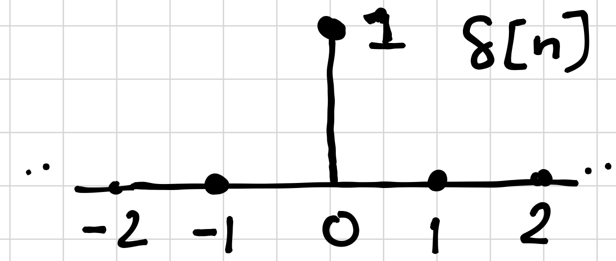
Impulse Function (Dirac-Delta Function)

DT Unit impulse

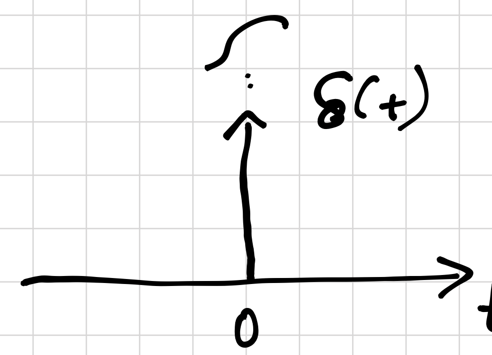
$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & \text{otherwise} \end{cases}$$

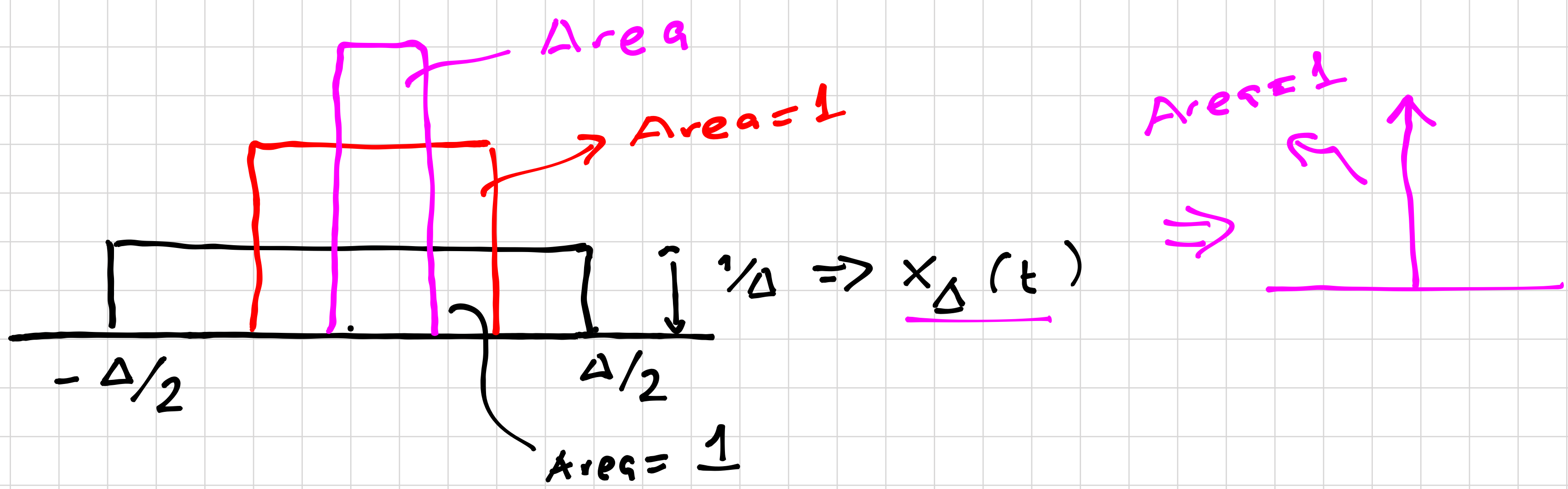


CT

CT unit impulse function has the following properties

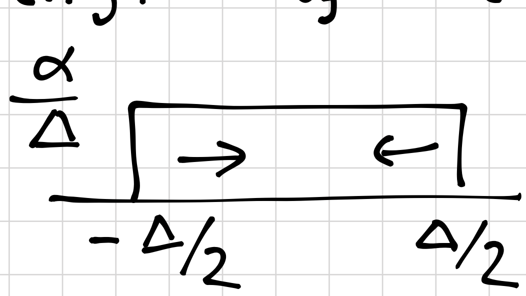
1. $\delta(t) = 0 \quad , \quad t \neq 0$
2. $\displaystyle\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$





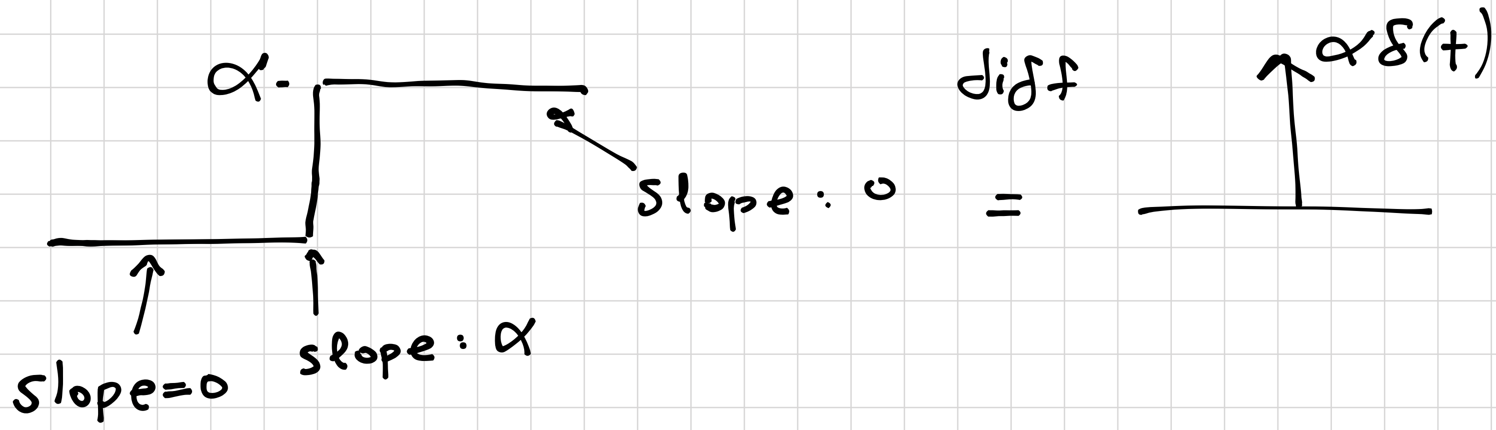
$$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$$

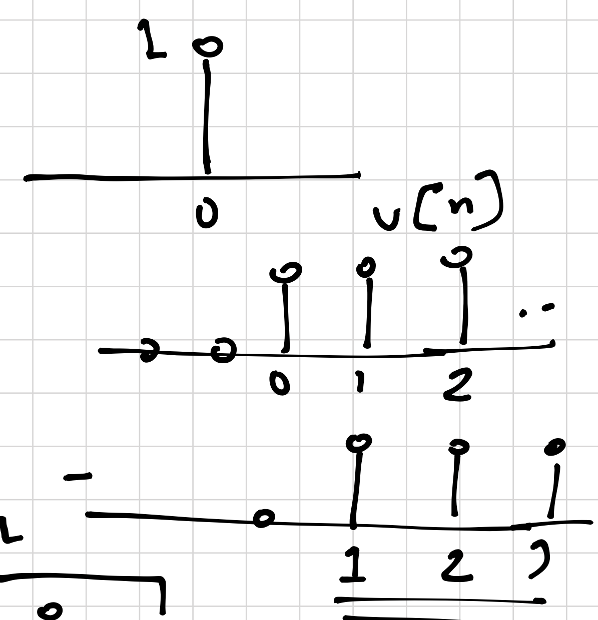
The area under the pulse is called the strength of the impulse



$\Rightarrow \alpha \cdot \delta(t)$

- $$\boxed{\delta(t) = \frac{d}{dt} u(t)}$$
- $$\boxed{u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau}$$

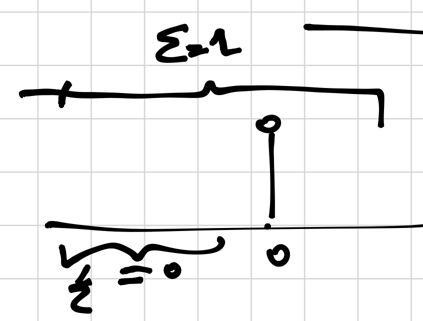




* Similarly for DT

- $\delta[n] = u[n] - u[n-1]$

- $u[n] = \sum_{k=-\infty}^{n} \delta[k]$

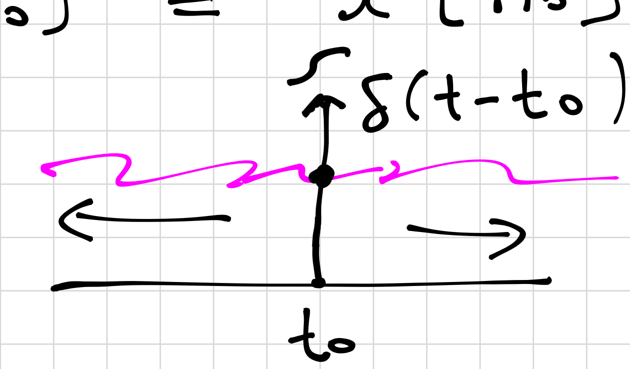


- Impulse function is an EVEN function.

$$\delta(t) = \delta(-t)$$
$$\delta[n] = \delta[-n]$$

- $$\int_{-\infty}^{+\infty} x(t)\, \delta(t-t_0)\, dt = \underline{x(t_0)}$$

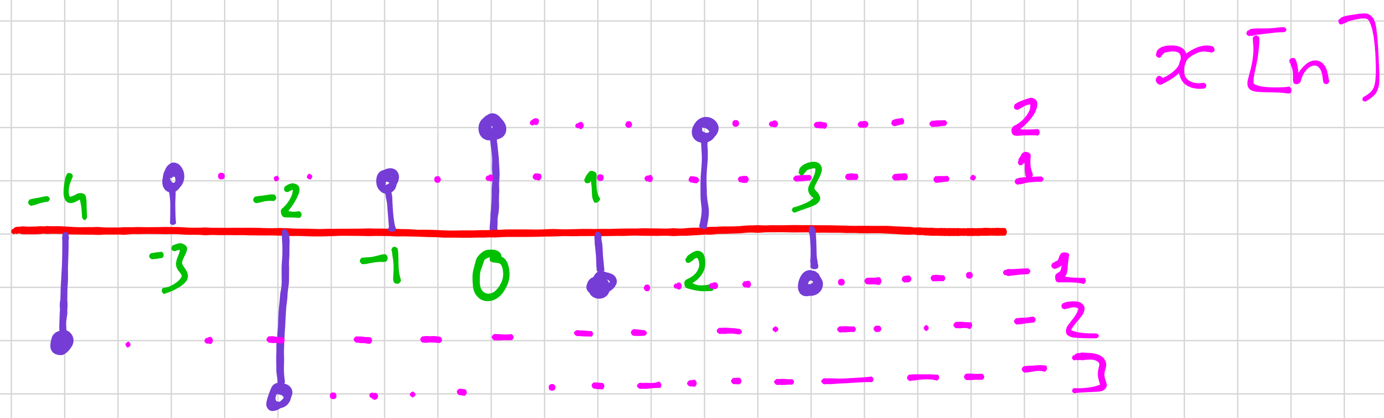
- $$\sum_{n=-\infty}^{+\infty} x[n]\, \delta[n-n_0] = x[n_0]$$



- $\delta(\alpha t) = \frac{1}{\alpha}\delta(t)$
  $[\alpha > 0]$

at t_0 $x(t) \to x(t_0)$

Ex



Sketch $\underbrace{x[2n+2]}_{x_1[n]} + \underbrace{x[3n-3]}_{x_2[n]} = y(n)$

	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	
$x(n+2)$	-2	+1	-3	1	2	-1	2	-1						
$x[2n+2]$				-2	-3	2	2							$x_1(n)$
$x[n-3]$						-2	1	-3	1	2	-1	2	-1	
$x[3n-3]$							1	2	-1					$x_2[n)$

$y[n]$ →

	-3	-2	-1	0	1	2
$y(n)$	-2	-3	2	3	2	-1

(b) is $y[n)$
- Power or energy signal ?

Let's check its energy,

$$E = \sum_{n=-\infty}^{+\infty} y^2[n] = (-2)^2 + (-3)^2 + 2^2 + 3^2 + 2^2 + (-1)^2$$
$$= 4 + 9 + 4 + 9 + 4 + 1$$
$$= \underline{31}$$

: ASSIGNMENT :

Find "problems.pdf" on Yandex "handouts" directory & Solve them !!!

Ex

Given two periodic CT signals, $x_1(t)$ and $x_2(t)$ with periods T_1 and T_2, respectively.

Under what condition $x_1(t) + x_2(t) = x(t)$ is periodic and if so what is the period?

Sol

$$x_1(t) = x_1(t + mT_1), \quad m \in \mathbb{Z}^+$$

$$x_2(t) = x_2(t + kT_2), \quad k \in \mathbb{Z}^+$$

For some T, if $x(t)$ is periodic

$$x(t+T) = x_1(t+T) + x_2(t+T)$$

should be equal to

$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = x_1(t+T) = x_1(t + mT_1)$$

$$x_2(t) = x_2(t+T) = x_2(t + kT_2)$$

$$T = m \cdot T_1 = k T_2$$

$$\frac{k}{m} = \frac{T_1}{T_2} \quad \therefore \quad \frac{k}{m} \text{ must be a RATIONAL number}$$

i) If k/m is not rational then $x(t)$ is not periodic.

ii) If k/m is rational then ~~then~~ the fundamental period of $x(t)$ is

$$T = \underset{(\text{okek})}{\text{LCM}}(T_1, T_2)$$

or $T = mT_1 = kT_2$ if m and k are relatively prime.

Ex

The period of $x(t) + c$, $c \in \mathbb{R}$ is the same as the period of $x(t)$!

Ex Give two DT signals, $x_1[n]$ and $x_2[n]$, with periods N_1 and N_2, respectively, under what condition $x[n] = x_1[n] + x_2[n]$ is periodic and if so, what is the period?

$$x_1[n] = x_1[n + mN_1], \quad m \in \mathbb{Z}^+$$

$$x_2[n] = x_2[n + kN_2], \quad k \in \mathbb{Z}^+$$

So if $x[n]$ is periodic, for some $N \in \mathbb{Z}^+$

$$x[n] = x[n+N] = x_1[n+N] + x_2[n+N]$$

$$= x_1[n + mN_1] + x_2[n + kN_2]$$

$\therefore$ $N = m \cdot N_1 = k \cdot N_2$ This equation will always be satisfied.

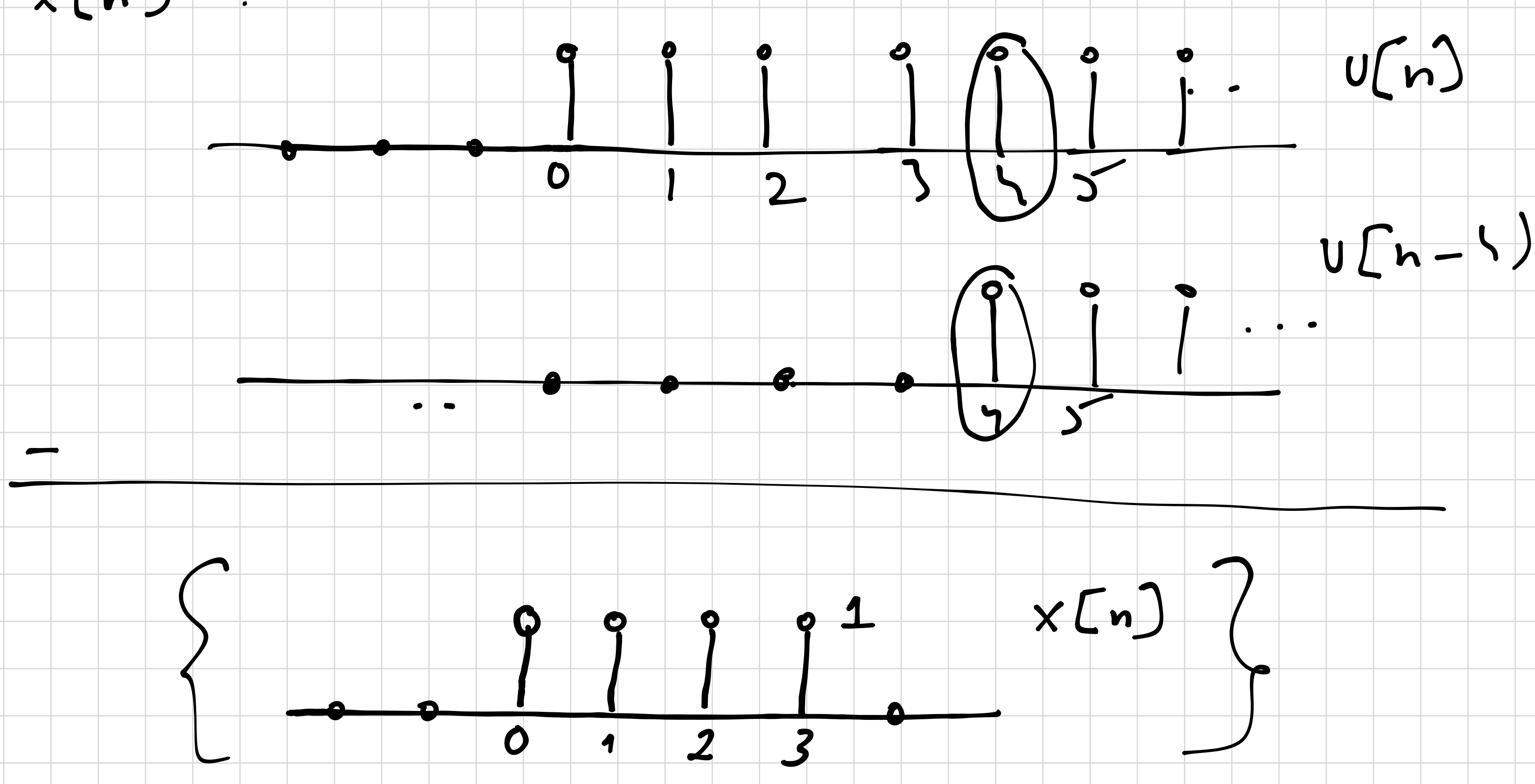
$\therefore$ $x[n]$ is periodic!

$N = \mathrm{LCM}[N_1, N_2]$!

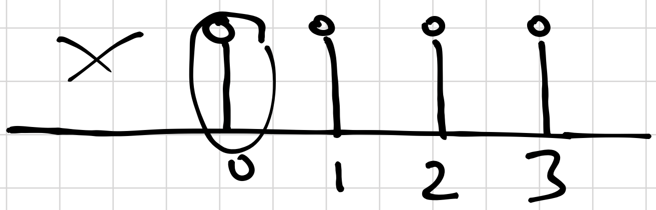
Ex

$$x[n] = u[n] - u[n-4]$$

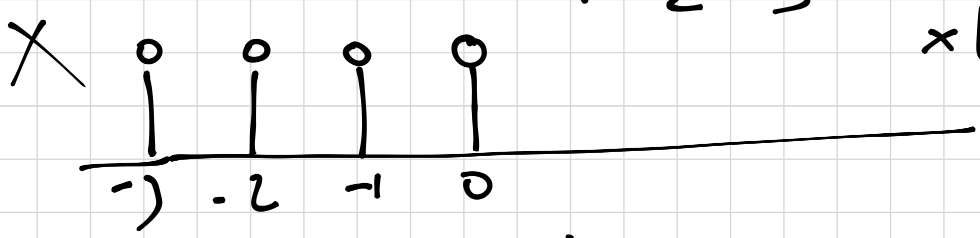
Find and sketch the even and odd components of $x[n]$.



$$xe[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$$

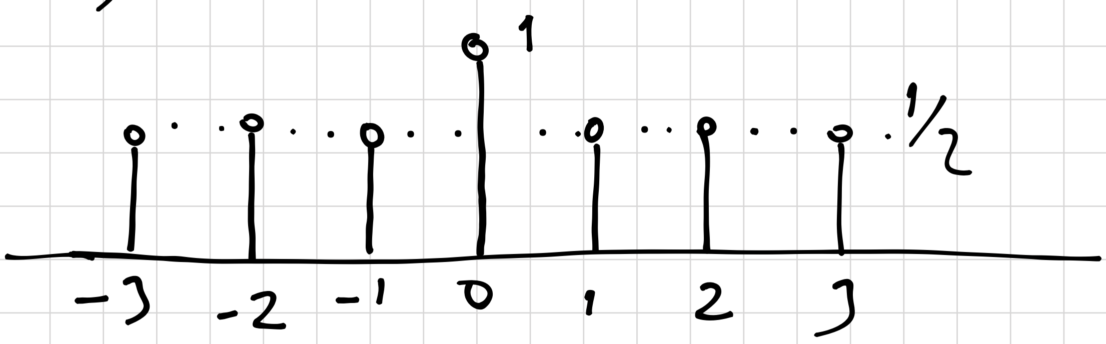


$x[n]$



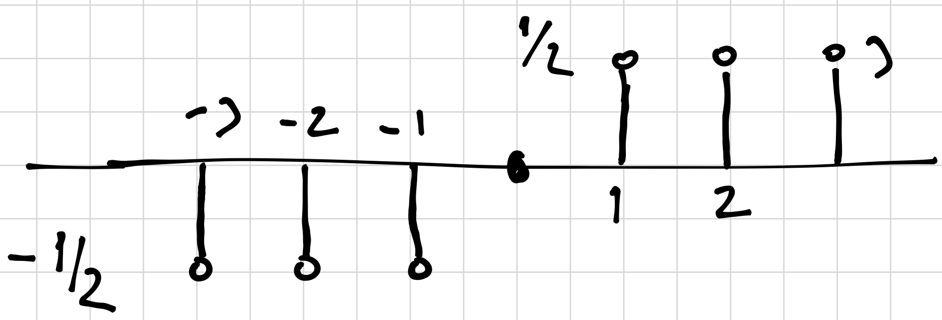
$x[-n]$

$x(n) = x_o(n) + xe(n)$



$xe[n]$

$$x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$$

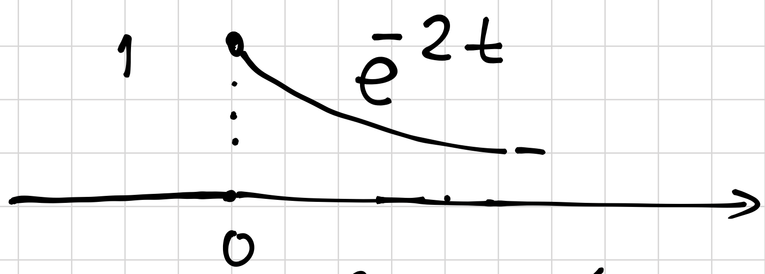


$x_o[n]$

Ex

$$x(t) = e^{-2t} \cdot u(t)$$

is this a power or energy signal?



$$E = \int_0^{\infty} \left(e^{-2t}\right)^2 \cdot dt = \int_0^{\infty} e^{-4t}\, dt$$

$$= \left(-\frac{1}{4}\right) e^{-4t}\Big|_0^{\infty} = \left(-\frac{1}{4}\right)(0 - 1)$$

$$= \frac{1}{4}$$

$0 < E < \infty$ $\therefore$ It is an energy signal.

Ex

$$x(t) = \cos^2\left(\frac{2}{3}\pi t\right)$$

/* $\cos^2\alpha = \frac{1}{2}\left[1+\cos 2\alpha\right]$ */

Is $x(t)$ periodic?

$$x(t) = \frac{1}{2}\left\{\cos\left(\underbrace{\frac{4}{3}\pi t}_{\omega t}\right) + 1\right\}$$

$$\omega = \frac{4}{3}\pi \qquad f = \frac{\omega}{2\pi} = \frac{2}{3}\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{3}{2} = 1.5 \ \text{seconds.}$$

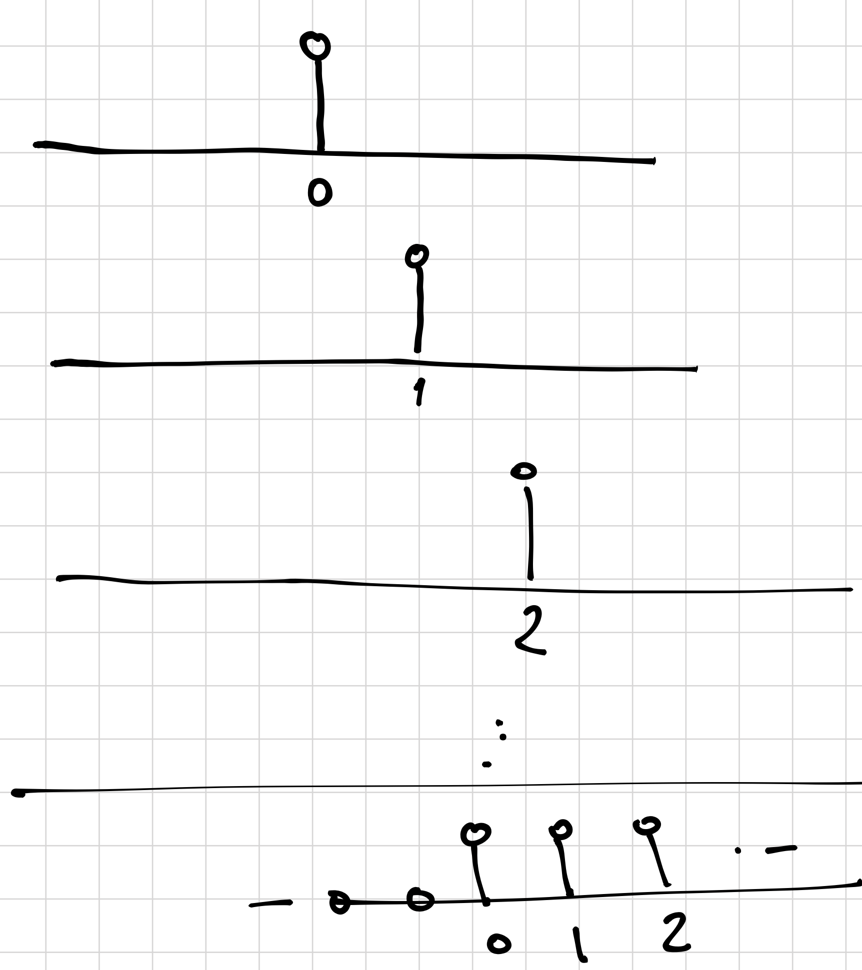
Even ? odd ?

$x(t) = x(-t)$ $\therefore$ even

?

Ex

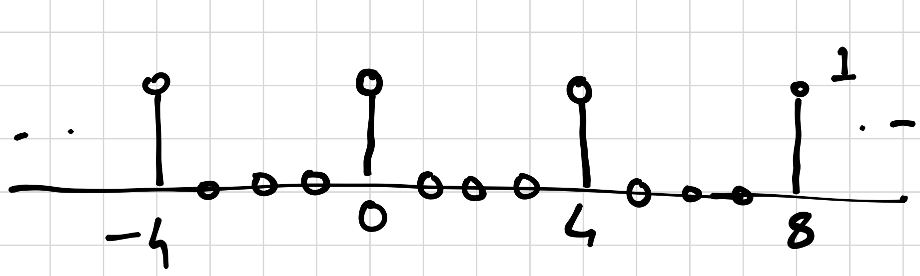
$$\sum_{k=0}^{\infty} \delta[n-k] = \ ..$$



$= u[n]$

Ex

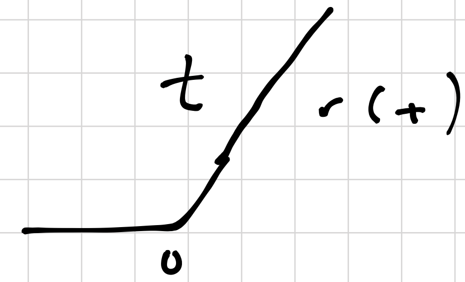
$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k]$$



Ramp Function

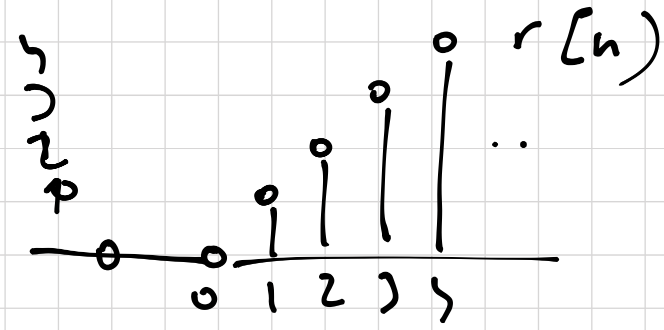
CT

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



DT

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



Systems

$$\underset{\text{input}}{x(t)} \longrightarrow \boxed{\text{CT-System}} \longrightarrow \underset{\text{output}}{y(t)}$$

$$y(t) = \mathcal{H}\{x(t)\}$$

$$x(t) \xrightarrow{\mathcal{H}} y(t)$$

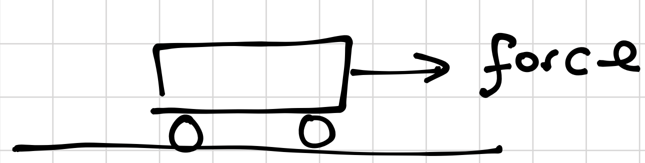
$$x[n] \longrightarrow \boxed{\text{DT-System}} \longrightarrow y[n]$$

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

$$x[n] \longrightarrow \boxed{\mathcal{H}} \longrightarrow y[n]$$

We use the operator $\mathcal{H}\{\cdot\}$ to denote the action of a system.

Ex



v: velocity

$\rho . v$: frictional force

m: mass

$$\frac{dv(t)}{dt} = \frac{1}{m} \cdot \left[f(t) - \rho\, v(t)\right]$$

$$f(t) \longrightarrow \boxed{\mathcal{H}} \longrightarrow v(t) = \mathcal{H}\{f(t)\}$$

Interconnection of Systems

We can view the systems as interconnections of operations. We can represent the systems with block diagrams.

Ex Moving average system

Consider a DT-system

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

Show a block diagram representation of this system.

$$x[n] \rightarrow \boxed{H} \rightarrow y[n]$$

Let the operator S^k denote the following

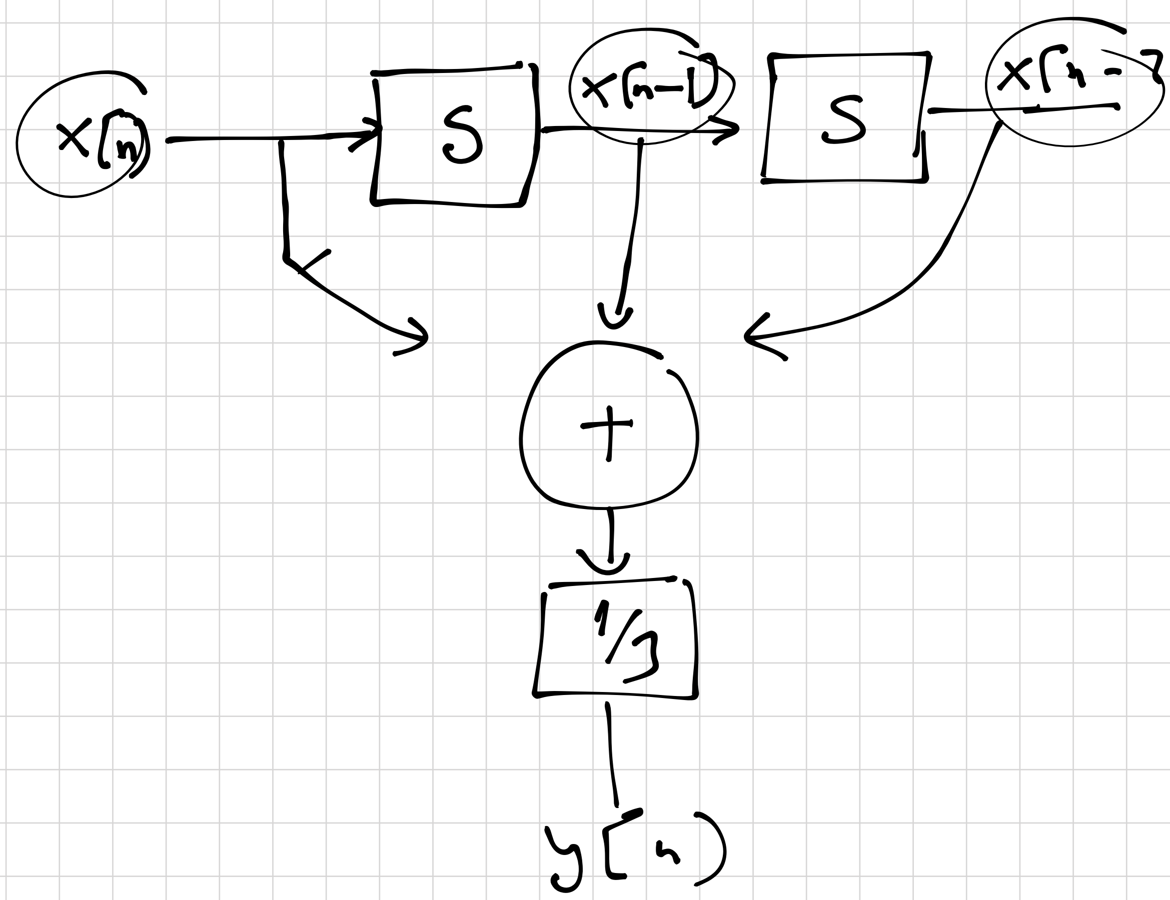
$$x[n] \rightarrow \boxed{S^k} \rightarrow x[n-k]$$

Then, the overall system would be

$$\mathcal{H} = \frac{1}{3}\left[S^0 + S^1 + S^2\right]$$

$$= \frac{1}{3}\left[1 + S + S^2\right]$$

Cascade form



Parallel form