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Robust distributed control of plantwide processes based on dissipativity

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ABSTRACT

This paper presents a novel robust distributed control approach for plantwide processes based on dissipativity. The model of individual process unit (subsystem) is uncertain but within a polytopic region. By adopting the systems behavioral approach, the dissipativity property of process units with model uncertainties can be directly determined from that of the models on the vertices of the polytopic region. The plantwide robust stability and performance conditions are represented in terms of the dissipativity condition of the plantwide system, which is the linear combination of the dissipativity of the individual process units and the controllers subject to process/controller network topology. Then the required dissipativity properties for individual controllers are obtained by solving a set of linear matrix inequalities. Controllers are synthesized individually based on their respective supply rates through *J*-factorization. The dissipativity properties (both the storage function and supply rate) in quadratic differential forms are used to capture detailed dynamic features of processes, which significantly reduces the conservatism in dissipativity based control analysis and synthesis. This work develops a single dissipativity-based framework to deal with both process model uncertainties and interactions within the process network, leading to a scalable and flexible robust distributed control approach that works with arbitrary process and controller network topologies. A case study is presented to illustrate the proposed method.

1. Introduction

Modern chemical plants often consist of many process units (reactors, heat exchangers distillation columns, etc.) interconnected through material recycle (where unreacted reactants are separated from the products and reused) and energy integration loops (where excessive heat generated in process units is utilized in another process unit) [1]. Such a process is referred to as a plantwide process [2]. While plantwide processes generally yield higher production rates and better energy efficiency, the complex interactions make the dynamics of the system much harder to understand than a standalone unit. From control point of view, material recycles and heat integration among process units can be understood as positive feedback loops within the process network, which have the potential to amplify the exogenous disturbances, leading to deteriorating effects on control performance [3]. Much work has been done in the field of plantwide control. Luyben and coworkers developed methods for plantwide control design and structure evaluation [4,5]. Georgakis and coworkers made contributions to the plantwide operability analysis [6,7] as well as the application to benchmark processes [8].

Control of plantwide systems is a large-scale problem, which can make centralized control approaches (which treat the entire plantwide system as a single complex multivariable process) prohibitively complex or even infeasible computationally as the scale of the problem escalates [9]. Decentralized control approaches [10,11], where each process unit is controlled by a local controller without the knowledge of the rest of plant, are much simpler to implement. In decentralized designs, additional stability conditions based on robust control theory need to be implemented to ensure plantwide stability. However, these approaches are very conservative and often lead to poor plantwide control performance because the known interactions are treated as uncertainties (e.g., [12,13]). In the past decade, distributed control strategies [14–16], which coordinate an array of controllers through a communication network to achieve the closed-loop plantwide objectives, started to attract much attention in both industry and academia. One of the approaches is to represent the plantwide process as

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a network of process units and design a network of local controllers to control the plantwide system [17,18]. These approaches can provide improved global performance while preserving the flexibility and fault tolerance of decentralized control. The key issue in distributed control design is to handle process interactions in a scalable and flexible way.

Another issue in process control is model uncertainty since accurate process dynamic models are often difficult to obtain [12]. In distributed control, the interactions between process units may exacerbate the effects of uncertainties, leading to poor plantwide performance and stability. There are a number of robust decentralized control approaches [12,19] for plantwide systems including multi-unit decentralized control (with block diagonal controllers) [20,21]. In these approaches, both interactions between process units and model-plant mismatch are modeled as uncertainties and dealt with using a robust control framework. To the best of our knowledge, little has been reported in the literature on robust distributed control for plantwide chemical processes.

To address above issues, we develop a novel robust distributed control approach for linear time-invariant systems based on dissipativity theory. In this approach, the plantwide robust stability, robust performance and distributed control (i.e., the model uncertainties, the interaction effects as well as the external disturbances) are achieved using a *single* framework based on the dissipativity theory. The rationale is as follows: (1) as an input-output property, the dissipativity is found to be useful in interaction analysis and stability design of network systems [22,18]; (2) $\lfloor H_\infty$ type of stability and performance conditions can be represented in terms of dissipativity [23,18]; (3) dissipativity based framework is suitable for distributed control due to its scalability – the supply rate of a plantwide process is the linear combination of the supply rates of its individual process units based on the network topology [24,17]. Furthermore, the link with thermodynamics allows the integration of process design and process control [25–27]. This approach is made possible by the use of supply rates and storage functions in quadratic difference forms (QDFs) [28], which capture much more detailed features of process dynamics compared to conventional QSR-dissipativity [18]. The plantwide process and distributed control system are represented as two interacting process and controller networks. For each uncertain process unit, we adopt the systems behavioral approach in [29] to describe the polytopic uncertainty. In this way, the kernel representation of the uncertain system is a convex combination of the kernel representations of the models on the vertices of polytopic region in the parameter space. Parametric dissipativity, which is also a convex combination of (QDF) dissipativity properties (for both supply rates and storage functions) of models on vertices, is proposed to describe the dynamic features of uncertain subsystems. The interaction effects among process units and distributed controllers in a plantwide system are captured by the dissipativity properties of subsystems (both process units and distributed controllers) and the network topologies of process and controller networks. To be specific, the plantwide stability and performance conditions are represented using a global dissipativity condition (the plantwide supply rate), which in turn are translated into the dissipativity conditions that individual controllers must satisfy. The control design is separated into two steps: firstly, feasible supply rates for individual process units and distributed controllers are obtained by solving a set of dissipation conditions described by linear matrix inequalities (LMIs); secondly, controllers are synthesized individually based on their respective supply rates through *J*-factorization [30].

The remainder of this paper is structured as follows. The following section provides preliminaries on behavior system with polytopic uncertainty and a network perspective of the plantwide control problem. The dissipativity analysis using quadratic differential forms together with its extension (parametric dissipativity) for uncertain systems are presented in Section 3. The plantwide analysis and control synthesis using parametric dissipativity is given in Section 4. Finally, an illustrative example is presented in Section 5.

2. Preliminaries and problem formulation

2.1. Notation

We use the standard notation $\mathbb{R}^{\mathrm{n}}$, $\mathbb{R}^{\mathrm{n}_1\times\mathrm{n}_2}$ etc. When a dimension is not specified (but finite), it is written as $\mathbb{R}^{\bullet}$, $\mathbb{R}^{\mathrm{n}\times\bullet}$, $\mathbb{R}^{\bullet\times\bullet}$, etc. We typically use the superscript w (for example in $\mathbb{R}^{\mathrm{w}}$) when generic elements of that space are denoted by w. The set of infinitely differentiable maps from $\mathbb{R}$ to $\mathbb{R}^{\mathrm{n}}$ is denoted by $\mathfrak{C}^\infty(\mathbb{R},\mathbb{R}^{\mathrm{n}})$. Let I_n and $0_{n\times m}$ be the n-dimensional identity matrix and $n\times m$ zero matrix, respectively. If no subscripts is applied, the dimensions of I and 0 are inferred from contexts. The operator $\mathrm{col}(v_1, v_2, \ldots, v_n)$ stacks vectors or matrices $v_1, v_2, \ldots, v_n$ and $\mathrm{diag}(D_1, D_2, \ldots, D_n)$ forms the block-diagonal matrix with $D_1, D_2, \ldots, D_n$ on the diagonal.

Let $\mathbb{R}[\xi]$ be the ring of polynomials in the indeterminate ξ with real coefficients. The spaces of vectors and matrices with components in $\mathbb{R}[\xi]$ are denoted by $\mathbb{R}^{\mathrm{n}}[\xi]$ and $\mathbb{R}^{\mathrm{n}\times\mathrm{m}}[\xi]$, respectively. Any $X(\xi)\in\mathbb{R}^{\mathrm{n}\times\mathrm{m}}[\xi]$ can be represented by $X(\xi)=\sum_{k=0}^{L}\tilde{X}_k\xi^k$ where $\tilde{X}_k\in\mathbb{R}^{\mathrm{n}\times\mathrm{m}}$. The *real polynomial matrix* $(\tilde{X}_0,\ldots,\tilde{X}_L)$ is called the *coefficient matrix* of $X(\xi)$ and is denoted by $\tilde{X}$. We use $\mathbb{R}^{\mathrm{n}\times\mathrm{m}}[\zeta,\eta]$ for the set of real polynomial matrices in the indeterminates ζ and η. Define the asterisk operator * by $\Phi^\star(\zeta,\eta)\stackrel{\Delta}{=}\Phi^T(\eta,\zeta)$. If $\Phi\in\mathbb{R}^{\mathrm{n}\times\mathrm{m}}[\zeta,\eta]$ satisfies $\Phi^\star=\Phi$, then Φ is *symmetric*. A symmetric element $\Phi\in\mathbb{R}^{\mathrm{n}\times\mathrm{n}}[\zeta,\eta]$ can be written as $\Phi(\zeta,\eta)=\sum_{k,l=0}^{L}\tilde{\Phi}_{kl}\zeta^k\eta^l=(\lfloor I_{\mathrm{w}}^L)^T(\zeta)\tilde{\Phi}\lfloor I_{\mathrm{w}}^L(\eta)$ where $\lfloor I_{\mathrm{w}}^L(\xi)=\mathrm{col}(I_{\mathrm{w}},\xi I_{\mathrm{w}},\ldots,\xi^L I_{\mathrm{w}})$ with L as the *order* of Φ, $\tilde{\Phi}=\begin{bmatrix}\tilde{\Phi}_{00}&\cdots&\tilde{\Phi}_{0L}\\ \vdots&\ddots&\vdots\\ \tilde{\Phi}_{L0}&\cdots&\tilde{\Phi}_{LL}\end{bmatrix}$ is called the *coefficient matrix* of Φ. Φ is said to be positive semidefinite, denoted by $\Phi(\zeta,\eta)\succ(\succeq)0$, if its coefficient matrix is positive (semi)definite, *i.e.*, $\tilde{\Phi}>(\geq)0$.

2.2. Behaviors of plantwide system with polytopic uncertainties

In the behavioral approach [31] to linear systems, a dynamical system is given by a triple $\Sigma=(\mathbb{R},\mathbb{R}^{\mathrm{w}},\mathfrak{B})$, where $\mathbb{R}$ is the time axis, $\mathbb{R}^{\mathrm{w}}$ is the signal space, and the *behavior* $\mathfrak{B}$ is a linear subspace of $\mathfrak{C}^\infty(\mathbb{R},\mathbb{R}^{\mathrm{w}})$ consisting of all solutions of the following LTI differential equations,

$$\sum_{k=0}^{L}\tilde{R}_k\frac{\mathrm{d}^k}{\mathrm{d}t^k}w=0, \tag{1}$$

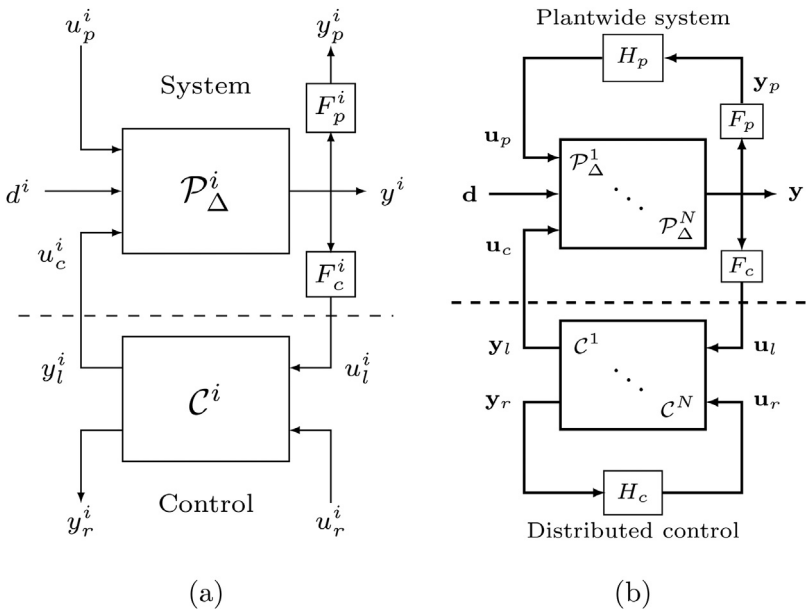


Fig. 1. Configurations of (a) a closed-loop subsystem and (b) distributed control of plantwide system.

with $\tilde{R}_k \in \mathbb{R}^{\bullet\times w}$. The *manifest variable* $w = \mathrm{col}(y, u) \in \mathbb{R}^w$ includes the input-output variables. By defining the differential operator $\xi \triangleq \frac{d}{dt}$, we can have the *kernel representation* of $\mathfrak{B}$,

$$R(\xi)w = 0, \tag{2}$$

where $R(\xi) = \sum_{k=0}^{L} \tilde{R}_k \xi^k \in \mathbb{R}^{\bullet\times w}[\xi]$. A system $\mathfrak{B}$ is said to be *controllable* if for each $w_1, w_2 \in \mathfrak{B}$ there exists a $w \in \mathfrak{B}$ and $t' \geq 0$ such that $w(t) = w_1(t)$ for $t < 0$ and $w(t) = w_2(t - t')$ for $t \geq t'$. It can be shown that $\mathfrak{B}$ is controllable if and only if its kernel representation satisfies $\mathrm{rank}(R(\lambda)) = \mathrm{rank}(R)$ for all $\lambda \in \mathbb{C}$. Controllable systems are exactly those that admit *image representations*, that is, there exists an $M \in \mathbb{R}^{w\times\bullet}[\xi]$ such that any $w \in \mathfrak{B}$ can be represented by

$$w = M(\xi)\ell \tag{3}$$

where ℓ is called the *latent variable* and possesses the properties of the state. A system with polytopic uncertainty yields a kernel representation [29]

$$R_\Delta(\xi)w = 0 \tag{4}$$

with

$$R_\Delta(\xi) = \sum_{j=1}^{M} \theta_j R_j(\xi), \quad \theta \in \Theta \triangleq \left\{ \theta \in [0,1]^M \mid \sum_{j=1}^{M} \theta_j = 1 \right\} \tag{5}$$

where θ is a weighting parameter and $R_j(\xi)w = 0$, $j = 1, \ldots, M$ represent the behaviors on the vertices.

In this work, a plantwide chemical process is represented as a network of process units interconnected through physical (mass and energy) flows [27]. As shown in Fig. 1a, each unit (subsystem) $\lfloor P^i_\Delta$, $i = 1, \ldots, N$ is described by behaviors with polytopic uncertainty

$$R^i_\Delta(\xi)w^i = 0 \tag{6}$$

where $w^i = \mathrm{col}(y^i, u^i_p, u^i_c, d^i)$ with output y^i, interconnecting input u^i_p, control input u^i_c, and external disturbance d^i. The process interconnecting output $y^i_p = F^i_p y^i$ denotes the physical flows to other units. We assume that each subsystem is equipped with a local controller $\lfloor C^i$, which is a dynamical system with the manifest variable $v^i = \mathrm{col}(y^i_l, y^i_r, u^i_l, u^i_r)$ where $y^i_l = u^i_c$ is the control action for $\lfloor P^i_\Delta$, $u^i_l = F^i_c y^i$ is the process measured output, u^i_r, y^i_r denote the exchanged input/output information between $\lfloor C^i$ and other controllers. The overall process and controller network are depicted in Fig. 1b. Matrices $F_p = \mathrm{diag}(F^1_p, \ldots, F^N_p)$ and $F_c = \mathrm{diag}(F^1_c, \ldots, F^N_c)$ represent "selectors" for the interconnecting and measured outputs. Matrix H_p with elements either 0 or 1 represents the process network topology. The interconnection relations among process units can be described by

$$\mathbf{u}_p = H_p \mathbf{y}_p = H_p F_p \mathbf{y} \tag{7}$$

where $\mathbf{y} = \mathrm{col}(y^1, y^2, \ldots, y^N)$ and $\mathbf{u}_p$ can be defined analogously. Similarly, H_c defines the controller communication network, which may have any structure. A common choice is the controller network to have the same topology as the process network, *i.e.*, $H_c = H_p$, $\mathbf{y}_r = \mathbf{y}_p$ and $\mathbf{u}_r = \mathbf{u}_p$. Another example is $H_c = 0$ representing the fully decentralized control configuration. Then the plantwide distributed control design problem can be stated as follows.

Problem 1. Given a plantwide chemical process with N units, a process network topology H_p as shown in Fig. 1, with the uncertain behavior of the i-th process unit described by

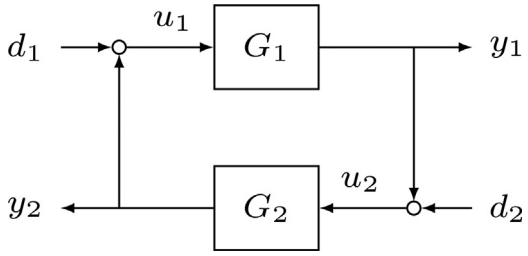


Fig. 2. Feedback interconnection of two systems.

$$\lfloor P_{\Delta}^{i}: \quad R_{\Delta}^{i}(\xi)w^{i}=0, \quad \theta^{i}\in\Theta^{i}=\left\{\theta^{i}\in[0,1]^{M_i}\,|\sum_{j=1}^{M_i}\theta_{j}^{i}=1\right\}, \tag{8}$$

where $w^i = \mathrm{col}\,(y^i, u_p^i, d^i, u_c^i)$ and θ^i are, respectively, the manifest variable and weighting parameters of the i-th subsystem, design N distributed controllers $\lfloor C^i$ with manifest variable $v^i = (y_l^i, y_r^i, u_l^i, u_r^i)$ and controller network topology H_c such that the closed-loop plantwide system is robustly stable and the frequency-weighted $\lfloor H_\infty$ norm from disturbances $\mathbf{d}$ to outputs $\mathbf{y}$ satisfies

$$\left\|\mathbf{W}\mathbf{T}_{\mathbf{yd}}\right\|_{\infty}\le\gamma, \tag{9}$$

where $\mathbf{W}(s)$ is a weighting function, $\mathbf{T}_{\mathbf{yd}}(s)$ is the sensitivity function form disturbance to output, and γ is the desired gain bound.

Remark 1. The proposed approach allows arbitrary choices of the topology matrix H_c to meet different design requirements. To achieve the best plantwide performance, H_c can be chosen such that the controller topology is the same as that of the plant. For other requirements (e.g., to reduce communication burden to suit a limited bandwidth), control design can be carried out repeatedly with different choices of H_c until a satisfactory result is obtained.

3. Dissipativity analysis

Dissipative systems are those for which the change in stored energy is bounded by the amount of energy supplied by the environment. This concept was extended into a general dynamical system setting by Willems in [32]. A continuous-time system

$$\dot{x}=f(x,u), \quad y=h(x,u) \tag{10}$$

is said to be dissipative if there exist a function defined on input and output, called the *supply rate* $s(y,u)$ and a positive semidefinite function defined on the state, called the storage function $V(x)$ such that [32]

$$V(x(T))-V(x(0))\le\int_0^T s(y(t),u(t))\,dt, \quad \forall T>0. \tag{11}$$

The following QSR-type supply rate [33] is commonly used

$$s(y,u)=\begin{bmatrix}y\\u\end{bmatrix}^{T}\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}\begin{bmatrix}y\\u\end{bmatrix} \tag{12}$$

where $Q^T = Q$, $R^T = R$, and $\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}$ is called the *supply rate matrix*. The QSR supply rate can be used to describe dynamic features of a system. For example, if $Q=0$, $S=1/2I$, $R=0$, then the system is passive. If $Q=-I$, $S=0$, $R=\rho^2 I$, then the $\lfloor L_2$ gain of (11) is bounded by ρ.

Another feature of QSR-type supply rate is that the total supply rate of a networked dynamics with arbitrary parallel, serial and feedback interconnected subsystem is still a quadratic form of the stacked input and output variables. For example, consider two systems ($G_1: u_1 \mapsto y_1$ and $G_2: u_2 \mapsto y_2$) in negative feedback configuration (as shown in Fig. 2) with supply rate matrix $\begin{bmatrix}Q_i & S_i\\ S_i^T & R_i\end{bmatrix}$, $i=1,2$. The net supply rate of the closed-loop system from $\mathrm{col}\,(d_1,d_2)$ to $\mathrm{col}\,(y_1,y_2)$ can be written as

$$s(y_1,y_2,d_1,d_2)=\begin{bmatrix}y_1\\y_2\\d_1\\d_2\end{bmatrix}^{T}\begin{bmatrix}Q_1+R_2 & S_1+S_2^T & S_1 & R_2\\ S_1^T+S_2 & Q_2+R_2 & R_1 & S_2\\ S_1^T & R_1 & R_1 & 0\\ R_2 & S_2^T & 0 & R_2\end{bmatrix}\begin{bmatrix}y_1\\y_2\\d_1\\d_2\end{bmatrix}. \tag{13}$$

Thus, QSR-type dissipativity is a scalable and flexible tool for analysis of large-scale networked systems [22].

3.1. QDF dissipativity

System analysis based on QSR-type supply rate can be conservative, as the supply rate in (12) can only provide a coarse description of the system's dynamic features [18]. The QDF was introduced in [28] to represent dissipativity property (storage functions and supply rates) of system behaviors. A QDF $Q_\Phi(w)$ takes the form

$$Q_\Phi(w) \triangleq \sum_{k,l=0}^{L} \left(\frac{\mathrm{d}^k}{\mathrm{d}t^k} w\right)^T \tilde{\Phi}_{kl} \left(\frac{\mathrm{d}^l}{\mathrm{d}t^l} w\right). \tag{14}$$

Defining $\zeta = \frac{\mathrm{d}}{\mathrm{d}t}^T$ and $\eta = \frac{\mathrm{d}}{\mathrm{d}t}$, a QDF can also be written more compactly as

$$Q_\Phi(w) = w^T \Phi(\zeta, \eta) w \tag{15}$$

and is said to be *induced* by the two-variable polynomial matrix $\Phi(\zeta, \eta)$. The dissipativity with respect to a QDF is given in the following definition.

Definition 1 *(QDF dissipativity [28]).* A behavior $\mathfrak{B}$ in (2) is said to be Φ-dissipative if there exists a storage function induced by $\Psi \succeq 0$ such that

$$\frac{\mathrm{d}}{\mathrm{d}t} Q_\Psi(w) \leq Q_\Phi(w), \tag{16}$$

where $Q_\Phi(w)$ is a QDF supply rate.

Since QDF dissipativity takes the extended manifest variable $\hat{w} \triangleq \mathrm{col}\,(w, w^{(1)}, \ldots, w^{(L)})$ into account, it can capture much more detailed features of the system dynamics compared to the QSR-type dissipativity, leading to a much less conservative dissipativity-based analysis [18]. Assuming that the manifest variable has an input-output partition $w = (y, u)$ with $u \in \mathbb{R}^{\mathrm{u}}$ and $y \in \mathbb{R}^{\mathrm{y}}$, the QDF $Q_\Phi(w)$ can be rewritten as

$$Q_\Phi(y,u) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} \lfloor Q(\zeta,\eta) & \lfloor S(\zeta,\eta) \\ \lfloor S^\star(\zeta,\eta) & \lfloor R(\zeta,\eta) \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} \hat{y} \\ \hat{u} \end{bmatrix} \begin{bmatrix} \lfloor \tilde{Q} & \lfloor \tilde{S} \\ \lfloor \tilde{S}^T & \lfloor \tilde{R} \end{bmatrix} \begin{bmatrix} \hat{y} \\ \hat{u} \end{bmatrix} \tag{17}$$

where $\lfloor Q \in \mathbb{R}^{\mathrm{y}\times\mathrm{y}}[\zeta,\eta]$, $\lfloor S \in \mathbb{R}^{\mathrm{y}\times\mathrm{u}}[\zeta,\eta]$, $\lfloor R \in \mathbb{R}^{\mathrm{u}\times\mathrm{u}}[\zeta,\eta]$. Since $Q_\Phi(y,u)$ can be understood as a dynamic QSR supply rate which utilizes the extended input-output trajectories, it provides a scalable and flexible dissipativity/stability analysis for large-scale systems [18]. Moreover, it can be used to describe the frequency-weighted $\lfloor H_\infty$ gain bound of LTI systems.

Proposition 1 ([18]). *Assuming that an LTI system $G : u \mapsto y$ with transfer function $G(s)$ is Φ-dissipative (i.e., dissipative with respect to the supply rate given in (17)) with $\lfloor Q(\zeta,\eta) \prec 0$ (i.e., $\lfloor \tilde{Q} \prec 0$), then the system G satisfies*

$$\left\| \frac{N}{\alpha} G \right\|_\infty \leq \gamma \tag{18}$$

where, $N(j\omega) = (-\lfloor Q)^{\frac{1}{2}}(-j\omega, j\omega)$ and a scalar $\alpha(\omega) > 0$ satisfying

$$\gamma^2 \alpha^2(\omega) I \geq \lfloor R(-j\omega, j\omega) - \lfloor S^T(-j\omega, j\omega) \lfloor Q^{-1}(-j\omega, j\omega) \lfloor S(-j\omega, j\omega), \quad \forall \omega. \tag{19}$$

The following lemma is used to determine the dissipativity properties of a behavior.

Lemma 2 ([34]). *Let $\mathfrak{B}$ be given by the kernel representation in (2). $\mathfrak{B}$ is Φ-dissipative if and only if there exist $\Psi \in \mathbb{R}^{\mathrm{w}\times\mathrm{w}}[\zeta,\eta]$ with $\Psi^\star = \Psi \succeq 0$, and $F \in \mathbb{R}^{\mathrm{w}\times\bullet}[\zeta,\eta]$ such that*

$$\Phi(\zeta,\eta) - \nabla\Psi(\zeta,\eta) + F^\star(\zeta,\eta) R(\eta) + R^T(\zeta) F(\eta,\zeta) \succeq 0 \tag{20}$$

where $\nabla\Psi(\zeta,\eta) = (\zeta + \eta)\Psi(\zeta,\eta)$.

In the above lemma, $F(\zeta,\eta)$ can be regarded as a Lagrange multiplier for the model in kernel representation (2). Since $\frac{\mathrm{d}}{\mathrm{d}t} Q_\Psi(w) = Q_{\nabla\Psi}(w)$, Condition (20) can be further converted into an LMI [34]:

$$\tilde{\Phi} - \tilde{\Psi}_\nabla + \tilde{F}^T \hat{R} + \hat{R}^T \tilde{F} \succeq 0 \tag{21}$$

with $\tilde{\Phi} \in \mathbb{R}^{(L_\Phi+1)\mathrm{w}\times(L_\Phi+1)\mathrm{w}}$, $\tilde{\Psi} \in \mathbb{R}^{L_\Phi \mathrm{w}\times L_\Phi \mathrm{w}}$, $\tilde{F} \in \mathbb{R}^{(L_\phi+1)\mathrm{w}\times(L_\Phi - L_R)\mathrm{p}}$ being the coefficient matrices of $\Phi(\zeta,\eta)$, $\Psi(\zeta,\eta)$ and $F(\zeta,\eta)$ respectively, and

$$\begin{aligned} \tilde{\Psi}_\nabla &= \begin{bmatrix} 0 & \tilde{\Psi} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \tilde{\Psi} & 0 \end{bmatrix} \in \mathbb{R}^{(L_\Phi+1)\mathrm{w}\times(L_\Phi+1)\mathrm{w}}, \\ \hat{R} &= \begin{bmatrix} R_0 & R_1 & \cdots & R_{L_R} & 0 & \cdots & 0 \\ 0 & R_0 & R_1 & \cdots & R_{L_R} & \ddots & 0 \\ \vdots & \ddots & \cdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & R_0 & R_1 & \cdots & R_{L_R} \end{bmatrix} \in \mathbb{R}^{(L_\Phi - L_R)\mathrm{p}\times(L_\Phi+1)\mathrm{w}}, \end{aligned} \tag{22}$$

where L_Φ, L_R are the orders of $\Phi(\zeta,\eta)$ and $R(\xi)$, respectively.

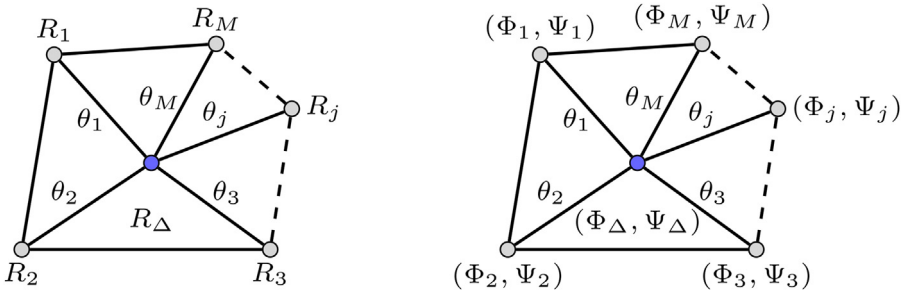


Fig. 3. Parametric dissipativity of a behavior with polytopic uncertainty.

3.2. Parametric dissipativity

For a behavior with polytopic uncertainty, the system model $R_\Delta(\xi)w = 0$ can be represented as convex combination of kernel representations

$$R_j(\xi)w = 0, \quad j = 1, \ldots, M. \tag{23}$$

Here we show that if the dissipativity properties, supply rates Φ_j and storage functions Ψ_j, $j = 1, \ldots, M$ of (23) admit a common F in (20), then the dissipativity property of $\mathfrak{B}_\Delta$ can also be a convex combination of Φ_j, Ψ_j with the same weighting parameter θ as $R_\Delta(\xi)$ (as shown in Fig. 3), that is,

$$\Phi_\Delta(\zeta, \eta) = \sum_{j=1}^{M} \theta_j \Phi_j(\zeta, \eta), \quad \Psi_\Delta(\zeta, \eta) = \sum_{j=1}^{M} \theta_j \Psi_j(\zeta, \eta), \tag{24}$$

where $(\Phi_\Delta, \Psi_\Delta)$ is called *parametric dissipativity* property. This proposed method allows for an arbitrary numbers of vertices, M, for each subsystem.

Proposition 3. *Let (Φ_j, Ψ_j) be a QDF dissipativity property of (23) and $\mathfrak{B}_\Delta$ be a convex combination of kernel representations in (23) with weighting parameter of $\theta \in \Theta$. $\mathfrak{B}_\Delta$ is Φ_Δ-dissipative with storage function $Q_{\Psi_\Delta}(w)$, where Φ_Δ, Ψ_Δ are defined in (24) with $\Psi_j \succeq 0$, if there exists an $F \in \mathbb{R}^{w \times \bullet}[\zeta, \eta]$ satisfying*

$$\Phi_j(\zeta, \eta) - \nabla\Psi_j(\zeta, \eta) + F^\star(\zeta, \eta)R_j(\eta) + R_j^T(\zeta)F(\eta, \zeta) \succeq 0, \quad j = 1, \ldots, M. \tag{25}$$

Proof. It is easy to verify that (25) leads to the condition

$$\Phi_\Delta(\zeta, \eta) - \nabla\Psi_\Delta(\zeta, \eta) + F^\star(\zeta, \eta)R_\Delta(\eta) + R_\Delta^T(\zeta)F(\eta, \zeta) \succeq 0. \tag{26}$$

Since $\Psi_\Delta = \sum_{j=1}^{M} \theta_j \Psi_j \succeq 0$, then by Lemma 2, $\mathfrak{B}_\Delta$ is dissipative with respect to the supply rate induced by Φ_Δ with storage function induced by Ψ_Δ. This completes the proof. □

Similar to (21), we can search for the parametric dissipativity property by solving the following LMIs

$$\tilde{\Psi}_j \succeq 0, \quad \tilde{\Phi}_j - \tilde{\Psi}_{\nabla_j} + \tilde{F}^T \hat{R}_j + \hat{R}_j^T \tilde{F} \succeq 0, \quad j = 1, \ldots, M. \tag{27}$$

4. Plantwide dissipativity synthesis and distributed control

4.1. Plantwide dissipativity synthesis

The central idea for plantwide analysis is to search for parametric dissipativity properties of individual subsystems and controllers such that the plantwide stability and performance condition is satisfied. The distributed controllers are then synthesized individually based on their respective supply rates through J-factorization algorithm from [30].

Denote the collective variables $\mathbf{w}_P$, $\mathbf{w}'_P$ and independent variable $\mathbf{w}_I$ of the plantwide system as follows:

$$\mathbf{w}_P = \mathrm{col}(\mathbf{w}, \mathbf{v}), \quad \mathbf{w}'_P = \mathrm{col}(\mathbf{y}, \mathbf{y}_l, \mathbf{y}_r, \mathbf{u}_p, \mathbf{d}, \mathbf{u}_c, \mathbf{u}_l, \mathbf{u}_r), \quad \mathbf{w}_I = \mathrm{col}(\mathbf{y}, \mathbf{y}_l, \mathbf{y}_r, \mathbf{d}). \tag{28}$$

Then we can have the following relationships:

$$\mathbf{w}_P = P_\pi \mathbf{w}'_P, \quad \mathbf{w}'_P = H_\pi \mathbf{w}_I, \tag{29}$$

where P_π is a permutation matrix which has exactly one entry of 1 in each row and each column and 0s elsewhere, and H_π is defined as follows

$$H_\pi = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ H_p F_p & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & I & 0 & 0 \\ F_c & 0 & 0 & 0 \\ 0 & 0 & 0 & H_c \end{bmatrix}. \tag{30}$$

Assume that the i-th process unit $\lfloor P^i_\Delta$ and the corresponding controller $\lfloor C^i$ are Φ^i_Δ- and Φ^i_c-dissipative, respectively. Then the net supply rate of the plantwide system can be written as

$$\begin{aligned} Q_{\Phi_I}(\mathbf{w}_I) &= \sum_{i=1}^{M} Q_{\Phi^i_\Delta}(w^i) + Q_{\Phi^i_c}(v^i) = \mathbf{w}_P^T \underbrace{\operatorname{diag}(\Phi^1_\Delta, \ldots, \Phi^N_\Delta, \Phi^1_c, \ldots, \Phi^N_c)}_{\Phi_P} \mathbf{w}_P \\ &= \mathbf{w}_I^T \underbrace{H_\pi^T P_\pi^T \Phi_P P_\pi H_\pi}_{\Phi_I} \mathbf{w}_I = \begin{bmatrix} \mathbf{y} \\ \mathbf{y}_l \\ \mathbf{y}_r \\ \mathbf{d} \end{bmatrix}^T \begin{bmatrix} \lfloor Q_{yy} & \lfloor Q_{yl} & \lfloor Q_{yr} & \lfloor S_{yd} \\ \lfloor Q^\star_{yl} & \lfloor Q_{ll} & \lfloor Q_{lr} & \lfloor S_{ld} \\ \lfloor Q^\star_{yr} & \lfloor Q^\star_{lr} & \lfloor Q_{rr} & \lfloor S_{rd} \\ \lfloor S^\star_{yd} & \lfloor S^\star_{ld} & \lfloor S^\star_{rd} & \lfloor R_{dd} \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{y}_l \\ \mathbf{y}_r \\ \mathbf{d} \end{bmatrix}. \end{aligned} \tag{31}$$

Now we present the main result of this work in the following theorem.

Theorem 4. *Consider a plantwide chemical system consisting of N interconnected uncertain subsystems and distributed controllers (as shown in Fig. 1b). Let $\lfloor P^i_\Delta$, $\lfloor C^i$ be Φ^j_Δ- and Φ^j_c-dissipative, respectively. If the net supply rate in (31) satisfies*

$$\begin{bmatrix} \lfloor Q_{yl}(\zeta,\eta) & \lfloor Q_{yr}(\zeta,\eta) \end{bmatrix} = 0 \tag{32a}$$

$$\begin{bmatrix} \lfloor S^\star_{yd}(\zeta,\eta) & \lfloor S^\star_{ld}(\zeta,\eta) & \lfloor S^\star_{rd}(\zeta,\eta) \end{bmatrix} = 0 \tag{32b}$$

$$\begin{bmatrix} \lfloor Q_{ll}(\zeta,\eta) & \lfloor Q_{lr}(\zeta,\eta) \\ \lfloor Q^\star_{lr}(\zeta,\eta) & \lfloor Q_{rr}(\zeta,\eta) \end{bmatrix} \preceq 0 \tag{32c}$$

$$\lfloor Q_{yy}(\zeta,\eta) + N^T(\zeta) N(\eta) \preceq 0 \tag{32d}$$

$$\gamma^2 p(\zeta) p(\eta) I - \lfloor R_{dd} \preceq 0 \tag{32e}$$

with p a Hurwitz polynomial, then the closed-loop plantwide system is robustly stable and satisfy the frequency-weighted $\lfloor H_\infty$ norm condition

$$\left\| \mathbf{W} \mathbf{T}_{\mathbf{yd}} \right\|_\infty \le \gamma \tag{33}$$

with weighting function $\mathbf{W}(s) = N(s)/p(s)$.

Proof. Condition in (32) implies that

$$\begin{aligned} Q_{\Phi_I}(\mathbf{w}_I) &\le \begin{bmatrix} \mathbf{y} \\ \mathbf{d} \end{bmatrix}^T \begin{bmatrix} -N^T(\zeta)N(\eta) & 0 \\ 0 & \gamma^2 p(\zeta) p(\eta) I \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{d} \end{bmatrix} + \begin{bmatrix} \mathbf{y}_l \\ \mathbf{y}_r \end{bmatrix}^T \begin{bmatrix} \lfloor Q_{ll}(\zeta,\eta) & \lfloor Q_{lr}(\zeta,\eta) \\ \lfloor Q^\star_{lr}(\zeta,\eta) & \lfloor Q_{rr}(\zeta,\eta) \end{bmatrix} \begin{bmatrix} \mathbf{y}_l \\ \mathbf{y}_r \end{bmatrix} \\ &\le \begin{bmatrix} \mathbf{y} \\ \mathbf{d} \end{bmatrix}^T \begin{bmatrix} -N^T(\zeta)N(\eta) & 0 \\ 0 & \gamma^2 p(\zeta) p(\eta) I \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{d} \end{bmatrix}. \end{aligned} \tag{34}$$

Thus, the plantwide closed-loop system is dissipative with respect to the supply rate induced by $\operatorname{diag}(-N^T(\zeta)N(\eta),\ \gamma^2 p(\zeta)p(\eta)I)$. From Proposition 1, the performance description in (33) is achieved. □

Remark 2. In the proposed approach, the parametric dissipativity of all process units (i.e., $\Psi^i_{p,j}$, Φ^i_p with $j = 1, \ldots, M$ for the i-th process unit) and supply rate matrices for all controllers (i.e., Φ^i_c of i-th controller) need to be solved simultaneously to satisfy the *plantwide robust performance* condition (33). The above dissipativity synthesis takes into account all interactions among process units/controllers to achieve a high level of plantwide robust performance and is carried out *offline*. Real-time control is implemented in a fully distributed manner.

Remark 3. As the weighting parameter θ^i is unknown, we need to check all possible combinations of process supply rates on the vertices, in which case the number of LMIs increases exponentially with the numbers of subsystems and polytopic vertices. One way to resolve this issue is to find *a common supply rate $Q_{\Phi^i_p}(w^i)$ shared by the models* (i.e., behaviors (23)) at all vertices rather than using an individual supply rate for each model on the vertices (i.e., $\Phi^i_{p,j}$ for the j-the vertex). This only requires to solve one set of LMIs for (32). From the plantwide viewpoint, this approach considers the common input-output dynamic features of individual subsystems under all possible

uncertain parameters. Furthermore, *different* storage functions (i.e., Ψ_j for the $j-$th vertex) are determined for different vertex models that validate the shared supply rates. This significantly reduces the conservativeness caused by the common supply rate, thereby preserving the flexibility of the polytopic conditions while keep the computational complexity within a manageable level.

Remark 4. Equations in (32) represent the conditions that ensure plantwide robust stability and robust performance requirements. Often the robust performance conditions competes with that of robust stability. If a robust performance requirement were not achievable, then the LMIs in (32) would not be feasible. In this case, the weighting function $\mathbf{W}(s)$ could be adjusted to relax the performance requirement.

Remark 5. Generally speaking, all robust control designs bear a certain level of conservatism. The proposed approach reduces the conservatism by (a) describing the uncertain region using a convex hull to provide a tighter bound, (b) adopting QDF type dissipativity to capture detailed dynamic features of the processes and (c) allowing the use of different storage functions for different vertices of the polytope as well as using the multiplier $F(\zeta, \eta)$.

4.2. Dissipativity-based robust distributed control synthesis

Denote $\mathfrak{L}_\Phi$ as the set of all controllable Φ-dissipative behaviors. The task of distributed control synthesis is to find the controller behaviors $\mathfrak{B}_c^i \in \mathfrak{L}_{\Phi_c^i}$ $(i = 1, \ldots, N)$ whose Φ_c^i satisfy the plantwide robust stability and performance condition in (32). The key to control synthesis lies in the parametrization of dissipative behaviors, as shown in the following theorem.

Theorem 5 ([35]). *Assume that the polynomial matrix $\Phi \in \mathbb{R}^{\mathrm{w}\times\mathrm{w}}[\zeta, \eta]$ admits the following J-factorization:*

$$\Phi(-\xi, \xi) = K^T(-\xi)JK(\xi), \tag{35}$$

where $J = \begin{bmatrix} I_{\sigma_+} & 0 \\ 0 & -I_{\sigma_-} \end{bmatrix}$ is a signature matrix of $(\sigma_+, 0, \sigma_-)$. Then the following statements hold:

(1) *$K(\xi)$ is a map from the set $\mathfrak{L}_\Phi$ to $\mathfrak{L}_J$, i.e., for any $\mathfrak{B} \in \mathfrak{L}_\Phi$ with image representation $w = M(\xi)\ell$, we have the behavior $\mathfrak{B}'$ with image representation $w' = K(\xi)M(\xi)\ell$ belonging to $\mathfrak{L}_J$.*
(2) *Let $L(\xi) = \mathrm{adj}\,(K(\xi))$, i.e., $L(\xi)K(\xi) = \det(K(\xi))I$. Then $L(\xi)$ parametrizes the set $\mathfrak{L}_\Phi$ through the set $\mathfrak{L}_J$, i.e., $L(\xi)$ is a map from the set $\mathfrak{L}_J$ to $\mathfrak{L}_\Phi$.*

From the above theorem, we can obtain a controller's image representation from a J-dissipative system, which is relatively easy to construct. The following proposition gives a sufficient condition for the existence of J-factorization.

Proposition 6. *If the supply rate $Q_\Phi(w) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} \lfloor Q(\zeta, \eta) & \lfloor S(\zeta, \eta) \\ \lfloor S^\star(\zeta, \eta) & \lfloor R(\zeta, \eta) \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}$ with $y \in \mathbb{R}^{\mathrm{y}}$, $u \in \mathbb{R}^{\mathrm{u}}$ satisfies*

$$\lfloor Q(\zeta, \eta) \prec 0, \quad \lfloor R(\zeta, \eta) \succ 0, \tag{36}$$

then Φ admits a J-factorization in (35).

$$\Phi(-\xi, \xi) = K^T(-\xi)JK(\xi), \quad K \in \mathbb{R}^{\mathrm{w}\times\mathrm{w}}[\xi]. \tag{37}$$

Proof. We prove the proposition by constructing the matrix $K(\cdot)$. From (36), we can have $\lfloor\tilde{Q} = -\Gamma^T\Gamma < 0$ and $\lfloor\tilde{R} = \Lambda^T\Lambda > 0$. Then we can obtain the J-factorization of $\Phi(\zeta, \eta)$ as follows:

$$\begin{aligned} \Phi(\zeta, \eta) &= \begin{bmatrix} \lfloor I_n^L(\zeta) \\ \lfloor I_m^L(\zeta) \end{bmatrix}^T \begin{bmatrix} \lfloor\tilde{Q} & \lfloor\tilde{S}^T \\ \lfloor\tilde{S} & \lfloor\tilde{R} \end{bmatrix} \begin{bmatrix} \lfloor I_n^L(\eta) \\ \lfloor I_m^L(\eta) \end{bmatrix} = \begin{bmatrix} \lfloor I_m^L(\zeta) \\ \lfloor I_n^L(\zeta) \end{bmatrix}^T \begin{bmatrix} \lfloor\tilde{R} & \lfloor\tilde{S}^T \\ \lfloor\tilde{S} & \lfloor\tilde{Q} \end{bmatrix} \begin{bmatrix} \lfloor I_m^L(\eta) \\ \lfloor I_n^L(\eta) \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} \lfloor I_m^L(\zeta) \\ \lfloor I_n^L(\zeta) \end{bmatrix}^T \begin{bmatrix} \Lambda & \Lambda^{-1}\tilde{S}^T \\ 0 & \Gamma \end{bmatrix}^T}_{K^T(\zeta)} \underbrace{\begin{bmatrix} I_m & 0 \\ 0 & -I_n \end{bmatrix}}_{J} \underbrace{\begin{bmatrix} \Lambda & \Lambda^{-1}\tilde{S}^T \\ 0 & \Gamma \end{bmatrix} \begin{bmatrix} \lfloor I_m^L(\eta) \\ \lfloor I_n^L(\eta) \end{bmatrix}}_{K(\eta)}. \end{aligned} \tag{38}$$

□

A more efficient and robust algorithm to compute the J-factorization [30] is adopted in this work:

(1) Find a solution $P = P^T \in \mathbb{R}^{(L+h)\mathrm{w}\times(L+h)\mathrm{w}}$ to the following algebraic Riccati equation (ARE)

$$PA(h) + A^T(h)P + \phi_{11}(h) - (\phi_{01}(h) + B^T(h)P)^T\phi_{00}^{-1}(\phi_{01}(h) + B^T(h)P) = 0 \tag{39}$$

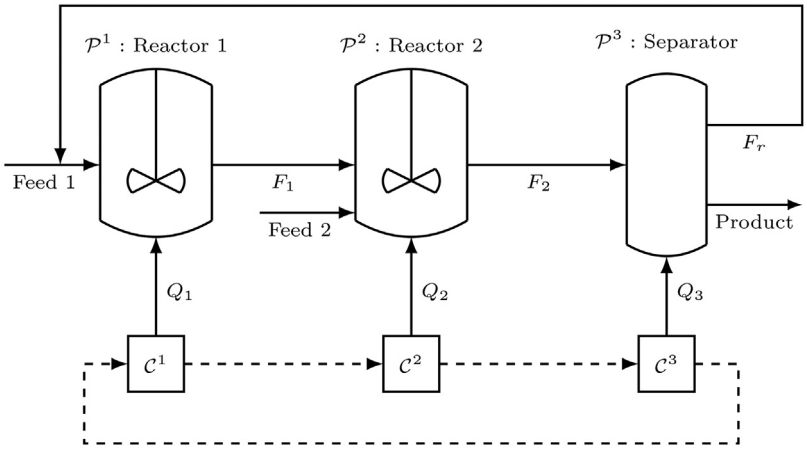


Fig. 4. Configuration of the reactor-separator process.

where $h \geq 0$ is sufficiently large, $\phi_{00} = \tilde{\Phi}_{00} \in \mathbb{R}^{w\times w}$,

$$\begin{aligned} \phi_{01}(h) &= \begin{bmatrix} \tilde{\Phi}_{01} & \cdots & \tilde{\Phi}_{0L} & 0_{w\times hw} \end{bmatrix}, \quad \phi_{11}(h) = \begin{bmatrix} \tilde{\Phi}_{11} & \cdots & \tilde{\Phi}_{1L} & 0_{w\times hw} \\ \vdots & \ddots & \vdots & \vdots \\ \tilde{\Phi}_{1L} & \cdots & \tilde{\Phi}_{LL} & 0_{w\times hw} \\ 0_{w\times hw} & \cdots & 0_{w\times hw} & 0_{w\times hw} \end{bmatrix}, \\ A(h) &= \begin{bmatrix} 0_{w(L+h-1)w} & 0_{w\times w} \\ I_{(L+h-1)w} & 0_{(L+h-1)w\times w} \end{bmatrix}, \quad B(h) = \begin{bmatrix} I_w \\ 0_{(L+h-1)w\times w} \end{bmatrix}. \end{aligned} \tag{40}$$

(2) Factorize $\phi_{00} = K_0^T J K_0$ with a non-singular $K_0 \in \mathbb{R}^{w\times w}$, and $K(\xi)$ can be constructed as

$$K(\xi) = K_0 \begin{bmatrix} I_w & \phi_{00}^{-1}(\phi_{01}(h) + B^T(h)P) \end{bmatrix} \lfloor I_w^L(\xi). \tag{41}$$

In summary, the proposed dissipativity-based robust distributed control approach includes the following two steps:

(1) Dissipativity synthesis: search for the parametric dissipativity $(\Psi_{p,j}^i, \Phi_p^i)$ with $j = 1, \ldots, M$ of i-th subsystem and supply rate matrix Φ_c^i of i-th controller by solving the following LMIs *simultaneously*:
   - the dissipativity condition in (27) (for an individual process unit) for all N process units;
   - plantwide stability and performance condition in (32);
   - controller feasibility condition in (36).
(2) Distributed robust control synthesis: To design each controller $\lfloor C^i$,
   - compute the J-factorization of Φ_c^i as $\Phi_c^i(-\xi, \xi) = {K^i}^T(-\xi) J^i K^i(\xi)$;
   - construct a J^i-dissipative system with image representation $z^i = M^i(\xi)\,\ell^i$;
   - obtain the image representation of the i-th controller as $v^i = L^i(\xi)M^i(\xi)\ell^i$.

5. Case study

A case study of a system of two chemical reactors and one separator with configuration depicted in Fig. 4 is carried out to illustrate the proposed controller design procedure. The chemical reaction involved is A → B → C where A is the reactant, B is the product and C is the undesirable by-product [36]. The aim is to control the temperature in the two reactors and the separator under the disturbances from the temperature of the feed streams 1 and 2 by manipulating the heat supplies Q_1, Q_2 and Q_3. The models governing the dynamics of each subsystem are given as follows:

$$\lfloor P^1 : \begin{cases} \dfrac{dx_{A_1}}{dt} = \dfrac{F_{1f}}{V_1}(x_{A_{1f}} - x_{A_1}) + \dfrac{F_r}{V_1}(x_{A_r} - x_{A_1}) - k_1 e^{\frac{-E_1}{RT_1}} x_{A_1} \\ \dfrac{dx_{B_1}}{dt} = \dfrac{F_{1f}}{V_1}(x_{B_{1f}} - x_{B_1}) + \dfrac{F_r}{V_1}(x_{B_r} - x_{B_1}) + k_1 e^{\frac{-E_1}{RT_1}} x_{A_1} - k_2 e^{\frac{-E_2}{RT_1}} x_{B_1} \\ \dfrac{dT_1}{dt} = \dfrac{F_{1f}}{V_1}(T_{1f} - T_1) + \dfrac{F_r}{V_1}(T_3 - T_1) + \dfrac{-\Delta H_1}{C_p} k_1 e^{\frac{-E_1}{RT_1}} x_{A_1} + \dfrac{-\Delta H_2}{C_p} k_2 e^{\frac{-E_2}{RT_1}} x_{B_1} + \dfrac{Q_1}{\rho C_p V_1} \end{cases} \tag{42}$$

Table 1
Operating point and parameters [36].

Operating Point		Parameters			
x_{A_1}	0.292	$x_{A_{1f}}$	0.846	k_1	2.77×10^3 s^{-1}
x_{B_1}	0.666	$x_{B_{1f}}$	0.154	k_2	2.5×10^3 s^{-1}
T_1	447.6 K	$x_{A_{2f}}$	0.610	E_1	5×10^4 kJ kmol^{-1}
x_{A_2}	0.287	$x_{B_{2f}}$	0.390	E_2	6×10^4 kJ kmol^{-1}
x_{B_2}	0.670	F_{1f}	5 m^3 h^{-1}	ΔH_1	-6×10^4 kJ kmol^{-1}
T_2	444.3 K	F_{2f}	5 m^3 h^{-1}	ΔH_2	7×10^4 kJ kmol^{-1}
x_{A_3}	0.114	V_1	1 m^3	α_A	3.5
x_{B_3}	0.800	V_2	0.5 m^3	α_B	1
T_3	449.3 K	V_3	1 m^3	α_C	0.5
T_{1f}	300 K	F_r	50 m^3		
T_{2f}	300 K	ρ	1000 kg m^{-3}		
Q_1	12.6×10^5 kJ h^{-1}	C_p	4.2 kJ kg^{-1} K^{-1}		
Q_2	16.2×10^5 kJ h^{-1}	R	8.314 kJ kmol^{-1} K^{-1}		
Q_3	12.6×10^5 kJ h^{-1}				

$$\lfloor P^2 : \begin{cases} \dfrac{dx_{A_2}}{dt} = \dfrac{F_1}{V_2}(x_{A_1} - x_{A_2}) + \dfrac{F_{2f}}{V_2}(x_{A_{2f}} - x_{A_2}) - k_1 e^{\frac{-E_1}{RT_2}} x_{A_2} \\ \dfrac{dx_{B_2}}{dt} = \dfrac{F_1}{V_2}(x_{B_1} - x_{B_2}) + \dfrac{F_{2f}}{V_2}(x_{B_{2f}} - x_{B_2}) + k_1 e^{\frac{-E_1}{RT_2}} x_{A_2} - k_2 e^{\frac{-E_2}{RT_2}} x_{B_2} \\ \dfrac{dT_2}{dt} = \dfrac{F_1}{V_2}(T_1 - T_2) + \dfrac{F_{2f}}{V_2}(T_{2f} - T_2) + \dfrac{-\Delta H_1}{C_p} k_1 e^{\frac{-E_1}{RT_2}} x_{A_2} + \dfrac{-\Delta H_2}{C_p} k_2 e^{\frac{-E_2}{RT_2}} x_{B_2} + \dfrac{Q_2}{\rho C_p V_2} \end{cases} \tag{43}$$

$$\lfloor P^3 : \begin{cases} \dfrac{dx_{A_3}}{dt} = \dfrac{F_2}{V_3}(x_{A_2} - x_{A_3}) - \dfrac{F_r}{V_3}(x_{A_r} - x_{A_3}) \\ \dfrac{dx_{B_3}}{dt} = \dfrac{F_2}{V_3}(x_{B_2} - x_{B_3}) - \dfrac{F_r}{V_3}(x_{B_r} - x_{B_3}) \\ \dfrac{dT_3}{dt} = \dfrac{F_2}{V_3}(T_2 - T_3) + \dfrac{Q_3}{\rho C_p V_3} \end{cases} \tag{44}$$

In the models, $x_{A_1}, x_{A_2}, x_{A_3}, x_{A_{1f}}$ and $x_{A_{2f}}$ denote, respectively, mass fraction of A in the first reactor, second reactor, separator, the first feed stream and the second feed stream. Similar counterparts can be defined for x_B and T, respectively, for the mass fraction of B and temperature. x_{A_r} and x_{B_r} denote, respectively, the mass fraction of A and B in the recycle stream and are computed as

$$x_{A_r} = \frac{\alpha_A x_{A_3}}{\alpha_A x_{A_3} + \alpha_B x_{B_3} + \alpha_C x_{C_3}} \tag{45a}$$

$$x_{B_r} = \frac{\alpha_B x_{B_3}}{\alpha_A x_{A_3} + \alpha_B x_{B_3} + \alpha_C x_{C_3}}, \tag{45b}$$

where α_A, α_B and α_C are the reflux ratios for A, B and C, respectively, in the separator. F_{1f} and F_{2f} denote, respectively, the volumetric flowrates in Feed 1 and Feed 2. Since the mass fraction of C, x_C, is not independent to x_A and x_B ($x_A + x_B + x_C = 1$ always holds for all streams), a model for it is not included. For this process, the manifest and interconnection variables can be defined as

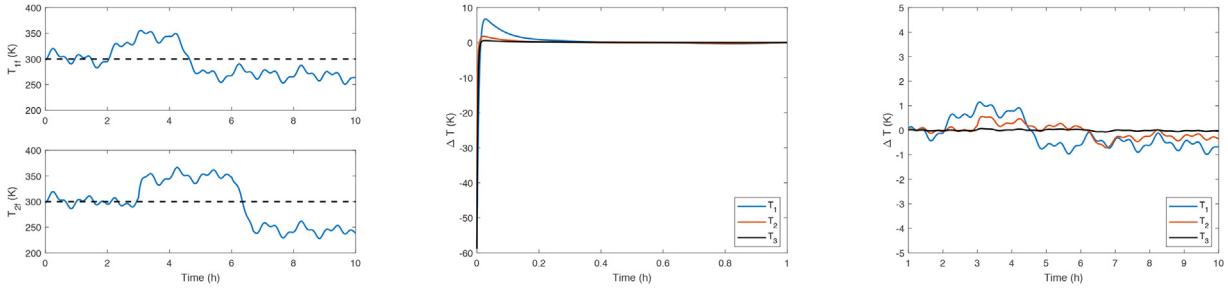
$$\begin{aligned} y^i = \text{col}\ \ & (x_{A_i}, x_{B_i}, T_i), \quad u_c^i = y_l^i = Q_i, \quad u_l^i = T_i\, d^i = T_{jf}, \quad i = 1, 2, 3, \quad j = 1, 2, \\ & u_p^1 = y^3, \quad u_p^2 = y^1, \quad u_p^3 = y^2, \quad u_r^1 = y_r^3, \quad u_r^2 = y_r^1, \quad u_r^3 = y_r^2. \end{aligned} \tag{46}$$

The operating point and parameters in the process are given in Table 1. For illustration purpose, we assume that there are uncertainties in the reaction rate coefficients k_1 and k_2 up to 10% and that reflex ratio for component C in the separator, α_C, is uncertain, between 0.5 and 0.7. The models are linearized around the operating point and transformed into kernel representation. For each process unit, the effects of uncertain parameters in the nonlinear process model are reflected by the uncertain linear models in a polytopic region. The detailed models of all vertices (in kernel representations) are presented in A.

The objective of the distributed control system is to control the purity of product B in all units. As T_3 is the control error of the temperature of the final product, higher level of disturbance attenuation should be achieved compared with T_1 and T_2. The weighting functions for plantwide performance (as in (33)) is therefore chosen as $\mathbf{W}(s) = \text{diag}\,(W_1(s), W_2(s), W_3(s))$ where

$$W_1(s) = W_2(s) = \frac{1}{0.2s+1}\,\text{diag}\,(0, 0, 20), \quad W_3(s) = \frac{1}{0.2s+1}\,\text{diag}\,(0, 0, 50) \tag{47}$$

are the weighting functions for process units 1 to 3 respectively, which requires a new steady state to be reached within approximately 1 hour after the introduction of the disturbance and the disturbance effect to be attenuated by 20 times in the reactors and 50 times in the separator. The details of constructing a QDF based on a weighting function is illustrated in B. Simulation studies were carried out for a "true" process behavior randomly chosen within the polytopic regions (i.e., with different θ^i, $i = 1, \ldots, 3$).



(a) disturbance (b) transition (c) steady-state

Fig. 5. Simulation result.

Following the two-step design, inequalities (27), (32) and (36) should be solved simultaneously, bringing a total of 39 LMIs. The J-factorization process (39)–(41) is then used to determine the final controllers that attain image representation

$$\begin{bmatrix} Q_i \\ y_r^i \\ T_i \\ u_r^i \end{bmatrix} = \begin{bmatrix} M_y^i(\xi) \\ M_u^i(\xi) \end{bmatrix} \ell^i \tag{48}$$

for some polynomial matrix $M^i(\xi)$ and latent variable ℓ^i. The simulation result is depicted in Fig. 5 and the performance specified by the weighting matrices (47) has been achieved.

6. Conclusion

Based on the dissipativity theory, a robust distributed control approach has been proposed for plantwide systems whose model-plant mismatch is described by polytopic uncertainties. The plantwide robust stability and performance conditions have been developed based on global dissipativity conditions, which in turn have been converted into the supply rate that each distributed control must satisfy. The controllers can be synthesis individually in parallel by computing J-factorization of their corresponding supply rates.

Acknowledgement

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Appendix A. Uncertain process models

This section provides the kernel representations to the subsystems. The manifest variables considered for each unit are

$$w^1 = \text{col}\left(x_{A_1}, x_{B_1}, T_1, x_{A_3}, x_{B_3}, T_3, Q_1, T_{1f}\right) \tag{A.1a}$$

$$w^2 = \text{col}\left(x_{A_2}, x_{B_2}, T_2, x_{A_1}, x_{B_1}, T_1, Q_2, T_{2f}\right) \tag{A.1b}$$

$$w^3 = \text{col}\left(x_{A_3}, x_{B_3}, T_3, x_{A_2}, x_{B_2}, T_2, Q_3\right). \tag{A.1c}$$

After linearization and setting up the polytopic region, the first process is within a region of 8 vertices, whose kernel representations are

$$R_1^1(\xi) = \begin{bmatrix} \xi+65.97 & 0 & 0.128 & -102.2 & 6.46 & 0 & 0 & 0 \\ -14.57 & \xi+55.90 & -0.106 & 77.83 & -27.3 & 0 & 0 & 0 \\ -2082 & 383.6 & \xi+45.97 & 0 & 0 & -50 & -0.0002 & -5 \end{bmatrix} \tag{A.2}$$

$$R_2^1(\xi) = \begin{bmatrix} \xi+65.97 & 0 & 0.128 & -102.2 & 6.46 & 0 & 0 & 0 \\ -16.03 & \xi+55.98 & -0.104 & 77.83 & -27.3 & 0 & 0 & 0 \\ -2290 & 422.0 & \xi+46.90 & 0 & 0 & -50 & -0.0002 & -5 \end{bmatrix} \tag{A.3}$$

$$R_3^1(\xi) = \begin{bmatrix} \xi+71.03 & 0 & 0.140 & -102.2 & 6.46 & 0 & 0 & 0 \\ -16.03 & \xi+55.90 & -0.119 & 77.83 & -27.3 & 0 & 0 & 0 \\ -2390 & 383.6 & \xi+44.15 & 0 & 0 & -50 & -0.0002 & -5 \end{bmatrix} \tag{A.4}$$

$$R_4^1(\xi) = \begin{bmatrix} \xi+71.03 & 0 & 0.140 & -102.2 & 6.46 & 0 & 0 & 0 \\ -14.57 & \xi+55.98 & -0.117 & 77.83 & -27.3 & 0 & 0 & 0 \\ -2082 & 422.0 & \xi+45.07 & 0 & 0 & -50 & -0.0002 & -5 \end{bmatrix} \tag{A.5}$$

$$R_5^1(\xi)=\begin{bmatrix}\xi+65.97 & 0 & 0.128 & -103.8 & 3.77 & 0 & 0 & 0\\ -14.57 & \xi+55.90 & -0.106 & 70.66 & -32.2 & 0 & 0 & 0\\ -2082 & 383.6 & \xi+45.97 & 0 & 0 & -50 & -0.0002 & -5\end{bmatrix} \tag{A.6}$$

$$R_6^1(\xi)=\begin{bmatrix}\xi+71.03 & 0 & 0.128 & -103.8 & 3.77 & 0 & 0 & 0\\ -16.03 & \xi+55.98 & -0.104 & 70.66 & -32.2 & 0 & 0 & 0\\ -2290 & 422.0 & \xi+46.90 & 0 & 0 & -50 & -0.0002 & -5\end{bmatrix} \tag{A.7}$$

$$R_7^1(\xi)=\begin{bmatrix}\xi+71.03 & 0 & 0.140 & -103.8 & 3.77 & 0 & 0 & 0\\ -16.03 & \xi+55.90 & -0.119 & 70.66 & -32.2 & 0 & 0 & 0\\ -2290 & 383.6 & \xi+44.15 & 0 & 0 & -50 & -0.0002 & -5\end{bmatrix} \tag{A.8}$$

$$R_8^1(\xi)=\begin{bmatrix}\xi+65.97 & 0 & 0.140 & -103.8 & 3.77 & 0 & 0 & 0\\ -14.57 & \xi+55.98 & -0.117 & 70.66 & -32.2 & 0 & 0 & 0\\ -2082 & 422.0 & \xi+45.07 & 0 & 0 & -50 & -0.0002 & -5\end{bmatrix}. \tag{A.9}$$

The second process is in a polytopic region with 4 vertices, namely

$$R_1^2(\xi)=\begin{bmatrix}\xi+133.2 & 0 & 0.115 & -110 & 0 & 0 & 0 & 0\\ -13.20 & \xi+120.8 & -0.096 & 0 & -110 & 0 & 0 & 0\\ -1886 & 340.8 & \xi+111.9 & 0 & 0 & -110 & -0.0005 & -10\end{bmatrix} \tag{A.10}$$

$$R_2^2(\xi)=\begin{bmatrix}\xi+133.2 & 0 & 0.115 & -110 & 0 & 0 & 0 & 0\\ -13.20 & \xi+120.9 & -0.094 & 0 & -110 & 0 & 0 & 0\\ -1886 & 374.8 & \xi+112.7 & 0 & 0 & -110 & -0.0005 & -10\end{bmatrix} \tag{A.11}$$

$$R_3^2(\xi)=\begin{bmatrix}\xi+134.5 & 0 & 0.127 & -110 & 0 & 0 & 0 & 0\\ -14.52 & \xi+120.8 & -0.107 & 0 & -110 & 0 & 0 & 0\\ -2074 & 340.8 & \xi+110.2 & 0 & 0 & -110 & -0.0005 & -10\end{bmatrix} \tag{A.12}$$

$$R_4^2(\xi)=\begin{bmatrix}\xi+134.5 & 0 & 0.127 & -110 & 0 & 0 & 0 & 0\\ -14.52 & \xi+120.9 & -0.105 & 0 & -110 & 0 & 0 & 0\\ -2074 & 374.8 & \xi+111.1 & 0 & 0 & -110 & -0.0005 & -10\end{bmatrix}. \tag{A.13}$$

The last process is within a 2-vertex region whose vertices are

$$R_1^3(\xi)=\begin{bmatrix}\xi+112.2 & -6.46 & 0 & -60 & 0 & 0 & 0\\ -77.83 & \xi+37.30 & 0 & 0 & -60 & 0 & 0\\ 0 & 0 & \xi+60 & 0 & 0 & -60 & -0.0002\end{bmatrix} \tag{A.14}$$

$$R_2^3(\xi)=\begin{bmatrix}\xi+113.8 & -3.77 & 0 & -60 & 0 & 0 & 0\\ -70.66 & \xi+42.15 & 0 & 0 & -60 & 0 & 0\\ 0 & 0 & \xi+60 & 0 & 0 & -60 & -0.0002\end{bmatrix}. \tag{A.15}$$

Appendix B. Construction of QDFs from weighting functions

This section provides the details of the construction of QDFs in (32) from the weighting functions (47). We use $W_1(s)$ to illustrate the procedure and the other ones follow analogously.

For $W_1(s)$ we can write it as $W_1(s)=N_1(s)/p_1(s)$ where $N_1(s)=\mathrm{diag}(0,0,20)$ and $p_1(s)=0.2s+1$. Since in linearized systems all variables are deviation variables, it then follows that $s=\xi$. Therefore, $N(\xi)=\mathrm{diag}(0,0,20)$ and $p(\xi)=0.2\xi+1$. We then have

$$N^T(\zeta)N(\eta)=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 400\end{bmatrix}, \tag{B.1}$$

$$p(\zeta)p(\eta)I_3=[1+0.2(\zeta+\eta)+0.04\zeta\eta]\,I_3=\begin{bmatrix}I_3\\ \zeta I_3\end{bmatrix}^T\begin{bmatrix}I_3 & 0.2I_3\\ 0.2I_3 & 0.04I_3\end{bmatrix}\begin{bmatrix}I_3\\ \eta I_3\end{bmatrix}. \tag{B.2}$$

In most cases, the order of the other QDFs in LMIs are not of the same order. To match the dimensions, these QDFs can be augmented. For example, to represent (B.2) with second order, one can write it as

$$p(\zeta)p(\eta)I_3=\begin{bmatrix}I_3\\ \zeta I_3\\ \zeta^2 I_3\end{bmatrix}^T\begin{bmatrix}I_3 & 0.2I_3 & 0_{3\times3}\\ 0.2I_3 & 0.04I_3 & 0_{3\times3}\\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3}\end{bmatrix}\begin{bmatrix}I_3\\ \eta I_3\\ \eta^2 I_3\end{bmatrix}. \tag{B.3}$$

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