4 Deterministic & Random Signals

(& → and)

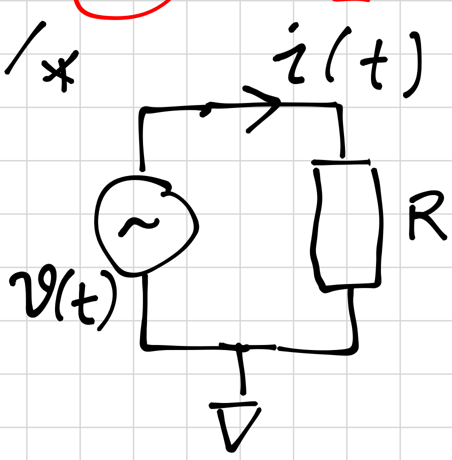
— If there is no uncertainty about the value of a signal at any time, this signal is called a "deterministic signal"

— A random signal is a signal about which there is uncertainty before it occurs.

&: and

5 Energy Signals and Power Signals

/*



Instantaneous Power dissipation

$$p(t) = \frac{v^2(t)}{R} = R \cdot i^2(t)$$

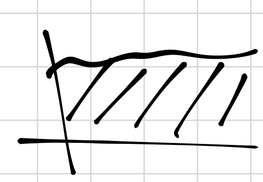
*/

For a signal $x(t)$ the instantaneous power is usually defined as

$$p(t) = x^2(t)$$

$\boxed{\text{The ENERGY of } x(t):}$

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$



$$\left\{ E = \int_{-\infty}^{\infty} x^2(t)\, dt \right\}$$

$\boxed{\text{Time-averaged power:}}$ (just average power)

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$ ***

For a periodic signal:

→ This simplifies to $P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$

(where T is the period)

DT- signals

Energy

$$E = \sum_{n=-\infty}^{+\infty} x^2[n]$$

Average power

$$P = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n]$$

If $x[n]$ is periodic

$$\Rightarrow \quad P = \frac{1}{N} \sum_{n=0}^{N} x^2[n]$$

- A signal is called an ENERGY signal if and only if its total energy satis this condition:

$$\boxed{0 < E < \infty}$$

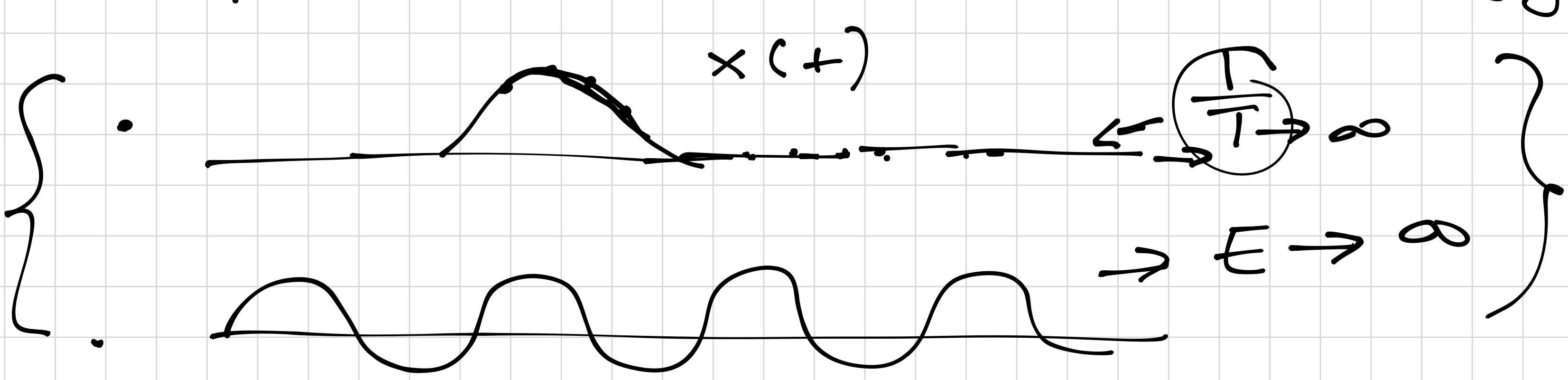
- A signal is called a POWER signal iff

$$\boxed{0 < P < \infty}$$

A signal CANNOT be both power and energy signals.

An energy signal $\rightarrow$ Zero power

A power " $\rightarrow$ Infinite energy.



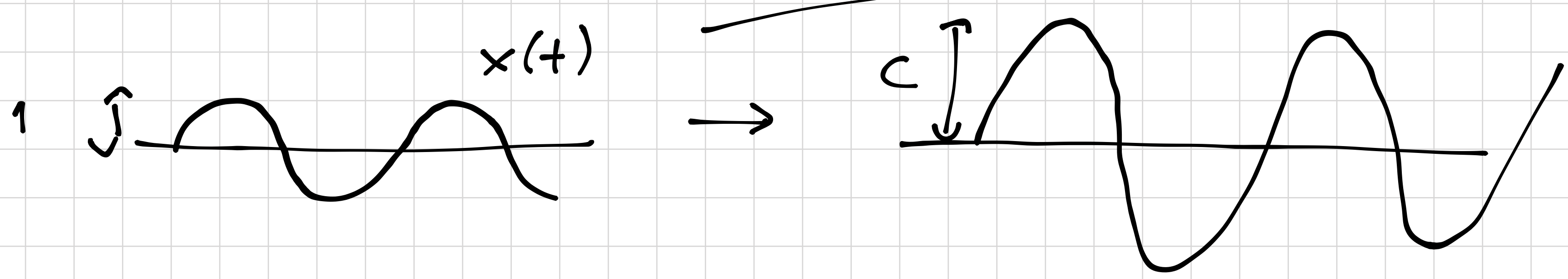
Basic Operations on Signals

$x(t)$ Independent variable

Dependent variable

→ Operations Performed on the Dependent Variable

- Amplitude scaling → $c \cdot x(t)$



c : Scaling factor

(Same for DT)

- Addition $x_1(t) + x_2(t)$

- Multiplication $x_1(t) \cdot x_2(t)$

- Differentiation → Only for CT signals

$$x(t) \rightarrow\rightarrow y(t) = \frac{d}{dt} x(t)$$

- Integration

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Operations Performed on the Independent Variable

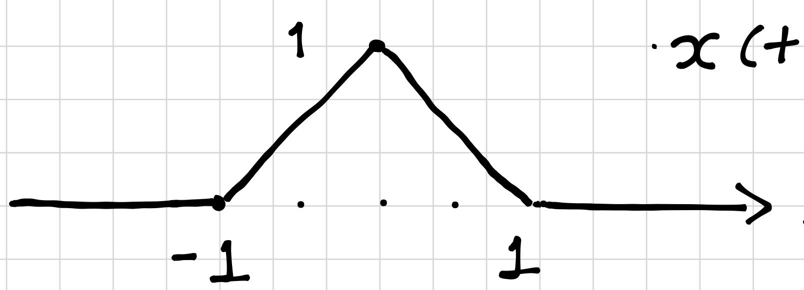
$x(t)$ → Independent variable

⊙ Time Scaling

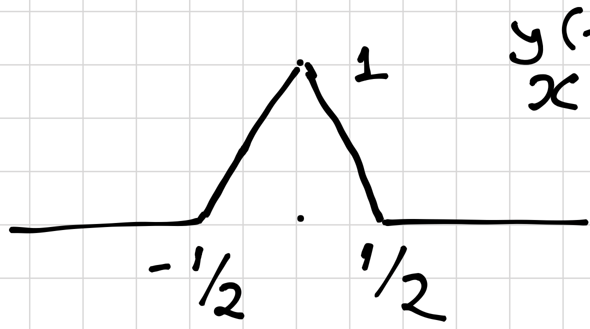
$$y(t) = x(\alpha t)$$

$\alpha > 1 \rightarrow$ $y(t)$ is compressed

$0 < \alpha < 1 \rightarrow$ " " expanded

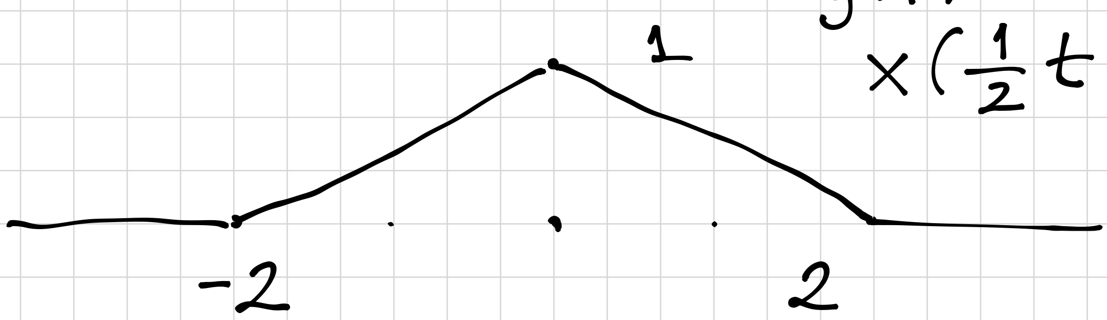


$x(t)$



$y(t) = x(2t)$

$y(-\frac{1}{2}) = x(-1)$
$y(\frac{1}{2}) = x(1)$
$y(0) = x(0)$

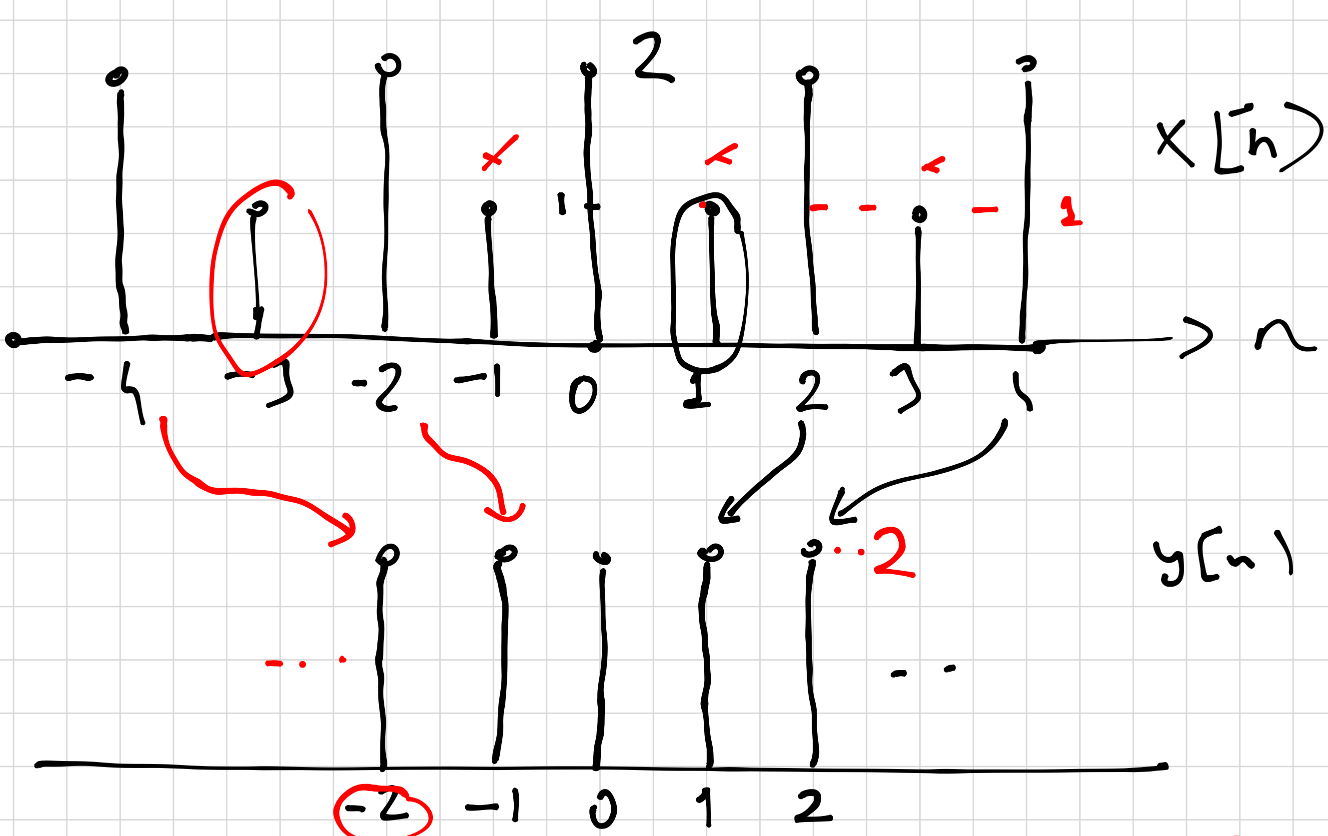


$y(t) = x(\frac{1}{2}t)$

$$y(-2) = x(\tfrac{1}{2} \cdot -2) = x(-1)$$
$$y(2) = x(\tfrac{1}{2} \cdot 2) = x(1)$$

For DT

$$y[n] = x[k \cdot n], \quad k > 0, \quad k \in \mathbb{Z}^+$$



$x[n]$

$y[n]$ $y[n] = x[2n]$

$y[0] = x[0]$
$y[1] = x[2]$
$y[2] = x[4]$

Ex

-3	-2	-1	0	1	2	3	4	5	
... 0 -4	-1	7	1	4	2	-2	0	3	0 ...

- $y[n] = x[2n]$

	-1	0	1		
... 0	-1	1	2	0	0 ...

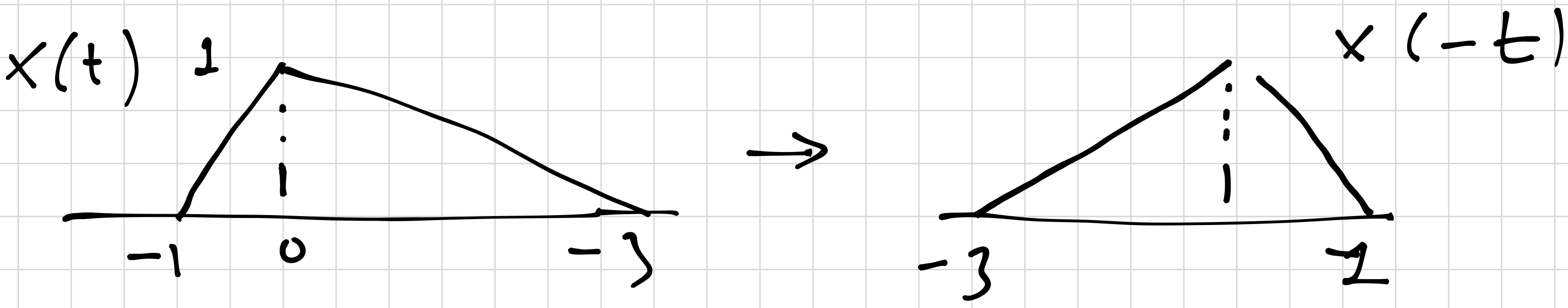
$y[n] = x[3n]$

	-1	0	1	
. 0 0 0	-4	1	-2	0 ...

Reflection

$$y(t) = x(-t)$$

$$y[n] = x[-n]$$

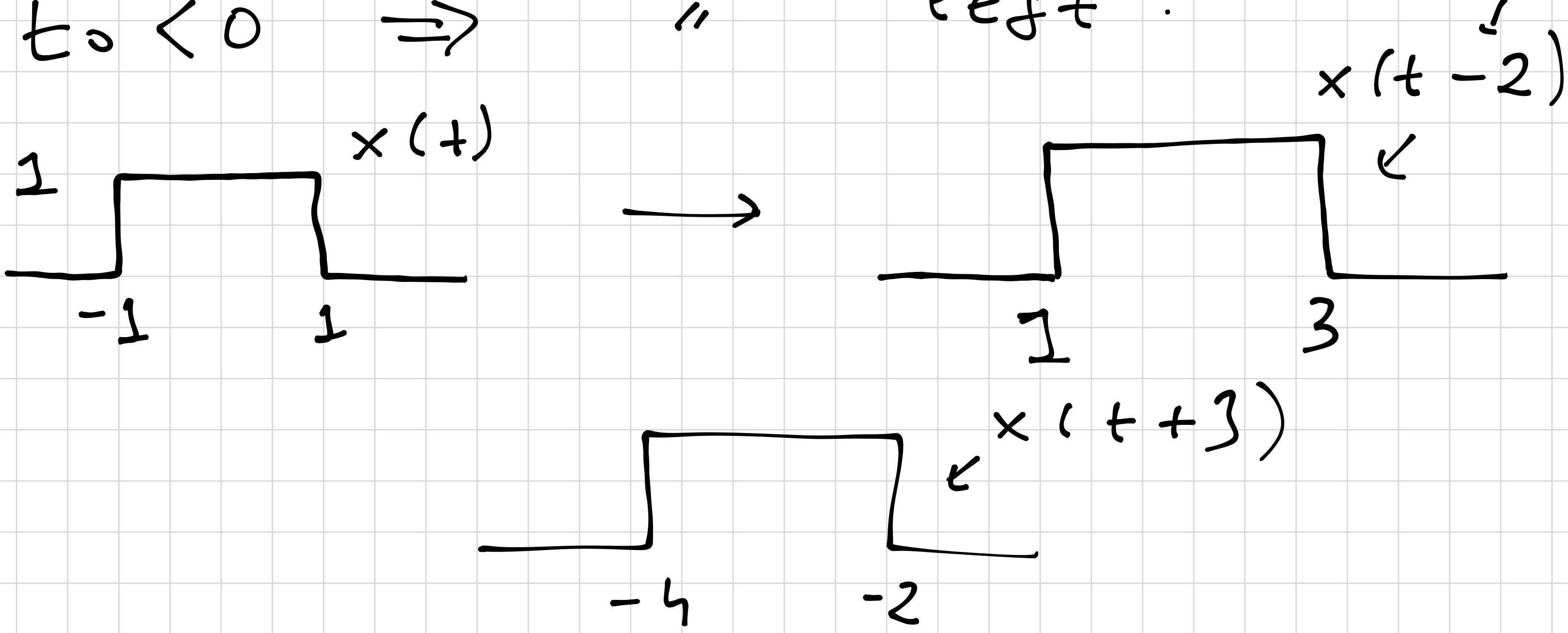


Time Shifting

$$y(t) = x(t - t_0), \quad t_0 \in \mathbb{R}$$

$t_0 > 0 \Rightarrow$ shift right

$t_0 < 0 \Rightarrow$ " left.



For DT.

$$y[n] = x[\underline{n-n_0}], \quad n_0 \in \mathbb{Z}$$

Precendence on Operations

$$y(t) = x(\underline{\underline{\alpha}}\, t - \underline{\underline{\beta}})$$

- $y(0) = x(-\beta)$
- $y\left(\frac{\beta}{\alpha}\right) = x(0)$

Shift

① $v(t) = x(t-\beta)$

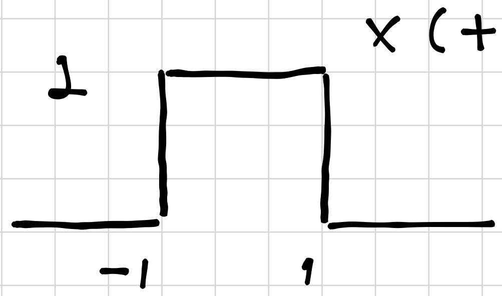
② $y(t) = v(\alpha t) = x(\alpha t - \beta)$

scale

$$y(t) = x(2t+3)$$

Ex

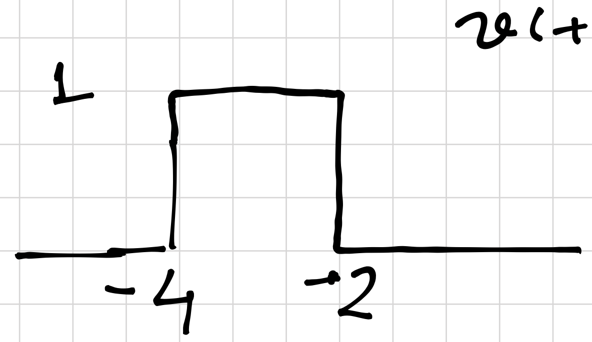
x(t)



Shift

$$v(t) = x(t+3)$$

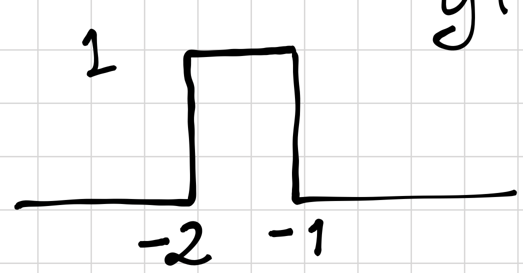
v(t)



Scale

$$y(t) = v(2t) = x(2t+3)$$

y(t)



$\alpha \longrightarrow$ Same for DT

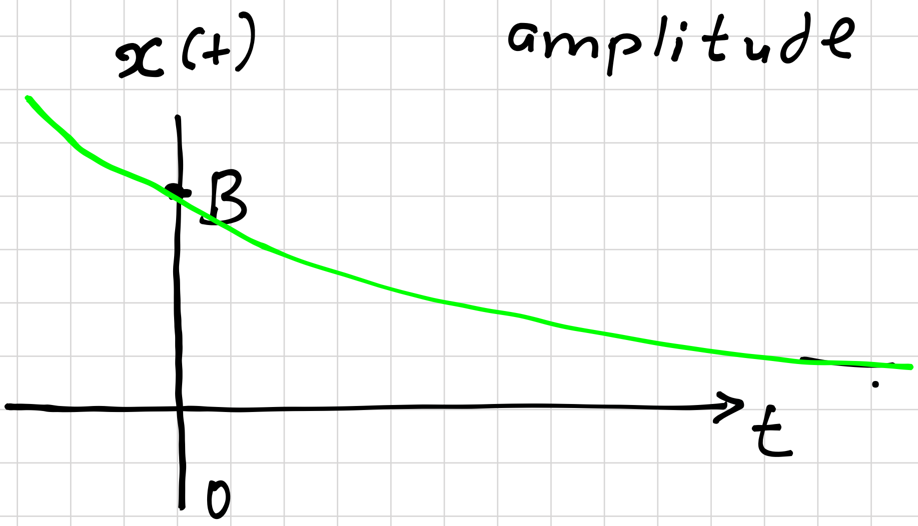
Elementary Signals

① Exponential Signals

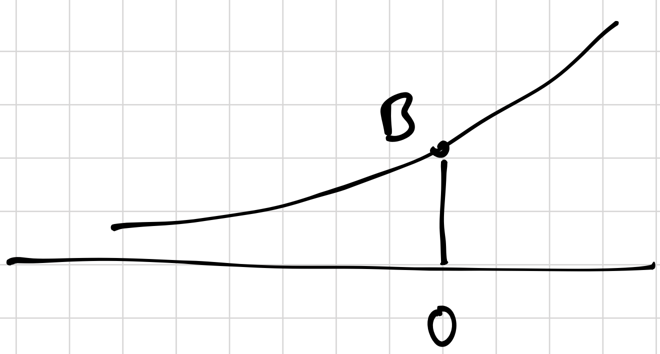
CT

$$x(t) = B \cdot e^{at} \qquad B, a \in \mathbb{R}$$

$\swarrow$ amplitude



If
$a < 0$
$\downarrow$
Decaying exponential



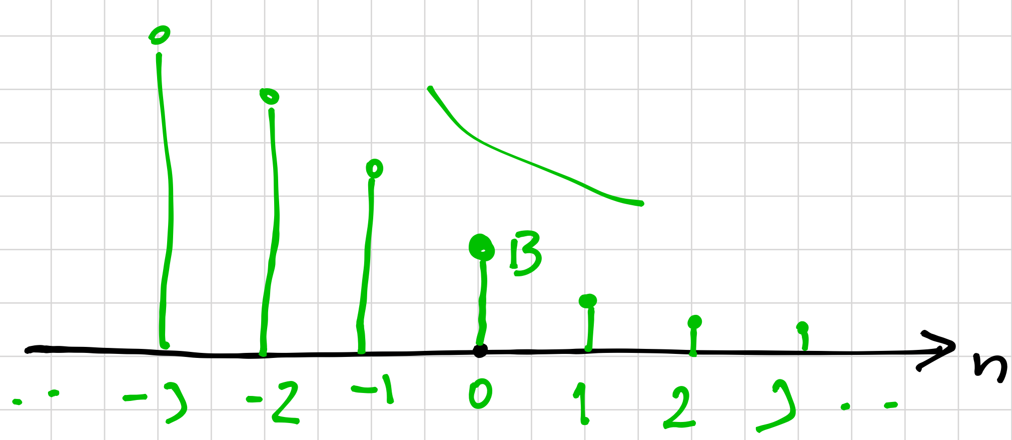
$a > 0$
Growing exponential.

DT

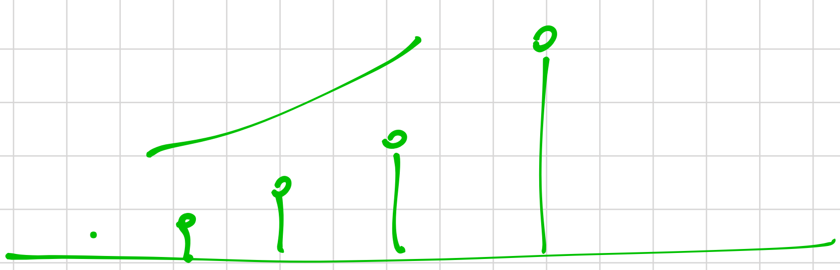
$$x[n] = B \cdot r^{n}$$

$0 < r < 1 \Rightarrow$ Decaying Exponential

$r > 1 \Rightarrow$ Growing exponential.

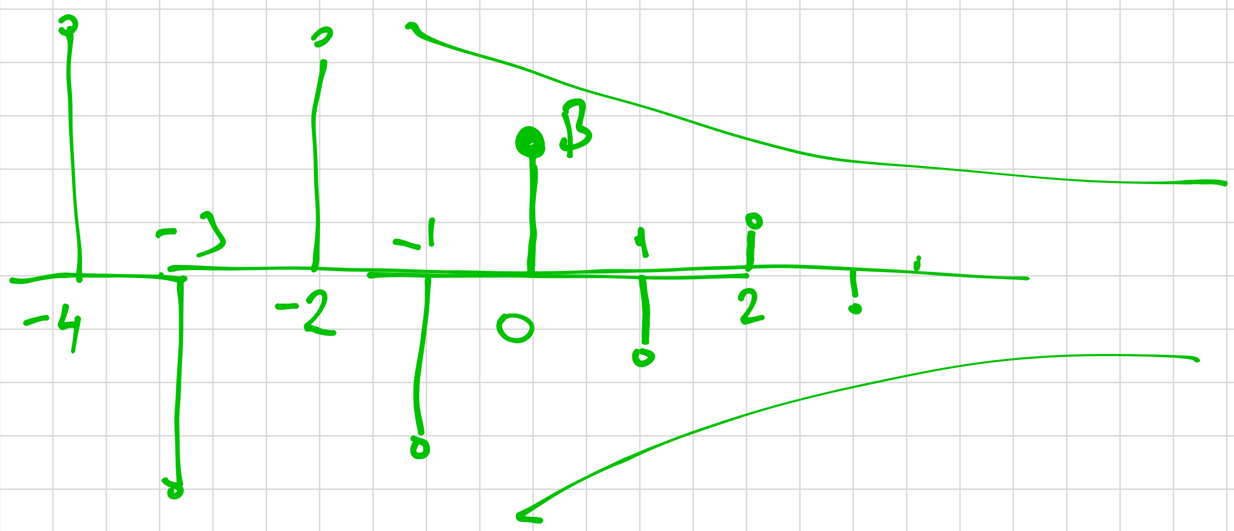


$0 < r < 1$



$r > 1$

$-1 < \beta < 0$



$r < -1$ ...

Sinusoidal Signal

CT

$$x(t) = A \cdot \underline{\cos}(\omega t + \phi)$$

amplitude (A), (angular) freq. (ω), phase angle (ϕ)

ω: angular freq.

$f = \frac{\omega}{2\pi}$: freq.

$T = \frac{2\pi}{\omega}$

(For CT) Always periodic.

DT

Not all DT sinusoidal signals are periodic.

$$x[n] = A \cdot \cos[\Omega n + \phi]$$

$\exists N \in \mathbb{Z}^+,\ x[n + \underline{N}] = x[n]$

$$x[n+N] = A \cdot \cos[\Omega n + \underline{\Omega N} + \phi]$$

/* $\cos(a) = \cos(a + 2\pi)$ ... */

$\Omega N = 2\pi m \qquad (m \in \mathbb{Z}^+)$

integers!

$\Omega = 2\pi \frac{m}{N}$ if we can find a (m, N) pair

$\Longrightarrow x[n]$ is periodic.

Ex

$$x[n] = \cos[0.4\pi n + 0.2]$$

Is $x[n]$ periodic?

$$\Omega = 0.4\pi = 2\pi \frac{m}{N} \Rightarrow \frac{m}{N} = 0.2 = \frac{1}{5}$$

$N = 5 \Rightarrow$ Periodic !

Ex $\quad x[n] = \sin[3n + 5]$

$$\Omega = 3 = 2\pi \cdot \frac{m}{N}$$

No integer pairs (m, N) exists!

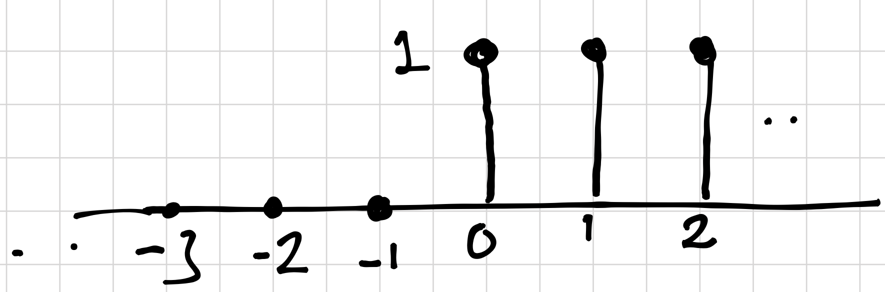
$\therefore x[n]$ is non-periodic !

$\boxed{\text{Step Function}}$

DT

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

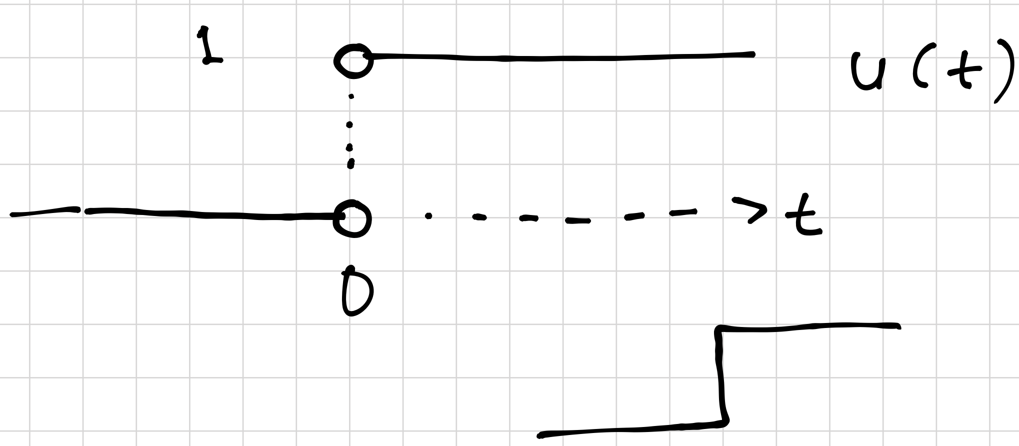
$\Rightarrow$ (unit) step function



$u[n]$

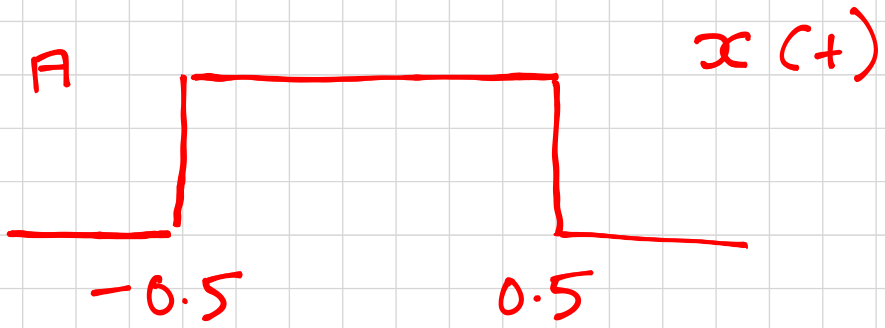
CT

$$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$$

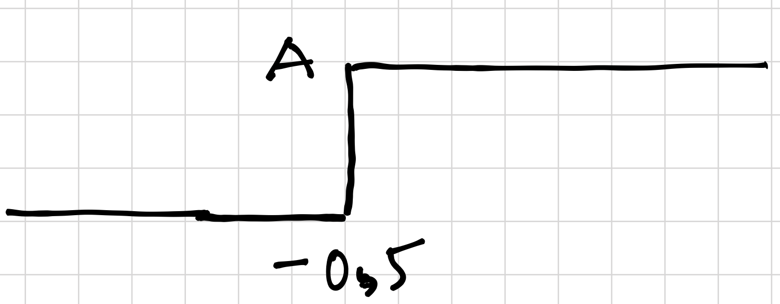


Unit step function can be used to construct other forms of discontinuous signals

Ex



Express $x(t)$ as a weighted sum of (two) step functions



$$x_1(t) = A \cdot u(t+0.5)$$

$$x_2(t) = A \cdot u(t-0.5)$$

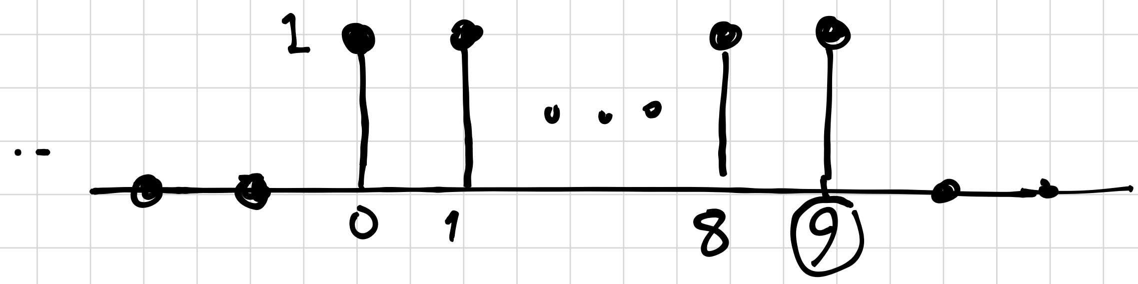
$$= x(t)$$

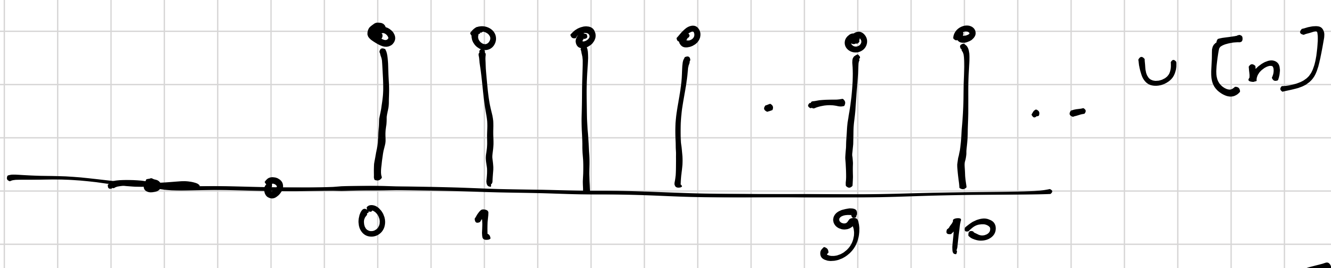
$$x(t) = x_1(t) - x_2(t)$$

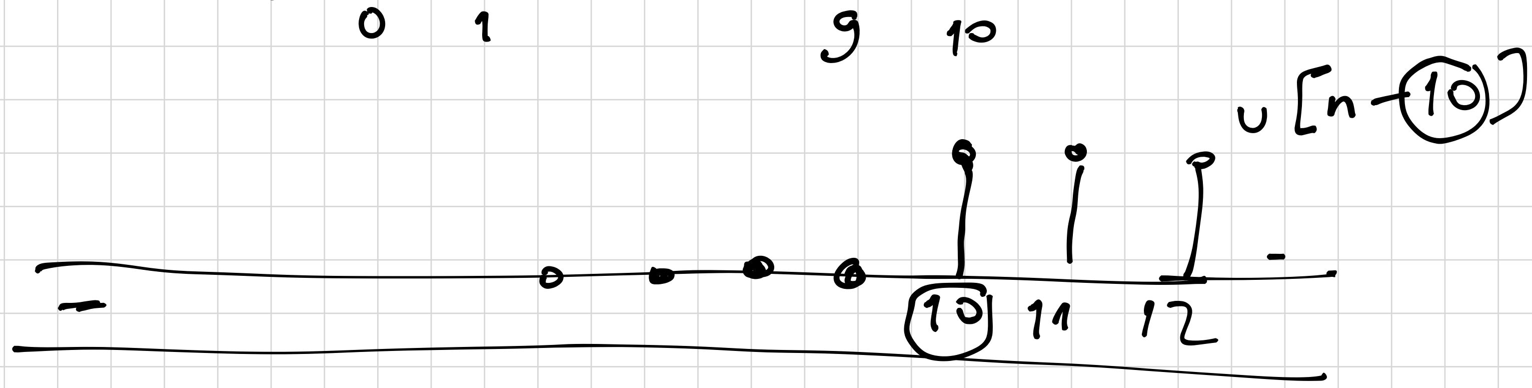
$$x_1(t) = A \cdot \left(u[t+0.5] - u[t-0.5]\right)$$

Ex

$$x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{otherwise} \end{cases}$$







$$x[n] = u[n] - u[n-10]$$