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Admissibility of h_3

My heuristic h_3 is defined as follows $h_3(anyState) = \lceil \frac{M(anyState)}{2} \rceil$, where the M(anyState) essentially just uses h_2 getting the sum of Manhattan distances of all the numbered tiles in State anyState from their goal positions.

Each single or double move can reduce the total Manhattan distance by at most 2, so any sequence of k moves can reduce our M value by at most 2k.
This works because:

- A single move can reduce total manhattan distance by at most 1
- A double move can reduce it by at most 2

Since a state with manhattan distance M must eventually reach 0, or goal state, any solution of length k must have $k \geq \lceil \frac{M}{2} \rceil$ true. We use ceil since if you are required to perform any single moves, it still counts as a full move, so an odd value must round up as an integer. So h_3 is always a lower bound on the true number of moves required. It never overestimates, so h_3 is admissible.