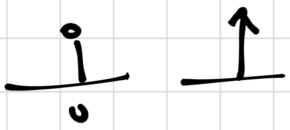
Summary

- Signals
- Systems
- Classification of Signals
  - → CT / DT signals
  - → Even/odd "
  - → Periodic / Non-Periodic ?
  - → Deterministis vs Random
  - → Energy vs Power Signals
- Basic Operations on Signals
  - → On Dependent variable
  - → On Independet V.
    - ~ Time scaling
    - ~ Reflection
    - ~ Time shifting

    Precedence → $\underset{\text{First}}{\underline{\text{Shift}}}$ then $\underline{\text{Scale}}$
- Elementary Signals
  - ~ Exponential Signals
  - ~ Sinusoidal "
    - Period ?
  - ~ ~~1.6.5~~ ~~1.6.5~~
  - ~ Step Function $\underline{u}[n]$ $u(t)$

  - ~ Impulse "



    $\delta(t)$ $\delta[n]$

    ~~1.6.7~~
  - ~ Ramp Function $r(t)$ $r[n]$
- Block Diagrams
- Properties of Systems
  - 1.8.1. Stability
  - 1.8.2 Memory
  - 1.8.5 Time Inv.
  - 1.8.3 Causality
  - 1.8.4. Invertibility
  - 1.8.6. Linearity.

~~1.9~~

~~1.10~~

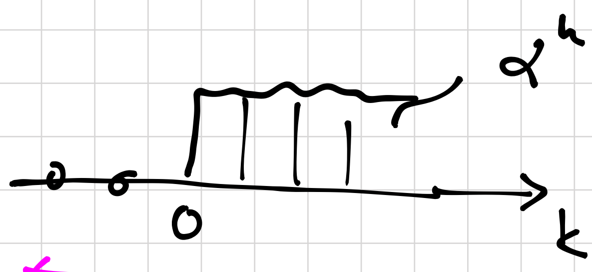
~~1.11~~

2. (LTI systems on Time Domain)

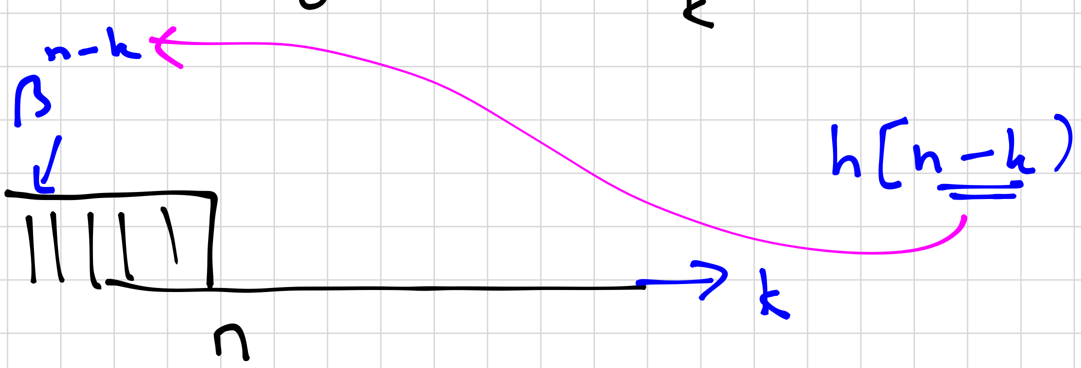
    2.1 Intro
    2.2 Convolution Sum
    2.3 " " Evaluation P.
    2.4. " Integral
    2.5 " " " P.
    2.6 (Interconnection of LTI systems)
    2.7 Impulse Response vs. LTI System Properties
    ~~2.7.4~~
    2.8 Step Response

Ex

$$\left.\begin{aligned} x[n] &= \alpha^n u[n] \\ h[n] &= \beta^n u[n] \end{aligned}\right\} \quad y[n] = x[n] * h[n] = ? \quad \text{for } \alpha \neq \beta$$



x[k]



$$\sum_{k=0}^{M-1} \beta^k = \begin{cases} \dfrac{1-\beta^M}{1-\beta}, & \beta \neq 1 \\ M, & \beta = 1 \end{cases}$$

$n < 0 \Rightarrow y[n] = 0$

$n \geq 0$

$$y[n] = \sum_{k=0}^{n} \alpha^k \cdot \beta^{n-k}$$

$$y[n] = \beta^n \cdot \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k$$

$$= \beta^n \cdot \left\{ \frac{1 - (\alpha/\beta)^{n+1}}{1 - \alpha/\beta} \right\}$$

$$= \left[\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right]$$

~~y[n]~~ =

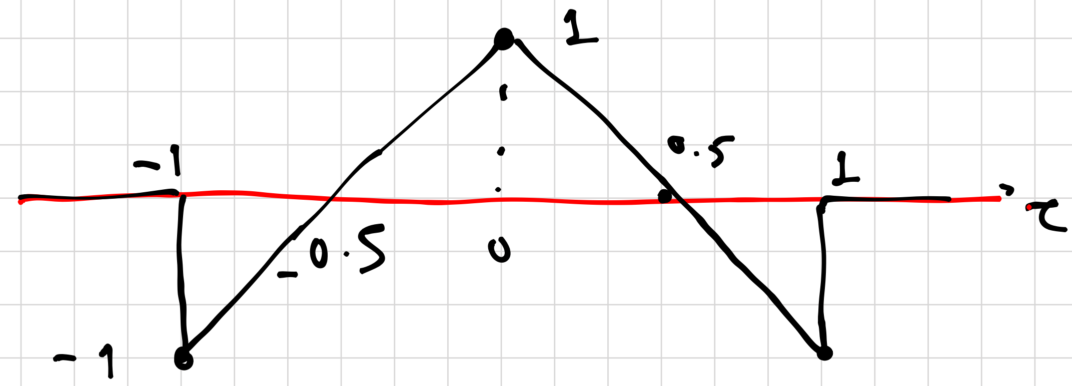
$$y[n] = \left[\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right] u[n]$$

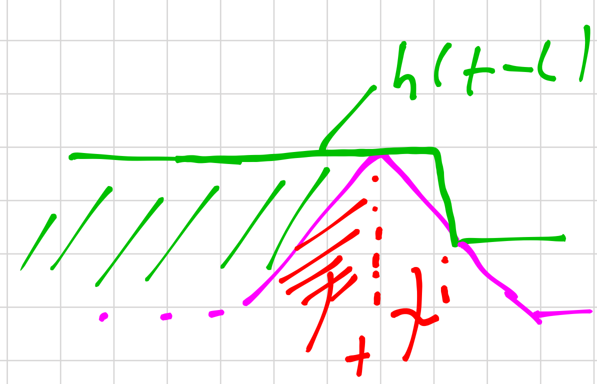
Ex Evaluate the CT convolution, $x(t) * h(t)$ for the following

$$x(t) = \begin{cases} 2t + 1, & -1 < t \leq 0 \\ -2t + 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$

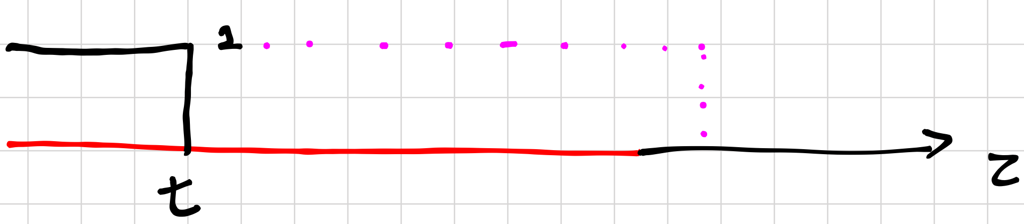
$$h(t) = u(t)$$

$x(\tau)$





$h(t-\tau)$



① $t < -1 \Rightarrow y(t) = 0$

② $-1 < t < 0 \Rightarrow y(t) = \displaystyle\int_{-1}^{t} \underbrace{(2\tau + 1)}_{x(\tau)} \cdot \overset{h(t-\tau)}{1} \cdot d\tau$

$$y(t) = \left[\tau^2 + \tau \right]_{-1}^{t} = (t^2 + t) - (\underbrace{(-1)^2 - 1}_{0}) = t^2 + t$$

③ $0 < t < 1$

$$y(t) = \underbrace{\int_{-1}^{0} (2\tau + 1)\, d\tau}_{0} + \int_{0}^{t} (-2\tau + 1)\, d\tau = (-\tau^2 + \tau) \Big|_{0}^{t} = t - t^2$$

④ $t > 1$

$$y(t) = \int_{-1}^{0} (2\tau + 1)\, d\tau + \int_{0}^{1} (-2\tau + 1)\, d\tau = 0.$$

Ex

$$x[n] = (-1)^{n+1} \cdot u[n]$$
$$h[n] = u[n] - u[n-1]$$
$$\Big\} \; x[n] * h[n]$$

$$h[n] = u[n] - u[n-1] = \delta[n]$$

$$\delta[n] * x[n] = x[n]$$

$$\boxed{y[n] = (-1)^{n+1} \cdot u[n]}$$

$$* \quad \delta[n-k] * x[n] = x[n-k] \; *$$

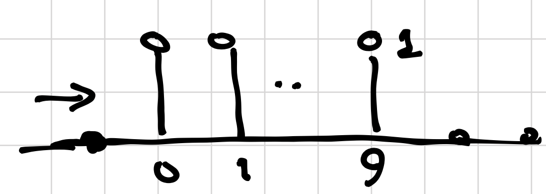
Ex

$$y[n] = \sum_{k=0}^{9} x[n-k]$$

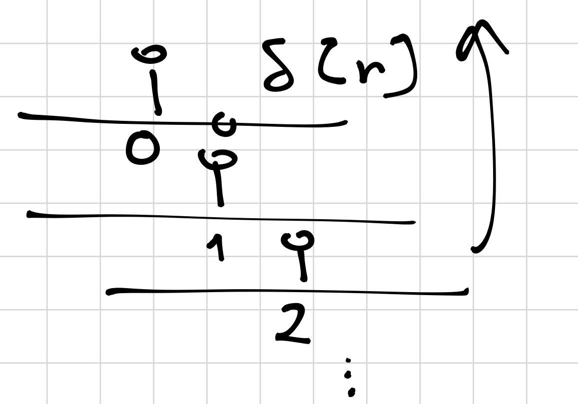
Determine the impulse response and the step response of this system.

__Impulse Response__

$$h[n] = \mathcal{H}\{\delta[n]\}$$
$$= \sum_{k=0}^{9} \delta[n-k]$$



__Step Response__



$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

- $n < 0 \Rightarrow \quad s[n] = 0$
- $0 \leq n \leq 9 \quad s[n] = \sum_{k=0}^{n} 1 = n+1$
- $n > 9 \quad s[n] = \sum_{k=0}^{9} 1 = 10$

Stable
$$\sum_{k=-\infty}^{+\infty} h[n] < \infty$$
$$\sum_{k=0}^{9} 1 = 10 \Rightarrow \text{STABLE!}$$

$$s[n] = \begin{cases} 0, & n < 0 \\ n+1, & 0 \leq n \leq 9 \\ 10, & n > 9 \end{cases}$$

- Causal?
$h[n-k] = 0$
$k < 0$
CAUSAL

- Memoryless? $\sim c\,\delta[n]$ $\quad h[n-k] = 0 \quad k \neq 0$

Not memoryless

Ex

$$y[n] = e^{-3n} \cdot x[n] \cdot u[n]$$

a) Is this system LINEAR?

b) " " " Time Invariant?

a) Superposition?

$$H\{x_1[n] + x_2[n]\} = e^{-3n}\{x_1[n] + x_2[n]\}u[n]$$

$$= e^{-3n}x_1[n] \cdot u[n]$$

$$+ e^{-3n}x_2[n] \cdot u[n]$$

$$= y_1[n] + y_2[n]$$

Homogenity

$$H\{\alpha \cdot x[n]\} = e^{-3n}\{\alpha \cdot x[n]\} u[n]$$

$$= \alpha \cdot y[n]$$

LINEAR

b) $H\{x(\cancel{t}-\cancel{t_0})\} = \underbrace{e^{-3n} x[n-n_0] u[n]}_{y_1[n]}$

$$y[n-n_0] = e^{-3(n-n_0)} \cdot x[n-n_0] u[n-n_0]$$

$$\neq y_1[n]$$

not T.I

Ex

$$y[n] = \begin{cases} x[2n], & n<0 \\ \dfrac{n}{n+1}, & n \geq 0 \end{cases}$$

n

a) memoryless? $y[-1] = x[-2] \Rightarrow$ not memoryless

b) causal $n<0 \Rightarrow 2n<n$ } causal.

c) stable? ⊛ $n>0$ $\frac{n}{n+1} < \frac{n+1}{n+1} < 1 <$ $< \infty$ $\forall n$.

$\frac{n}{n+1} < \frac{n+1}{n+1} < 1$ $|x[n]| \leq M_x < \infty$ for all n } Stable.

$n<0 \Rightarrow$ $|y[n]| = \underbrace{|x[2n]|}$

STABLE.

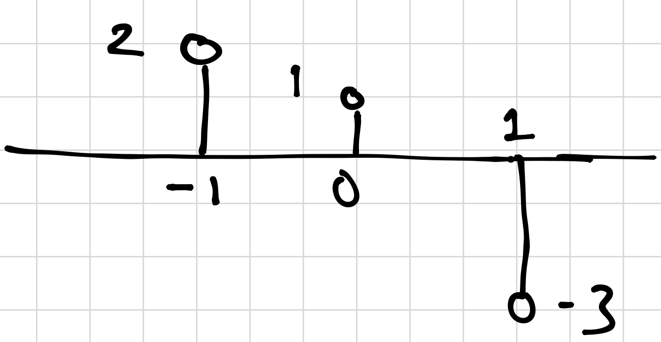
$|y[n]| \leq M_x < \infty$

Ex

$$y[n] = 2x[n+1] + x[n] - 3x[n-1]$$

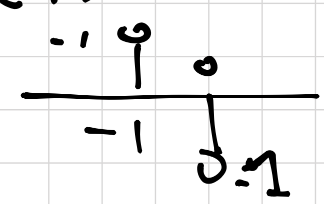
a) Impulse response.

$$h[n] = 2 \cdot \delta[n+1] + \delta[n] - 3\delta[n-1]$$



$h[n]$

b) $x[n] = \delta[n+1] - \delta[n]$ $\quad y[n] = ?$



$$\delta[n-k] * x[n] = x[n-k]$$

$$y[n] = 2x[n+1] + x[n] - 3x[n-1]$$

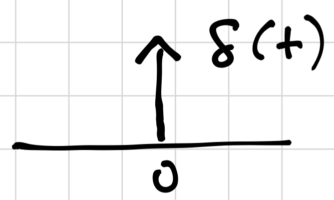
-2	-1	0	1	2	
2	-2				$2 \cdot x[n+1]$
	1	-1			$x[n]$
		-3	3		$-3x[n-1]$

+

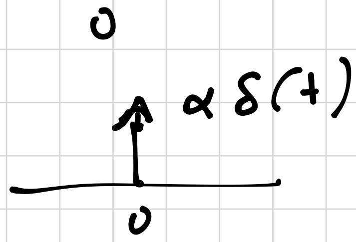
-2	-1	0	1
2	-1	-4	3

$$\Rightarrow y[n] = 2\delta[n+2] - \delta[n+1] - 4\delta[n] + 3\delta[n-1]$$

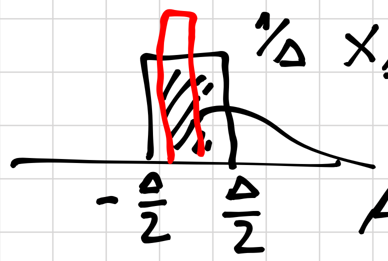
· Impulse Function (CT)



$$\left.\int_{-\infty}^{+\infty} \delta(t)\,dt = 1 \right\}$$



α: strength



$$\left\{ \delta(t) = \lim_{\Delta \to 0} x_\Delta(t) \right\}$$