CSP-Example Problems

1-You are scheduling a robot assigned to schedule professors and computer science classes that meet M/W/F.

- There are 5 classes to schedule and 3 professors to teach these classes.
- Your requirements are:

(1) each professor only teaches one class at a time;

(2) each class is taught by only one professor; and

(3) some professors can only teach some of the classes.

You must produce a complete and consistent schedule.

- You decide to formulate this task as a CSP in which classes are the variables (named C1 through C5) and professors are the domain values (named A, B, and C).
- After you have solved the CSP, each class (variable) will be assigned one professor (value), and all constraints will be satisfied.

- The classes (variables) are:

C1, Class 1 - Intro to Programming: meets from 8:00-8:50am

C2, Class 2 - Intro to Artificial Intelligence: meets from 8:30-9:20am

C3, Class 3 - Natural Language Processing: meets from 9:00-9:50am

C4, Class 4 - Computer Vision: meets from 9:00-9:50am

C5, Class 5 - Machine Learning: meets from 9:30-10:20am

- The professors (domain values) are:

A, Professor A, who is available to teach Classes C3 and C4.

B, Professor B, who is available to teach Classes C2, C3, C4, and C5.

C, Professor C, who is available to teach Classes C1, C2, C3, C4, C5.

a) For each variable C1-C5, write down its domain as a subset of the values {A, B, C}.
b) Write below all constraints that are associated with this CSP. Write each constraint **implicitly** as **Ci ≠ Cj** for all classes C_i and C_j that overlap in time and so cannot be taught by the same professors.
c) Draw the constraint graph associated with your CSP.

d) Run Arc Consistency (AC-3) on the domains in a), the constraints in b), and the constraint graph in c). Write down the reduced domains that result when all inconsistent domain values are removed by Arc Consistency (AC-3).

e) Give one solution to this CSP.

2- SUDOKU AS A CONSTRAINT SATISFACTION PROBLEM

- A Sudoku board consists of $n \times n$ squares, some of which are initially filled with digits from 1 to n.
- The objective of the Sudoku puzzle is to fill in all the remaining squares such that no digit appears twice in any row, column, or $\sqrt{n} \times \sqrt{n}$ box.
- **Consider the case in which $\boldsymbol{n}$=4.**
- The Sudoku puzzle with n=4 can be formulated as a constraint satisfaction problem with n^2=16 variables, where every variable stands for one of the 16 squares on the 4x4 Sudoku board.

- If we denote rows with letters A-D and columns with numbers 1-4, we end up with 16 variables: A1, A2, A3, A4, B1, B2, …, D3, D4.
- Each variable has domain values {1,2,3,4}.
- To specify the constraints of the problem, we could use binary constraints for each pair of variables, but might end up with a large number of constraints.
- A better alternative is to use global *AllDiff()* constraints, where *AllDiff*$(X1, X2, \ldots, Xn)$ means that all n variables, X_1, X_2, …, X_n, must have different values.

a) How many *AllDiff()* constraints are required to specify that no digit may appear twice in any row, column, or $\mathbf{2} \times \mathbf{2}$ box?

b) FORWARD CHECKING

- Consider the 4×4 Sudoku board below. Possible domain values for each variable are shown in each square. Variable B3 has just been assigned value 3 (circled).
- Cross out all values from the domains of the remaining unassigned variables that now would be eliminated by Forward Checking.

	1	2	3	4
A	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
B	1 2 3 4	1 2 3 4	3	1 2 3 4
C	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
D	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4

	1	2	3	4
A	1 2 3 4	1 2 3 4	1 2 ~~3~~ 4	1 2 ~~3~~ 4
B	1 2 ~~3~~ 4	1 2 ~~3~~ 4	3	1 2 ~~3~~ 4
C	1 2 3 4	1 2 3 4	1 2 ~~3~~ 4	1 2 3 4
D	1 2 3 4	1 2 3 4	1 2 ~~3~~ 4	1 2 3 4

c) ARC CONSISTENCY.

- Consider the 4×4 Sudoku board below. Possible domain values for each variable are shown in each square. Variables B3, C2, and D4 have been assigned values, but no constraint propagation has been done.
- **Cross out all values from the domains of the remaining unassigned variables that now would be eliminated by Arc Consistency.**

	1	2	3	4
A	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
B	1 2 3 4	1 2 3 4	3	1 2 3 4
C	1 2 3 4	2	1 2 3 4	1 2 3 4
D	1 2 3 4	1 2 3 4	1 2 3 4	4

	1	2	3	4
A	$1\ 2\ 3\ \cancel{4}$	$1\ \cancel{2}\ 3\ \cancel{4}$	$\cancel{1}\ \cancel{2}\ \cancel{3}\ \boxed{4}$	$1\ 2\ \cancel{3}\ \cancel{4}$
B	$1\ 2\ \cancel{3}\ \cancel{4}$	$1\ \cancel{2}\ \cancel{3}\ 4$	3	$1\ 2\ \cancel{3}\ \cancel{4}$
C	$\cancel{1}\ \cancel{2}\ \cancel{3}\ \boxed{4}$	2	$\boxed{1}\ \cancel{2}\ \cancel{3}\ \cancel{4}$	$\cancel{1}\ \cancel{2}\ \boxed{3}\ \cancel{4}$
D	$1\ \cancel{2}\ 3\ \cancel{4}$	$1\ \cancel{2}\ 3\ \cancel{4}$	$\cancel{1}\ \boxed{2}\ \cancel{3}\ \cancel{4}$	4

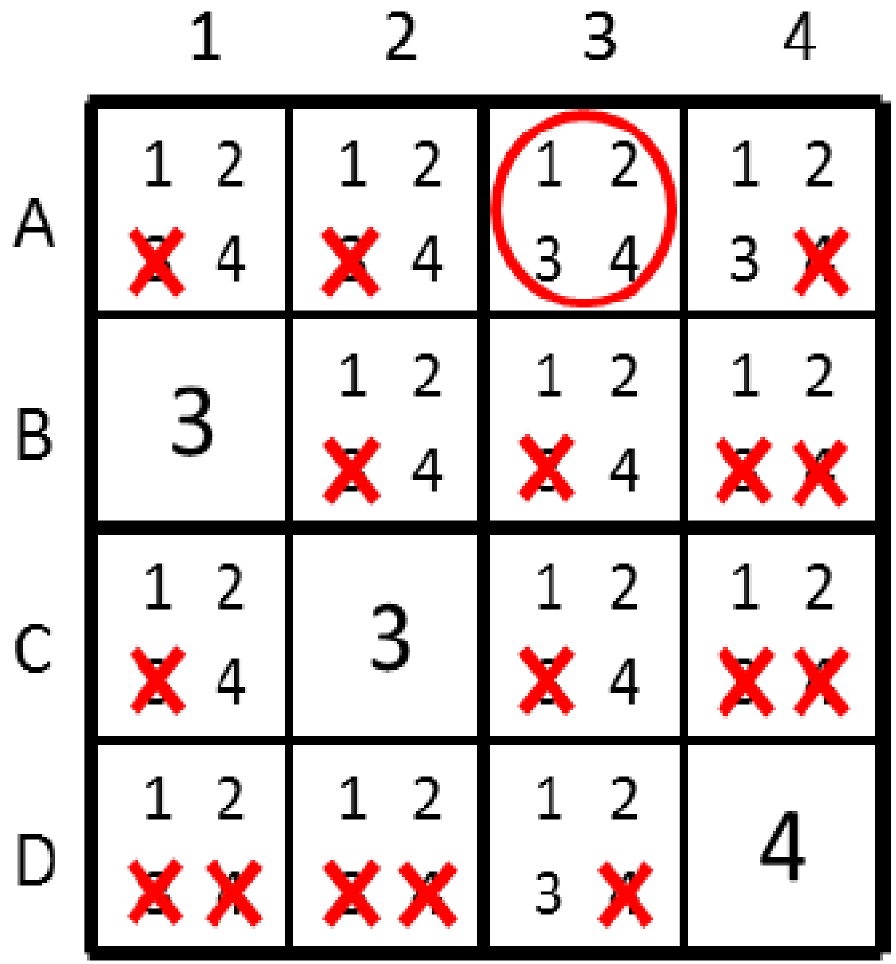
d) MINIMUM-REMAINING-VALUES (MRV) HEURISTIC

- Consider the 4×4 Sudoku board below. Possible domain values for each variable are shown in each square. Variables A1 and D4 are assigned and constraint propagation has been done.
- **List all unassigned variables (in any order) that now might be selected by the Minimum-Remaining-Values (MRV) Heuristic**

	1	2	3	4
A	3	1 2 ~~3~~ 4	1 2 ~~3~~ 4	1 2 ~~3~~ ~~4~~
B	1 2 ~~3~~ 4	1 2 ~~3~~ 4	1 2 3 4	1 2 3 ~~4~~
C	1 2 ~~3~~ 4	1 2 3 4	1 2 3 ~~4~~	1 2 3 ~~4~~
D	1 2 ~~3~~ ~~4~~	1 2 3 ~~4~~	1 2 3 ~~4~~	4

e) LEAST-CONSTRAINING-VALUE (LCV) HERIUSTIC.

- Consider the 4×4 Sudoku board on the right. Possible domain values for each variable are shown in each square. Variables B1, C2, and D4 are assigned and constraint propagation has been done. A3 (circled) is chosen for assignment next.
- **List all values (in any order) for variable A3 (circled) that now might be selected by the Least-Constraining-Value (LCV) Heuristic:**



f) DEGREE HEURISTIC (DH)

- Consider the 4×4 Sudoku board on the left. Possible domain values for each variable are shown in each square. Variables A3, B2, and C3 are assigned and constraint propagation has been done.
- **List all unassigned variables (in any order) that now might be selected by the Degree Heuristic (ignore MRV for this problem):**

	1	2	3	4
A	~~1~~ 2 ~~3~~ 4	~~1~~ 2 ~~3~~ 4	1	~~1~~ 2 3 4
B	1 2 ~~3~~ 4	3	~~1~~ 2 ~~3~~ 4	1 2 ~~3~~ 4
C	1 2 ~~3~~ 4	1 2 ~~3~~ 4	3	1 2 ~~3~~ 4
D	1 2 3 4	1 2 ~~3~~ 4	~~1~~ 2 ~~3~~ 4	1 2 ~~3~~ 4

g) MIN-CONFLICTS HEURISTIC.

- Consider the 4×4 Sudoku board on the left showing a complete but inconsistent assignment. Current values for each variable are shown in each square. Variable D3 (circled) has just been selected to be assigned a new value during local search for a complete and consistent assignment.
- **What new value for variable D3 (circled) would be chosen now by the Min-Conflicts Heuristic?**

	1	2	3	4
A	2	4	1	3
B	3	1	4	2
C	1	2	3	2
D	4	3	?	1

3) You are a map-coloring robot assigned to color this Midwest USA map. Adjacent regions must be colored a different color (R=Red, B=Blue, G=Green).

TN=Tennesse
IN=Indiana
IA=Iowa
IL=Illinois
MO=Missouri
KY=Kentucky

a) Draw the constraint graph.

b) FORWARD CHECKING. Cross out all values that would be eliminated by Forward Checking, after variable TN has just been assigned value R as shown:

TN	IN	IA	IL	MO	KY
R	R G B	R G B	R G B	R G B	R G B

c) ARC CONSISTENCY. TN and MO have been assigned values, but no constraint propagation has been done. Cross out all values that would be eliminated by Arc Consistency

TN	IN	IA	IL	MO	KY
R	R G B	R G B	R G B	G	R G B

d) MINIMUM-REMAINING-VALUES HEURISTIC. Consider the assignment below. IA is assigned and constraint propagation has been done. List all unassigned variables that might be selected by the Minimum-Remaining-Values (MRV) Heuristic.

TN	IN	IA	IL	MO	KY
R G B	R G B	G	R B	R B	R G B

e)DEGREE HEURISTIC. Consider the assignment below. (It is the same assignment as in problem 4c above.) IA is assigned and constraint propagation has been done. List all unassigned variables that might be selected by the Degree Heuristic

TN	IN	IA	IL	MO	KY
R G B	R G B	G	R B	R B	R G B

f)MIN-CONFLICTS HEURISTIC. Consider the complete but inconsistent assignment below. IA has just been selected to be assigned a new value during local search for a complete and consistent assignment. What new value would be chosen below for IA by the Min-Conflicts Heuristic?

TN	IN	IA	IL	MO	KY
B	G	**?**	G	B	B