NEURAL NETWORKS

Bidirectional Associative Memory Neural Networks

Lecture 9

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BIDIRECTIONAL ASSOCIATIVE MEMORY(BAM) NEURAL NETWORKS

- BAM Neural Networks were first introduced by Bart Kosko (1987, 1988).
- BAM Neural Networks are recurrent network designed to work as heteroassociative memory.

- A BAM Neural Network is composed of neurons arranged in two layers.

- The neurons in one layer are fully interconnected to the neurons in the second layer.

- There is no interconnection among neurons in the same layer.

- The weight from layer A to layer B is same as the weights from layer B to layer A.

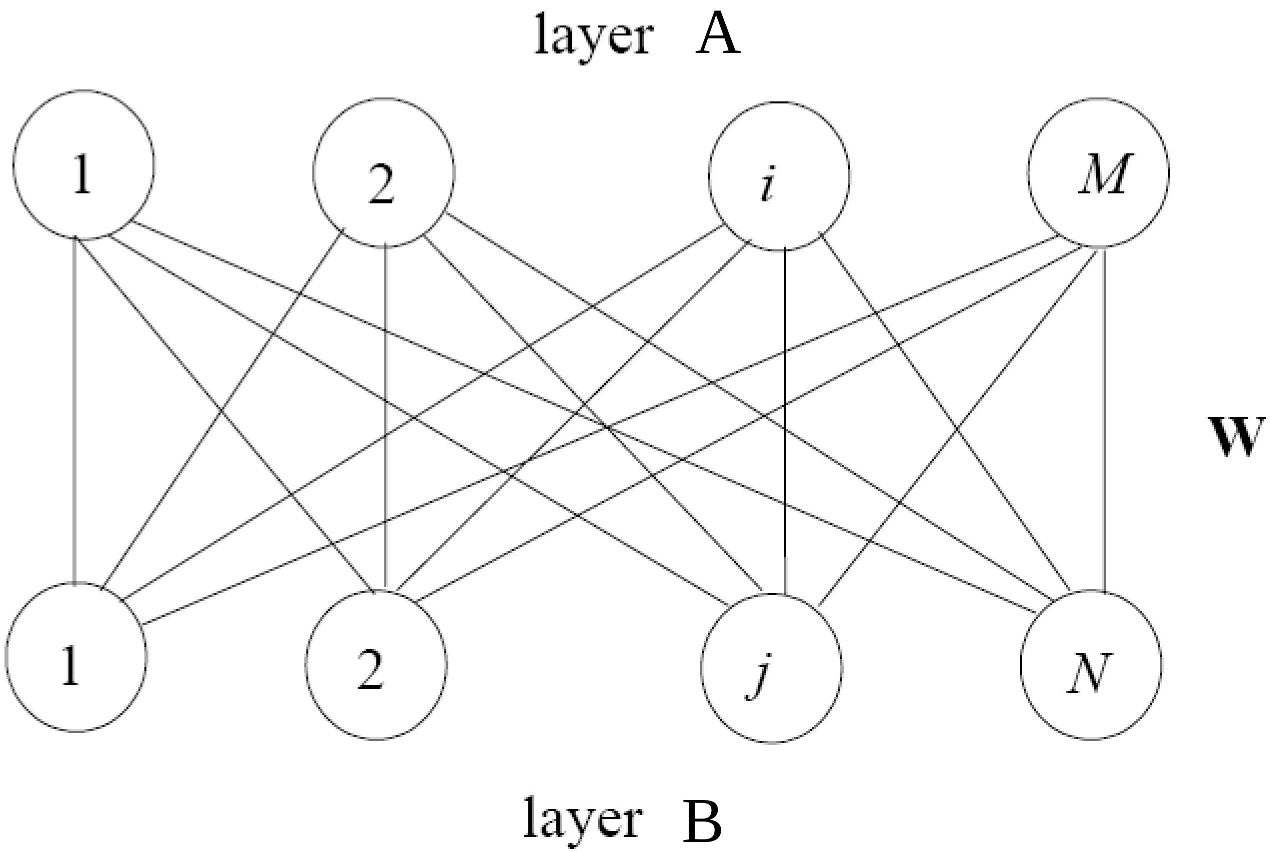


Figure 1. *Bidirectional Associative Memory: Basic Simple Diagram*

- It is a heteroassociative, content-addressable memory consisting of two layers.
- It uses the forward and backward information flow to produce an associative search for stored stimulus-response association.
- The network's dynamics involves two layers of interaction.
- The stability corresponds to a local energy minimum.
- The basic architectural diagram of the Bidirectional associative memory is shown in Figure 2.
- The patterns are

$$\{(a^{(1)}, b^{(1)}), (a^{(2)}, b^{(2)}), \ldots,(a^{(p)}, b^{(p)})\}$$

- Let us assume that an initializing vector **b** is applied at the input to the layer A of neurons.

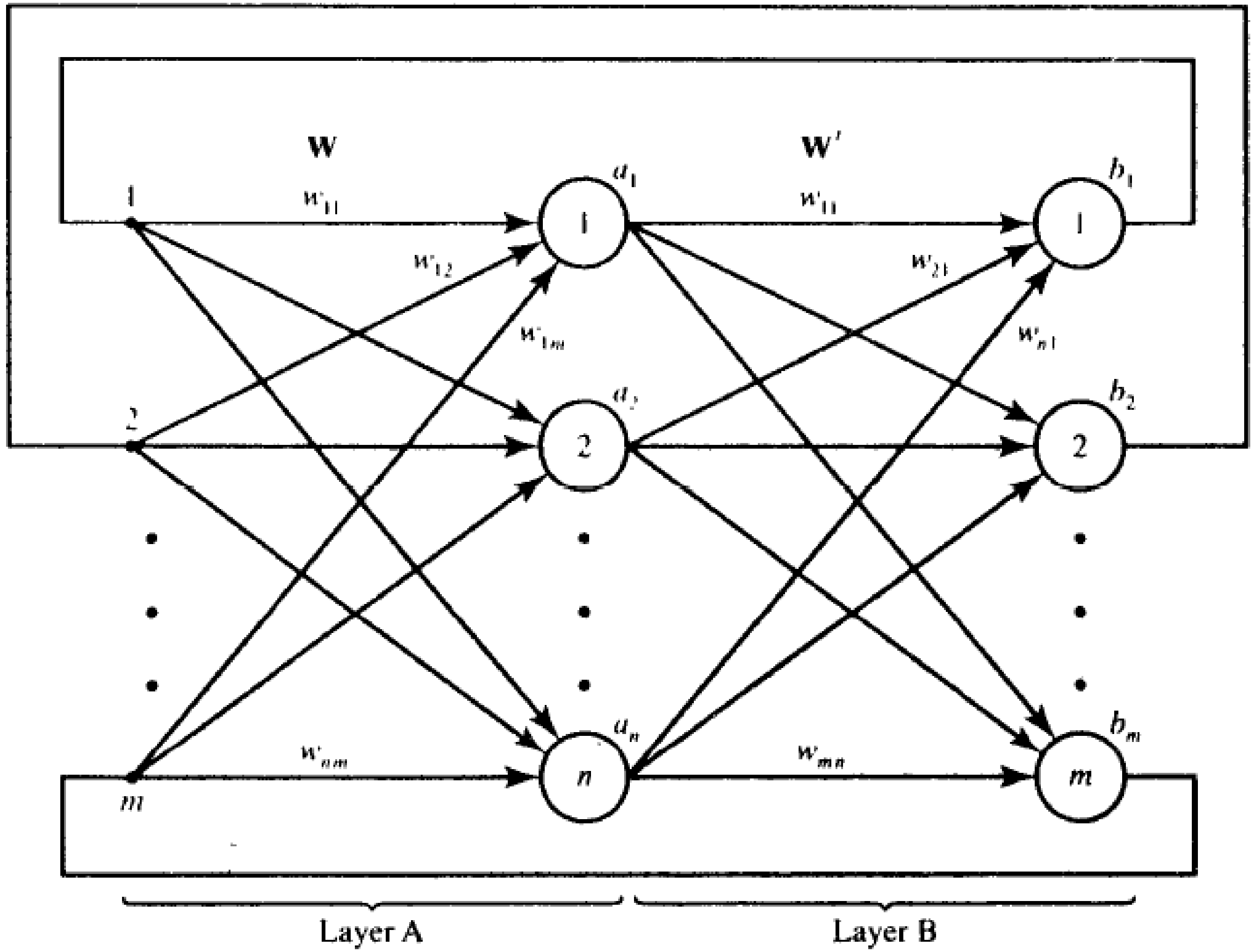


Figure 2. *Bidirectional Associative Memory: General Arcitectural Graph*

- Figure 3. shows the simplified diagram of the Bidirectional associative memory often encountered in the literature.
- Layer A and B operate in an alternate fashion-first transferring the neuron's output signals towards the right by using matrix **W**, and then toward the left by using the matrix $\mathbf{W}^T$, respectively.

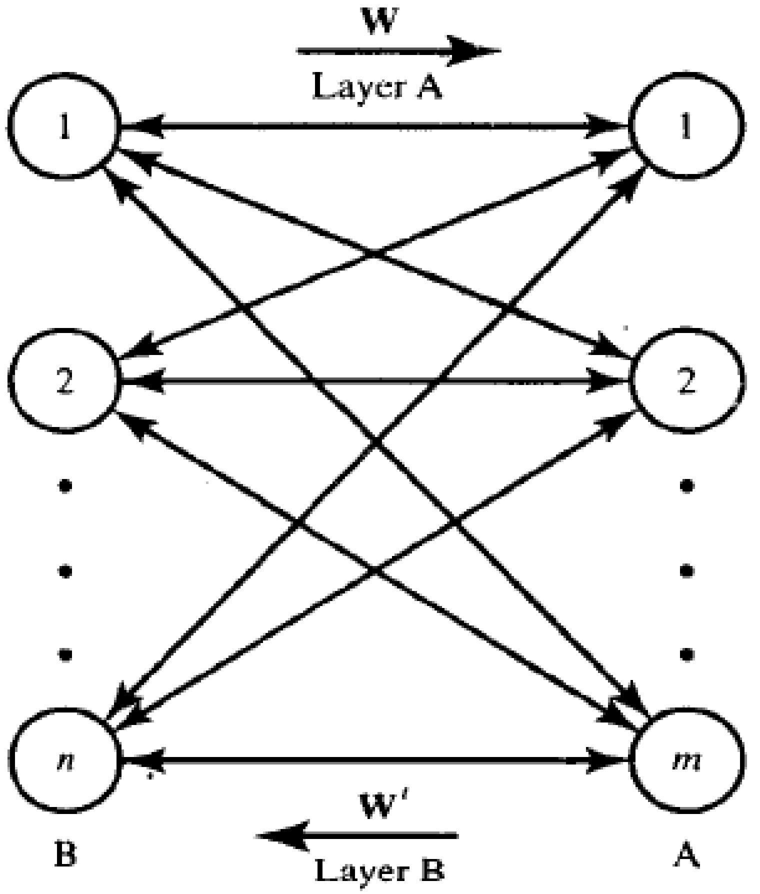


Figure 3. *Bidirectional Associative Memory: Simplified Arcitectural Graph*

- The Hopfield unidirectional auto-associators have been discussed in previous lecture.

- Kosko extended this network to two layer, bidirectional structure which can achieve hetero-association.

Definition : If the associated pattern pairs (X,Y) are different and if the model recalls a pattern Y given a pattern X or vice-versa, then it is termed as hetero-associate memory.

- The Bidirectional associative memory maps bipolar binary vectors

  $a^k = [a_1\ a_2\ \ldots\ a_n]^t,\ \ a_i = \pm 1\ ,\ i = 1,\ 2, \ldots,\ n,$

  into vectors

  $b^k = [b_1\ b_2\ \ldots b_m]^t,\ \ b_i = \pm 1,\ i = 1,\ 2, \ldots,\ m,$

  or vise-versa.

- Where $\boldsymbol{a}^{k}$ and $\boldsymbol{b}^{k}$ are bipolar binary vectors, which are members of the *k*'th pair.

How to Use a BAM?

Phase 1 : *Storage Process*

Store all pattern pairs into the network by finding the weight matrix.

Phase 2 : *Retrieval Process*

Recall a pattern when an input is given as the initial state.

Storage Process

Step 1: The associations between pattern pairs are stored in the memory in the form of bipolar binary vectors with entries -1 and 1.

$$\{(a^{(1)}, b^{(1)}), (a^{(2)}, b^{(2)}), \ldots,(a^{(p)}, b^{(p)})\}$$

Vector a store pattern and is n-dimensional, **b** is m-dimensional which stores associated output.

Step 2: The Weight Matrix is calculated by

$$W = \sum_{i=1}^{p} a^{(i)} (b^{(i)})^T$$

Retrieval Process

Input: Pattern (often noisy/corrupted)

Output: Corresponding pattern (complete / relatively noise-free)

Process:

1. Load input pattern onto core group of highly-interconnected neurons.
2. Run core neurons until they reach a steady state.
3. Read output off of the states of the core neurons.

Retrieval Process

- Assume an initializing vector b is applied at the input (layer A). So,

First Forward Pass $a^1 = \Gamma[Wb^0]$

- Now vector a^1 is applied to Layer B. So,

First Backward Pass $b^2 = \Gamma[W^T a^1]$

Here,

$$a^{k+1} = \Gamma(Wb^k)$$

corresponds the equation:

$$a^{k+1} = sgn(Wb^k)$$

- So the procedure can be shown as

First Forward Pass	$a^1 = \Gamma[Wb^0]$
First Backward Pass	$b^2 = \Gamma[W^T a^1]$
Second Forward Pass	$a^3 = \Gamma[Wb^2]$
Second Backward Pass	$b^4 = \Gamma[W^T a^3]$
.	
.	
.	
k/2 th Backward Pass	$b^k = \Gamma[W^T a^{k-1}]$

- If the network is stable the procedure stop at an equilibrium pair like $(a^{(i)}, b^{(i)})$.

- This means that the updates continue and the memory comes to its equilibrium if

$$a^{k+2} = a^k$$

for the updates

$$(a^k \rightarrow b^{k+1} \rightarrow a^{k+2})$$

- This corresponds to the energy function reaching one of its minima.

Summary

Retrieve the nearest of (A_i , B_i) pattern pair, given any pair (α , β) .

The methods and the equations for retrieve are :

– start with an initial condition which is any given pattern pair (α , β),

– determine a finite sequence of pattern pairs (α' , β') , (α'' , β'') . . . .

until an equilibrium point (α_f , β_f) is reached, where

$$\beta' = \Phi(\alpha M) \quad \text{and} \quad \alpha' = \Phi(\beta' M^T)$$

$$\beta'' = \Phi(\alpha' M) \quad \text{and} \quad \alpha'' = \Phi(\beta'' M^T)$$

$$\Phi(F) = G = g_1 , g_2 , \ldots , g_r ,$$

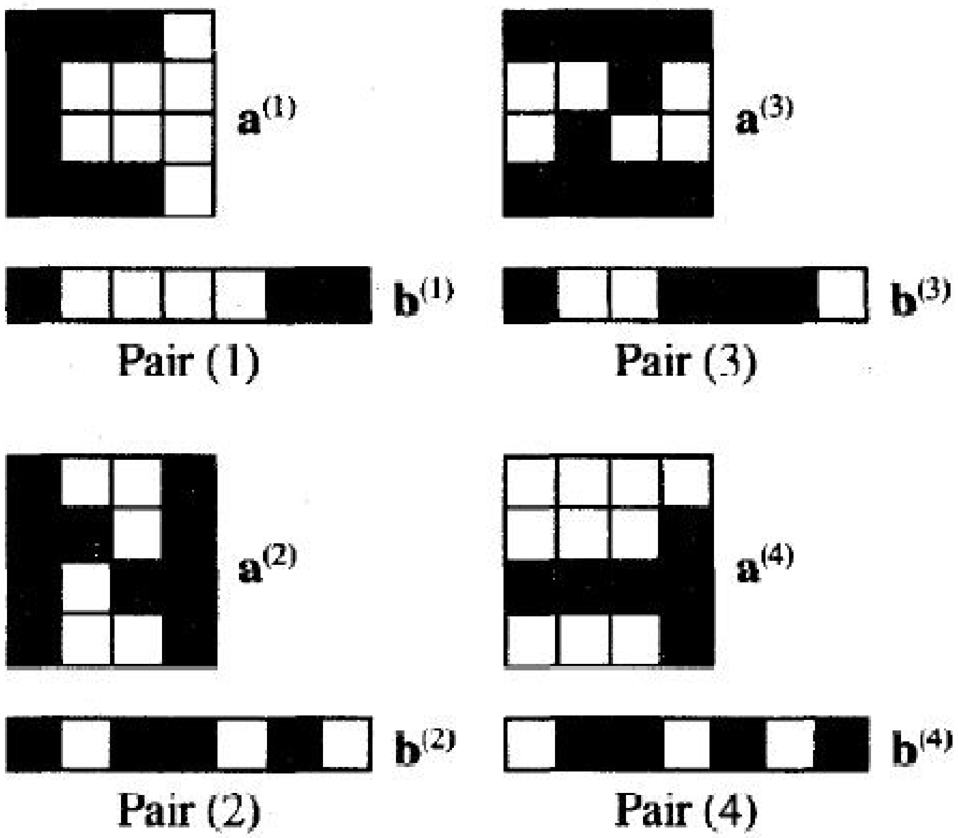
$$F = (f_1 , f_2 , \ldots , f_r)$$

M is correlation matrix

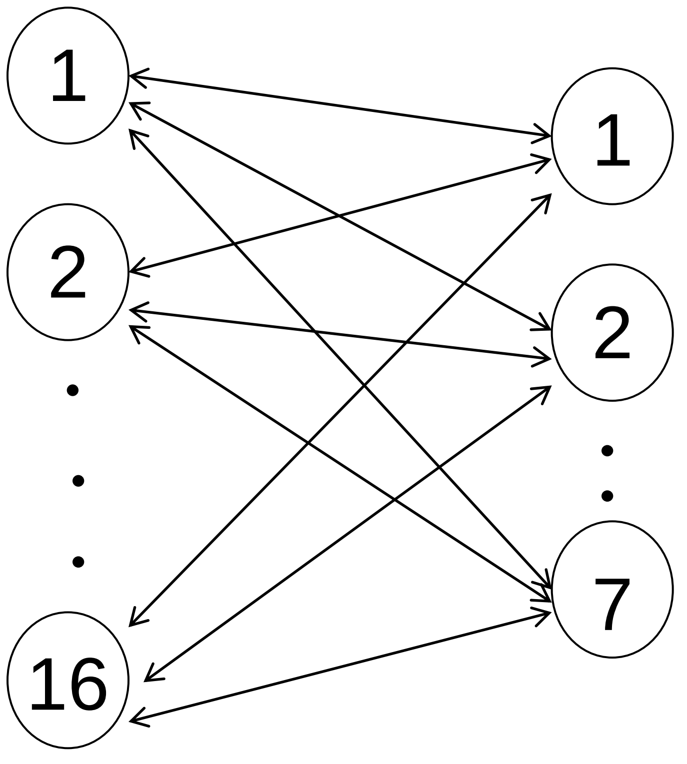
$$g_i = \begin{cases} 1 & \text{if } f_i > 0 \\ \begin{cases} 0 \text{ (binary)} \\ -1 \text{ (bipolar)} \end{cases} , & f_i < 0 \\ \text{previous } g_i , & f_i = 0 \end{cases}$$

➢ Example for Storage Phase

- Stored association pairs:



- a_i , $i=1,\ldots,n$. $n = 16$
- b_i , $i=1,\ldots,m$. $m = 7$



- Storing four pair of associations as vector pairs :

$$
\begin{aligned}
a^{(1)} &= [1\ 1\ 1\ -1\ 1\ -1\ -1\ -1\ 1\ -1\ -1\ -1\ 1\ 1\ 1\ -1]^T, \\
b^{(1)} &= [1\ -1\ -1\ -1\ -1\ 1\ 1]^T, \\
a^{(2)} &= [1\ -1\ -1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ 1\ 1\ 1\ -1\ -1\ 1]^T, \\
b^{(2)} &= [1\ -1\ -1\ 1\ 1\ 1\ -1]^T, \\
a^{(3)} &= [1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ 1]^T, \\
b^{(3)} &= [1\ -1\ 1\ 1\ -1\ 1\ -1]^T, \\
a^{(4)} &= [-1\ -1\ -1\ -1\ -1\ -1\ -1\ 1\ 1\ 1\ 1\ 1\ -1\ -1\ -1\ 1]^T \\
b^{(4)} &= [-1\ 1\ 1\ -1\ 1\ -1\ 1]^T,
\end{aligned}
$$

- $W = \sum_{i=1}^{p} a^{(i)} b^{(i)T}$

- Weight matrix is obtained as:

$$W = \begin{bmatrix} 4 & -4 & -2 & 2 & -2 & 4 & -2 \\ 2 & -2 & 0 & 0 & -4 & 2 & 0 \\ 2 & -2 & 0 & 0 & -4 & 2 & 0 \\ 2 & -2 & 0 & 4 & 0 & 2 & -4 \\ 2 & -2 & -4 & 0 & 0 & 2 & 0 \\ 0 & 0 & -2 & 2 & 2 & 0 & -2 \\ 0 & 0 & 2 & 2 & -2 & 0 & -2 \\ -2 & 2 & 0 & 0 & 4 & -2 & 0 \\ 0 & 0 & -2 & -2 & 2 & 0 & 2 \\ -2 & 2 & 4 & 0 & 0 & -2 & 0 \\ -2 & 2 & 0 & 0 & 4 & -2 & 0 \\ -2 & 2 & 0 & 0 & 4 & -2 & 0 \\ 4 & -4 & -2 & 2 & -2 & 4 & -2 \\ 2 & -2 & 0 & 0 & -4 & 2 & 0 \\ 2 & -2 & 0 & 0 & -4 & 2 & 0 \\ 0 & 0 & 2 & 2 & 2 & 0 & -2 \end{bmatrix}_{16\times 7}$$

Example for Retrieval Phase

- Assume a key vector a^1 at the memory input is a distorted prototype of $a^{(2)}$, $HD(a^{(2)}, a^1) = 4$:

$$a^1 = [-1\ -1\ -1\ 1\ -1\ 1\ -1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ 1\ 1]^T,$$

- Therefore:

$$\begin{aligned}
b^2 &= \Gamma[-16\ 16\ 0\ 0\ 32\ -16\ 0]^T = [-1\ 1\ 0\ 0\ 1\ -1\ 0]^T, \\
a^3 &= [-1\ -1\ -1\ -1\ -1\ 1\ -1\ 1\ 1\ 1\ 1\ 1\ 1\ -1\ -1\ 1]^T, \\
b^4 &= [-1\ 1\ 1\ -1\ 1\ -1\ 1]^T, \\
a^5 &= [-1\ -1\ -1\ -1\ -1\ -1\ -1\ 1\ 1\ 1\ 1\ 1\ -1\ -1\ -1\ 1]^T, \\
b^6 &= b^4,\ a^7 = a^5
\end{aligned}$$

- $HD(a^{(1)}, a^1) = 12, HD(a^{(2)}, a^1) = 4,\ HD(a^{(3)}, a^1) = 10$ $, HD(a^{(4)}, a^1) = 4$

➢ Example for Retrieval Phase

- Four steps of retrieval of pair 4: