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A Structural Analysis of a Game: The Liverpool v Manchester United Cup Final of 1977*

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A team is more than the sum of its individual players, and so implies a structure of relations on the set. The q*-analysis, or polyhedral dynamics, of Atkin is chosen to define and operationalise intuitive notions of structure in a soccer match between Liverpool and Manchester United. The injection of* q*-holes, or obtrusive objects, by the defense of one team appears to contribute to the fragmentation and loss of the other.*

Introduction

It is curious that the structure of team games has gone virtually unexamined (Reep and Benjamin, 1968). The very idea of a team implies some sort of structure, since a team is supposed to be more than the sum of the individual players. In this paper we shall explore the structure of a soccer match, although it is clear that the formal analysis is applicable to many other sports.

Anyone who has observed a soccer match will readily acknowledge the extraordinary complexity and fluid dynamics of the contemporary game, and the difficulty of defining in a clear, operational way what a knowledgeable spectator would imply by the phrase "the structure of the game". At the same time, most devotees would acknowledge that soccer matches between different sets of players may have a very different 'feel' about them. People might report that "team X were much better organized", that "team Y were falling apart", that "team Z played a very tight defensive game", and so on. We would like to suggest that contained in these intuitive, descriptive phrases are very powerful and important notions of structure. Our task, therefore, is to find operational definitions of these intuitive

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ideas, and see whether a structural examination can lead to insights not readily obtainable by simply watching the game itself, or by observing a replay recorded on a television tape.

The Liverpool–Manchester Cup Final, 1977

Consider the players of the Liverpool and Manchester United soccer teams taking part in the cup final of 1977 as a set of 22 elements. Define a relation in this set according to the number of times the ball passes from one player to another. Thus, a matrix can be constructed, the rows of which are considered as 'senders', while the columns are 'receivers'. Binary relations may be defined on the entire set by choosing a variety of slicing parameters; that is, certain values or levels must be equalled or exceeded before the interaction between a particular pair of players is part of the defining relation and consequent structure. Alternatively, we may examine relations defined on particular subsets. For example, relations may be defined on the Liverpool and Manchester teams alone, or they may be defined between the Liverpool and Manchester subsets. We shall refer to the former as 'passing' relations, for they define the internal structures of the respective teams. The latter, the relations between the two sets of players, are defined as 'stealing' relations, for they imply the loss of the ball by one team to the other.

It is worth noting that to investigate anything requires the use of up to three languages of enquiry; verbal description using words; graphic description in the form of pictures, maps and diagrams; and algebraic description in symbols. We shall employ all three languages interchangeably, and sometimes simultaneously. The 'language of structure' or polyhedral dynamics, developed by the mathematician R. H. Atkin, has been chosen, and it is important to emphasise that it is an algebraic language for the discussion, definition, and description of structure. Although a computer algorithm is used as a form of prosthetic intelligence, q-analysis is an interactive process, rather than the usual sort of multivariate, generally linear, approaches and techniques characteristic of much social science research today. In contrast, and in a tradition of 18th-Century science, such algebraic topological analysis keeps the investigator very close to the original data, in no way imposing a linear, or perhaps unspecified, filter between the original data set and the acts of description and analysis.

q-analysis: an introductory sketch

A substantial literature, across a wide spectrum of fields, has employed q-analysis or polyhedral dynamics over the past ten years[1]. Many items in

[1] Areas of application include social and economic analysis, particularly unemployment; regional and urban planning; urban aesthetics and design; the physical and institutional structure of a

the literature contain introductory statements, and we only have room here for a thumbnail sketch, mainly to introduce terms and concepts[2]. It is important to note that q-analysis represents an entire methodology or approach to structural analysis, rather than a technique through which data are cranked in the hope that some highly filtered and anaemic residue will appear at the other end upon which some 'interpretation' may be imposed. Despite the apparently eosteric mathematical notation to be found in theoretical works[3], the heart of q-analysis is not mathematical, but conceptual and philosophical. It is the extreme intellectual discipline imposed by q-methodology that distinguishes it from more conventional and highly constrained approaches, nearly all of which end up borrowing inappropriate mathematical structures from the physical sciences, and trying to map the complexity of human affairs upon them.

It is difficult to convey the sense of discipline imposed, for the reader is likely to nod her head at a series of assertions, many of which appear intuitively obvious, and are assumed to characterise most scientific work. In the social sciences they seldom do. To assert, for example, that it is crucial to define rigorously and unambiguously the sets of elements under enquiry, is to assert an apparent banality. The social science literature over all fields provides copious evidence to the contrary. To note the critical and logical importance of not confusing elements and sets of elements appears to be noting the obvious. The same reference literature denies such an assertion. Our verbal language of everyday discourse is highly structured into a hierarchy of cover sets, in which the partition appears to be the exception rather than the rule. If the algebraic structure inherent within ordinary language is not disclosed prior to an analysis, grave logical difficulties, ambiguities and paradoxes will obscure the actual structures characterising a data set.

Apart from the fundamental task of defining the sets of elements and their hierarchical structure, q-analysis rests upon the equally important task of defining relations between sets. For those familiar with conventional approaches, this is a distinction of first importance: a function is a mapping is a relation – the reverse is not true. If we look at a problem through the

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university; international television; taxonomy; geology; aesthetics, in painting and poetry; clinical psychology; industrial relations; large organisations and corporations, such as the multinational firm; communication structures; agricultural development; diffusion of innovations; team games; chess; gender research; transportation structure and circulation; ecological disruption; medical diagnostics; physics, particularly quantum theory; decision making; and the structure of periodic markets. In the course of investigating such structures, requiring the development of original mathematics, considerable contributions have been made by changing our views of social dynamics, and even the concept of time itself. Philosophers have commented that q-analysis may well represent a primary example of the emancipatory tradition of enquiry as outlined by Jürgen Habermas.

[2] A full bibliography would include well over 100 items, including formal texts (Atkin, 1974), scholarly books, introductions for the general reader, over a dozen research monographs, whole issues of international journals, and substantial reviews in English, Portuguese and Japanese; see, for example, P. Gould, *Dimâmica de Poliedros: Uma Introducao para Cientistas Sociais, Géografos e Planeadores* (Lisboa: Estudos para o Planeamento Regional e Urbano, 1979).

[3] Although considered completely standard and well-defined in algebraic topology.

small and constrained keyhole of the function, we actually [illegible]ish rich dimensionality and information out of its characterising data se[illegible] All relations are not functions. On the other hand, if we expand our approach, and assume the structure between two sets of elements is a relation, we will always find a function if it happens to characterise the actual structure. All functions are relations. It is numbing to see how the social sciences have blindly followed the functional methodologies of the physical sciences, and how tortuous statistical assumptions have had to be made to get around the obvious descriptive difficulties.

A third characteristic of q-analysis is the very careful conceptual distinction made between backcloth and traffic, and the point is perhaps best made with a couple of examples. Backcloth refers to a supporting structure upon which traffic can exist; backcloth can exist without traffic, but traffic requires backcloth for support. For example, in agriculture the backcloth might consist of a relation between a set of farmers F and a set of elements S, such as land, water, instruments of cultivation, irrigation equipment, literacy, technical aid, credit institutions, and so on. Formally, we could write such a relation $\lambda \subseteq F \otimes S$, in which each farmer is a simplex, a multidimensional polyhedron defined by vertices in the set S. Farmers sharing certain characteristics are connected to form a complex – the simplicial complex $K_F(S; \lambda)$. Or we could consider the conjugate $K_S(F; \lambda^{-1})$, in which each element in the set of agricultural characteristics is a simplex defined by farmer vertices. The backcloth is defined as the union of a complex and its conjugate. Traffic on such a backcloth would be the crops and animals raised by the farmers, and it is clear that the structure and dimensionality of the underlying geometry of the backcloth can allow or forbid the existence and magnitude of the traffic supported by it. For example, in a recent pilot study of an irrigated region in Portugal, it was clear that traffic such as fruit trees (an innovative crop for which there were high hopes of rapid diffusion), was severely constrained by the low degree of land ownership and the high levels of renting from absentee landlords.

A second example of the backcloth-traffic distinction would be a relation between a set of branch plants, say B, and a set of locations L, so that the backcloth consists of the relation $\lambda \subseteq B \otimes L$. Traffic might consist of the products produced by such a large-scale organisation, and it has become clear in a number of studies that management tends to view things from the viewpoint of the complex, while the unions regard the situation from the conjugate. Different pieces of a usually ill-defined structure are perceived by different parties, and it is not surprising that communication between the two groups is often difficult and at apparent cross-purposes.

The term traffic is a technical one, and does not necessarily imply movement on the backcloth. Such questions characterise q-transmission. For example, in a system of periodic markets in India, price changes may be transmitted from one system to another through the connective structure of the backcloth formed by relations between small villages and the markets themselves.

The simplicial complexes have an intuitive geometric representation derived from the binary relation defined by the incidence matrix. However, q-analysis is not constrained by 1-0 binary matrices as the initial data: more familiar matrices, usually employing elements of the sets Z and Q (integers and rational numbers) may be the starting point. The real numbers, R, are never employed, and despite illusions to the contrary, no example of their use in the social sciences exists. Data matrices employing Z or Q may be converted to binary relations by choosing appropriate slicing parameters – either a single value θ, a slicing row or column vector, or a set $\{\theta_{ij}\}$ as the investigator considers appropriate. Relations defined by such slicing values define, in turn, the structures to be interpreted and analysed.

Various measures are available to describe both local and global structural properties. Each simplex in the structure has a degree of eccentricity that is close to the normal and intuitive meaning of the word; namely, the degree to which it is embedded in the complex or has vertices unique to itself. Global properties of the complex are described by a structure vector $\boldsymbol{Q}$, whose elements describe the number of pieces, or degree of fragmentation, into which a complex falls at various dimensional levels. For example, a complex consisting of two three-dimensional simplices, say σ^3_{A} and σ^3_{B}, may only connect at $q = 0$ (one vertex of zero dimensionality in common), so the structure vector $\boldsymbol{Q} = \{\overset{3}{2}\ 2\ 2\ \overset{0}{1}\}$. An obstruction vector $\hat{\boldsymbol{Q}} = \boldsymbol{Q} - 1$ may be thought of intuitively as describing the number of 'gaps' in the backcloth, so obstructing the transmission of traffic.

If a backcloth describes the multidimensional space upon which traffic can exist and move, then certain topological properties may be given important substantive interpretation within the context of a particular analysis. For example, simplices may be so connected that they form a q-hole in the structural geometry, and such holes are analogous to solid objects in ordinary E^3. We may not normally think of a tree as a hole in our everyday Euclidean space – until we try to move through instead of around it. Thus, q-holes are obstructive objects in our topological space, and may well be traffic generators. For example, in a study of the committee structure of a university, a large q-hole generated considerable 'buck passing' traffic in the form of reports and memoranda. Only a vice-chancellor, at the $N + 1$ level in the hierarchy, could fill such a hole and destroy such an obstruction. He was accused of acting in an arbitrary and authoritarian way, but another view is that the structure, the very geometry of the space, forced him to act in that apparent way.

At various dimensional or q-levels, a simplicial complex may fragment into components consisting of simplices sharing a face at that particular dimensional level. Within such components various local structures can appear, either very compact and tightly connected structures or long chains. The length of such q-chains may provide important insights. For example, in a recent study of interpersonal communication amongst farmers in a rural area of Portugal, it was quite clear that q-chain lengths were considerable, even within components of a highly fragmented backcloth. Such local analysis added greater weight to the more general global finding of high values in the obstruction vector, and led directly to well-supported policy statements regarding the need for agricultural extension services.

Although the focus has been upon intuitive geometric notions in this brief and necessarily highly compressed introduction, it is important to realise that the complex and multidimensional nature of most problems requires algebraic expression. Every complex can be expressed as a polynomial, each term of which may have associated with it traffic values which form naturally graded patterns. Such an algebraic expression of backcloth and traffic as a pattern polynomial reminds us that change may be much more complex in the human sciences than the physical sciences, where the backcloth appears to be much simpler and stable. The difference between two pattern polynomials, expressing the change in structures at different times, may record alterations not only in graded traffic patterns, but changes in backcloth structure as well These algebraic expressions, known as 'strain pairs', are analogous to the more familiar differentials.

We now turn to a relatively simple and straightforward problem of describing the structure of a game. A total structural analysis can quickly generate a study of monograph size. We shall present here only a few, but interesting and important, details from the entire structural analysis simply as an example of how certain forms of micro-spatial behaviour might be examined in the future. We are not aware of any prior analytical work of this nature on games, with the exception of a simple graphical demonstration (Gatrell and Gould, 1979), and the pioneering work of Atkin (1972) on the positional description of chess. We examine first the 'internal' structures of the two teams, defined by their own passing relations.

The internal structure of the Liverpool team

In the Liverpool submatrix, we can define a relation on the set of 11 players by choosing a succession of slicing parameters between 8, the highest value, and 1 (Table 1). As a first, simple example, we define the relation $\lambda \subseteq L \otimes L$ when $\theta \geqslant 8$, at which point only the player σ^0_{KK} appears in the simplicial complex, defined by the 'receiver' vertex $\langle SH \rangle$. In the conjugate, σ^0_{SH} is obviously defined by $\langle KK \rangle$. Although this example is the simplest that could be imagined, it does indicate that the simplex σ^0_{KK} may well appear as an important piece of the 'connective tissue' in any simplicial complex defined by successively smaller slicing parameters.

If the complex is now defined as $K_S(R; \lambda; \theta \geqslant 7)$, the structure is still extremely simple, but notice how $\sigma^0_{KK} = \langle SH \rangle$ is both a simplex in the sender structure (Table 2), as well as defining the second simplex $\sigma^0_{PN} = \langle KK \rangle$. This Liverpool player also appears in the conjugate, or receiver structure, even when the slicing parameter defines the relation at this extremely high dimensional level.

Table 1. *The Liverpool v Manchester United interaction matrix of senders and receivers*

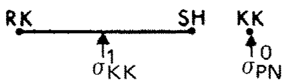
		Liverpool											Manchester United										
		RC	JJ	PH	TS	PN	RK	TM	JC	SH	DJ	KK	AS	JN	BG	MB	AA	SC	LM	SM	GH	SP	JG
Liverpool	RC		2		1	1	1	1	1	1	3				1		3		1		1		
	JJ	1		1			5	2		5		4	2	1	1			2	1	1			
	PH	2	2				1	2	1	2	1	2		1	2	2	1	2			1		1
	TS	1	1			2			1	1	1	1				2	1			1	1		1
	PN	4			2			1	2	1	2	7	1				3		1				
	RK	1		3	1			2	2	2		1	3	2			1	2	1	1			
	TM		3			3	1		3		4	5	3						1				
	JC					3	2	4		2	1	3	3	2	1		3		1	1			
	SH		2			1	3	4	2		4	1		7		2			1	1			
	DJ					1		1	1	3		3		2	1	1	4		1		1		
	KK		2	1	1	1	6	3	8	8	2		1		3	2	2		1			1	
Manchester United	AS		1	2		2	1			2		1		1	2		3	1				4	3
	JN		2	2	1	1	1	2					3		1			2	2			2	5
	BG		1	1						1		1	3	3		3	3		1	2		1	
	MB				1	2							1	1	1		1	1	2	2	1		2
	AA			1	4	1			3		1		1			1		2		2		4	
	SC		3	2			4							4	2					1	1	1	3
	LM		1	1	1			1	1			1		1	2			1		2	2	2	2
	SM		2	1	2			1	1			1			2						6	2	2
	GH	2	1			2	2		1	1	1	1				3	1		1			2	1
	SP	2	2	2	1	1			1									3		3	1		1
	JG	1	1	2	2	1	1	1		1				1				3	2		1		

Table 2. *The complex and conjugate structures of Liverpool at $\theta \geqslant 7$*

	λ	λ^{-1}
$q = 0$	{PN}, {KK}	{SH}, {KK}

The prominence of player KK on the Liverpool team is emphasised when $\theta \geqslant 6$. If we put on our 0-dimensional glasses, we see our simplicial complex $K_S(R;\lambda;\theta \geqslant 6)$ as:

Figure 1. *The structure of the Liverpool complex $K_S(R;\lambda)$ when $\theta \geqslant 6$*



so under this particular slicing definition, the Liverpool team consists of two 'systems' with no connection between them. In the conjugate σ^0_{RK} and σ^0_{SH} are both defined on the vertex ⟨KK⟩, while σ^0_{KK} is defined on the vertex ⟨PN⟩.

Figure 2. *The structure of the Liverpool conjugate $K_R(S;\lambda^{-1})$ when $\theta \geqslant 6$*

In brief, σ^0_{RK} and σ^0_{SH} are faces of each other by virtue of sharing the zero-dimensional face or vertex ⟨KK⟩.

As the slicing parameter θ is reduced, it is clear that a more complex and interesting structure emerges. For example, at $\theta \geqslant 5$ (Table 3) σ^1_{JJ} and σ^1_{KK} are faces of each other, sharing the receivers ⟨RK, SH⟩ who defined them. In the conjugate, all the simplices are 1-dimensional, but even at the $q = 0$ level, the complex falls into two components, indicating some obstruction between these two receiving systems that are *internal* to the Liverpool team.

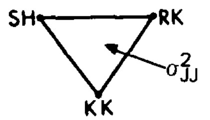
Table 3. *The complex and conjugate structures of Liverpool at $\theta \geqslant 5$*

	λ	λ^{-1}
$q = 1$	{JJ, KK}	{RK, SH}, {KK}
0	{JJ, KK}, {PN, TM}	{RK, SH}, {KK}

Even though we are deliberately presenting extremely simple structures at the moment, it is worth asking whether such information about the Liverpool structure would be of value to the coach of an opposing team. If the purpose of defense is to break up and fragment the structure of the opposing team (a structure defined by the passing relations between them), it appears that the Liverpool team is already developing two, quite disconnected, subsystems.

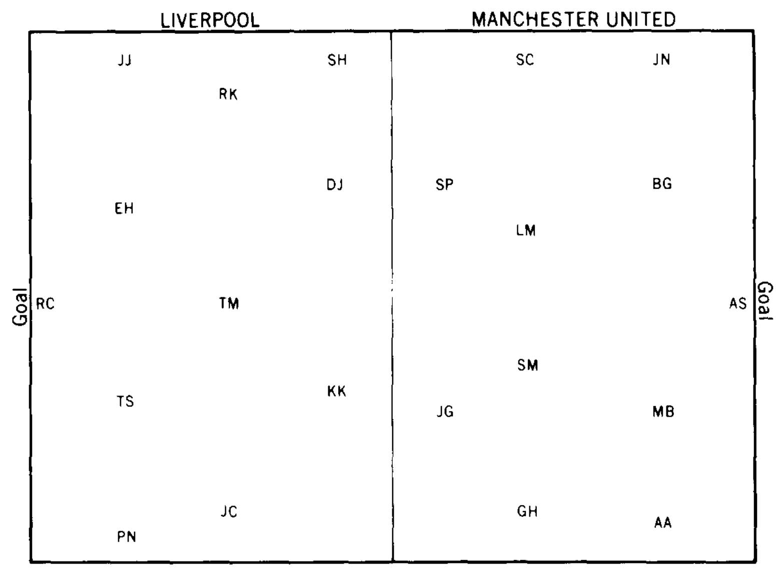
When the relation on the set of Liverpool players is defined by $\theta \geqslant 4$, the first two-dimensional simplex occurs in the sender structure. It is worth

Figure 3. *The structure of the Liverpool complex $K_S(R;\lambda)$ when $\theta \geqslant 4$*



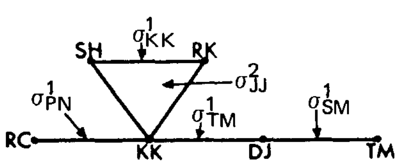
noting that σ^2_{JJ} is defined by the vertices ⟨RH, SH, KK⟩, which might be considered a rather different, and perhaps unusual structure when the positions on the actual field of play are considered. Player JJ is clearly an

Figure 4. *Starting positions on the field of play in E^3*



important, high-dimensional sender, and it is understandable that players RK and SH should define him, since they are both on the left side of the field. However, KK is also one of the defining vertices, and the prominence of KK

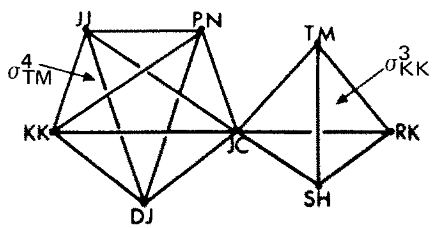
Figure 5. *The prominence of player KK in the Liverpool complex when $\theta \geqslant 4$ at the* q = *1 dimensional level.*



in the emerging Liverpool structure is emphasised at $q = 1$. Player KK is a crucial vertex in the structure. If he were removed, this embryonic Liverpool structure would fall into three pieces of reduced dimensionality, because "things fall apart, the centre [?] cannot hold" (Yeates in Gardener, 1972). Player KK is the linchpin of Liverpool, and notice how this is now a hard, well-defined statement, since we can both see and describe the structure in our verbal, graphic and algebraic languages. Although the player simplices are connected at $q = 0$, our obstruction vector indicates a sudden jump in disconnection at $q = 1$. In a sense, we can think of these four separate components as distinct systems or channels in which the ball moves. At this particular dimensional level, the geometry of the structure expresses the disconnection of the Liverpool senders into four, quite separate systems. In the conjugate structure, σ^2_{KK} still emerges as the most prominent structure in the receiver complex.

When we define the relation at $\theta \geqslant 3$, we obtain some quite different insights: Liverpool player σ^4_{TM} now emerges as the highest dimensional simplex, and at $q = 3$ σ^4_{TM} and σ^3_{KK} share only the zero-dimensional face ⟨JC⟩. Notice that these four- and three-dimensional simplices are defined

Figure 6. *The structure of the Liverpool complex $K_S(R; \lambda)$ when $\theta \geqslant 3$*



by vertices that really bear little relation to each other in spatial terms on the actual football field (Figure 4). The passing relations of Liverpool create certain substructures, or channels, that define a space in which the normal metric properties of the football field appear virtually irrelevant. This would imply that conventional positions on the field mean little in terms of how player structures actually emerge during the course of a particular game. It is legitimate to ask which is the 'real' space in which the game takes place. Often, the actual playing field appears irrelevant to a hard and well-defined scientific description of the game.

It is worth emphasising that the sudden emergence of σ^4_{TM} does not contradict any of our previous statements about the importance of player KK. The statements we make about structures cannot be oversimplified, particularly by forcing our data sets through linear, or perhaps even unspecified filters, which are often deployed through illegitimate, or undefined, algebraic operations. The scientific statements we make depend strictly upon our definitions, which must be as clear and explicit as possible. In the conjugate, that is the receiving structure, σ^4_{KK} is still a high-dimensional receiver, getting at least three passes from five other team members. However, there is a high degree of obstruction at $q = 2$, indicating that the Liverpool team has fragmented into five separate components. If we put on our two-dimensional lenses, we would see a disconnected structure in which none of the five players shared a 'two-face' with any other.

At $\theta \geqslant 2$, σ^5_{KK} again emerges as the most prominent sender, but the degree of fragmentation in the team remains high until $q = 2$ (Table 4).

Table 4. *The complex and conjugate structures of Liverpool at $\theta \geqslant 2$*

	λ	λ^{-1}
$q = 5$	{KK}	{TM}, {SH}, {KK}
4	{EH}, {PN}, {TM}, {JC}, {SH, KK}	{JJ}, {TM, SH}, {JC}, {DJ}, {KK}
3	{JJ, JC}, {EH}, {PN}, {RK}, {TM}, {SH, KK}	{JJ, JC, DJ}, {RK, TM, SH, KK}
2	{JJ, EH, PN, RK, TM, JC, SH, KK}	{JJ, RK, TM, JC, SH, DJ, KK}, {PN}
1	{RC, JJ, EH, PN, RK, TM, JC, SH, DJ, KK}	{RC, JJ, PN, RK, TM, JC, SH, DJ, KK}
0	{RC, JJ, EH, TS, PN, RK, TM, JC, SH, DJ, KK}	{RC, JJ, EH, TS, PN, RK, TM, JC, SH, DJ, KK}
	$\hat{Q} = \{\overset{5}{0}\ 4\ 5\ 0\ 0\ \overset{0}{0}\}$	$\hat{Q} = \{\overset{5}{2}\ 4\ 1\ 1\ 0\ \overset{0}{0}\}$

It is notable that σ^0_{TS} only appears in the simplex at all when $q = 0$, and we must again raise the question as to whether such a structural analysis would be of use to a coach – this time, the coach of the Liverpool team. In essence, the virtual disconnection of σ^0_{TS} means that Liverpool is trying to play the game one man short, a player who is simply not well-connected to his team mates.

The internal structure of the Manchester team

We shall not examine the internal structure of the Manchester team in such detail, but merely point out some possibly significant comparisons with their opponents. For example, at $\theta \geqslant 5$, the Manchester team has only σ^0_{JN} and σ^0_{SM} in the sender complex, while the Liverpool team were already connecting the one-simplices σ^1_{JJ} and σ^1_{KK}, with σ^0_{PM} and σ^0_{TM} sharing the zero-face $\langle KK \rangle$.

The difference in internal structure of the two teams is emphasised at $\theta \geqslant 4$. At the $q = 0$ level, six of the Liverpool team were already connected into a single component, while the Manchester team remains fragmented (Table 5). Such 'channelling' or fragmentation may be a deliberate strategy on the part of the Manchester team, or it could be that they are being broken up by the defensive play of Liverpool. We shall examine such questions below.

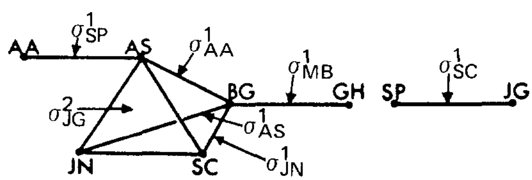
As the slicing parameter is lowered to $\theta \geqslant 3$, the degree of fragmentation in the Manchester team still appears higher than Liverpool, but it is interesting

Table 5. *The complex and conjugate structures of Manchester at $\theta \geqslant 4$*

	λ	λ^{-1}
$q = 1$		{SP}
0	{AS, AA}, {JN}, {SC}, {SM}	{SP}, {JN}, {GH}, {JG}

to examine graphically the conjugate, or receiving structure, and compare it with that of Liverpool at the same slicing and q-level. Although there is a fragmented pattern at $q = 1$, the structure is totally different from that of Liverpool. The player simplex σ^2_{JG} has the highest dimensionality, but all

Figure 7. *The structure of the Manchester conjugate $K_R(S; \lambda^{-1})$ when $\theta \geqslant 3$*



the other players have zero connection with each other, with the exception of disconnected σ^1_{SC}, defined by the vertices ⟨SP, JG⟩. Notice that this structure is completely different from that of Liverpool for the same slicing level. Liverpool, we recall, depended much more upon the single, high-dimensional receiver σ^4_{KK}. Can we interpret this well-defined empirical fact as evidence of somewhat better balance in the Manchester team, a team which seems to construct its receiving structure out of low-dimensional simplices, rather than one, high-dimensional player?

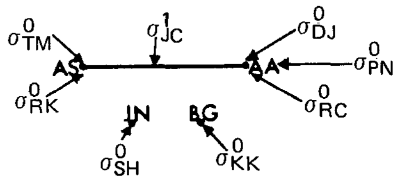
Stealing relations: Manchester from Liverpool

We now turn to an examination of the structure of the third submatrix, whose elements describe a relation between the sets of Liverpool and Manchester players. These are stealing relations, for we can think of Liverpool sending to Manchester receivers, or, alternatively, the Manchester thieves stealing from the Liverpool victims.

If we first slice at $\theta \geqslant 7$, a very simple structure appears consisting of the Liverpool player σ^0_{SH} defined by the Manchester vertex ⟨JN⟩. In the conjugate stealing structure, σ^0_{JN} of Manchester is defined by the vertex ⟨SH⟩, because he stole the ball seven times from SH during the game. It appears that σ^0_{JN} may develop as an important Manchester simplex, breaking up the structure of the Liverpool team. We recall, from the analysis of the structural relations on the Liverpool team alone, that the Liverpool player SH was a fairly high-dimensional receiver, but not as high a sender. In brief, he appeared sooner in the conjugate structures, and we begin to see the reason; he is losing the ball frequently to JN of Manchester.

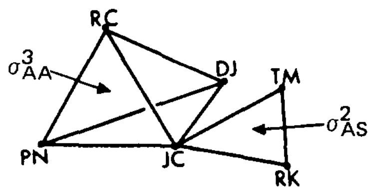
At $\theta \geqslant 4$, σ^0_{SH} is joined by σ^0_{DJ}, but they are in disconnected components, σ^0_{SH} being still defined by the Manchester 'thief' ⟨JN⟩, while σ^0_{DJ} is defined by ⟨AA⟩. However, at $\theta \geqslant 3$, some important and interpretable structure emerges. At $q = 0$, six of the Liverpool players are defined as the 1-simplex

Figure 8. *The complex $K_S(R; \lambda)$ when $\theta \geqslant 3$ defining the 'axis' of the Manchester defense.*



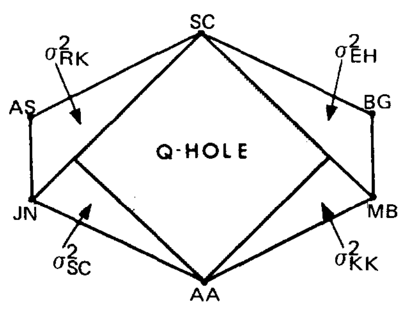
⟨AS, AA⟩, or individual faces of it. In this structure, we can see the emergence of a well-defined axis of the Manchester defense, aided by two disconnected components consisting of the players σ^0_{SH} on ⟨JN⟩, and σ^0_{KK} on ⟨BG⟩. In the conjugate structure, this defensive axis appears as the relatively high-dimensional Manchester player σ^3_{AA}, joined by σ^2_{AS} through the vertex ⟨JC⟩.

Figure 9. *The Manchester defense as the conjugate structure $K_R(S; \lambda^{-1})$ when $\theta \geqslant 3$*



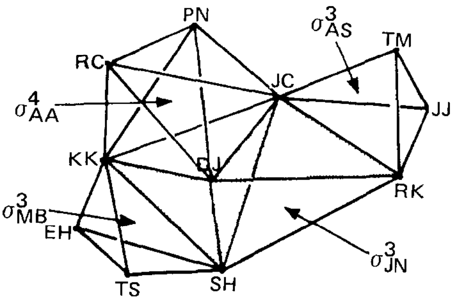
At $\theta \geqslant 2$, a quite extraordinary structure appears, because at $q = 2$ four Liverpool players are each defined in terms of three defensive Manchester players to form a structure that produces a q-hole in the geometry of the stealing space. Such holes in the backcloth or geometry are usually interpreted

Figure 10. *A* q*-hole, or obstructive object, in the stealing space $K_S(R; \lambda)$ of Liverpool → Manchester when $\theta \geqslant 2$*



analogously as solid objects in our ordinary E^3. In essence, the Manchester team has defined four Liverpool players in such a way that they have inserted an obstructive object in this portion of the stealing space. Thus σ^2_{RK} has been disconnected from σ^2_{KK}, since they are defined by disjoint sets of Manchester players, namely ⟨JN, AS, SC⟩, and ⟨AA, MB, BG⟩. It is as though the Manchester team were playing "here we go round the q-mulberry bush", which must have been a very frustrating tune for the Liverpool players to hear.

Figure 11. *The conjugate structure $K_R(S; \lambda^{-1})$ of the Liverpool → Manchester stealing relations when $\theta \geq 2$*



In the conjugate, we have a structure of stealing relations that centres, at $q = 3$, around σ^4_{AA}, with σ^3_{JN}, σ^3_{MB}, and σ^3_{AS}, sharing 0-dimensional faces respectively. In this defensive structure, the positions of the Manchester players, defined in terms of Liverpool vertices, do bear some relation to their respective positions on the field. For example, σ^3_{MB} is connected to σ^4_{AA} in the microgeographic space of the playing field. Perhaps we have some indication here that defensive play is more constrained and tighter in E^3, compared to the more open, roving and fluid play of the attacking players.

Stealing relations: Liverpool from Manchester

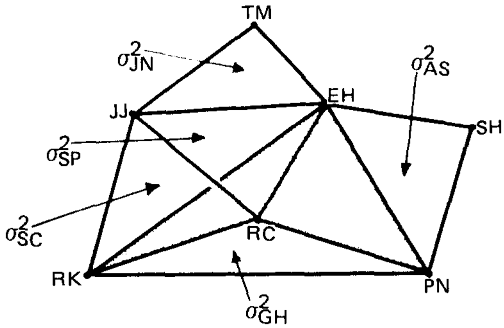
Turning now to the fourth submatrix, in which the Liverpool columns are stealing from the Manchester rows, we slice first at $\theta \geq 3$. In the conjugate structure, we have our first indication of the Liverpool defense structure, as two vertices, each defining two Liverpool players, make up the complex.

Figure 12. *The conjugate structure $K_R(S; \lambda^{-1})$ of the Manchester → Liverpool stealing relations when $\theta \geq 3$*

Do we have, perhaps, an indication of some 'ganging up' as σ^0_{JJ} and σ^0_{RK} steal the ball three times from ⟨SC⟩, and σ^0_{JC} and σ^0_{TS} take the ball away from ⟨AA⟩?

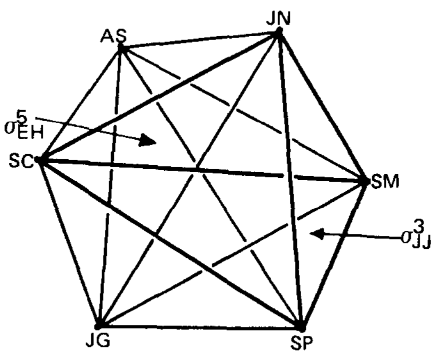
At $\theta \geqslant 2$, it is of interest to compare the Manchester and Liverpool structures at $q = 2$: two very different configurations emerge, indicating that the stealing relations of the respective teams produce quite different structures

Figure 13. *The stealing space $K_S(R; \lambda)$ of Manchester → Liverpool when $\theta \geqslant 2$*



of defense. In the former case, where Liverpool players were defined by Manchester defensive vertices, each player was connected to two others through the stealing relations to produce a q-hole. In contrast, the Manchester players, defined over a set of defensive Liverpool players, are all connected to each other. The Manchester defensive vertices ⟨EH, JJ⟩, contribute to the definition of three Liverpool players each, but others only contribute to one or two. In the conjugate, we can see how Liverpool relies very much on their two players σ^3_{JJ} and σ^5_{EH}, and such an emphasis on structural specialisation is highlighted by a comparison with a similar Manchester structure at $\theta \geqslant 2$ and $q = 3$.

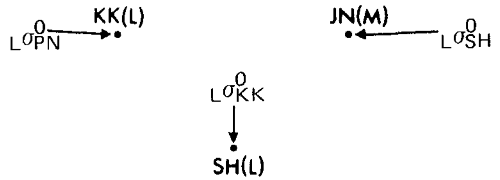
Figure 14. *The conjugate structure $K_R(S; \lambda^{-1})$ of the Manchester → Liverpool stealing relations when $\theta \geqslant 2$*



The structure of the entire game

To tear apart the structure of the entire game into four smaller and separate analyses is an artificial, and somewhat distorting, device to gain some initial insight and interpretation about a highly complex structure. We face now the question of linking these four separate analyses, by examining the interrelations between the two teams defined by all the passing and stealing relations. At the very high value of $\theta \geqslant 7$, very little structure emerges, but it is noticeable that even this simple, disconnected complex

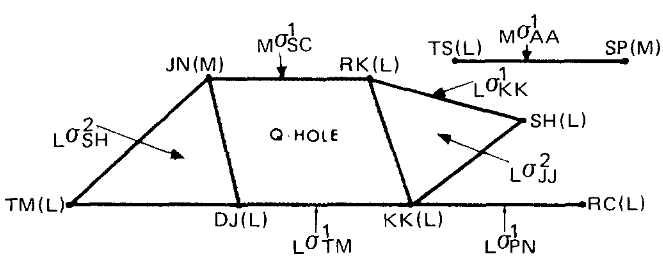
Figure 15. *The complex $K_S(R; \lambda)$ of the entire game when $\theta \geqslant 7$*



is composed entirely of Liverpool players. Significantly, however, σ^0_{SH} is already defined on the *Manchester* vertex ⟨JN⟩, to whom he loses the ball seven times. Perhaps we are seeing the beginning of the Manchester defense disconnecting the Liverpool offense structure? In brief, every injection of a vertex by Manchester into the Liverpool structure means one less connection that Liverpool can define on its own set of players.

We next slice at $\theta \geqslant 4$, noting still the prominence of the Manchester vertex ⟨JN⟩ in what is, essentially, a Liverpool structure. At $q = 1$, we have

Figure 16. *A* q-*hole in the game complex $K_S(R; \lambda)$, caused by incorporating the Manchester player SC and the Manchester vertex JN when $\theta \geqslant 4$*

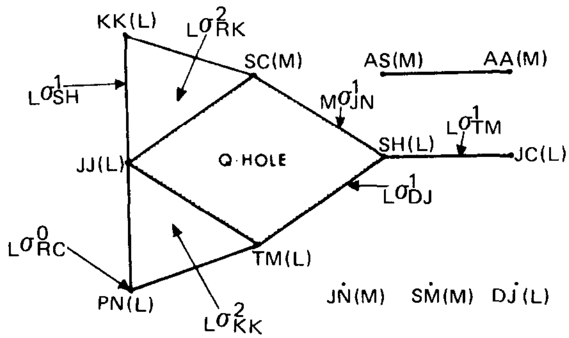


some evidence for the first time of the way in which the structure of one team may be augmented or confounded by the other. The Manchester vertex ⟨JN⟩ simultaneously defines the Liverpool player σ^2_{SH} and the Manchester player σ^1_{SC}, while the latter is also defined by the Liverpool vertex ⟨RK⟩. In this structure, ⟨JN⟩ has forced his way into the Liverpool structure, but Liverpool, in its turn, has managed to incorporate Manchester's σ^1_{SC} by the ability of ⟨RK⟩ to steal the ball away, putting σ^1_{SC} in the awkward position of half-playing for his opponents! But we must emphasise the word *half*-playing. Only partially incorporating an opposing player into your structure can be a dangerous business, because the Manchester simplex σ^1_{SC} has created a q-hole in the Liverpool geometry. This object in the topology must be felt somehow by the Liverpool team as an obstruction: Liverpool simplices σ^1_{KK},

σ_{JJ}^2, σ_{PN}^1 are no longer cleanly connected to their team mates σ_{TM}^1 and σ_{SH}^2, but must sometimes work around the q-hole via the dangerous Manchester vertex ⟨JN⟩.

This is exactly what we observe in the conjugate structure at $q = 0$. There are some peripheral simplices, but note that the Manchester vertex ⟨SC⟩

Figure 17. *A* q*-hole in the game conjugate $K_R(S; \lambda^{-1})$ caused by the intrusion of Manchester player JN when $\theta \geqslant 4$*



has been swallowed and incorporated into the Liverpool structure. Manchester player ⟨SC⟩ must be extremely popular at this point with the Liverpool team, since he is frequently giving up the ball to them. However, the fly in the Liverpool ointment is the Manchester simplex $\sigma_{JN}^1 = \langle SH, SC \rangle$. Although Manchester's σ_{JN}^1 is partially defined by his team mate and 'honorary' Liverpool player ⟨SC⟩, the intrusion of this Manchester simplex again produces a q-hole in the Liverpool structure. He has managed to loosen up Liverpool, disconnecting, for example, σ_{DJ}^1 from σ_{RK}^1 and σ_{SH}^1, as well as disconnecting σ_{TM}^1 almost entirely. We have a very simple analogy here to some of the deeper insights Atkin gained in the game of chess by describing it at the positional level (Atkin and Witten, 1975). In the game of chess, where the microspatial behaviour of the players is defined by extremely strict rules, it is possible to see how the structure of, say, White works its way into, and eventually destroys, the structural connectivity of Black, tearing it apart, shattering it, until Black is forced to resign. In our current analysis, a good defensive player like Manchester's σ_{JN}^1 is constantly stealing from the Liverpool team and breaking their structure apart.

At $\theta \geqslant 3$, the dimensionality of the simplicial complex explodes, and we are virtually forced to abandon our graphical language. Although one can array the players of the two teams around the circumference of a circle, and connect such vertices to represent the multidimensional polyhedra in the simplicial complex, the projection from higher dimensional spaces onto the plane obliterates any meaningful visual insight about the structure. However, at $q = 1$ and $q = 0$, we have our first real evidence of the way in which the two team structures are connecting up (Table 6). In the conjugate, particularly, we can really begin to see how the Manchester receivers are

Table 6. *The complex and conjugate structures defined on the whole game when $\theta \geqslant 3$*

	λ	λ^{-1}
q = 5	{TM}	{AA}
4	{TM}, {JC}, {KK}	{AA}, {KK}, {AS}
3	{TM}, {JC}, {SH}, {KK}, {BG}, {SC}	{RK}, {KK}, {AS}, {AA}
2	{JJ}, {PN}, {TM, JC}, {SH}, {DJ}, {KK}, {AS}, {BG}, {AA}, {SC}	{RK}, {TM}, {JC}, {SH}, {DJ}, {KK, AA}, {AS}, {JN}, {JG}
1	{RC}, {JJ, PN, TM, JC, SH, DJ, KK, BG, SC} {RK}, {AS}, {JN}, {AA}, {SP}	{JJ}, {PN, RK, TM, SH, KK, AS, JN, AA}, {JJ}, {MB}, {SC}, {CP}, {JG}
0	{RC, JN, PN, RK, TM, JC, SH, DJ, KK, AS, JN}, {BG, AA, SC, GH}, {SM}, {SP, JG}	{RC, JJ, EH, TS, PN, RK, TM, JC, SH, DJ, KK, AS, JN}, {BG, MB, AA, SP, JG}, {SC, SM}, {GH}
	$\hat{Q} = \{\overset{5}{0}\ 2\ 5\ 9\ 6\ \overset{0}{2}\}$	$\hat{Q} = \{\overset{5}{0}\ 2\ 3\ 8\ 7\ \overset{0}{2}\}$

beginning to enter the Liverpool component. It is worth noting that the idea of one structure entering, breaking up, and destroying another is a common one. In positional descriptions of classical games of chess, for example, the dissolution of one player's position is the direct and purposeful result of the entry of the opponent's pieces into a complex, and in fact multidimensional, structure (Atkin *et al.*, 1976). The idea has also been used in music: for example, the entry of the Russian theme in Shostakovitch's seventh symphony (the Leningrad) to break up the Nazi theme — a device previously used by Tchaikovsky with the 1812 Overture.

Simply to note the presence of Manchester players AS, JN, and AA in the second component of the conjugate at q = 1 (Table 6) says very little. After all, we could place the reverse interpretation upon this, as the Liverpool players PN, RK, TM, SH, and KK enter and obliterate a Manchester structure. However, by examining the vertices that define the five Liverpool and three Manchester players (Table 7), we can see how it is the latter who

Table 7. *The defining vertices or players in the conjugate structure of the game $K_R(S; \lambda^{-1})$ when $Q \geqslant 3$ and $q = 1$*

${}_{L}\sigma^{1}_{PN}$ = ⟨TM, JC⟩	${}_{L}\sigma^{4}_{KK}$ = ⟨JJ, PN, TM, JC, DJ⟩
${}_{L}\sigma^{3}_{RK}$ = ⟨JJ, SH, KK, *SC**⟩	${}_{M}\sigma^{4}_{AS}$ = ⟨RK, TM, JG, *JN*, *BG*⟩
${}_{L}\sigma^{2}_{TM}$ = ⟨JC, SH, KK⟩	${}_{M}\sigma^{2}_{JN}$ = ⟨SH, *BG*, *SC*⟩
${}_{L}\sigma^{2}_{SH}$ = ⟨JJ, DJ, KK⟩	${}_{M}\sigma^{5}_{AA}$ = ⟨RC, PN, JC, DJ, *AS*, *BG*⟩

*Manchester vertices italicised.

are incorporating and swallowing up the Liverpool players into their structures. To turn it around, these three Manchester simplices are crashing into the Liverpool structure and breaking it apart.

At lower slicing values, the structural complexity is overwhelming. Liverpool player KK, for example, is an eight-dimensional sender-simplex, and we lose our graphic language totally. We are obliged to stay with algebraic expressions, and rely increasingly upon global measures of structure. For example, the obstruction vector:

$$\hat{Q} = \{\overset{8}{0}\ 3\ 6\ 10\ 13\ 11\ 3\ 0\ \overset{0}{0}\}$$

indicates zero obstruction at $q = 0$ and 1, implying that all 22 players are at least one-connected to someone else, so that the simplicial complex is a single, coherent structure. However, the game fragments quickly after that: for example, at $q = 2$ the obstruction is 3, while at $q = 3$ the complex fragments into 12 separate components. Similarly, at $\theta \geqslant 1$, the obstruction vector:

$$\hat{Q} = \{\overset{14}{1}\ 1\ 4\ 11\ 14\ 12\ 7\ 2\ \overset{6}{0}\ 0\ 0\ 0\ 0\ 0\ \overset{0}{0}\}$$

indicates that everything is very highly connected up to $q = 6$. In a sense, our analysis has gone too far, for we are unable to see the trees for the wood, while at very high θ's we cannot see the wood for the individual trees. Our comprehension appears to rest at some middle slicing level, where we can clear the wood a bit to see the structure of relations between individual trees – namely, the players. In brief, when the slicing parameter θ is too high, the structural analysis is fairly dull with little detail. When the slicing parameter is too low, the structure is less interpretable because of the enormous amount of detail present. We have a clear demonstration of how human analytic abilities, which try to clarify and comprehend, rest crucially upon the choice of the slicing parameters that define the relations on and between sets to produce the actual structures.

Conclusions: What happened in E^3?

Observers, sportswriters, and commentators of the game in E^3 generally rated the play of Liverpool as superior, and expressed some surprise that Manchester won the game by 2 goals to 1. Some noted in commentaries of the match:

> "Liverpool were clearly the better side..."
>
> "... for balance and teamwork I prefer the Liverpool trio..."
>
> "Liverpool were playing like champions, stroking the ball about with masterful precision".

In the structural analysis we can make hard, well-defined statements which confirm these comments to a degree. But only to a degree: Liverpool did form a tight, internal structure, in the sense that many of their players were well-defined in terms of other Liverpool vertices. One commentator noted:

> "Liverpool also occasionally lose, but they don't fall apart"

But what good does it do to "stroke the ball around with masterful precision" between yourselves, putting on displays of footwork and precision passing, if the simplices in your complex that are meant to connect with the goal fail to do so?

And fail they did, and the reason is clear: the Manchester defense also incorporated Liverpool players into their structure, made the latter lose the ball repeatedly, and created q-holes in the geometry, obstructions that the Liverpool team had to work around. Such working around in the multi-dimensional space, where the real game was played, was more difficult and less spectacular compared to the "masterful stroking" in the illusory E^3 that brought the crowd to its feet. But there is little question, either in the result or in the structural analysis, why the connections to the goals were ultimately in favour of Manchester. As one commentator noted:

"Liverpool suffered the rare irony of playing well and losing".

But perhaps the less spectacular Manchester simplices σ^4_{AS}, σ^2_{JN} and σ^5_{AA} in $K_R(S; \lambda^{-1}; \theta \geqslant 3; q = 1)$ had something to do with it, despite the high-dimensional player KK of Liverpool. This was quite an achievement, since we can now reveal that KK was Kevin Keegan, since voted Europe's Football Player of the Year, and now a member of the powerful Hamburg team.

In conclusion, we would like to raise a number of questions that have arisen during this first, crude, and unrefined attempt to provide a well-defined structural analysis of a team sport. First, the approach of q-analysis (polyhedral dynamics) allows us to evaluate very precisely the structural effects of player-removal through injury, dismissal (by the referee), or substitution. What does the loss of player X, for example, mean in terms of disconnection in the internal structure of a team, and the injection of the other team's simplices into that portion of the complex? As we saw repeatedly, the loss of Kevin Keegan (KK) would have been devastating to the structure of Liverpool.

Secondly, we speculate that it would be descriptively useful to incorporate the two goals as pseudo-players in the game. After all, the games are actually won or lost depending upon how these critical vertices are connected into the team structures. Whether these stationary vertices will be incorporated or not is an interesting question, but one which we believe is not answerable *a priori.* On any one day, teams can play in desultory or in inspired fashions, and at this point we do not look forward to a predictive model of the bookmaker or logical positivist. We acknowledge the intuitive model of Nick the Greek, but dare not presume to emulate it.

Thirdly, we wish to raise an important question that occurred to both of us and a reader independently in the course of reviewing the analysis: is there a deeper structure of relations on the set of players that defines a backcloth upon which the passage of the ball may be considered as traffic? Do players have private evaluations and preferences forming a real, but still undefined and invisible, structure that actually underpins the sort of structural description we have provided here?

Some intriguing evidence comes from recent research by sports psychologists and scholars in physical education on leadership and team interaction (Klein and Christiansen, 1969; Loy *et al.*, 1978; Tropp and Landers, 1979).

We are beginning to get some indication and evidence that team interactions are not independent of other structures defined by leadership and interpersonal attractions, and it may be these which form part of the deeper structure, or geometric backcloth, upon which the traffic of the game can exist.

Finally, and almost as a fortuitous post-scriptum, we note that Liverpool and Manchester met twice this year (1979) in their attempts to reach the Cup Final again – once in a tie, and then in a second, deciding game. Will similar structures emerge as we analyse these 'repeated experiments'? And what would 'similar' mean in such a context? It is likely that the traffic on the backcloth will change, but with the loss of Kevin Keegan to Hamburg, who can doubt that powerful forces have acted on the backcloth to change the very geometrical property of the multidimensional space against which the games were played? Such forces will be graded, and may be expressed as strain pairs recording the differences between the pattern polynomials of each game.

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