Symbol and Superbaud Timing Recovery in Multi-H Continuous-Phase Modulation

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***Abstract*— This letter is concerned with the estimation of two synchronization parameters that play a key role in multi-h continuous-phase modulation receivers—the ordinary symbol timing phase and the so-called superbaud timing phase. The recovery of symbol and superbaud timing is implemented by means of feedforward nondata-aided algorithms. The novelty of the proposed method is that it can be applied to any modulation format, either full or partial response, with binary or multilevel symbols and with arbitrary modulation indices.**

***Index Terms*—Continuous phase, timing.**

I. Introduction

CONTINUOUS-PHASE modulation (CPM) is a signaling method with good power and bandwidth efficiency [1]. Furthermore, it generates constant envelope waveforms which are attractive in applications involving nonlinear amplifiers. An interesting generalization of CPM is the so-called multi-h modulation, which differs from the ordinary single-h format in that it uses a set of H modulation indices in a cyclic manner [2]. This results in delayed merging of neighboring phase trellis paths and, ultimately, in improved error performance [3]. Optimum coherent detection of multi-h CPM requires knowledge of the beginning of each index cycle, which is referred to as *superbaud* timing [4]. A coherent multi-h receiver also requires symbol timing and carrier synchronization.

Two symbol and superbaud synchronization schemes have been proposed in the literature. In [4] the received signal is fed to a suitable nonlinearity to generate tones separated by the superbaud frequency. Superbaud timing is then derived using a pair of phase-locked loops, a mixer, and a low-pass filter. Unfortunately, false locks are likely to occur at low SNR.

Another method is described by Premji and Taylor in [5]. Here, carrier phase and symbol timing are tracked by two decision-directed loops. Superbaud information is implicitly derived taking advantage of the different loop behavior, depending on whether the receiver's index cycle is or is not "in phase" with the transmitter's. When it is "in phase," the loops have good tracking performance. In the "out of phase" condition (vice versa), they exhibit large errors and undergo cycle slips. As a slip corresponds to the advancement or retardation of the receiver's index cycle by one step, the slipping process eventually leads to superbaud acquisition. A limitation of Premji and Taylor method is that the tracking loops are prone to false locks when operating with multilevel partial response pulses.

In this letter we propose a maximum-likelihood (ML) based synchronization scheme in which symbol and superbaud timing are derived independently of the carrier phase. The algorithms have a feedforward structure and, as such, are free from false locks or other impairments associated with feedback systems.

II. Signal Model

The received signal is first passed through a low-pass filter and then sampled at some rate $1/T_s$ which is a multiple N of the (nominal) symbol rate $1/T$. We assume a rectangular filter with a bandwidth $B_F = 1/2T_s$ somewhat larger than the signal bandwidth B_s. The samples from the filter at the times $t = kT_s$ are given by

$$x(k) = \exp\left\{ j\left[\theta + 2\pi \sum_i h_{[i]}\alpha_i q(kT_s - iT - \eta T - \varepsilon T)\right]\right\} + n(k) \qquad (2.1)$$

where $q(t)$ is the *phase-shaping pulse* and $\{h_0, h_1, \cdots, h_{H-1}\}$ are the modulation indices. The notation $[i]$ means that i is taken modulo H. In numerical examples discussed later, we consider two shapes for the derivative of $q(t)$—rectangular and raised cosine of length LT. The former is referred to as LREC, the latter LRC. In (2.1), $\{\alpha_i\}$ is the transmitted symbol sequence with elements taken independently from the alphabet $\{\pm 1, \pm 3, \cdots, \pm(M-1)\}$ and θ is the channel phase shift. The symbol timing parameter ε is a real number in the range [−1/2, 1/2), whereas the superbaud timing parameter η is an integer belonging to the set $\{0, 1, \cdots, H-1\}$. The noise samples $n(k)$ are zero-mean independent Gaussian random variables with real and imaginary components of variance N_0/T_s. Our aim is to derive estimates of ε and η based on the samples $\{x(k);\, 0 \le k \le NL_0 - 1\}$, L_0 being the observation length expressed in symbol intervals.

III. Symbol Timing Estimation

A symbol timing estimator is easily derived by extending the methods developed in [6] for single-h CPM signals. It turns out that the estimate $\hat{\varepsilon}$ of ε has still the expression indicated in [6], except that the discrete-time filter $h_1(k)$ in [6] is now

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replaced by

$$\overline{h}_1(k) = \frac{1}{HT}\, e^{j\pi k/N} \sum_{\tilde{\eta}=0}^{H-1} \int_0^T F[-kT_s,\, u - \tilde{\eta}T] e^{j2\pi u/T}\, du \tag{3.1}$$

where

$$F[\Delta t, t] \triangleq \prod_{i=-\infty}^{+\infty} \frac{1}{M} \frac{\sin[2h_{[i]}\pi M q(\Delta t,\, t-iT)]}{\sin[2h_{[i]}\pi q(\Delta t,\, t-iT)]} \tag{3.2}$$

and $q(\Delta t,\, t) \triangleq q(t) - q(t-\Delta t)$. It is worth noting that the estimator is nondata-aided and independent of the carrier phase and superbaud timing.

IV. Superbaud Timing Detector

Turning our attention to the superbaud phase η, we assume that the symbol timing has already been estimated and, accordingly, we replace ε by $\hat{\varepsilon}$. Reasoning as in [6], the likelihood function for η is found to be

$$\Lambda(\tilde{\eta}) = \sum_{k_1=0}^{NL_0-1} \sum_{k_2=0}^{NL_0-1} x(k_1)x^*(k_2)F[(k_2-k_1)T_s,\; k_2T_s - \hat{\varepsilon}T - \tilde{\eta}T] \tag{4.1}$$

where $\tilde{\eta}$ indicates a tentative value for η. The ML estimate of η is the location of the maximum of $\Lambda(\tilde{\eta})$ as $\tilde{\eta}$ varies in the set $\mathcal{H} \triangleq (0,\, 1,\, 2,\, \cdots,\, H-1)$. From (4.1), after some manipulations, we obtain

$$\hat{\eta} = \arg\max_{\tilde{\eta}\in\mathcal{H}} \{\mathrm{Re}[W_1 e^{-j2\pi\hat{\varepsilon}/H} e^{-j2\pi\tilde{\eta}/H}]\} \tag{4.2}$$

where

$$W_1 \triangleq \sum_{k=0}^{NL_0-1} [x(k)e^{j2\pi k/NH}]^* z_1(k) \tag{4.3}$$

and $z_1(k)$ is the output of a filter with impulse response

$$p_1(k) = \frac{1}{HT} \int_0^T F[kT_s,\, u] e^{-j2\pi u/HT}\, du \tag{4.4}$$

when driven by $x(k)$.

Application of (4.4) provides a new superbaud estimate every L_0T seconds. The question then arises of how to put these estimates together. A possible solution is as follows. Suppose we are at time k and have the *rough* estimates $\hat{\eta}(l)(l = 0,\, 1,\, \cdots,\, k)$ at our disposal, along with a *final* superbaud estimate $\hat{\hat{\eta}}(k)$. We want to compute $\hat{\hat{\eta}}(k+1)$ as the next $\hat{\eta}(k+1)$ comes in. To this end, we introduce a counter whose content $C(k)$ gives an indication of the reliability of $\hat{\hat{\eta}}(k)$. The higher $C(k)$ is, the more trusty $\hat{\hat{\eta}}(k)$ is.

Let us distinguish between two cases, according to whether $\hat{\eta}(k+1) = \hat{\hat{\eta}}(k)$ or $\hat{\eta}(k+1) \neq \hat{\hat{\eta}}(k)$. The first instance is seen as a confirmation that $\hat{\hat{\eta}}(k)$ is reliable. Accordingly, we set $\hat{\hat{\eta}}(k+1) = \hat{\hat{\eta}}(k)$ and increment $C(k)$ by one [unless $C(k)$ is at its maximum, $N_c - 1$]. The second instance indicates that $\hat{\hat{\eta}}(k)$ is not that reliable, and our reaction depends on whether $C(k) > 0$ or $C(k) = 0$. If $C(k) > 0$, we set $\hat{\hat{\eta}}(k+1) = \hat{\hat{\eta}}(k)$ but we decrease $C(k)$ by one. If $C(k) = 0$ (vice versa), we set $\hat{\hat{\eta}}(k+1) = \hat{\eta}(k+1)$ and we leave $C(k)$ equal to zero. In summary, the algorithm runs as follows.

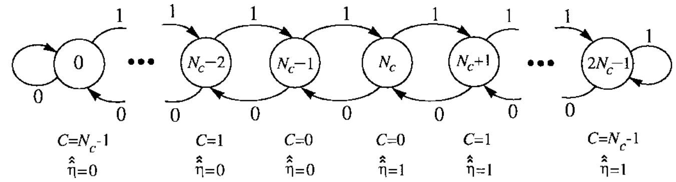

Fig. 1. State transition diagram for the postprocessing algorithm.

Initial condition: $\hat{\hat{\eta}}(0) = \hat{\eta}(0), C(0) = 0$

Generic step $(k \geq 0)$*:* If $\hat{\eta}(k+1) = \hat{\hat{\eta}}(k)$, set $\hat{\hat{\eta}}(k+1) = \hat{\hat{\eta}}(k)$ and $C(k+1) = C(k) + 1$ (unless $C(k) = N_c - 1$).

If $\hat{\eta}(k+1) \neq \hat{\hat{\eta}}(k)$ and $C(k) > 0$, set $\hat{\hat{\eta}}(k+1) = \hat{\hat{\eta}}(k)$ and $C(k+1) = C(k) - 1$.

If $\hat{\eta}(k+1) \neq \hat{\hat{\eta}}(k)$ and $C(k) = 0$, set $\hat{\hat{\eta}}(k+1) = \hat{\eta}(k+1)$ and $C(k+1) = 0$.

The above postprocessing scheme can be modeled as a homogeneous Markov chain. Taking the pair $C(k), \hat{\hat{\eta}}(k))$ as the state of the system, it is recognized that the chain has $N_c \times H$ states. The state transition diagram is reproduced in Fig. 1 for a 2-h modulation scheme. Edges are labeled by the value of $\hat{\eta}(k)$ (0 or 1). Note that the initial state is either $N_c - 1$ or N_c, depending on the value of $\hat{\eta}(0)$. The states 0 and $2N_c - 1$ represent the *normal* operating conditions when η equals 0 or 1, respectively.

V. Performance

All the conclusions about the performance of the symbol timing estimator in [6] still hold here for multi-h formats. The fact that the superbaud parameter is unknown has only marginal effects on the steady-state error variance of the timing estimator at high SNR. Consequently, we only concentrate on the superbaud detector. The main problem is to choose the detector parameters so that, in the steady-state, $\hat{\hat{\eta}}(k)$ is correct with high probability. To this end, we observe that the probability that $\hat{\hat{\eta}}(k)$ is in error, $P(\hat{\hat{\eta}} \neq \eta)$, is related to two distinct quantities: 1) the probability ρ that the rough estimates $\hat{\eta}(k)$ are in error; and 2) the size N_c of the counter. Unfortunately, no analytical method has been found to evaluate ρ. Thus, we shall rely on computer simulations.

Figs. 2 and 3 show the probability ρ versus the signal-to-noise ratio E_s/N_0 and $L_H \triangleq L_0/H$ for some modulation formats. In these results, the effects of the symbol timing errors on the superbaud estimator performance are taken into account. We see that, as expected, ρ is a decreasing function of E_s/N_0 and L_0. Also, it increases with the size of the symbol alphabet and the number of modulation indices. Note that satisfactory performance is obtained even with partial response signals.

To assess the influence of N_c, we first concentrate on a 2-h scheme. With no loss of generality, assume that the true superbaud phase is zero ($\eta = 0$). Under these conditions, a diagram of the type in Fig. 1 applies, in which the labels 0 and 1 to the edges can be replaced by the one-step transition probabilities, $1 - \rho$ and ρ, that $\hat{\eta}(k)$ is either 0 or 1. Let $\pi(n)$ be the probability that the system occupies the state $n(0 \leq n \leq 2N_c - 1)$ at equilibrium. Then, the probability

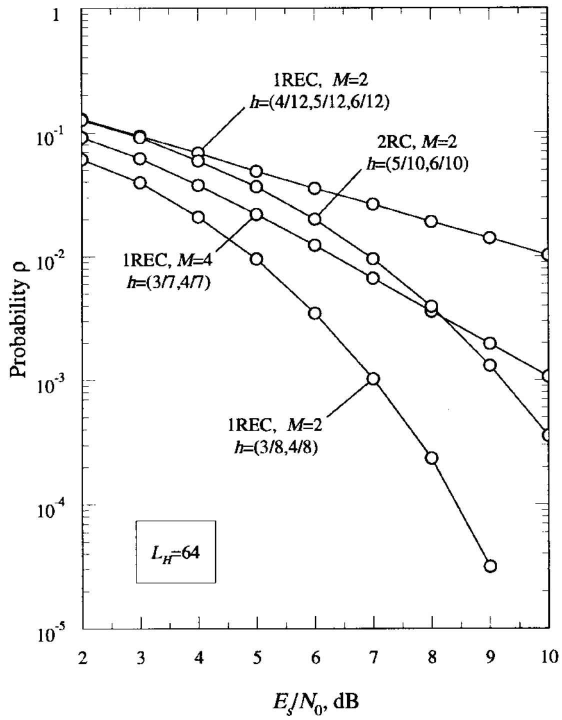

Fig. 2. Probability ρ versus E_s/N_0 for $L_H = 64$ and some modulation formats.

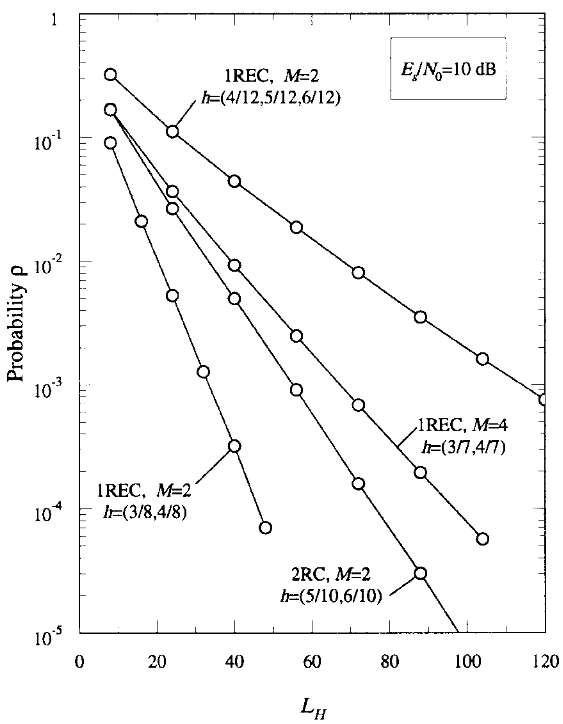

Fig. 3. Probability ρ versus L_H for $E_s/N_0 = 10$ dB and some modulation formats.

$P(\hat{\hat{\eta}} \neq \eta)$ is the sum of $\pi(n)$ over the states such that $\hat{\hat{\eta}}(k) = 1$

$$P(\hat{\hat{\eta}} \neq \eta) = \sum_{n=N_c}^{2N_c-1} \pi(n). \tag{5.1}$$

The computation of $\pi(n)$ is carried out with standard techniques [7] and is not reported here. Substituting the result into (5.1) produces (for $\rho \ll 1$, as happens in normal operating conditions)

$$P(\hat{\hat{\eta}} \neq \eta) \approx \rho^{N_c}. \tag{5.2}$$

Thus, the error probability of the superbaud detector decreases exponentially with N_c.

The above discussion has been concerned with a 2-h scheme. The general case is more complex. Nevertheless, if ρ is much less than unity, it is found that $P(\hat{\hat{\eta}} \neq \eta)$ is still given by (5.2). In other words, this equation is approximately true with any number of modulation indices.

An interesting question is the acquisition time of the superbaud detector. By this we mean the average time needed to achieve the *first* correct superbaud estimate. For simplicity let us concentate on a 2-h scheme, but all we say is approximately valid in general. Consider Fig. 1 and assume that the true superbaud phase is $\eta = 0$. The most likely acquisition occurs (with probability $1 - \rho$) for $\hat{\eta}(0) = 0$ and has a duration of L_0 symbols. The second most likely acquisition lasts $2L_0$ symbols and occurs [with probability $\rho(1-\rho)$] for $\{\hat{\eta}(0) = 1, \hat{\eta}(1) = 0\}$. The third most likely acquisition lasts $4L_0$ symbols and occurs [with probability $\rho^2(1-\rho)^2$] for $\{\hat{\eta}(0) = 1, \hat{\eta}(1) = 1, \hat{\eta}(2) = 0, \hat{\eta}(3) = 0\}$. In summary, the average acquisition is

$$L_{\text{acq}} = L_0(1-\rho) + 2L_0\rho(1-\rho) + 4L_0\rho^2(1-\rho)^2 + \cdots. \tag{5.3}$$

Under normal operating conditions, ρ is much less than unity and L_{acq} is only slightly greater than L_0.

Note that if an out-of-lock occurs in the tracking mode [when $C(k) = N_c - 1$], the reacquisition time lasts a minimum of $N_c L_0$ symbol intervals. Thus, selecting a large N_c to reduce the error probability of the superbaud detector [see (5.2)] may result in long reacquisitions

VI. Conclusions

We have proposed nondata-aided algorithms for symbol and superbaud timing recovery in multi-h CPM modulations. They operate independently of the carrier phase and have a feedforward structure that can be implemented in a fully digital form. Their application involves no limitations on pulse shape, symbol alphabet, and modulation indices. Also, contrary to what happens with feedback schemes, they are not prone to false locks caused by interactions with the carrier recovery circuit.

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