{T_s}} \sum_{k=0}^{N-1} \sum_i b_{k,i} g_{k,\underline{i}}(t - iT_s) \tag{3}$$

where $\{b_{k,i}\}$ are the pseudo-symbols which are a nonlinear function of the real symbol sequence $\{\alpha_i\}$, $\{g_{k,\underline{i}}(t)\}$ is a set of $N_h \cdot N$ PAM signal pulses, $N = 2^{\log_2 \mathrm{M}(L-1)}(\mathrm{M}-1)$ when M is an integer power of 2. A detailed description of such pseudo-symbols and the PAM pulses can be found in [3]-[5].

An important property of PAM representation is that only a few PAM pulses count for the most energy of the signal [3]-[5]. Exploiting this property, we can discard the less significant pulses with optimal approximation technique [3]-[5]. As a result, Equation (3) turns out to be the reduced-complexity form

$$s(t;\alpha) \cong \sqrt{\frac{2E_s}{T_s}} \sum_{k=0}^{K-1} \sum_i b_{k,i} h_{k,\underline{i}}(t - iT_s) \tag{4}$$

where $\{h_{k,\underline{i}}(t)\}$ are is the approximated PAM signal pulses, the number of $\{h_{k,\underline{i}}(t)\}$ is $N_h \cdot K$, and $K \leq N$.

### B. Channel model

Here, the channel is modeled as the additive white Gaussian noise (AWGN) channel undergoing timing and phase offsets. Consequently, the signal observed at the receiver can be described as

$$r(t) = s(t - \tau - \eta T_s; \alpha) e^{j\theta} + w(t) \tag{5}$$

where $w(t)$ is the complex-valued additive white Gaussian noise with zero mean and power spectral density $N_0$. The variables $\tau \in [0, T_s)$, $\eta \in \{0, 1, \cdots, N_h - 1\}$ and $\theta \in [0, 2\pi)$ represent the symbol timing, superbaud timing and phase offset, respectively.

## III. Proposed PAM-based Superbaud Timing Recovery Method

In Subsection A, the PAM-based maximum likelihood sequence detection (MLSD) is first reviewed. Then the PAM-based superbaud timing recovery method is presented in Subsection B.

### A. PAM-based MLSD

The symbol sequence $\alpha$ can be recovered using MLSD. In [6], it was shown that the likelihood function of the hypothesized symbol sequence $\tilde{\alpha}$ over the observation $0 \leq t \leq L_0 T_s$ is

$$\Lambda(\tilde{\alpha}, \theta, \tau, \eta) = \mathrm{Re}[\int_0^{L_0 T_s} r(t) s^*(t - \tau - \eta T_s; \tilde{\alpha}) e^{-j\theta} dt] \tag{6}$$

where we assume for the moment that $\tau$, $\eta$ and $\theta$ are known. Inserting Equation (4) into the expression of Equation (6), we have

$$\Lambda(\{\tilde{b}_{k,i}\}, \theta, \tau, \eta) \cong \mathrm{Re}\{\int_0^{L_0 T_s} [r(t) e^{-j\theta} \sum_{k=0}^{K-1} \sum_i \tilde{b}_{k,i}^* h_{k,\underline{i}}(t - iT_s - \tau - \eta T_s)] dt\} \tag{7}$$

Changing the order of integration and summation, we have

$$\Lambda(\{\tilde{b}_{k,i}\}, \theta, \tau, \eta) \cong \sum_{i=0}^{L_0-1} \mathrm{Re}[y_i(\{\tilde{b}_{k,i}\}, \theta, \tau, \eta)] \tag{8}$$

which is a function that can be maximized efficiently using the Viterbi algorithm (VA). The metric increment is defined as [6]

$$y_i(\{\tilde{b}_{k,i}\}, \theta, \tau, \eta) = e^{-j\theta} \sum_{k=0}^{K-1} \tilde{b}_{k,i}^* x_{k,i}(\tau + \eta T_s) \tag{9}$$

with

$$x_{k,i}(\tau + \eta T_s) = r(t) \otimes h_{k,\underline{i}}(-t)_{|t = \tau + iT_s + \eta T_s} \tag{10}$$

Here, the time-reversed PAM pulses $\{h_{k,\underline{i}}(-t)\}$ serve as the impulse response of the MF bank, and $x_{k,i}(\tau + \eta T_s)$ is the outputs of the MF bank sampled at $\tau + iT_s + \eta T_s$.

### B. PAM-based superbaud timing estimation

In this subsection, we are going to derive the proposed PAM-based superbaud timing recovery method. Here, we assume that $\eta$, $\theta$ are unknown, and $\tau$ has been perfect estimated by symbol timing estimator [10] [11]. Consequently, Equation (8) turns out to be

$$\Lambda(\{\tilde{b}_{k,i}\}, \tilde{\theta}, \tilde{\eta}) = \mathrm{Re}[e^{-j\tilde{\theta}} \sum_{k=0}^{K-1} \tilde{b}_{k,i}^* x_{k,i}(\hat{\tau} + \tilde{\eta} T_s)] \tag{11}$$

Averaging the likelihood function with respect to $\theta$, we have

$$\Lambda(\{\tilde{b}_{k,i}\}, \tilde{\eta}) = I_0[\sum_{k=0}^{K-1} \tilde{b}_{k,i}^* x_{k,i}(\hat{\tau} + \tilde{\eta} T_s)] \tag{12}$$

where $I_0$ is the modified zero-order Bessel function of the first kind. Here we assume a low signal to noise ratio and make the approximation, i.e., $I_0(\lambda) \cong 1 + \lambda^2/4$. As a result, Equation (12) turns out to be

$$\Lambda(\{\tilde{b}_{k,i}\}, \tilde{\eta}) = \left| \sum_{k=0}^{K-1} \tilde{b}_{k,i}^* x_{k,i}(\hat{\tau} + \tilde{\eta} T_s) \right|^2 \tag{13}$$

Equation (13) can be rearranged in the following form

$$\Lambda(\{\tilde{b}_{k,i}\},\tilde{\eta}) \cong \sum_{i=0}^{L_0-1}\sum_{k_1=0}^{K-1}\sum_{k_2=0}^{K-1}\sum_{p=-i}^{L_0-1-i}\tilde{b}^*_{k_1,i}\tilde{b}_{k_2,i+p} \cdot x_{k_1,i}(\hat{\tau}+\tilde{\eta}T_s)x^*_{k_2,i+p}(\hat{\tau}+\tilde{\eta}T_s) \tag{14}$$

Averaging the likelihood function with respect to $\{b_{k,i}\}$, we have

$$\Lambda(\tilde{\eta}) = \sum_{i=0}^{L_0-1}\sum_{k_1=0}^{K-1}\sum_{k_2=0}^{K-1}\sum_{p=-i}^{L_0-1-i} A_{k_1k_2}(p)x_{k_1,i}(\hat{\tau}+\tilde{\eta}T_s)x^*_{k_2,i+p}(\hat{\tau}+\tilde{\eta}T_s) \tag{15}$$

where $A_{k_1,k_2}(p) = E\{b^*_{k_1,i}b_{k_2,i+p}\}$ are the autocorrelation of the pseudo-symbols. The expression and property of this autocorrelation functions are well-known and the interested reader can refer to [3]-[5] for details. Particularly, it is shown that absolute value of the autocorrelation functions is a decreasing function of $|p|$ [3]-[5]. This property enables us to further reduce the complexity of the proposed method by truncating the likelihood function with respect to $p$. Consequently, Equation (15) turns out to be

$$\Lambda(\tilde{\eta}) \cong \sum_{i=0}^{L_0-1}\sum_{k_1=0}^{K-1}\sum_{k_2=0}^{K-1}\sum_{p=-d}^{d} A_{k_1k_2}(p)x_{k_1,i}(\hat{\tau}+\tilde{\eta}T_s)x^*_{k_2,i+p}(\hat{\tau}+\tilde{\eta}T_s) \tag{16}$$

where $d$ is a design parameter. We can infer that the performance of the proposed method will improve as $d$ increases. This inference is further verified in numerical simulations.

Finally, the maximum likelihood estimation of $\tilde{\eta}$ is the location of the maximum of $\Lambda(\tilde{\eta})$ as $\eta$ varies in the set $\mathrm{H} = \{0,1,2,\cdots,N_h-1\}$, i.e.,

$$\hat{\eta} = \arg\max_{\tilde{\eta}\in\mathrm{H}}\{\Lambda(\tilde{\eta})\} \tag{17}$$

Application of Equation (17) provides a superbaud estimate every $L_0T_s$ seconds. The question then arises of how to put these estimates together. To this end, a postprocessing scheme proposed in [18] is used as the solution and described as follows.

Suppose that we are at time $n$ and have the rough estimates $\hat{\eta}(l)(l=0,1,\cdots,n)$ at our disposal, along with a final estimate $\hat{\hat{\eta}}(n)$. We want to compute $\hat{\hat{\eta}}(n+1)$ as next $\hat{\eta}(n+1)$ comes in. For this purpose, we introduce a counter $C(n)$ gives an indication of the reliability of $\hat{\hat{\eta}}(n)$. The higher $C(n)$ is, the more trusty $\hat{\hat{\eta}}(n)$ is. Moreover, the maximum value of $C(n)$ is set to be $N_c$. In summary, the algorithm runs as follows.

Initial condition: $\hat{\hat{\eta}}(0) = \hat{\eta}(0)$, $C(0) = 0$;

Generic step ($n \geq 0$): If $\hat{\eta}(n+1) = \hat{\hat{\eta}}(n)$, set $\hat{\hat{\eta}}(n+1) = \hat{\hat{\eta}}(n)$ and $C(n+1) = C(n)+1$ (unless $C(n) = N_c - 1$);

If $\hat{\eta}(n+1) \neq \hat{\hat{\eta}}(n)$, and $C(n) > 0$, set $\hat{\hat{\eta}}(n+1) = \hat{\hat{\eta}}(n)$ and $C(n+1) = C(n) - 1$;

If $\hat{\eta}(n+1) \neq \hat{\hat{\eta}}(n)$, and $C(n) = 0$, set $\hat{\hat{\eta}}(n+1) = \hat{\eta}(n+1)$ and $C(n+1) = 0$.

Define $\rho$ as the probability that superbaud timing estimate $\eta$ obtained over the observation length $L_0T_s$ is in error. According to [18], the final error probability is

$$P(\hat{\hat{\eta}} \neq \eta) \approx \rho^{N_c} \tag{18}$$

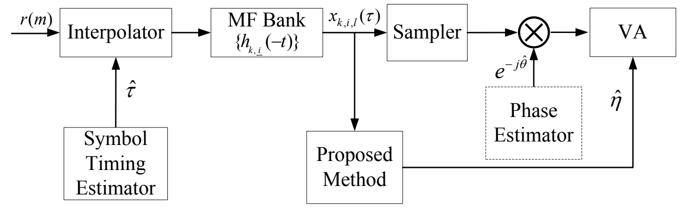


Fig. 1. Digital implementation of the PAM-based superbaud timing recovery system.

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON BETWEEN THE PROPOSED METHOD AND METHOD PRESENTED IN [18].

| Algorithm, Modulation schemes | Additions per symbol | Multiplications per symbol |
|---|---|---|
| Proposed Method, M=2,L=1,$N_h$=3 schemes | 8 | 12 |
| Method presented in [18], M=2,L=1,$N_h$=3 schemes | 1107 | 2214 |
| Proposed Method, M=4,L=2,$N_h$=2 schemes | 60 | 108 |
| Method presented in [18], M=4,L=2,$N_h$=2 schemes | 1618 | 3226 |

and average acquisition time is

$$L_{acq} = L_0(1-\rho) + 2L_0\rho(1-\rho) + 4L_0\rho^2(1-\rho)^2 + \cdots \tag{19}$$

Under normal operation conditions, $\rho$ is much smaller than 1 and thus $L_{acq}$ is only slightly large than $L_0$.

## IV. Digital Implementation

Fig. 1 shows a digital implementation of the proposed PAM-based superbaud timing recovery system. First, the discrete signal $r(m)$ is sampled from $r(t)$ at an oversample rate $Q$ samples per symbol. The most recent estimate of $\tau$ is generated by the symbol timing estimator [10] [11], and sent to a sample interpolator to synchronize the samples. Then the synchronized samples are fed to the MF bank. The output of MF bank are denoted as $x_{k,i,l}(\tau+\eta T_s)$, where $i = \mathrm{int}(m/Q)$ and $l = m \bmod Q$.

$x_{k,i}(\tau+\eta T_s)$ can be obtained by proper combinations of $x_{k,i,l}(\tau+\eta T_s)$ and then used to generate the superbaud timing estimate. At the same time, the MF outputs $x_{k,i,l}(\tau+\eta T_s)$ are sampled at the symbol rate and then sent to PAM-based viterbi algorithm (VA) [6]-[9] to obtain branch metrics.

It is worth noting that, as the same MF outputs are utilized in both the proposed method and PAM-based detectors, the proposed method is reduced-complexity in nature, i.e., it does not need additional front-end filters when applied to the PAM-based CPM receivers. Moreover, it is seen that the proposed method can be applied to both coherent and noncoherent CPM receivers as it works independent of phase recovery.

## V. Computational Complexity

In Table I, the computational complexity of the proposed method and the method presented in [18] are compared with two typical modulation schemes, i.e., M=2, L=1, $N_h$=3

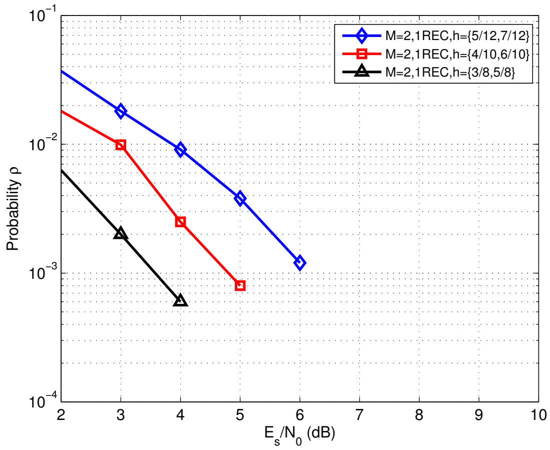


Fig. 2. Effect of modulation indices on the performance of the proposed method.

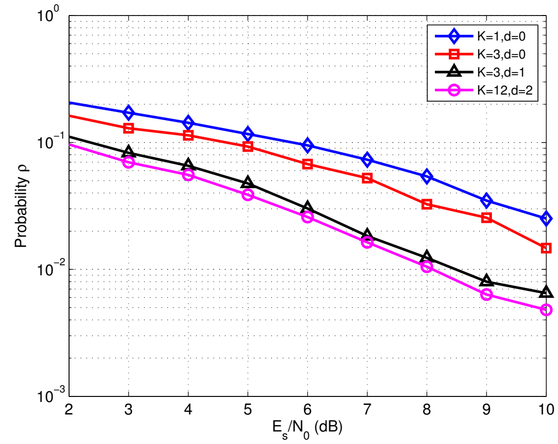


Fig. 3. Effects of $K$ and $d$ on the performance of the proposed method.

schemes and M=4, L=2, $N_h$=2 schemes. Here, the oversample rate $Q$ is set to be 8, and the multiplication and additions referred are all real. In consideration of the reduced-complexity nature of the method, MFs are not taken into account when we calculate the complexity of the proposed method. As shown in the table, compared with [18], the proposed method reduces the computational complexity remarkably when applied to PAM-based CPM receivers.

## VI. Numerical Results

In this section, the performance of the proposed method is evaluated by Monte-Carlo simulations. We first investigate the effects of parameters and signal types on the performance of the proposed method, and then gives the performance comparisons between the proposed method and the method presented in [18]. Here, the performance of the superbaud timing estimation is evaluated by the error probability $\rho$ versus $E_s/N_0$. The parameter $L_h$ is set to be 64 in simulations, where $L_h = L_0/N_h$. Furthermore, unless stated otherwise, the number of MFs $K$ is set to be 1 and 3 for M=2 schemes and M=4 schemes, respectively, and the parameter $d$ is set to be 0 and 1 for $L = 1$ and $L > 1$ schemes, respectively in the following.

In Fig. 2, we investigate the effect of the modulation indices on the performance of the proposed method. The modulation formats are M=2, 1REC schemes. From the figure, it is seen that $h_d$ has strong effect on the performance of the estimation, where $h_d$ is defined as the difference between the modulation indices, e.g., $h_d = h_{\underline{0}} - h_{\underline{1}}$ for $h = \{h_{\underline{0}}, h_{\underline{1}}\}$. As it increases, the performance improves remarkably. This result can be explained by an extreme counterexample, i.e., if $h_d \to 0$, the signal turns out to be single-h and thus it becomes impossible to derive the superbaud timing information.

In Fig. 3, we investigate the effects of the numbers of MFs $K$ and the parameter $d$ on the performance of the proposed method. Here, the modulation formats are M=4, 2RC scheme with $h = \{2/8, 3/8\}$. From the figure, it is seen that

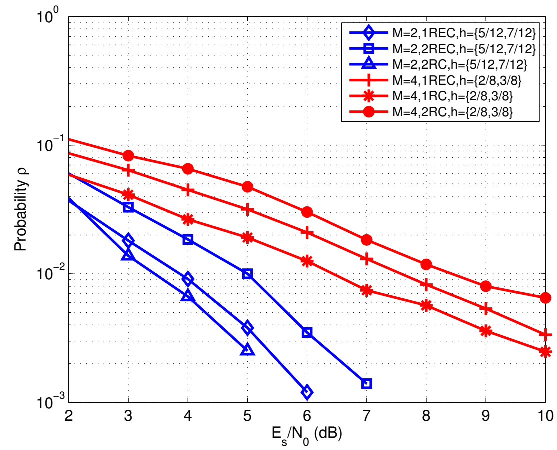


Fig. 4. Effect of frequency pulse shapes on the performance of the proposed method.

- The performance of the proposed method improves as $K$ increases. This phenomenon corresponds to what has been found with PAM-based detectors [6]-[9] and symbol timing estimator [10][11].
- As we have expected, the performance of the proposed method is an decreasing function of $d$. This result also corresponds to what has been found in [11].
- Minor improvement is seen if $K$ and $d$ exceed certain values. Moreover, the complexity of the proposed method increases as $K$ and $d$ grows. Therefore, we can infer that optimal values of $K$ and $d$ exist for a special modulation scheme, which minimize the complexity while satisfactory performance is still maintained.

In Fig. 4, we investigate the effect of frequency pulse shapes on the performance of the proposed method. Here, the modulation formats are M=2, $h = \{5/12, 7/12\}$ schemes, and M=4, $h = \{2/8, 3/8\}$ schemes. From the figure, it is seen that

- The performance of the proposed method is a decreasing function of frequency pulse length $L$. This phenomenon

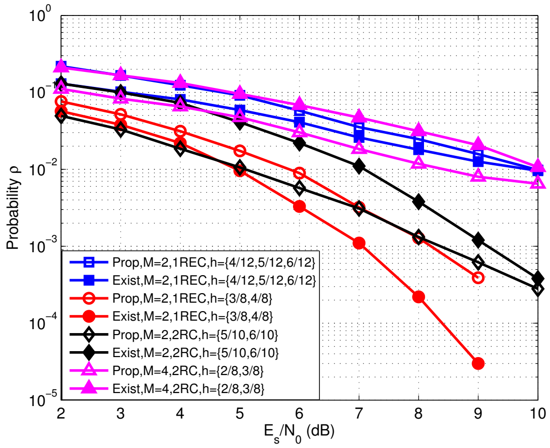


Fig. 5. Performance comparisons between the proposed method and the method presented in [18].

can be explained as follows. As the likelihood function is averaged over data, non-data-aided timing recovery methods suffer self-noises from the intersymbol interference (ISI) of the signals. Thus, limitations arise with partial response schemes with which the ISI is more severe.

- Similarly, the performance with LRC schemes is better than LREC schemes, due to the fact that ISI is more severe with LREC schemes than with LRC schemes.

Fig. 5 compares the performances of the proposed method and the method presented in [18] with several modulation formats, i.e., M=2, 1REC, $h = \{4/12, 5/12, 6/12\}$ scheme, M=2, 1REC, $h = \{3/8, 4/8\}$ scheme, M=2, 2RC, $h = \{5/10, 6/10\}$ scheme, and M=4, 2RC, $h = \{2/8, 3/8\}$ scheme. In the figures the proposed method is called Prop for short, while the method presented in [18] is called Exist for short. From the figure, it is seen that

- As same as the method presented in [18], the proposed method is also suitable for M-ary, multi-h CPM in general, including partial response schemes.
- The proposed method is able to outperform [18] with some modulation schemes, e.g., M=2, 2RC, $h = \{5/10, 6/10\}$ scheme and M=4, 2RC, $h = \{2/8, 3/8\}$ scheme. This phenomenon can be explained as follows. On one hand, as we have mentioned, non-data-aided methods suffer self-noises from the ISI of the signals when the likelihood function is averaged over data. On the other hand, the proposed method and the method presented in [18] average the likelihood function over data in different manners, i.e., for the proposed method the likelihood function is averaged over pseudo-symbols $\{b_{k,i}\}$, while in [18] it is averaged over actual symbols $\{\alpha_i\}$. As a result, the two methods suffer self-noises in different manners and thus it is possible that the proposed method suffers less self-noises than [18] with some modulation schemes.

## VII. Conclusion

In this paper, we propose a reduced-complexity non-data-aided superbaud timing recovery method for PAM-based multi-h CPM receivers. Compared with the existing method presented in [18], the proposed method is reduced-complexity in nature, due to the fact that it shares the MF bank with PAM-based CPM detectors. Furthermore, numerical results show that the performance of the proposed method is better than the existing method presented in [18] with some modulation schemes. In conclusion, the proposed method provides an important synchronization component for the PAM-based CPM receivers.

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