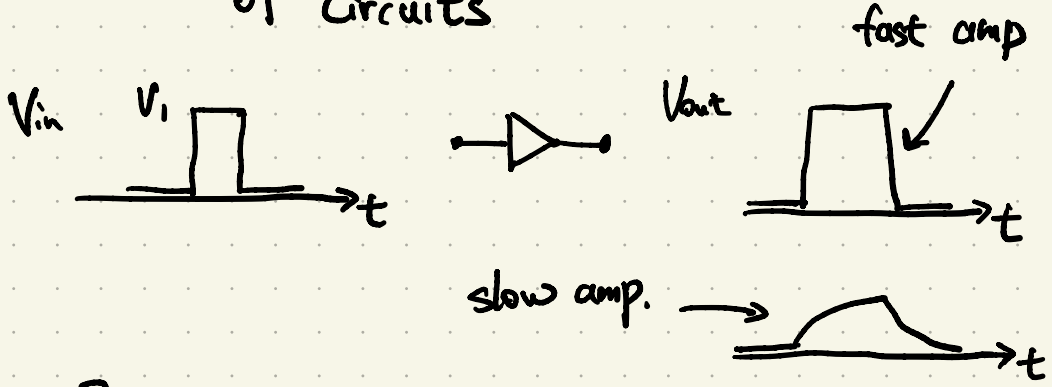
Lec 17 Frequency Response

- Objective & Motivation
- Circuit Theory Concepts

Objective: Determine the speed limitation of circuits



Why?

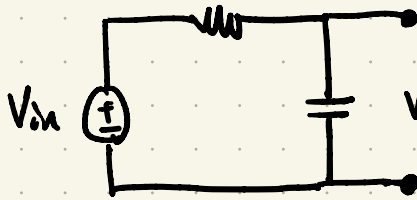
High speed circuits are everywhere:

WiFi: 2.4GHz, 5GHz Bluetooth: 2.4GHz

Gps: 1.5GHz, automotive radars: 26 GHz
77 GHz

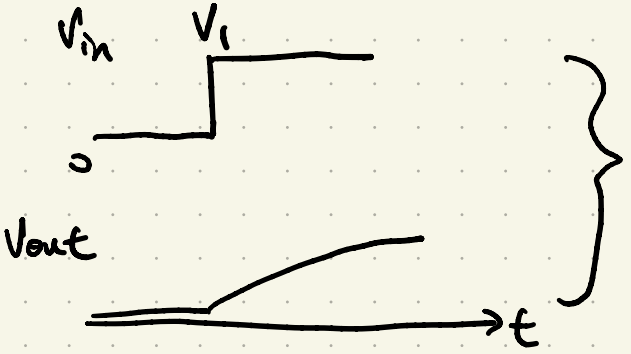
Microprocessors: 4-5GHz

Example



$V_{out} = V_1 (1 - \exp\frac{-t}{\tau})$

$\tau = R_1 C_1$

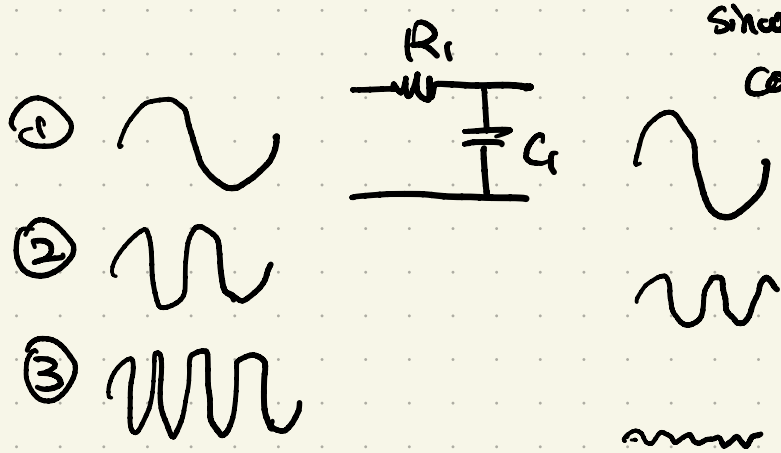


(a) Time-Domain Analysis

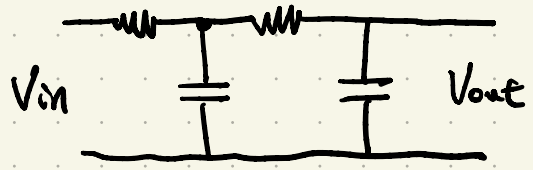
To improve the speed, we must minimize $R_1 C_1$

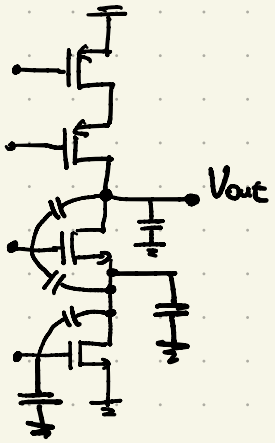
(b) Frequency-Domain Analysis

Formulates the response to sinusoidal sinωt or cosωt

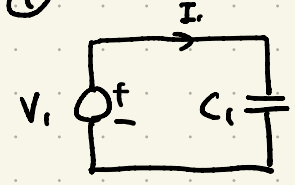


Need for Frequency Response





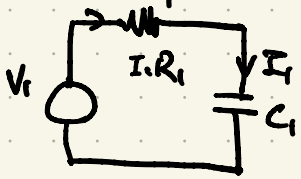
Circuit Theory Concepts

① 

Time Domain: $I_1 = C_1 \frac{dV_1}{dt}$

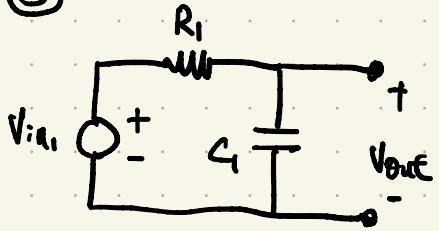
Freq. Domain: $I_1 = C_1 s \cdot V_1 \Rightarrow \frac{V_1}{I_1} = \frac{1}{C_1 s}$

"Impedance"

② 

$V_1 = R_1 I_1 + \frac{1}{C_1}\int I_1 \, dt$

$V_1 = R_1 I_1 + \frac{I_1}{C_1 s}$

③ 

Voltage Divider:

$$V_{out} = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R_1} V_{in}$$

Transfer Function

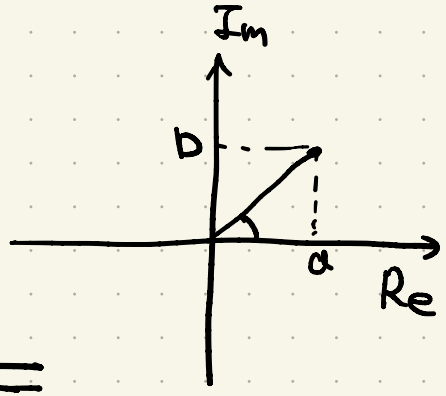
$$= \frac{1}{1 + sCR_1} V_{in}$$

$$R_1 C_1 \frac{dV_{out}}{dt} + V_{out} = V_{in}$$

④ s is generally a complex frequency. But we can assume $s = j\omega$ if the signals of interest are sinusoids.

eg: $H(s) = \frac{1}{1 + R_1 C_1 s}$ if V_{in} is a sinusoid, $s = j\omega \Rightarrow H(j\omega) = \frac{1}{R_1 C_1 j\omega + 1}$

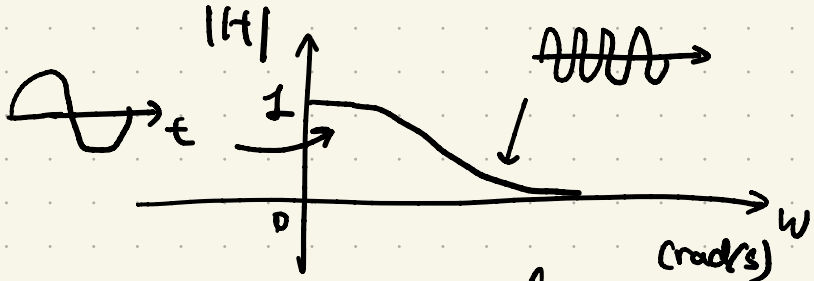
⑤ a complex number $a + jb$ can be represented in the polar form:



$$\text{Magnetude} = \sqrt{a^2 + b^2}$$

$$\text{phase} = \tan^{-1}\frac{b}{a}$$

$$|H(j\omega)| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$



$\omega = 2\pi f$

How to compute the freq. response?

① Determine the transfer function, $H(s)$

② Replace s with $j\omega$ $\Rightarrow H(j\omega)$

③ Compute $|H(j\omega)|$ = Freq. Resp.

Example

$$H(j\omega) = \frac{1}{\sqrt{1+\omega^2 R_1^2 C_1^2}}$$