The PAM Decomposition of CPM Signals With Integer Modulation Index

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***Abstract*—This letter presents the solution for decomposing continuous-phase modulated (CPM) signals with integer modulation index into pulse-amplitude modulated components. The notion of main complex pulse is also introduced. A simplified demodulator for CPM signals with integer modulation index is proposed as an application example and simulation results using a quaternary 2 raised cosine (RC) scheme are given.**

***Index Terms*—Continuous phase modulation (CPM), integer modulation index, pulse-amplitude modulated (PAM) decomposition.**

I. Introduction

Decomposing a continuous-phase modulated (CPM) signal into pulse-amplitude modulated (PAM) components [1], [2] is an effective tool to overcome the nonlinearity inherent with the CPM scheme. It has been used to construct reduced-complexity CPM demodulators [3]–[5] and even perform signal parameter estimation [4], [6].

Laurent decomposition is only valid for binary CPM signals with noninteger modulation index [1]. Mengali and Morelli dealt with this limitation by expressing a CPM signal with integer modulation index as the product of two CPM signals with noninteger modulation indexes [2]. Since a CPM signal with integer modulation index has been used in communications systems (such as Motorola FLEX paging protocols), finding a general and unique PAM representation for a CPM signal with arbitrary integer modulation index is of both theoretical and practical significance.

In this letter, the exact PAM representation for a binary CPM signal with integer modulation index is presented in Section II. Signal approximation through PAM decomposition for M-ary CPM signal with integer modulation index is formulated, and a notion of main complex pulse is introduced in Section III. Application example and simulation results are given in Section IV.

II. Binary CPM Signal Decomposition

Let us consider the M-ary CPM signal with complex envelope

$$S(t) = e^{j\sum_{n=-\infty}^{\infty} a_n \varphi(t-nT)} \tag{1}$$

where $a_n \in [\pm 1, \pm 3, \ldots, \pm(M-1)]$ is the M-ary data symbol, T is the symbol interval, and $\varphi(t)$ is the phase-shift function, which is assumed to be zero for negative value of time and $h\pi$ (h denotes the modulation index) for time greater than L symbol intervals. Laurent has shown that, for any binary CPM signal (i.e., $M=2$ and $a_n = \pm 1$) with noninteger modulation index, (1) can be exactly expressed as a sum of 2^{L-1} PAM components [1].

To derive the PAM representation for a binary CPM signal with integer modulation index, (1) can be rewritten in the time interval $[NT, (N+1)T]$, where N is a time index, as

$$\begin{aligned} S(t) &= e^{jh\pi \sum_{n=-\infty}^{-L} a_n} \\ &\quad \cdot e^{j\left[h\pi\sum_{n=-L+1}^{N-L} a_n + \sum_{n=N-L+1}^{N} a_n\varphi(t-nT)\right]} \\ &= e^{jh\pi \sum_{n=-\infty}^{-L} a_n} \cdot \prod_{n=-L+1}^{N-L} e^{jh\pi a_n} \prod_{n=N-L+1}^{N} e^{ja_n\varphi(t-nT)} \\ &= e^{jh\pi \sum_{n=-\infty}^{-L} a_n} \cdot \cos^N(h\pi) \prod_{n=N-L+1}^{N} [\cos\varphi(t-nT) \\ &\quad + ja_n \sin\varphi(t-nT)], \qquad NT \le t \le (N+1)T \end{aligned} \tag{2}$$

and further expanded into a sum of 2^L different terms, as has been addressed in [8]. However, by carefully analyzing and grouping these terms, it is found that $S(t)$ is exactly the sum of one data-independent periodic component and 2^{L-1} data-dependent PAM components. Ignoring the factor $\exp[jh\pi\sum_{n=-\infty}^{-L} a_n]$ since it only introduces an initial phase shift, $S(t)$ is finally expressed as

$$S(t) = \sum_{n=-\infty}^{\infty} J^n h_0(t-nT) + \sum_{n=-\infty}^{\infty} J^n \sum_{k=1}^{2^{L-1}} a_{k,n} h_k(t-nT) \tag{3}$$

where $J = \cos(h\pi)$, which takes on value $+1$ (for h even) or -1 (for h odd), $h_0(t)$ is a pulse of duration T, which represents one period of the periodic component if $J=+1$, or a half period if $J=-1$, and is defined as

$$h_0(t) = \prod_{i=-L+1}^{0} \cos\varphi(t-iT), \qquad 0 \le t \le T. \tag{4}$$

$h_k(t)$ is the kth PAM pulse, defined by

$$\begin{aligned} h_k(t) = \prod_{i=1}^{L-1} \cos\varphi(t-iT) \prod_{i=0}^{L-1} &[(1-\beta_{k,i})\cos\varphi(t+iT) \\ &+ \beta_{k,i}\sin\varphi(t+iT)], \qquad 0 \le t \le L_k T \end{aligned} \tag{5}$$

where $\beta_{k,i} \in [0,1]$ is the ith digit in the radix-2 representation of $2k-1$, i.e., $2k-1 = \sum_{i=0}^{L-1} \beta_{k,i} 2^i$, L_k is the pulse duration

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(in symbol interval) determined by $L_k = L - \max_{\beta_{k,i}\neq 0} i$, i.e., $L_1 = L$, $L_2 = L-1$, $L_3 = L_4 = L-2$, $L_5 = L_6 = L_7 = L_8 = L-3, \ldots, L_{2^{L-2}+1} = \ldots = L_{2^{L-1}} = 1$. The complex coefficient $a_{k,n}$ associated with $h_k(t)$ is given by

$$a_{k,n} \prod_{i=0}^{L-1} (1 - \beta_{k,i} + \beta_{k,i} j a_{n-i}). \tag{6}$$

Assuming independent data symbols, the first-order moment of the coefficient $a_{k,n}$ is zero, according to (6), and the second-order one is $E\{a_{k,n}a_{i,m}^*\} = \begin{cases} 1, & m=n, i=k \\ 0, & \text{otherwise} \end{cases}$. Therefore, by defining (7) as shown at the bottom of the page, the time-averaged auto-correlation function of the binary CPM signal with integer modulation index can be expressed as

$$R_{SS}(\tau) = \sum_{n=-\infty}^{\infty} h_{00}(\tau - 2nT) + \frac{1}{T}\sum_{k=1}^{2^{L-1}} \int_{-\infty}^{\infty} h_k(t+\tau)h_k(t)dt \tag{8}$$

and the power spectral density has an impulse spectrum part $((1)/(2T))\sum_{m=-\infty}^{\infty} H_{00}((m)/(2T))\delta(f - (m/2T))$ and a continuous spectrum part $(1/T)\sum_{k=1}^{2^{L-1}} |H_k(f)|^2$, where $H_{00}(f)$ and $H_k(f)$ are the Fourier transforms of $h_{00}(t)$ and $h_k(t)$, respectively. Note that for the noninteger modulation index case, no impulse spectrum exists, and the power spectral density can be expressed as the sum of the spectra of the PAM components only when the modulation index takes half-integer values [1].

III. Main Complex Pulse

Assuming that M is an integer power of 2, i.e., $M = 2^P$, the M-ary CPM signal with integer modulation index can be expressed as the product of P binary CPM signals with integer modulation indexes, and thus, can be decomposed into a sum of one data-independent periodic component and $2^{(L-1)P}(2^P - 1)$ PAM components. However, taking all these components into consideration is neither easy nor necessary in practice. Our study shows that the most significant part of signal energy is carried by the periodic component and the first $2^P - 1$ main PAM components associated with the $2^P - 1$ main PAM pulses of the longest duration LT. Therefore, the modulated signal waveform can be well approximated by

$$S(t) \approx \sum_{n=-\infty}^{\infty} J^n h_0(t - nT) + \sum_{n=-\infty}^{\infty} J^n \sum_{k=1}^{2^P-1} a_{k,n} h_k(t - nT) \tag{9}$$

where

$$h_0(t) = \prod_{i=-L+1}^{0} \frac{\sin M\varphi(t - iT)}{M\sin\varphi(t - iT)}, \qquad 0 \le t \le T \tag{10}$$

$$\begin{aligned} h_k(t) = &\prod_{\substack{i=-L+1 \\ i\neq 0}}^{L-1} \frac{\sin M\varphi(t - iT)}{M\sin\varphi(t - iT)} \\ &\times \prod_{\rho=0}^{P-1} \left[(1-\gamma_{k,\rho})\cos 2^\rho \varphi(t) + \gamma_{k,\rho}\sin 2^\rho\varphi(t)\right], \\ &0 \le t \le LT \end{aligned} \tag{11}$$

$$a_{k,n} = \prod_{\rho=0}^{P-1}\left(1 - \gamma_{k,\rho} + \gamma_{k,\rho} j a_n^{(\rho)}\right). \tag{12}$$

In (11) and (12), $\gamma_{k,\rho} \in [0,1]$ is the ρth digit in the radix-2 representation of the integer k, i.e., $k = \sum_{\rho=0}^{P-1}\gamma_{k,\rho}2^\rho$ for $1 \le k \le 2^P - 1$. In (12), $a_n^{(\rho)} = \pm 1$ is the ρth binary bit in the radix-2 representation of the M-ary symbol a_n, i.e., $a_n = \sum_{\rho=0}^{P-1} a_n^{(\rho)} 2^\rho$.

Further study also reveals that a segment of the periodic component and the first $2^P - 1$ main PAM pulses of the longest duration can be incorporated into one complex waveform with simpler expression, i.e.

$$\begin{aligned} S_{a_N}(t) &= \sum_{n=0}^{L-1} J^n h_0(t - nT) + \sum_{k=1}^{2^P-1} a_{k,N}h_k(t) \\ &= \prod_{\substack{i=-L+1 \\ i\neq 0}}^{L-1} \frac{\sin M\varphi(t - iT)}{M\sin\varphi(t - iT)} \cdot e^{ja_N\varphi(t)}, \\ &\quad 0 \le t \le LT \end{aligned} \tag{13}$$

which can be also interpreted as a "windowed" complex exponential with window function $w(t) = \prod_{\substack{i=-L+1 \\ i\neq 0}}^{L-1}((\sin M\varphi(t - iT))/(M\sin\varphi(t - iT)))$ and phase-shift function $a_N\varphi(t)$. We call $S_{a_N}(t)$ *the main complex pulse* associated with a_N, which has the symmetry property $S_{a_N}(t) = S_{-a_N}^*(t)$.

IV. Application Example

A simplified matched-filter demodulator for an M-ary CPM signal with integer modulation index over additive white Gaussian noise (AWGN) channel is constructed by making use of the above main complex pulse. The demodulator consists of a matched-filter bank of M filters, each matched to the main complex pulse associated with one symbol from the set $[\pm 1, \pm 3, \ldots, \pm(M-1)]$, followed by a decision maker. The M decision variables are $D_{a_N} = J^N \Re[C^* \int_0^{LT} r(t+NT)S_{a_N}^*(t)dt]$ for coherent receiving, or $D_{a_N} = |\int_0^{LT} r(t+NT)S_{a_N}^*(t)dt|$ for noncoherent receiving, where C is a complex coefficient representing the

$$h_{00}(\tau) = \begin{cases} \dfrac{1}{T}\left[\displaystyle\int_0^{T-\tau} h_0(t+\tau)h_0(t)dt + J\int_{T-\tau}^{T} h_0(t - T + \tau)h_0(t)dt\right], & 0 \le \tau \le T \\ h_{00}(-\tau), & -T \le \tau \le 0 \end{cases} \tag{7}$$

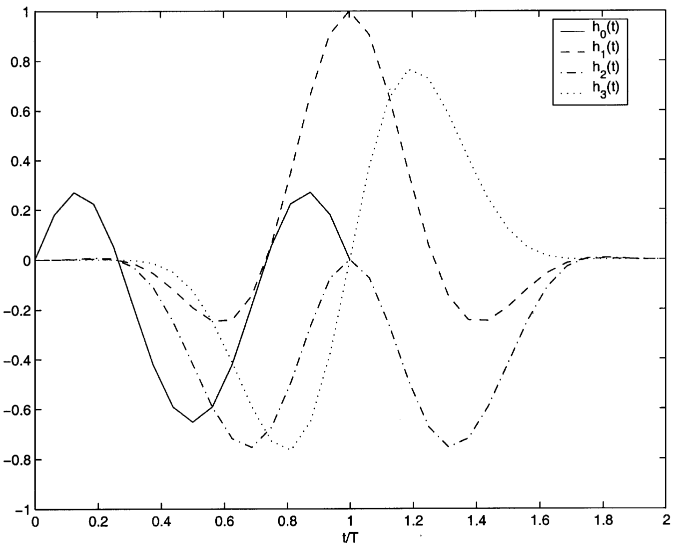

Fig. 1. Waveforms of $h_0(t)$ and the first three main PAM pulses for quaternary 2RC scheme with $h = 1$.

carrier phase and $r(t)$ is the received signal. The output symbol is simply decided according to the largest value among the decision variables computed by the demodulator. In the case of coherent receiving, the complex coefficient C can be blindly estimated by the recursion $C^{(N)} = \lambda C^{(N-1)} + (1-\lambda)J^N((\int_0^T r(t+NT)h_0(t)dt)/(h_{00}(0)))$, where $C^{(N)}$ is an estimate of C at time $t = NT$, and $0 < \lambda < 1$ is the "forgetting factor."

The quaternary 2RC (raised cosine frequency pulse with $L = 2$) CPM scheme with $h = 1$ is used to demonstrate the PAM decomposition and signal demodulation. Fig. 1 shows the waveforms of $h_0(t)$ and the first three main PAM pulses. The total power of the periodic component and the first three main PAM components is 0.9671, which means only 0.0329 of the signal power is ignored by signal approximation. The window function and the main complex pulses are shown in Fig. 2 ($S_{-1}(t)$ and $S_{-3}(t)$ are not displayed because of the symmetry property). We see that the window function takes the maximum value in the center, remains almost flat for nearly one symbol interval, and is then rapidly attenuated toward its edges. This window function ensures the matched filters collect the most significant part of signal energy and, at the same time, prevents the decision variable from being seriously affected by symbols adjacent to the one the demodulator is detecting. The bit-error rates (BERs) obtained by simulation for both coherent and noncoherent receiving are shown in Fig. 3, which indicates that the performance of the simplified coherent demodulator ($\lambda = 0.94$ is used for channel estimation) is very close to that of the optimal coherent receiver implemented using a four-state Viterbi algorithm. The degradation for noncoherent receiving is less than 2 dB as compared with the optimal coherent receiving when the BER is below 0.01. These results show that the proposed demodulator is very effective for $L = 2$, which is the most practical case. However, because of its symbol-by-symbol detection nature, the performances of this simplified matched-filter demodulator may degrade as L increases.

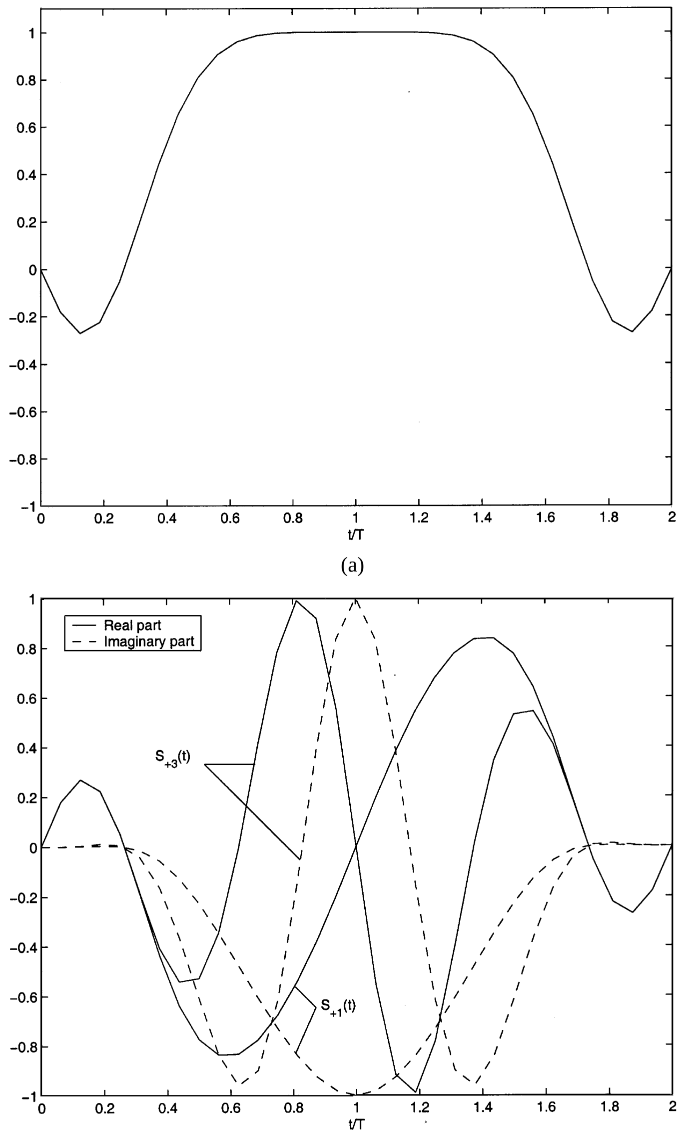

Fig. 2. Waveforms of (a) the window function and (b) the main complex pulses for quaternary 2RC scheme with $h = 1$.

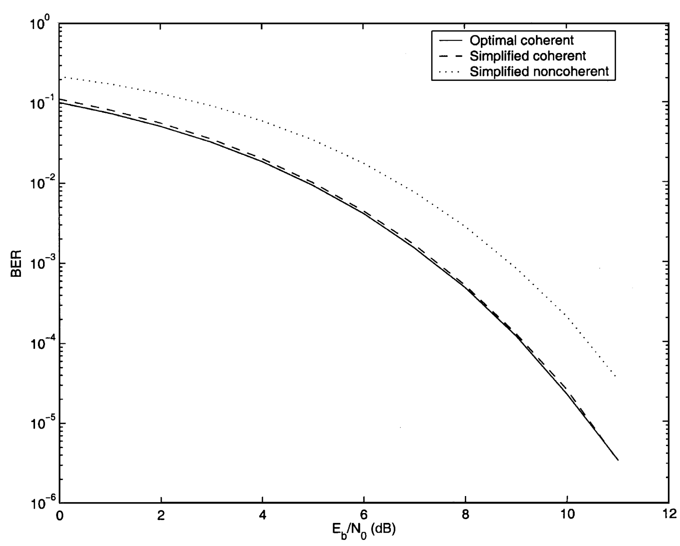

Fig. 3. Performances of the simplified matched-filter demodulator for quaternary 2RC scheme with $h = 1$.

V. Conclusions

Complementary to Laurent decomposition, the M-ary CPM signal with integer modulation index can be exactly decomposed into one data-independent periodic component and a finite number of data-dependent PAM components, and well approximated as the sum of the periodic component and the first $M-1$ main PAM components associated with the longest duration main PAM pulses. A segment of this periodic component and the first $M-1$ main PAM pulses can be incorporated into a main complex pulse with very simple expression. Application of the main complex pulse leads to a simplified matched-filter demodulation scheme. The data-independent periodic component can be exploited to blindly estimate channel parameters.

References

[1] P. A. Laurent, "Exact and approximate construction of digital phase modulation by superposition of amplitude-modulated pulses (AMP)," *IEEE Trans. Commun.*, vol. COM-34, pp. 150–160, Feb. 1986.

[2] U. Mengali and M. Morelli, "Decomposition of M-ary CPM signals into PAM waveforms," *IEEE Trans. Inform. Theory*, vol. 41, pp. 1265–1275, Sept. 1995.

[3] G. K. Kaleh, "Simple coherent receivers for partial-response continuous-phase modulation," *IEEE J. Select. Areas Commun.*, vol. 7, pp. 1427–1436, Dec. 1989.

[4] G. Colavolpe and R. Raheli, "Reduced-complexity detection and phase synchronization of CPM signals," *IEEE Trans.Commun.*, vol. 45, pp. 1070–1079, Sept. 1997.

[5] ——, "Noncoherent sequence detection of continuous-phase modulations," *IEEE Trans. Commun.*, vol. 47, pp. 1303–1307, Sept. 1999.

[6] A. N. D'Andrea, A. Ginesi, and U. Mengali, "Frequency detectors for CPM signals," *IEEE Trans. Commun.*, vol. 43, pp. 1828–1837, Feb.-Apr. 1995.

[7] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

[8] A. Napolitano and C. M. Spooner, "Cyclic spectral analysis of continuous-phase modulated signals," *IEEE Trans. Signal Processing*, vol. 49, pp. 30–44, Jan. 2001.