

Understanding euler angles

Name:

Due September 30th, in lab

This worksheet is intended as a short 30 min exercise to be done in groups of 2-3 people.

Euler angles

Euler angles allow us to specify rotations as a series of rotations around each of the X, Y, and Z axes.

Euler angles describe rotations as a sequence of rotations around each axis. Any sequence is valid: we can rotate first around X, then Y, then Z; or we can rotate first around Y, then X, and then Z.

In this lab exercise, we will convert between euler angles and rotation matrices.

Converting from XYZ euler angles to a matrix

1) Give the 3 rotation matrices corresponding to rotations around the X, Y, and Z axes respectively. Use the following notation to represent the sine and cosine angles in around each axis: c_x , s_x , c_y , s_y , c_z , s_z . For example, c_x will be the $\cos(\text{XAngle})$ and s_x will be $\sin(\text{XAngle})$.

2) What product of matrices gives the matrix corresponding XYZ euler angles below (you do not need to show their multiplication)

$$\begin{bmatrix} cy*cz & -cy*sz & sy \\ cx*sz + cz*sx*sy & cx*cz - sx*sy*sz & -cy*sx \\ sx*sz - cx*cz*sy & cz*sx + cx*sy*sz & cx*cy \end{bmatrix}$$

Extracting the XYZ euler angles back from a matrix

Given a rotation matrix, we can extract the angles back out of the matrix. In this section, we will go through the process of extracting euler angles from the matrix we computed in the last section.

1) What matrix element would be the easiest to get an angle back?

2) Recall the $\tan = \sin/\cos$. How could we use this to get the other angles back?

3) What happens when the rotation around the Y axis is 90 degrees? Compute the new euler

matrix for this case.

4) Use cosine and sine identities to simplify the matrix.

$$\sin(z + x) = \sin(z)\cos(x) + \cos(z)\sin(x)$$

$$\cos(z + x) = \cos(z)\cos(x) - \sin(z)\sin(x)$$

$$\sin(z - x) = \sin(z)\cos(x) - \cos(z)\sin(x)$$

$$\cos(z - x) = \cos(z)\cos(x) + \sin(z)\sin(x)$$

5) Think about the angles. We could split the rotation between the X and Z axes. Let's set the rotation around Z to zero. What should the rotation around X be?

6) Now consider the case where the Y rotation is -90. How could we compute the other angles?

7) Consider the following matrix. Extract the XYZ angles corresponding to this rotation.

0.9511	0	0.3090
0.2185	0.7071	-0.6725
-0.2185	0.7071	0.6725