

CS 383: Machine Learning

Prof Adam Poliak

Fall 2024

09/25/2024

Lecture 09

Announcements

HW03 is due Tuesday night

- **Reading quiz: Thursday**
 - Duame 9.3 (2 pages)

Proposed updated schedule

Midterm 1 was Thursday October 3rd

3 lectures this week

lecture on Wednesday 10/02 but no lecture on Thursday 10/03 (was supposed to be midterm 1)

No lecture Wednesday 10/09

HW02 decision trees due tonight, HW03 polynomial regression due next Tuesday 10/01, HW04 naive Bayes due Tuesday 10/08 (it'll be a shorter assignment)

Midterm 1 on Thursday 10/10

Outline

Normal equations vs SGD

Regularization

Probability

Naive Bayes

Pros and Cons

Gradient Descent

- Requires multiple iterations
- Need to choose η
- Works well when n is large
- Can support online learning

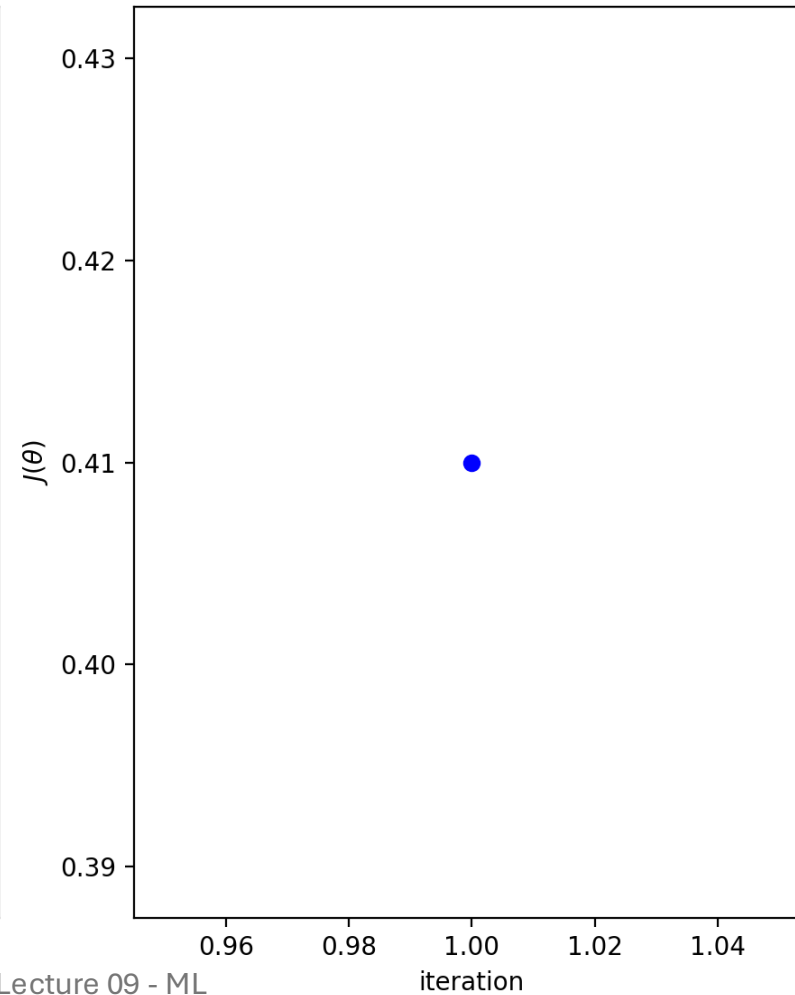
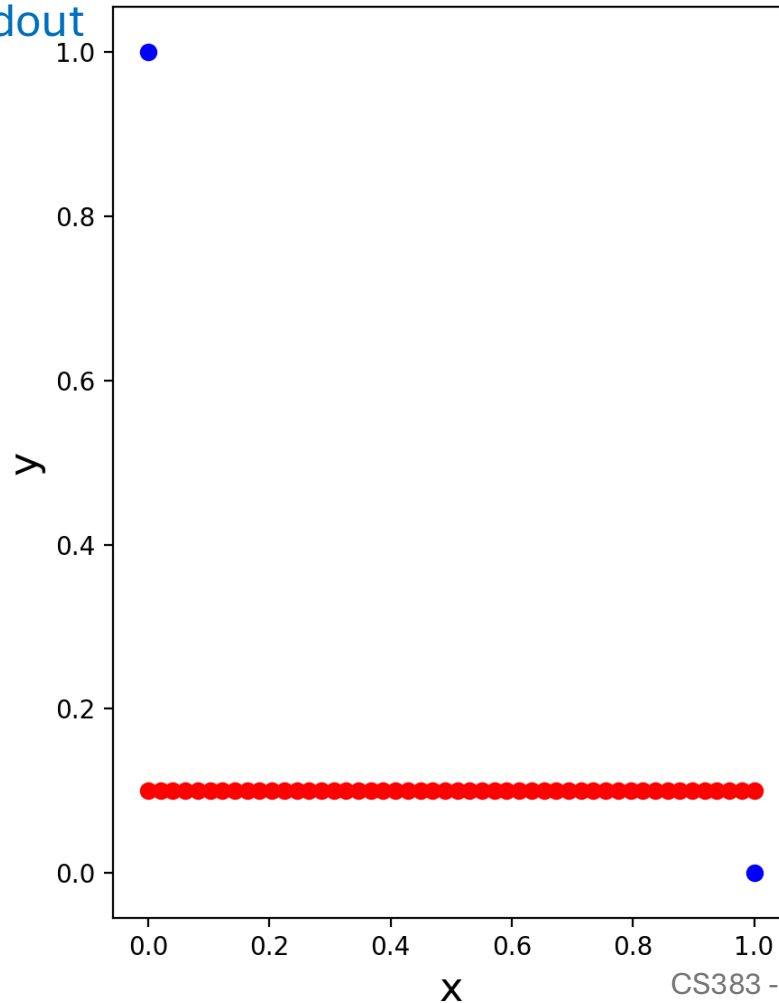
Normal Equation

- Non-iterative
- No need to choose η
- Slow if p is large
 - Matrix inversion is $O(p^3)$

Toy example, iteration 1

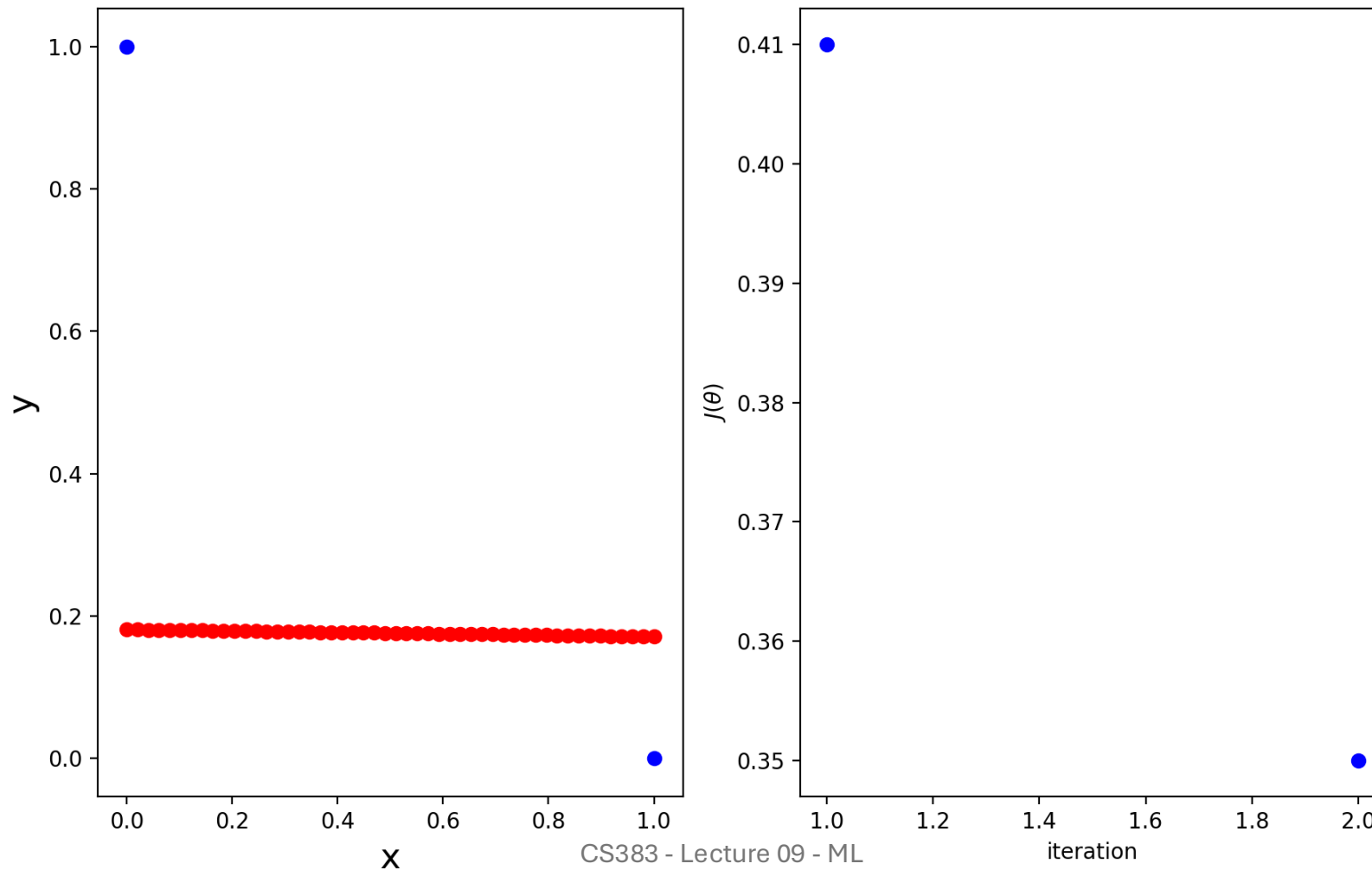
This is what you
should have
obtained in Handout
7!

iteration: 1, cost: 0.410000



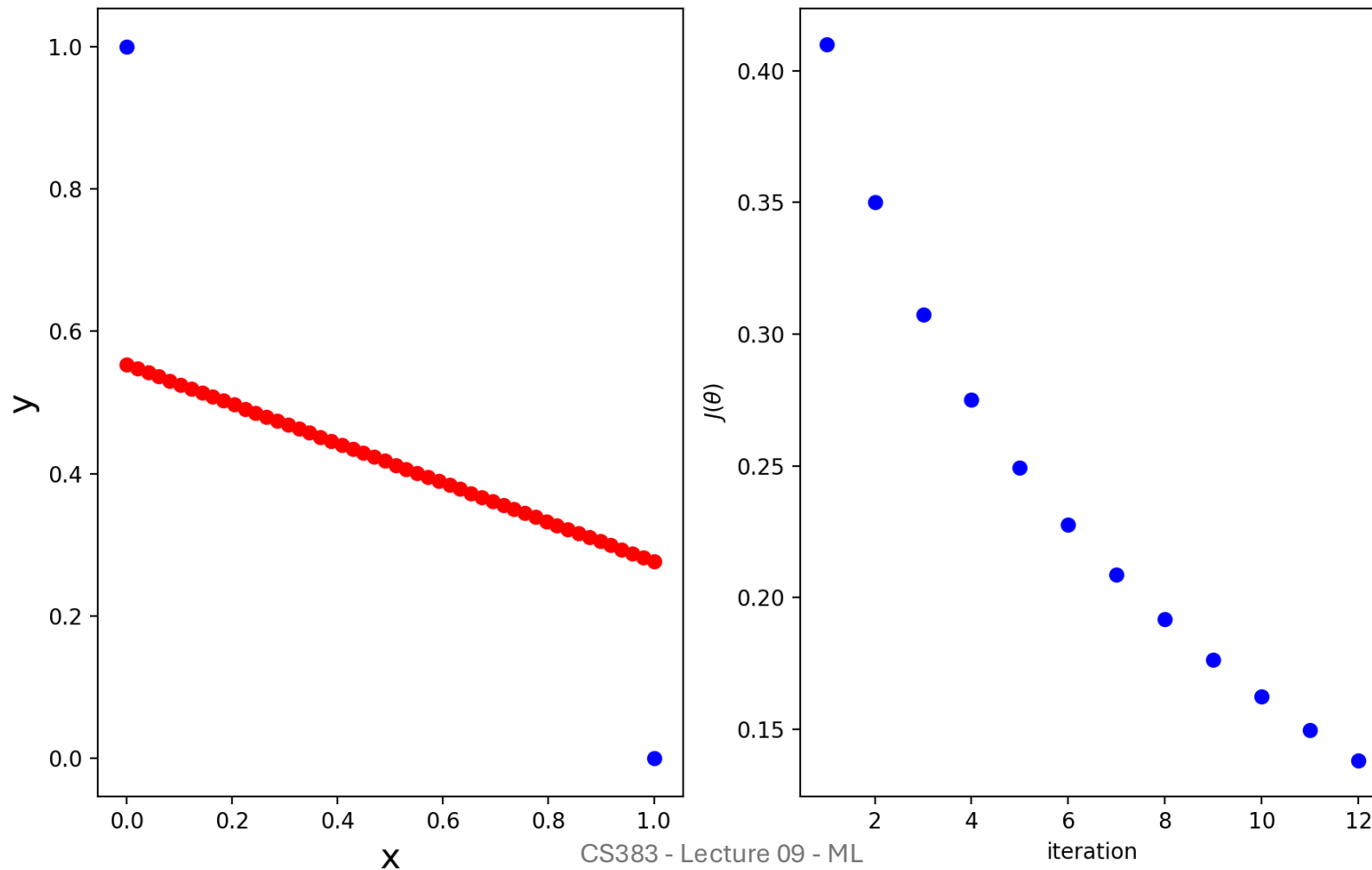
Toy example, iteration 2

iteration: 2, cost: 0.350001



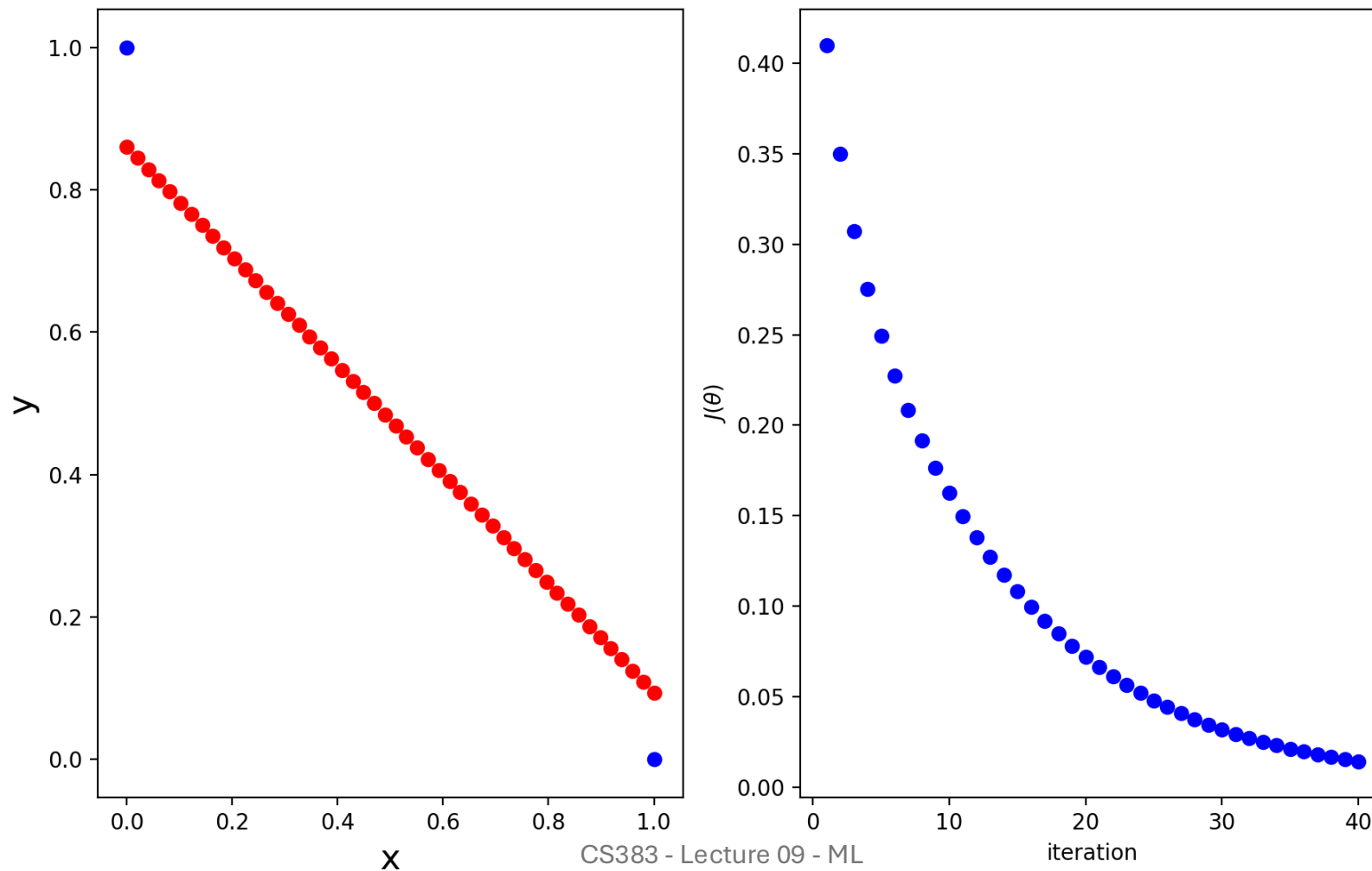
Toy example, iteration 12

iteration: 12, cost: 0.138047



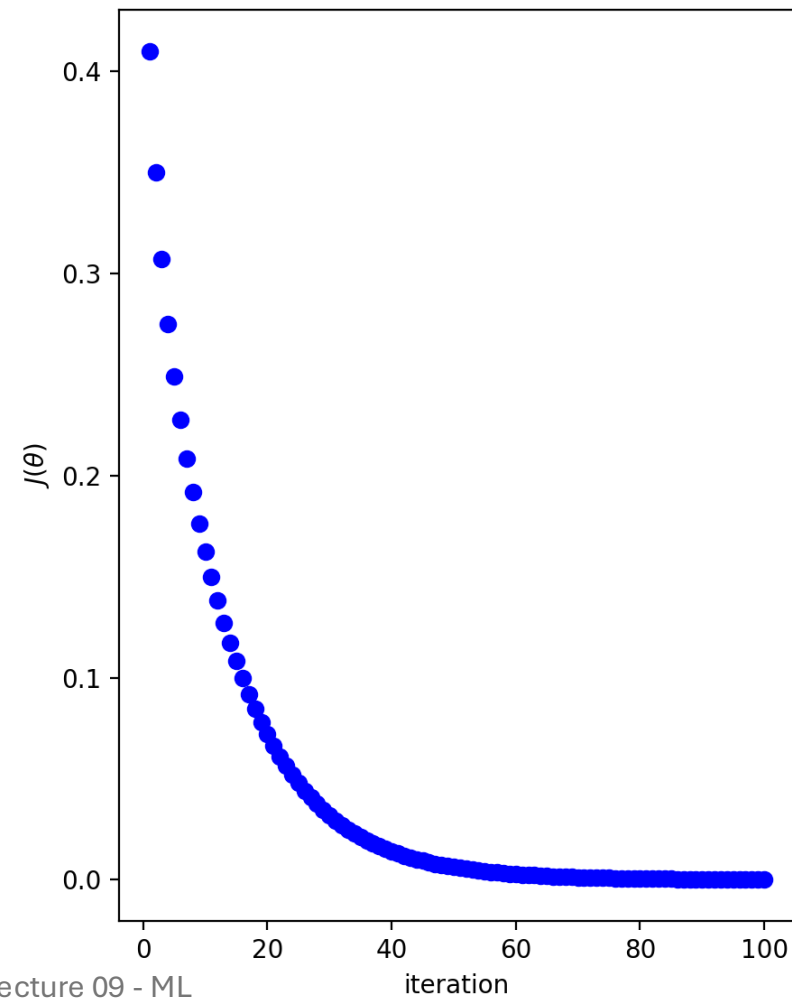
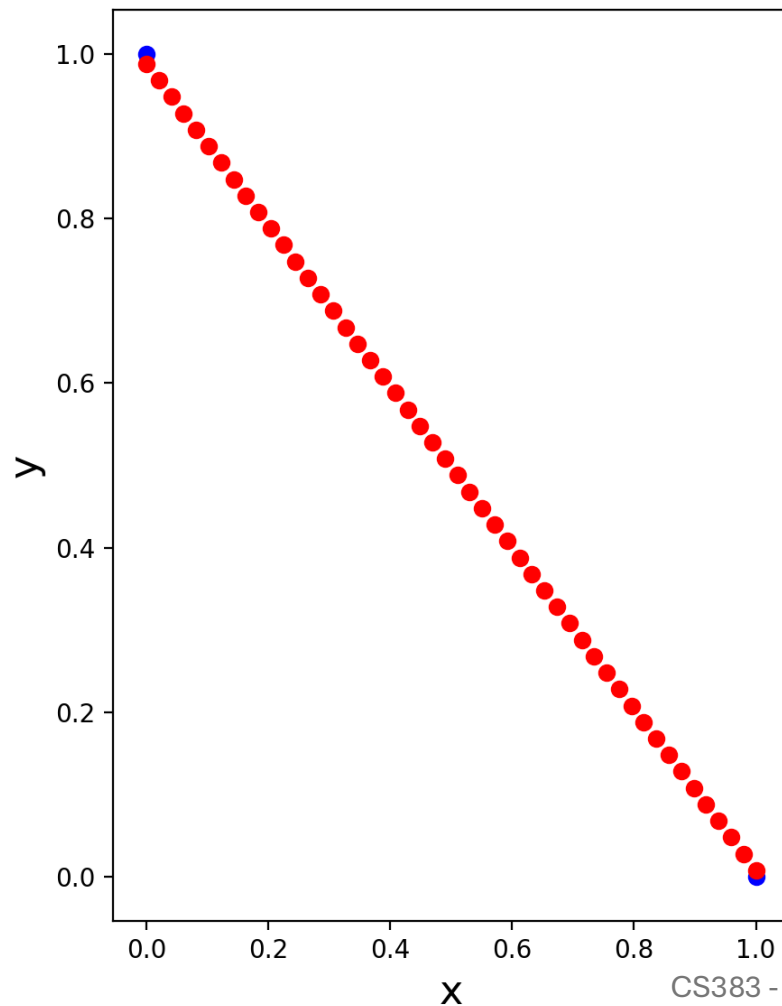
Toy example, iteration 40

iteration: 40, cost: 0.014064



Toy example, iteration 100

iteration: 100, cost: 0.000105



Outline

Normal equations vs SGD

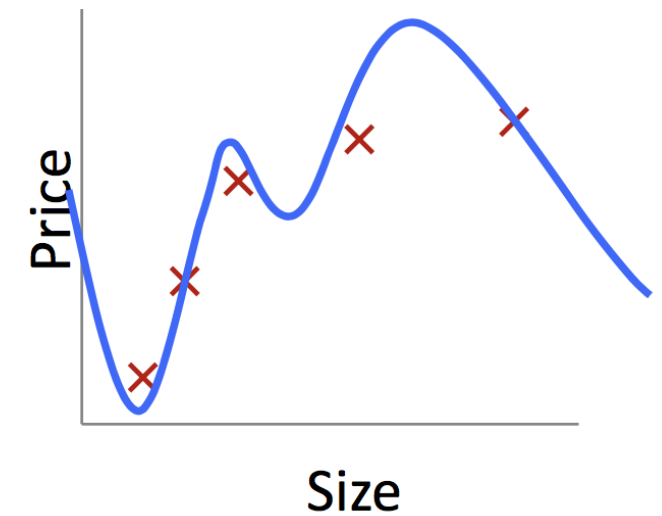
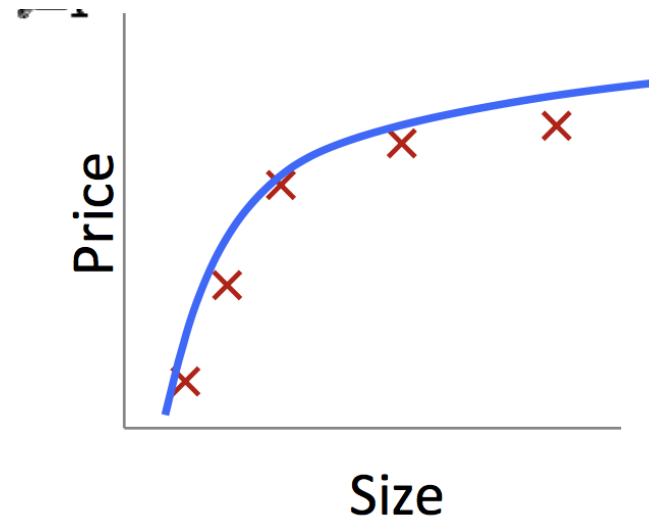
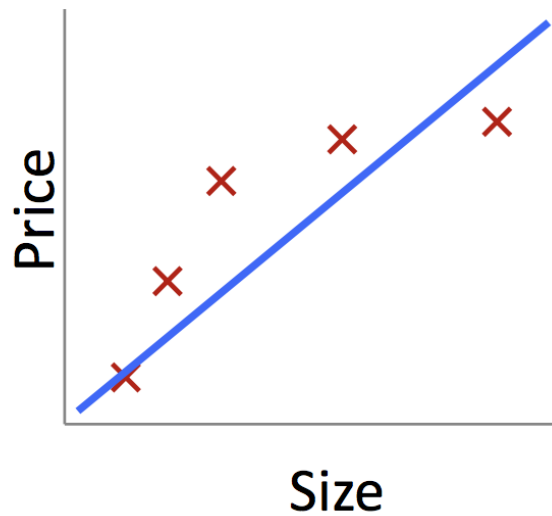
Regularization

Probability

Naive Bayes

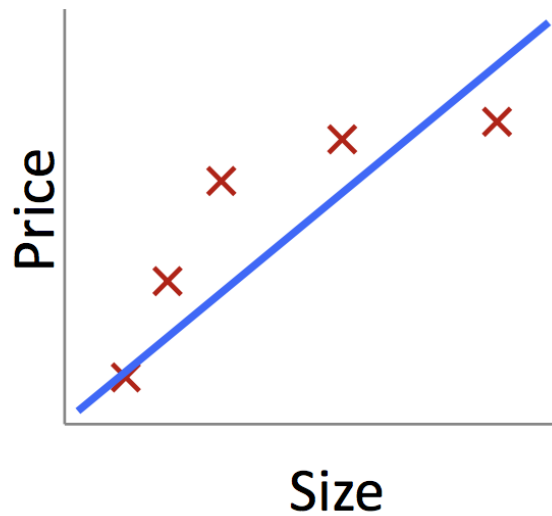
Generalization Error

Example: price vs. size (i.e. of a house or car)

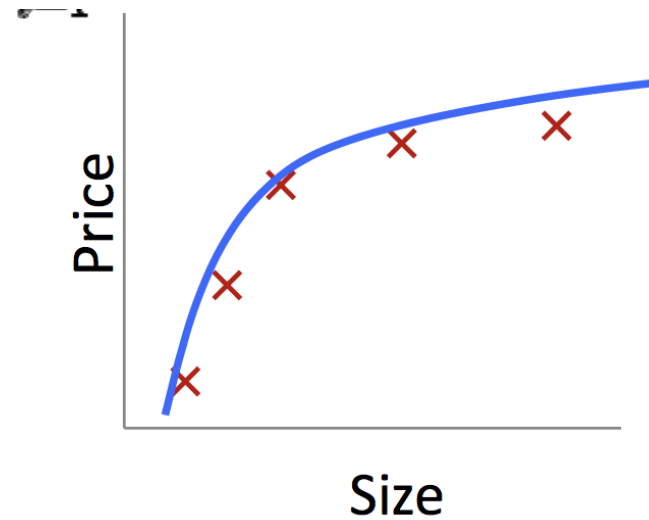


Generalization Error

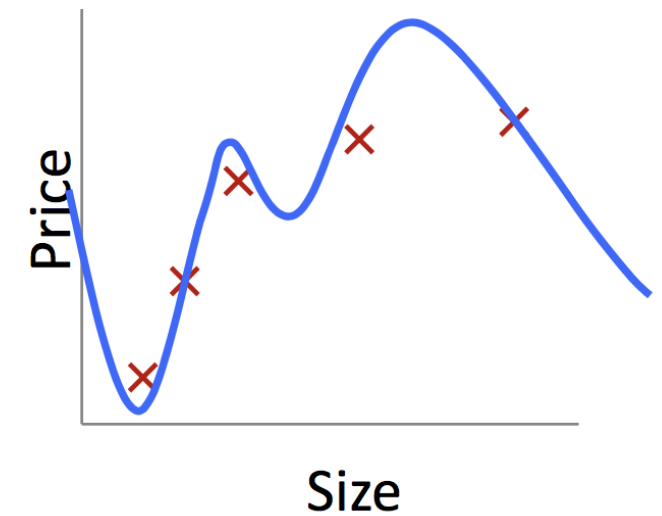
Example: price vs. size (i.e. of a house or car)



underfitting
(high bias)



correct fit



overfitting
(high variance)

Generalization Error

Structural error:

Hypothesis space cannot model true relationship

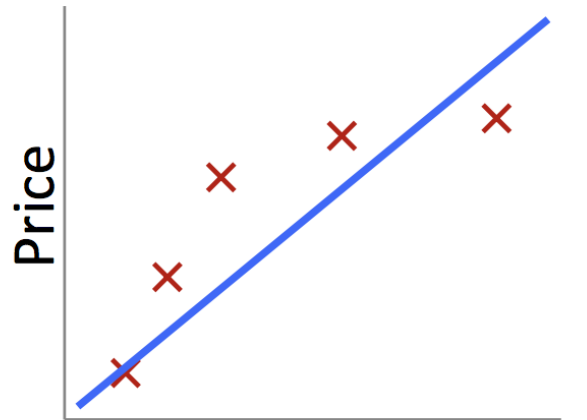
- More data doesn't help
- Need a more flexible model

Estimation (approximation) error:

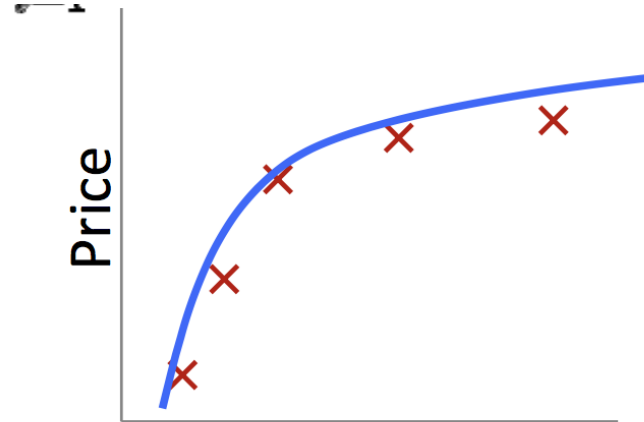
Hypothesis space *can* model true relationship, BUT hard to identify correct model due to large hypothesis space, small n , or noise

- ☐ Reduce hypothesis space
- ☐ Add more data

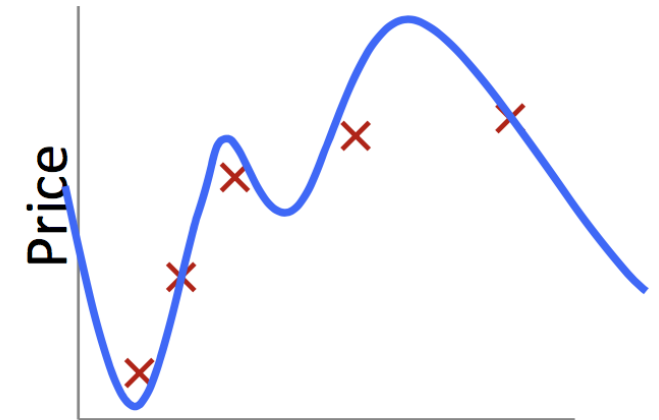
balance
↔



Size
underfitting
(high bias)



Size
correct fit



Size
overfitting
(high variance)

Regularization

What if ...

- we have a limited # of training examples ($n < p$), or
- we want to automatically control the complexity of the learned hypothesis?

Regularization

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Idea: penalize large values of w_j

Why prefer small weights?

- if large weights, small change in feature can result in large change in prediction
- prevent giving too much weight to any one feature
- might prefer zero weight for useless features

Common Regularizers

$$||\vec{w}||_0 = \sum_{j:w_j \neq 0} 1$$

L_0 norm

- Number of non-zero entries
- Minimizing L_0 norm is NP hard

$$||\vec{w}||_1 = \sum_{j=1}^p |w_j|$$

L_1 norm

- Sum of magnitude of weights
- Not differentiable

$$||\vec{w}||_2 = \sqrt{\sum_{j=1}^p w_j^2}$$

L_2 norm

- Sum of squared weights
- Differentiable

Outline

Normal equations vs SGD

Regularization

Probability

Naive Bayes

Probability & Bayes Derivation

Bayes Rule

Conditional Probability

Marginal Probability

Bayes Rule

$$\begin{aligned}P(A, B) &= \\&= P(A)P(A|B) \\&= P(B)P(B|A)\end{aligned}$$

Hence:

$$P(A|B) = \frac{P(A|B)P(A)}{P(B)}$$

Joint & Conditional Probability

Joint Probability of Multiple Variables $P(A, B, C) =$

$$\begin{aligned} &= P(C)P(A, B | C) \\ &= P(C)P(B|C)P(A|B, C) \end{aligned}$$

If A and B are independent:

$$P(A, B) = P(A)P(B)$$

If A and B are conditionally independent given C

$$P(A, B | C) = P(A | C)P(B | C)$$

If A, B, C are independent:

$$P(A, B, C) = P(A)P(B)P(C)$$