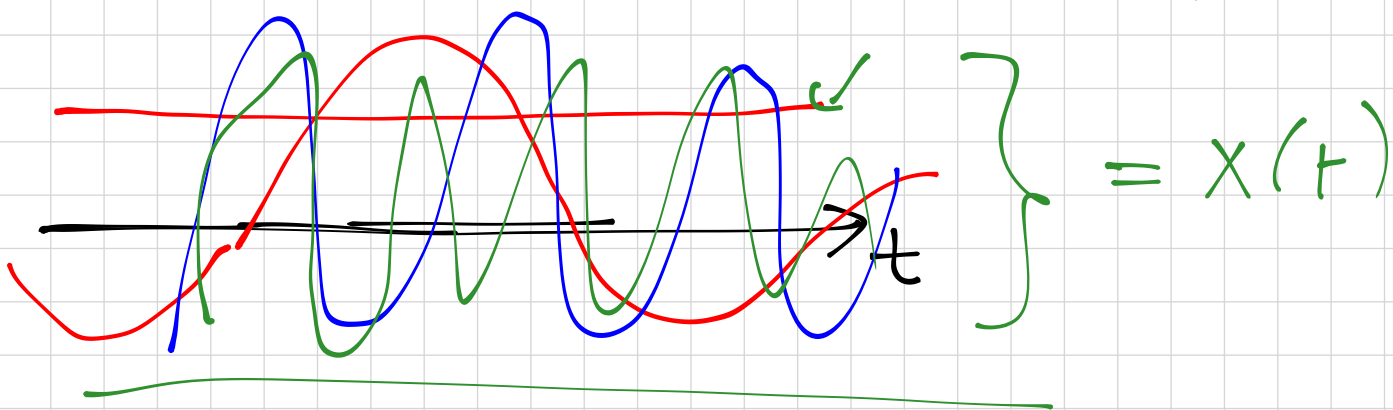


// Time Domain  $\longrightarrow$  Freq. domain.

/\*  $x(t) = a_1 + a_2 \cdot \sin(\omega_1 t) + a_3 \sin(\omega_2 t) + \dots$



## Fourier Representation of LTI signals

⊛ Frequency Response

DT

$$H\{e^{j\Omega}\} = \sum_{k=-\infty}^{+\infty} h[k] \cdot e^{-j\Omega k}$$

The figure shows the Discrete-Time Fourier Transform (DTFT) of the impulse response  $h[k]$ . The summation is from  $k=-\infty$  to  $+\infty$ . The term  $e^{-j\Omega k}$  is circled, and a diagram below it shows a periodic rectangular pulse train, indicating the periodic nature of the frequency response.

$$y[n] = H\{e^{j\Omega}\} \cdot e^{j\Omega n}$$

/\*  $H(\Omega)$  ⊛  $H\{e^{j\Omega}\}$  :

CT

$$x(t) = e^{j\omega t} \Rightarrow$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau) \cdot \underbrace{e^{j\omega(t-\tau)}}_{x(t-\tau)} d\tau$$

$$= e^{j\omega t} \int_{-\infty}^{+\infty} h(\tau) \cdot e^{-j\omega \tau} d\tau$$

/\* Bazi katepler

$H(j\omega) \longrightarrow H(\omega)$

$y(t) = \underbrace{H(j\omega)}_{\text{kullanıyor}} \cdot \underbrace{e^{j\omega t}}_{\text{1}}$

The figure shows the relationship between the continuous-time Fourier transform  $H(j\omega)$  and the discrete-time Fourier transform  $H(\omega)$ . It also shows the final expression for  $y(t)$  as the product of  $H(j\omega)$  and  $e^{j\omega t}$ , with a note indicating that  $H(j\omega)$  is the one used in the calculation.

Table 2.1

	Periodic	Nonperiodic
CT [t]	Fourier Series (FS) (Dis)	Fourier Trans. (FT) (W) (Continuous)
DT [n]	Discrete-Time F.S. (DTFS) (Dis)	Discrete-Time Fourier Tr. (DTFT) (W) (Continuous)

### ① Discrete-Time Fourier Series

$$n, N \xrightarrow{\text{DTFS}} \Omega, k$$

$x[n]$  is periodic,  $N_0$  is the fundamental period ( $\Omega_0 = \frac{2\pi}{N_0}$ ),  $\Omega_0$  is the fundamental freq.

DTFS of  $x[n]$

$$x[n] = \sum_{k=0}^{N-1} \boxed{X[k]} \cdot e^{jk\Omega_0 n}$$

DTFS coefficients

where

$$X[k] = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\Omega_0 n}$$

- $X[k]$  are called "DTFS coefficients"
- $X[k]$  is also termed "frequency domain" representation of  $x[n]$ . Each DTFS coefficient is associated with a different frequency.
- The limits of the summation can be chosen to be different as long as the summation is over  $N$  samples.

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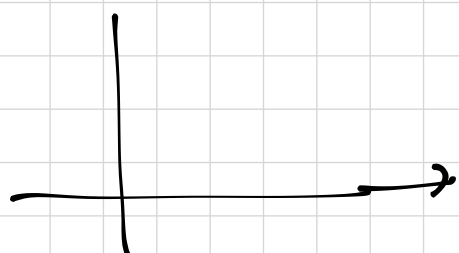
$$c = a + jb$$

$\arg(c)$  : angle

$$c = |c| \cdot e^{j \arg(c)} \leftarrow \text{Euler's formula}$$

$$|c| = \sqrt{a^2 + b^2}$$

$$\arg(c) = \arctan\left(\frac{b}{a}\right)$$



$$(e^{j\theta} = \cos \theta + j \sin \theta)$$

### DTFS by inspection

If the signal is sinusoidal then we can find the DTFS coefficients by inspection.

Ex

$$x(n) = \cos\left(\frac{\pi n}{3} + \theta\right)$$

$$* \cos(\alpha) = \frac{1}{2} (e^{j\alpha} + e^{-j\alpha}) *$$

$$\text{period: } \Omega = \frac{\pi}{3} \quad N = \frac{2\pi}{\Omega} = 2\pi / (\pi/3) = 6$$

$$x[n] = \frac{1}{2} \left[ e^{j(\frac{\pi}{3} \cdot n + \theta)} + e^{-j(\frac{\pi}{3} \cdot n + \theta)} \right]$$

$$= \underbrace{\frac{1}{2} \cdot e^{-j\theta}}_{X[-1]} \cdot \underbrace{e^{-j\frac{\pi}{3}n}}_{\substack{e^{+jk\Omega n} \\ k=-1}} + \underbrace{\frac{1}{2} e^{j\theta}}_{X[1]} \cdot \underbrace{e^{j\frac{\pi}{3}n}}_{k=1}$$

$$* x = \sum_{k=-\infty}^{\infty} X[k] \cdot e^{-jk\frac{\pi}{3}n}$$

$$X[-1] = \frac{1}{2} e^{-j\theta}$$

$$X[1] = \frac{1}{2}$$