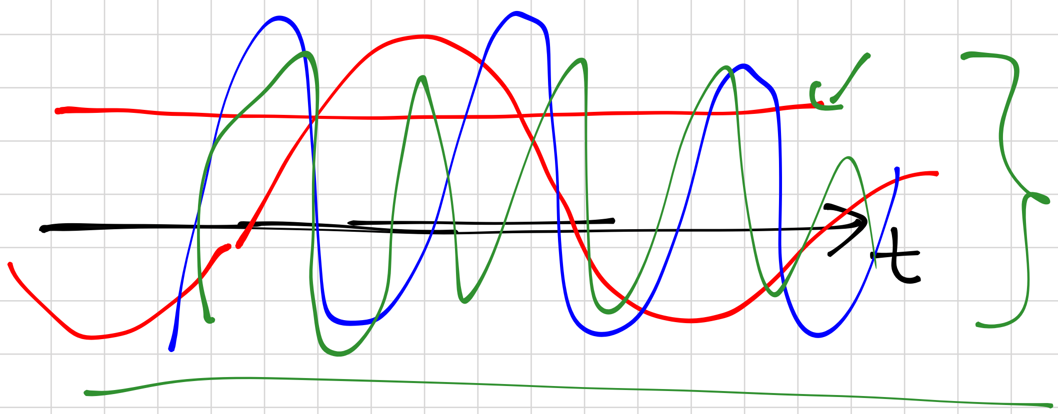
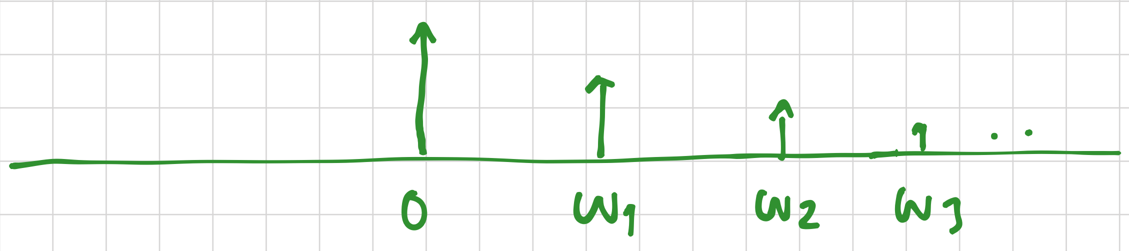
// Time domain ⟶ Frq. domain.

/* $x(t) = a_1 + a_2 \cdot \sin(w_1 t) + a_3 \sin(w_2 t) + \dots$



$= X(t)$



*/

Fourier Representation of LTI signals

$x[n] = e^{+j\Omega n}$ $\quad y[n] = \sum_{k=-\infty}^{+\infty} h[k]\cdot x[n-k]$

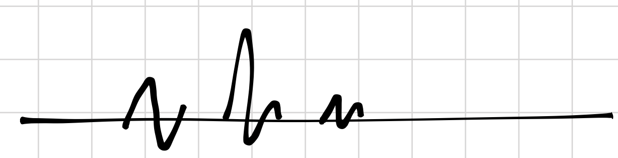
(*) Frequency Response

DT

$$H\{e^{j\Omega}\} \triangleq \sum_{k=-\infty}^{+\infty} h[k]\cdot e^{-j\Omega k}$$

$$y[n] = H\{e^{j\Omega}\}\cdot e^{j\Omega n}$$

/* $H(\Omega)$ */ $\leftarrow H\{e^{j\Omega}\}$:



CT

$$x(t) = e^{jwt} \Rightarrow$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\cdot \underbrace{e^{jw(t-\tau)}}_{x(t-\tau)}\, d\tau$$

$$= e^{jwt}\int_{-\infty}^{\infty} h(\tau)\cdot e^{-jw\tau}\, d\tau$$

$$H(jw) \Longrightarrow H(w)$$ /* Bazı kitapler $H(w)$ kullanıyor */

$$y(t) = H(jw)\cdot e^{jwt}$$

$\cancel{H(jw)}$ 1 $\rightarrow$ $H(w)$

Table 2.1

	Periodic	Nonperiodic
CT [t]	Fourier Series (FS) Dist Ω	Fourier Trans. (FT) ω (Continous)
DT [n]	Discrete-Time F.S. (DTFS) Dis Ω	Discrete-Time Fourier Tr. (DTFT) ω (Continuous)

① Discrete-Time Fourier Series

$n, N \xrightarrow{DTFS} \Omega, k$

$x[n]$ is periodic, N_0 is the fundamental period ($\Omega_0 = \frac{2\pi}{N_0}$), Ω_0 is the fundamental fre.

DTFS of $x[n]$

$$x[n] = \sum_{k=0}^{N-1} \boxed{X[k]} \cdot e^{jk\Omega_0 n}$$

DTFS coefficients

where

$$X[k] = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\Omega_0 n}$$

- $X[k]$ are called "DTFS coefficients"
- $X[k]$ is also termed "frequency domain" representation of of $x[n]$. Each DTFS coefficient is associated with a different frequency.

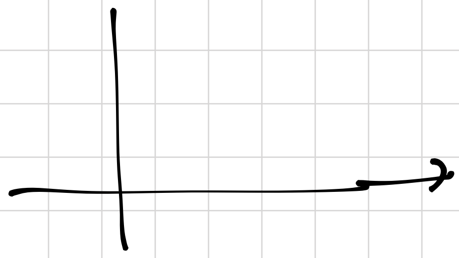
- The limits of the summation can be chosen to be different as long as the summation is over N samples.

/* $c = a + jb$ * $\quad$ arg(c) : angle

$c = |c| \cdot e^{j\,\arg(c)}$ $\quad$ } Euler's formula

$|c| = \sqrt{a^2+b^2}$

$\arg(c) = \arctan\left(\frac{b}{a}\right)$



$$\left(e^{j\theta} = \cos\theta + j\sin\theta\right)$$

DTFS by inspection

If the signal is sinoisoidal then we can find the DTFS coefficients by inspection.

Ex

$$x(n) = \cos\left(\frac{\pi n}{3} + \theta\right)$$

/* $\cos(\alpha) = \frac{1}{2}\left(e^{j\alpha} + e^{-j\alpha}\right)$ */

period : $\Omega = \frac{\pi}{3}$ $\quad$ $N = \frac{2\pi}{\Omega} = 2\pi / \pi/3 = 6$

$$x[n] = \frac{1}{2}\left[e^{j\left(\pi/3 \cdot n + \theta\right)} + e^{-j\left(\frac{\pi}{3}\cdot n + \theta\right)}\right]$$

$$= \underbrace{\frac{1}{2}\cdot e^{-j\theta}}_{X[-1]} \cdot \underbrace{e^{-j\frac{\pi}{3}n}}_{\substack{\| \\ e^{+jk\Omega n} \\ \downarrow \\ k=-1}} + \underbrace{\frac{1}{2}\, e^{j\theta}}_{X[1]}\; \underbrace{e^{j\frac{\pi}{3}n}}_{k=1}$$

/* $x = \sum_{k=\langle N\rangle} X[k] \cdot e^{-jk\,\pi/3\, n}$

$X[-1] = \frac{1}{2}\, e^{-j\theta}$

$X[1] = \frac{1}{2}$