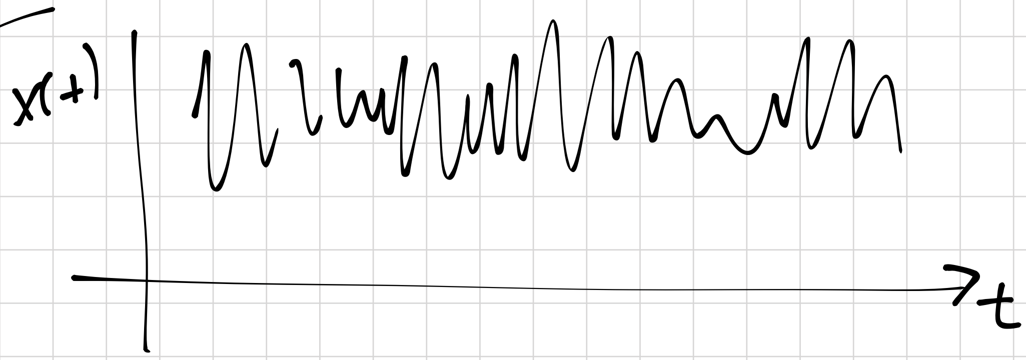
Özet



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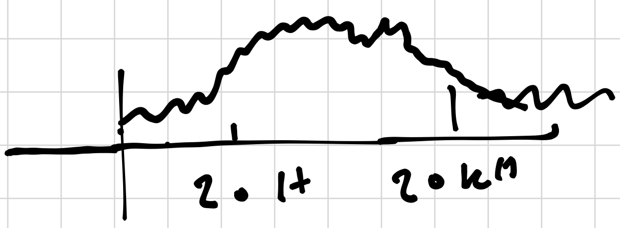
Assume this is an audio file



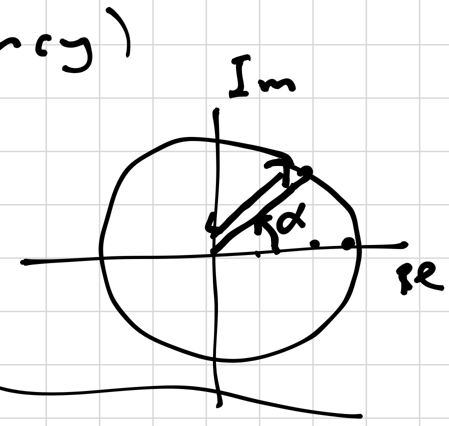
–	Periodic	Non-Periodic
Discrete Time	DTFS – Discrete Periodic. | at the freq. dom	DTFT → Continuous in the freq. domain.
→ Continuous Time	FS – Discrete	FT – Cont →

DTFS Periodic signal on time domain.
Discrete " representation " freq. domain
for $x[n]$, (period $= N$)

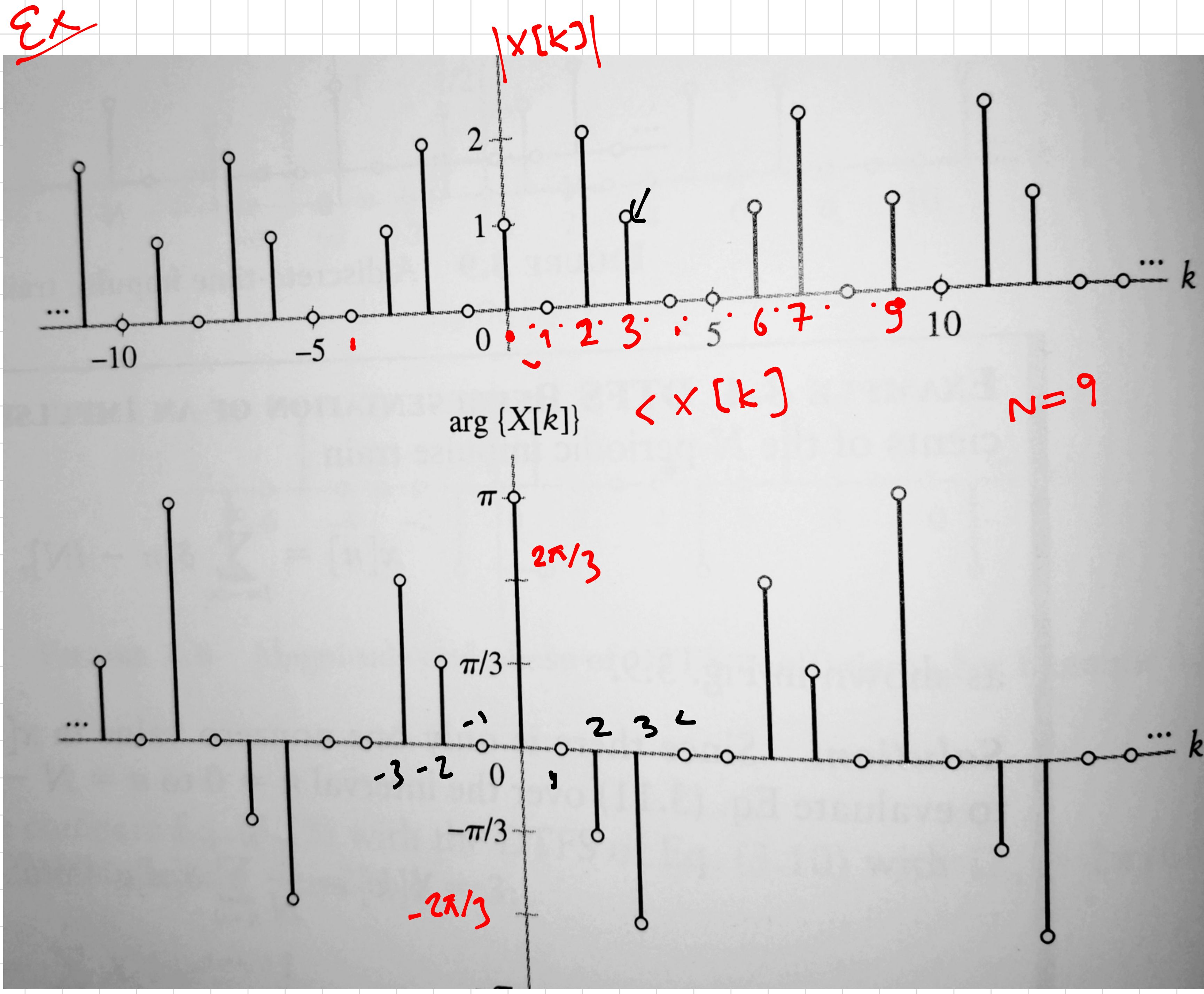
DTFS : $x[n] = \sum_{k=\langle N\rangle} X[k]\, e^{jk\Omega n}$

$\Omega = \frac{2\pi}{N}$ (fundamental frequency)

$X[k] = \frac{1}{N} \sum_{n=\langle N\rangle} x[n] \cdot e^{-jk\Omega n}$



Ex



$N = 9 \qquad \Omega = \frac{2\pi}{9}$

let's evaluate it between $k = (-4, 4)$

$$x[n] = \sum_{k=-4}^{4} X[k]\, e^{jk\Omega n} \qquad \Big|\; e^{j\theta} = \cos\theta + j\sin\theta$$

$$X[k] = |X[k]| \cdot e^{j\arg\{X[k]\}}$$

$X[-4] = 0 \qquad X[-3] = 1 \cdot e^{j\,2\pi/3}$

$X[-2] = 2 \cdot e^{j\,\pi/3}$

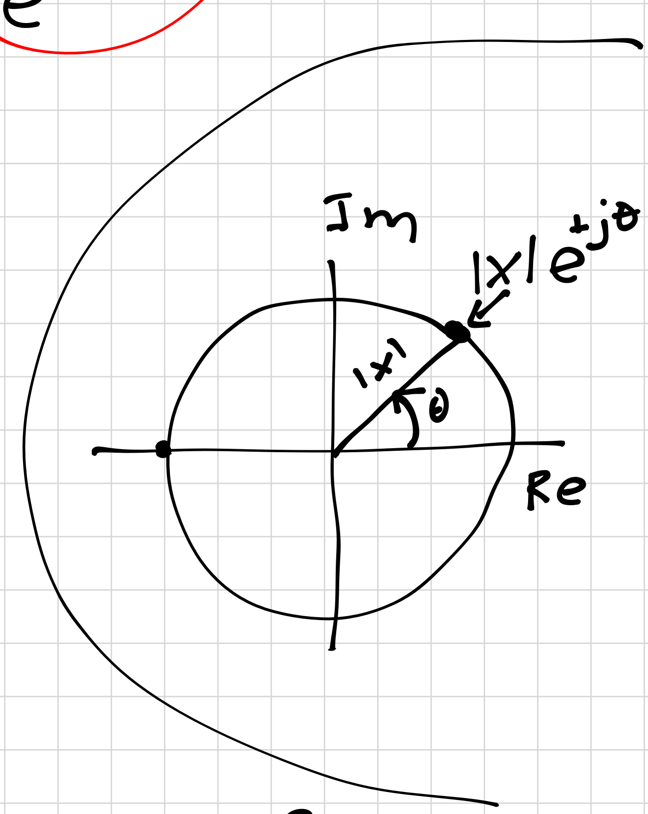
$X[-1] = 0$

$X[0] = \underbrace{1 \cdot e^{j\pi}}_{-1} \qquad X[1] = 0$

$X[2] = 2 \cdot e^{-j\,\pi/3}$

$X[3] = 1 \cdot e^{-j\,2\pi/3}$

$X[4] = 0$



-3

$x[n] = e^{+j\,2\pi/3} \cdot e^{j\cdot(-3)\cdot\left(\frac{2\pi}{9}\right)\cdot n}$

$\quad + \; 2\, e^{j\pi/3} \cdot e^{j(-2)\frac{2\pi}{9} n} \; + \; \ldots$ Sonraki sayfa

$N = 9$

$\Omega = \frac{2\pi}{9}$

$$x[n] = e^{j2\pi/3} \cdot e^{-j6\pi n/9} + 2e^{j\pi/3} \cdot e^{-j4\pi n/9} - 1 + 2e^{-j\pi/3} e^{j4\pi n/9} + e^{-j2\pi/3} \cdot e^{j6\pi n/9}$$

$$\boxed{x[n] = 2 \cdot \cos\left(\frac{6\pi}{9}n + \frac{2\pi}{3}\right) + 4\cos\left(\frac{4\pi n}{9} - \frac{\pi}{3}\right) - 1}$$

Problem

$$X[k] = \left(\frac{1}{2}\right)^k \quad \text{periodic on } 0 \le k \le 9$$

$N = 10$, find $x[n]$.

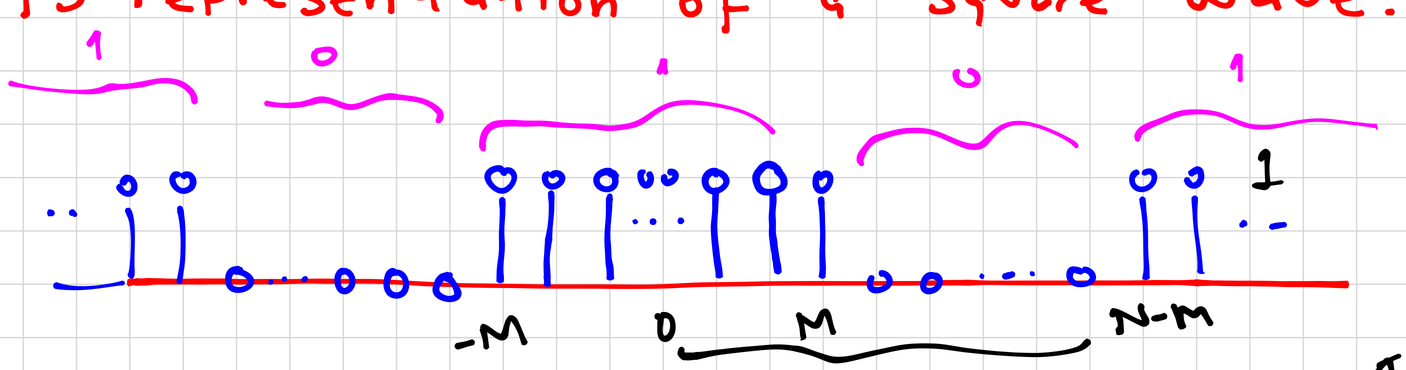
$$N = 10, \quad \Omega = \frac{2\pi}{10} = \frac{\pi}{5}$$

$$x[n] = \sum_{k=0}^{9} \left(\frac{1}{2}\right)^k e^{+jk\frac{\pi}{5}n}$$

appendix

$$= \sum_{k=0}^{9} \left(\frac{1}{2} e^{j\pi/5 n}\right)^k = \frac{1 - (1/2)^{10}}{1 - \frac{1}{2} e^{j\pi n/5}}$$

Ex DTFS representation of a square wave.



$$x[n] = \begin{cases} +1, & -M \le n \le M \\ 0, & M < n < N-M \end{cases}$$

1 period

$N \Rightarrow$ period

$N > 2M+1$

Find the DTFS coefficients.

$$N = N \qquad \Omega = \frac{2\pi}{N}$$

$$X[k] = \frac{1}{N} \sum_{n=-M}^{N-M-1} x[n] \cdot e^{-jk\Omega n}$$

$$= \frac{1}{N} \sum_{n=-M}^{M} 1 \cdot e^{-jk\Omega n}$$

Change variable $m = n + M$

$$\therefore \quad X[k] = \frac{1}{N} \sum_{m=0}^{2M} e^{-jk\Omega(m-M)} \qquad \Omega = \frac{2\pi}{N}$$

$$= \frac{1}{N} \cdot e^{jk\Omega M} \cdot \sum_{m=0}^{2M} e^{-jk\Omega m}$$

For $k = 0, \mp N, \mp 2N \ldots \qquad e^{-jk\Omega} = e^{jk\Omega} = 1$

$$X[k] = \frac{1}{N} \sum_{m=0}^{2M} 1 = \frac{2M+1}{N}$$

For $k \neq 0, \mp N, \mp 2N$

$$X[k] = \frac{e^{jk\Omega M}}{N} \left[\frac{1 - e^{-jk\Omega(2M+1)}}{1 - e^{-jk\Omega}} \right] \cdot \frac{e^{jk\Omega/2}}{e^{jk\Omega/2}}$$

$$= \frac{1}{N} \left[\frac{e^{jk\Omega\left(\frac{2M+1}{2}\right)}}{e^{jk\Omega/2}} \right]$$

$$= \frac{1}{N} \left[\frac{\exp\left(jk\Omega \frac{2M+1}{2}\right) - \exp\left(-jk\Omega \frac{2M+1}{2}\right)}{e^{jk\Omega/2} - e^{-jk\Omega/2}} \right]$$

/* $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ */

$$= \frac{1}{N} \frac{\sin\left(k\Omega \frac{2M+1}{2}\right)}{\sin(k\Omega/2)} \qquad N = \frac{2\pi}{\Omega}$$

$$X[k] = \begin{cases} \frac{1}{N} \frac{\sin(k\pi(2M+1)/N)}{\sin(k\pi/N)}, & k \neq 0, \mp N, \mp 2N \ldots \\ \frac{2M+1}{N}, & k = 0, \mp N, \mp 2N \ldots \end{cases}$$

We can also show that using L'Hôpital's rule

$$\lim_{k \to 0, \mp N, \mp 2N} \left[\frac{1}{N} \cdot \frac{\sin(k\pi(2M+1)/N)}{\sin(k\pi/N)} \right] = \frac{2M+1}{N}$$

Therefore we can simply write

$$\boxed{X[k] = \frac{1}{N} \frac{\sin\left(k\pi \frac{2M+1}{N}\right)}{\sin(k\pi/N)}}$$

DTFT $\longrightarrow$ Discrete Time Fourier Transform.

- DT Non-periodic signals.
- DTFT involves a continuum of frequencies on the interval $-\pi < \Omega < \pi$

Book uses a different notation

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega) \cdot e^{j\Omega n} \, d\Omega$$

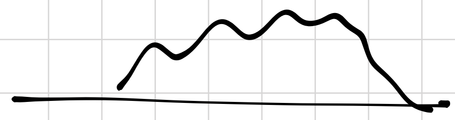
$$X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n] \cdot e^{-j\Omega n}$$

- $X(\Omega)$ is called the "frequency domain representation" of $x[n]$
- We say that $x[n]$ and $X(\Omega)$ are a DTFT pair

$$x[n] \xleftrightarrow{\text{DTFT}} X(\Omega)$$

- DTFT is usally used to analyze the action of DT systems on DT signals.

- The infinite sum converges if $x[n]$ has a finite duration and is finite valued.



- If $x[n]$ has an infinite duration then the sum converges if

$$\sum_{n=-\infty}^{+\infty} |x[n]| < \infty \quad (x[n] \text{ is absolutely summable})$$

- If $x[n]$ is not absolutely summable ~~then~~ but it does satisfy

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 < \infty$$

- then the infinite sum converges in a "mean-squared" sense but it does not converge "pointwise"

Example

$x[n] = \alpha^n u[n]$ Find the DTFS coefficients

$$X(\Omega) = \sum_{n=-\infty}^{+\infty} \alpha^n . u[n] . e^{-j\Omega n}$$

$$= \sum_{n=0}^{+\infty} \alpha^n . e^{-j\Omega n}$$

The sum converges only if $|\alpha| < 1$

In that case

$$X(\Omega) = \sum_{n=0}^{\infty} (\alpha e^{-j\Omega})^n = \frac{1}{1-\alpha e^{-j\Omega}} \qquad |\alpha| <$$

If α is real valued then

$$X(\Omega) = \frac{1}{1-\alpha\cos\Omega + j\alpha\sin\Omega}$$

$$|X(\Omega)| = \frac{1}{\left[(1-\alpha\cos\Omega)^2 + \alpha^2 \sin\Omega\right)^{1/2}}$$

$$= \frac{1}{\left[\alpha^2 + 1 + 2\alpha\cos\Omega\right]^{1/2}}$$

$$\arg\{X(\Omega)\} = -\arctan\left[\frac{\alpha\sin\Omega}{1-\alpha\cos\Omega}\right]$$

The figures are located at p <u>232</u>