1 Replacing the calculator with a computer

Let us consider a basic calculator with buttons to add, subtract, multiply, divide, and take square roots. Using such a simple thing is certainly familiar for any reader of these notes. Indeed, a familiarity with a *graphing* calculator is expected. `Julia` makes these familiar tasks just as easy, offering numerous conveniences along the way. In this section we describe how.

The following image is the calculator that Google presents upon searching for "calculator."

This calculator should have a familiar appearance with a keypad of numbers, a set of buttons for arithmetic operations, a set of buttons for some common mathematical functions, a degree/radian switch, and buttons for interacting with the calculator: `Ans`, `AC` (also `CE`), and `=`.

The goal here is to see the counterparts within `Julia` to these features.

For an illustration of *really* basic calculator, have some fun watching this video:

...unable to display raw html...

1.1 Operations

Performing a simple computation on the calculator typically involves hitting buttons in a sequence, such as "1", "+", "2", "=" to compute 3 from adding 1 + 2. In `Julia`, the process is not so different. Instead of pressing buttons, the various values are typed in. So, we would have:

```
1 + 2
```

3

Sending an expression to `Julia`'s interpreter - the equivalent of pressing the "=" key on a calculator - is done at the command line by pressing the `Enter` or `Return` key, and in `IJulia` using the "play" icon, or the keyboard shortcut `Shift-Enter`. If the current expression is complete, then `Julia` evaluates it and shows any output. If the expression is not complete, `Julia`'s response depends on how it is being called. Within `IJulia`, a message about "premature" end of input is given. If the expression raises an error, this will be noted.

The basic arithmetic operations on a calculator are "+", "-", "×", "÷", and "x". These have parallels in `Julia` through the *binary* operators: `+`, `-`, `*`, `/`, and `^`:

```
1 + 2, 2 - 3, 3 * 4, 4 / 5, 5 ^ 6
```

```
(3, -1, 12, 0.8, 15625)
```

On some calculators, there is a distinction between minus signs - the binary minus sign and the unary minus sign to create values such as -1.

In `Julia`, the same symbol, "`-`", is used for each:

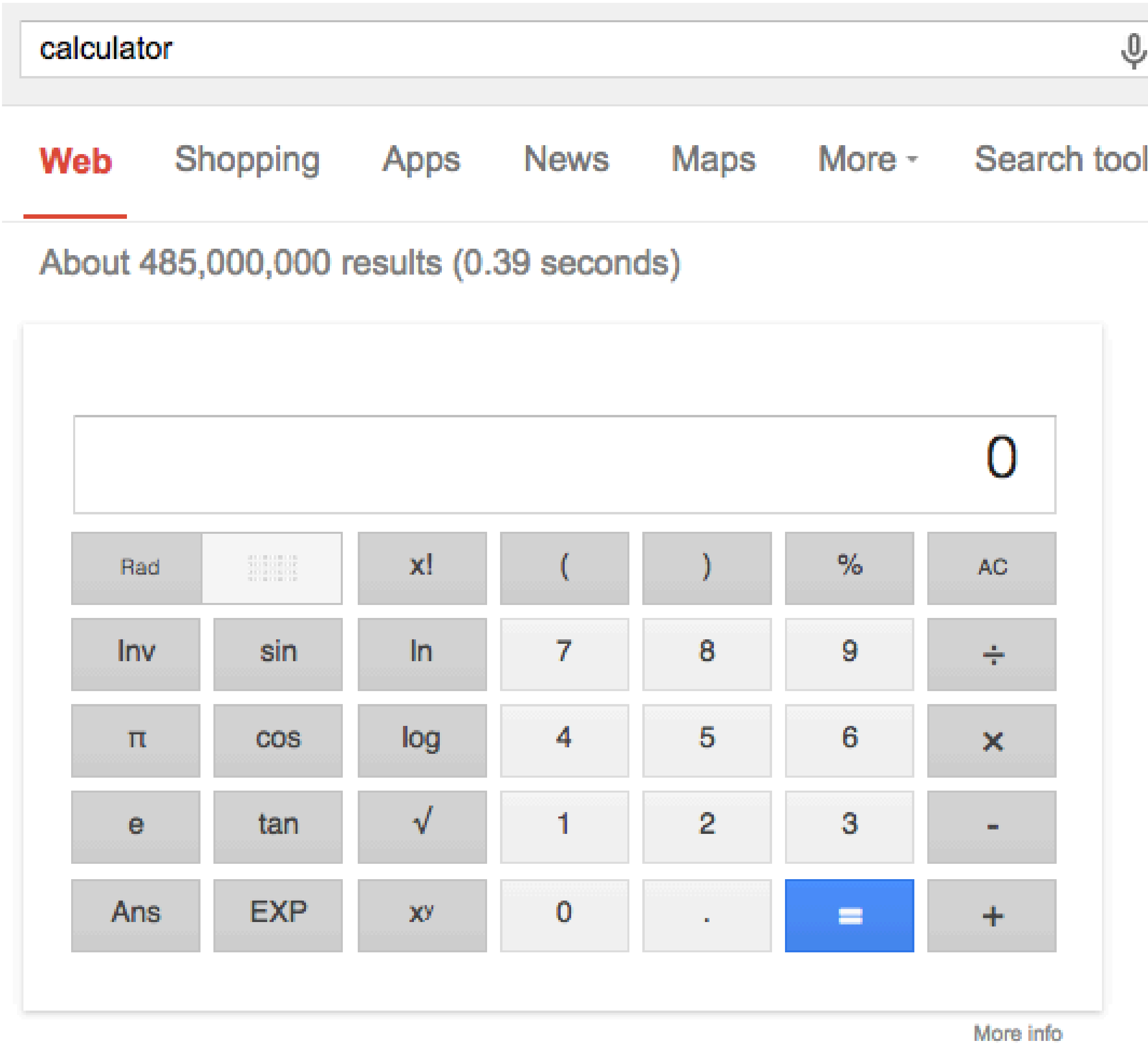

Figure 1: Screenshot of a calculator provided by the Google search engine

```
-1 - 2
```

-3

An expression like $6 - -3$, subtracting minus three from six, must be handled with some care. With the Google calculator, the expression must be entered with accompanying parentheses: $6 - (-3)$. In `Julia`, parentheses may be used, but are not needed. However, if omitted, a space is required between the two minus signs:

```
6 - -3
```

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(If no space is included, the value ”`--`” is parsed like a different, undefined, operation.)

> `Julia` only uses one symbol for minus, but web pages may not! Copying and pasting an expression with a minus sign can lead to hard to understand errors such as: `invalid character "-"`. There are several Unicode symbols that look similar to the ASCII minus sign, but are different. These notes use a different character for the minus sign for the typeset math (e.g., $1 - \pi$) than for the code within cells (e.g. `1 - 2`). Thus, copying and pasting the typeset math may not work as expected.

1.1.1 Examples

Example For everyday temperatures, the conversion from Celsius to Fahrenheit ($9/5C + 32$) is well approximated by simply doubling and adding 30. Compare these values for an average room temperature, $C = 20$, and for a relatively chilly day, $C = 5$:

For $C = 20$:

```
9 / 5 * 20 + 32
```

68.0

The easy to compute approximate value is:

```
2 * 20 + 30
```

70

The difference is:

```
(9/5*20 + 32) - (2 * 20 + 30)
```

-2.0

For $C = 5$, we have the actual value of:

```
9 / 5 * 5 + 32
```

4 1 . 0

and the easy to compute value is simply $40 = 10 + 30$. The difference is

```
(9 / 5 * 5 + 32) - 40
```

1 . 0

Example Add the numbers $1 + 2 + 3 + 4 + 5$.

```
1 + 2 + 3 + 4 + 5
```

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Example How small is $1/2/3/4/5/6$? It is about $14/10,000$, as this will show:

```
1/2/3/4/5/6
```

0 . 0 0 1 3 8 8 8 8 8 8 8 8 8 8 8 8 8 8 9

Example Which is bigger 4^3 or 3^4? We can check by computing their difference:

```
4^3 - 3^4
```

-1 7

So 3^4 is bigger.

Example A right triangle has sides $a = 11$ and $b = 12$. Find the length of the hypotenuse squared. As $c^2 = a^2 + b^2$ we have:

```
11^2 + 12^2
```

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1.2 Order of operations

The calculator must use some rules to define how it will evaluate its instructions when two or more operations are involved. We know mathematically, that when $1 + 2 \cdot 3$ is to be evaluated the multiplication is done first then the addition.

With the Google Calculator, typing `1 + 2 x 3 =` will give the value 7, but *if* we evaluate the `+` sign first, via `1 + 2 = x 3 =` the answer will be 9, as that will force the addition of `1+2` before multiplying. The more traditional way of performing that calculation is to use *parentheses* to force an evaluation. That is, `(1 + 2) * 3 =` will produce `9` (though one must type it in, and not use a mouse to enter). Except for the most primitive of calculators, there are dedicated buttons for parentheses to group expressions.

In `Julia`, the entire expression is typed in before being evaluated, so the usual conventions of mathematics related to the order of operations may be used. These are colloquially summarized by the acronym PEMDAS.

> **PEMDAS**. This acronym stands for Parentheses, Exponents, Multiplication, Division, Addition, Subtraction. The order indicates which operation has higher precedence, or should happen first. This isn't exactly the case, as "M" and "D" have the same precedence, as do "A" and "S". In the case of two operations with equal precedence, *associativity* is used to decide which to do. For the operations `+`, `-`, `*`, `/` the associativity is left to right, as in the left one is done first, then the right. However, `^` has right associativity, so `4^3^2` is `4^(3^2)` and not `(4^3)^2`. (Be warned that some calculators - and spread sheets, such as Excel - will treat this expression with left associativity.)

With rules of precedence, an expression like the following has a clear interpretation to `Julia` without the need for parentheses:

```
1 + 2 - 3 * 4 / 5 ^ 6
```

```
2.999232
```

Working through PEMDAS we see that `^` is first, then `*` and then `/` (this due to associativity and `*` being the leftmost expression of the two) and finally `+` and then `-`, again by associativity rules. So we should have the same value with:

```
(1 + 2) - ((3 * 4) / (5 ^ 6))
```

```
2.999232
```

If different parentheses are used, the answer will likely be different. For example, the following forces the operations to be `-`, then `*`, then `+`. The result of that is then divided by `5^6`:

```
(1 + ((2 - 3) * 4)) / (5 ^ 6)
```

```
-0.000192
```

1.2.1 Examples

Example The percentage error in x if y is the correct value is $(x - y)/y \cdot 100$. Compute this if $x = 100$ and $y = 98.6$.

```
(100 - 98.6) / 98.6 * 100
```

```
1.4198782961460505
```

Example The marginal cost of producing one unit can be computed by finding the cost for $n+1$ units and subtracting the cost for n units. If the cost of n units is n^2+10, find the marginal cost when $n = 100$.

```
(101^2 + 10) - (100^2 + 10)
```

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Example The average cost per unit is the total cost divided by the number of units. Again, if the cost of n units is n^2+10, find the average cost for $n = 100$ units.

```
(100^2 + 10) / 100
```

1 0 0 . 1

Example The slope of the line through two points is $m = (y_1 - y_0)/(x_1 - x_0)$. For the two points $(1, 2)$ and $(3, 4)$ find the slope of the line through them.

```
(4 - 2) / (3 - 1)
```

1 . 0

1.2.2 Two ways to write division - and they are not the same

The expression $a + b/c + d$ is equivalent to $a + (b/c) + d$ due to the order of operations. It will generally have a different answer than $(a + b)/(c + d)$.

How would the following be expressed, were it written inline:

$$\frac{1+2}{3+4}?$$

It would have to be computed through $(1+2)/(3+4)$. This is because unlike `/`, the implied order of operation in the mathematical notation with the *horizontal division symbol* (the vinicula) is to compute the top and the bottom and then divide. That is, the vinicula is a grouping notation like parentheses, only implicitly so. Thus the above expression really represents the more verbose:

$$\frac{(1+2)}{(3+4)}.$$

Which lends itself readily to the translation:

```
(1 + 2) / (3 + 4)
```

0 . 4 2 8 5 7 1 4 2 8 5 7 1 4 2 8 5 5

To emphasize, this is not the same as the value without the parentheses: