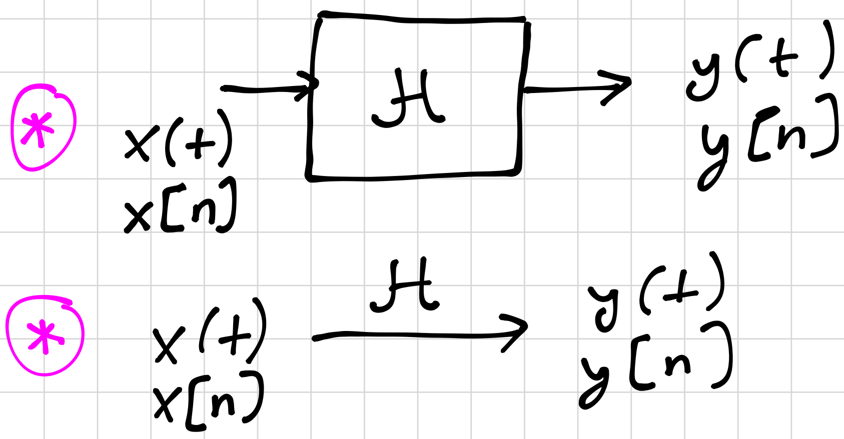
Systems ...



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$$\mathcal{H}\{x(t)\} = y(t)$$
$$\mathcal{H}\{x[n]\} = y[n]$$

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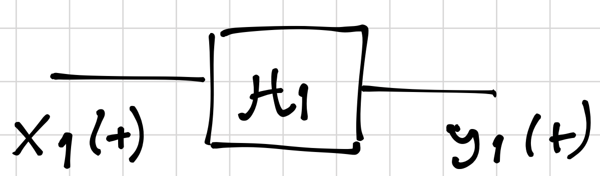
$$y_1(t) = \mathcal{H}_1\{x_1(t)\} = x_1(t) \cdot x_1(t-1)$$
$$y_2(t) = \mathcal{H}_2\{x_2(t)\} = |x_2(t)|$$
$$y_3(t) = \mathcal{H}_3\{x_3(t)\} = 1 + 2 \cdot x_3(t)$$
$$y_4(t) = \mathcal{H}_4\{x_4(t)\} = \cos[x_4(t)]$$



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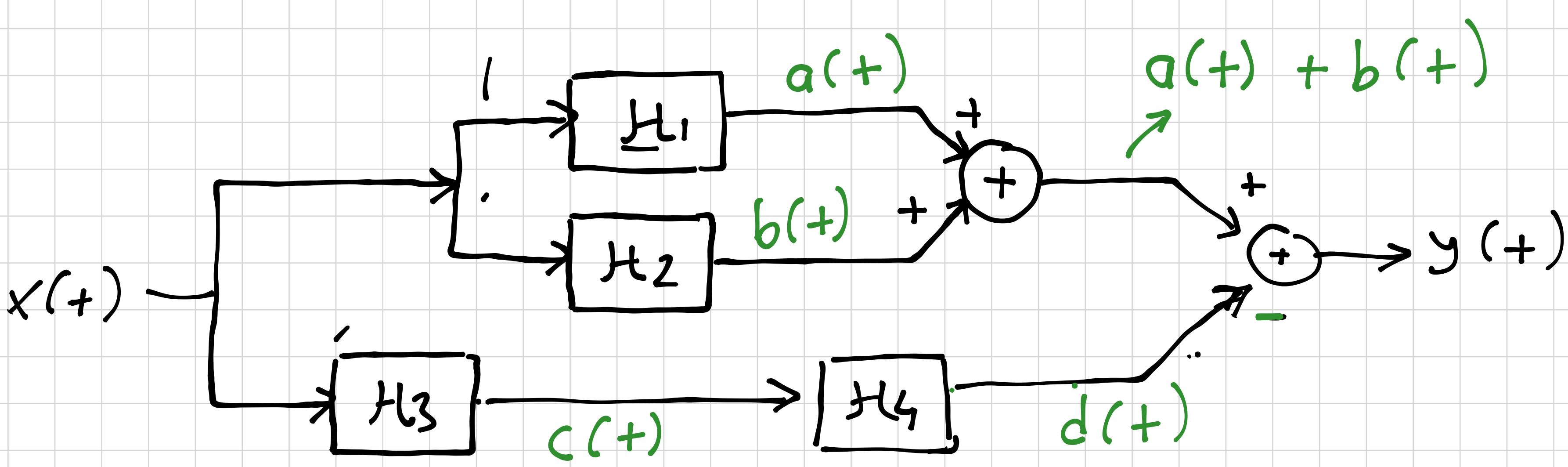


$$y(t) = a(t) + b(t) - d(t)$$
$$a(t) = x(t)\, x(t-1)$$
$$b(t) = |x(t)|$$
$$c(t) = 1 + 2 \cdot x(t)$$
$$d(t) = \cos[c(t)] = \cos[1 + 2x(t)]$$

$$\boxed{y(t) = x(t) \cdot x(t-1) + |x(t)| - \cos[1 + 2x(t)]}$$

PROPERTIES of SYSTEMS

① **Stability** — A system is called bounded-input bounded-output (BIBO) stable if and only if every bounded input results in a bounded output. Formally, let's $y(t) = \mathcal{H}\{x(t)\}$

$$y(t) \le M_y < \infty \quad , \forall t, \ \exists! M_y \in \mathbb{R}^+$$

when

$$x(t) \leq M_x < \infty \ , \forall t \ , \ M_x \in \mathbb{R}^+$$

→ Same rule applies to DT systems.

Ex

$$y[n] = \mathcal{H}\{x[n]\} = \begin{cases} x[2n] \ , & n < 0 \\ \dfrac{n}{n+1} \ , & n \geq 0 \end{cases}$$

Is $\mathcal{H}$ stable ?

. Assume $|x[n]| \leq M_x < \infty$ for $\forall n \in \mathbb{Z}$

✓ For $n < 0 \rightarrow |y[n]| = |x[2n]| \leq M_x$

$$|y[n]| \leq M_x < \infty$$

✓ For $n \geq 0 \rightarrow |y[n]| = \left|\frac{n}{n+1}\right| < \left|\frac{n+1}{n+1}\right| \leq 1$

$$|y[n]| < 1$$

$\therefore$ This system is BIBO-stable ✓

Ex

$$y(t) = (t+1)^2 \, x(t) \qquad \text{BIBO-STABLE ?}$$

Assume $|x(t)| \leq M_x < \infty$

$$|y(t)| = |(t+1)^2| \ |x(t)|$$
$$\leq (t+1)^2 \cdot M_x$$

Since $|y(t)|$ must be a finite value for $\forall t \in \mathbb{R}$, when $t \rightarrow \infty$, $(t+1)^2 \rightarrow \infty$

$\therefore$ this system is NOT-STABLE !

(2) Memory — A system is said to be memoryless if its output depends $\boxed{\text{only}}$ on the $\boxed{\text{current}}$ values of its input.

Ex $\quad y[n] = \left(2\cdot[n+1]\cdot x[n] - x^2[n]\right)^2$

If the system's output depends only on the past and/or future values of the input the system is said to be non-memoryless

Ex : $y[n] = \underbrace{x[n-1]}_{\text{at 1 cycle past.}}$ → has memory.

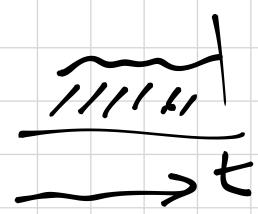
Ex $\quad y(t) = \underbrace{(t+1)}\cdot x(t)$ } memoryless.

(3) Causality — A causal system's output depends only on the current and/or past values of the input.

Ex : $y[n] = \underbrace{x^2[n-2]}_{\text{past}} + \underbrace{x[n]}_{\text{current}}$: causal.

Ex $\quad y(t) = \underbrace{x(t+1)}_{\text{future}}$: non-causal.

Ex : $y(t) = \underbrace{\int_{-\infty}^{t} x(\tau)\, d\tau}_{\text{past + current}}$? : causal.



Ex : $y(t) = \underbrace{(t+1)^2}\, \underbrace{x(t-1)}$ ⇒ causal.

(4) Invertibility — A system is invertible if distints inputs lead to distinct outputs, that is, if the input of the system can be recovered from the output of the system.

$$x(t) \longrightarrow \boxed{\mathcal{H}} \xrightarrow{y(t)} \boxed{\mathcal{H}^{inv}} \longrightarrow x(t)$$

$$\mathcal{H}^{inv}\left\{\mathcal{H}\{x(t)\}\right\} = (\mathcal{H}^{inv}\mathcal{H})\{x(t)\} = x(t)$$

$$\mathcal{H}^{inv}\cdot\mathcal{H} = I \quad : \text{ idendity system.}$$

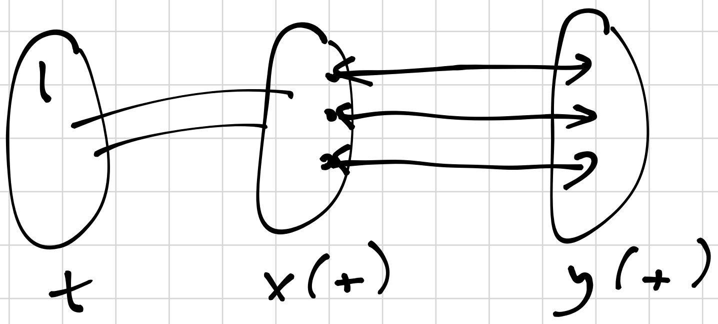
Ex

$$\left.\begin{aligned} y(t) &= 2\cdot x(t) = \mathcal{H}\{x(t)\} \\ x(t) &= \frac{1}{2}y(t) = \mathcal{H}^{inv}\{y(t)\} \end{aligned}\right\} \mathcal{H} \text{ is invertible}$$

Ex

$$y(t) = x^2(t) \rightarrow$$

$x(t)$ and $-x(t)$ will produce the same output. this system is non-invertible.



(5) Time Invariance — A system is said to be time invariant if a time delay or time advance of the input signal leads to an identical time shift in the output signal.

$$x(t) \rightarrow \boxed{\mathcal{H}} \rightarrow y(t)$$

$$x(t-t_0) \rightarrow \boxed{\mathcal{H}} \rightarrow y(t-t_0)$$

Otherwise, the system is called "time variant".

Ex

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \quad , \text{ is it T.I.?}$$

Let's first shift the input input in time.

$$? \left\langle \begin{aligned} y_2(t) &= \int_{-\infty}^{t} x(\tau - t_0)\, d\tau \quad \left\{ \overset{?}{=} y(t-t_0) \right\} \\ y_1(t) &= y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau \end{aligned}\right.$$

$$\tau' = \tau - t_0$$

$$\tau \to t \to \tau' = t - t_0$$

$$d\tau' = d\tau$$

$$y_2(t) = \int_{-\infty}^{t-t_0} x(\tau')\, d\tau'$$

$=$ Time invariant!!

1'

Ex

$$y[n] = r^n \, x[n] \qquad \text{T.I.?}$$

$$y_1[n] = \mathcal{H}\{x[n-n_0]\} = r^n \cdot x[n-n_0]$$

$$y_2[n] = y[n-n_0] = r^{n-n_0} \cdot x[n-n_0]$$

$$y_1[n] \neq y_2[n]$$ Time-variant system

6 Linearity

A system is said to be linear if it satisfies the following properties.

→ 1 Superposition

Let $x_1(t) \xrightarrow{\mathcal{H}} y_1(t)$,

$x_2(t) \xrightarrow{\mathcal{H}} y_2(t)$,

$x(t) \xrightarrow{\mathcal{H}} y(t)$

then: $x_1(t) + x_2(t) = x(t) \xrightarrow{\mathcal{H}} y(t) = y_1(t) + y_2(t)$

→ 2 Homogenity

$x(t) \xrightarrow{\mathcal{H}} y(t)$

then $\alpha x(t) \xrightarrow{\mathcal{H}} \alpha y(t)$

[Same applies to DT systems]

→ One could check both properties by checking :

$x_1 \xrightarrow{\mathcal{H}} y_1 \qquad x_2(t) \xrightarrow{\mathcal{H}} y_2(t)$

$$\alpha \cdot x_1(t) + \beta x_2(t) \xrightarrow{\mathcal{H}} \alpha y_1(t) + \beta y_2(t)$$

Ex

$$y(t) = (t+1)^2 \cdot x(t) \qquad \text{Linear?}$$

Homogenity

$$\mathcal{H}\{\alpha \cdot x(t)\} = (t+1)^2 \{\alpha \cdot x(t)\}$$
$$= \alpha \{(t+1)^2 \cdot x(t)\}$$
$$= y(t)$$

homogenity is satisfied ✓

Superposition

$$\mathcal{H}\{x_1(t) + x_2(t)\}$$
$$= (t+1)^2 \cdot \{x_1(t) + x_2(t)\}$$
$$= (t+1)^2 \cdot x_1(t) + (t+1)^2 \cdot x_2(t)$$
$$= y_1(t) + y_2(t)$$

Superposition satisfied ✓

$\therefore$ This system is LINEAR ! ■

Examples (Ch I)

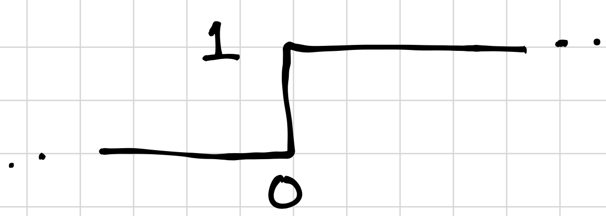
Ex

$$x(t) = u(2t-4)$$

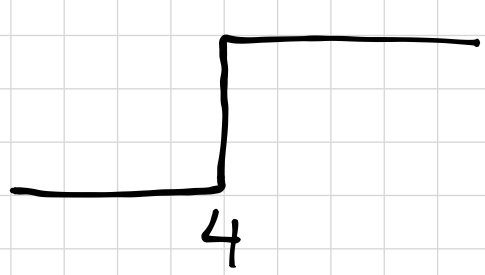
) Classify this signal.

{NOT- Geçen sene yanlış yapmışız}

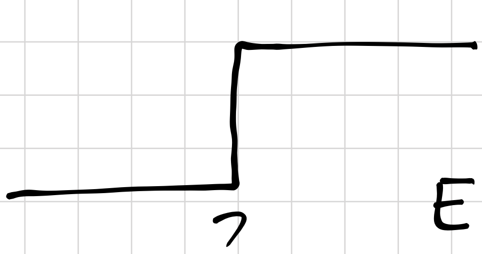
$u(t)$



$u(t-4)$



$u(2t-4)$



1. Periodic / nonperiodic } ? Non-periodic
2. CT - DT ?
3. ~~Even / odd~~
4. Power or energy ?

$$E = \int_2^{\infty} 1^2 \cdot dt = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_2^T 1^2 \, dt = \lim_{T\to\infty} \left(\frac{1}{T} \, t \Big|_2^T \right) = \lim_{T\to\infty} \frac{T-2}{T} = \lim_{T\to\infty} \left(1 - \frac{2}{T}\right) = 1$$

(Power signal)

Ex

$$x[n] = 2\cdot\cos\Big(\underbrace{\tfrac{\pi}{4}}_{\Omega_1} n\Big) + \sin\Big(\underbrace{\tfrac{\pi}{8}}_{\Omega_2} n\Big) - 2\cdot\cos\Big(\underbrace{\tfrac{\pi}{2}}_{\Omega_3} n - \tfrac{\pi}{8}\Big)$$

Periodic?
or non-"

$$\Omega_1 = \frac{\pi}{4} = 2\pi\cdot\frac{m_1}{N_1} \Rightarrow$$

$$\left.\frac{m_1}{N_1} = \frac{1}{8}\right\} \quad m_1 = 1 \quad N_1 = 8$$

$$\left.\Omega_2 = \frac{\pi}{8} = 2\pi\cdot\frac{m_2}{N_2}\right\} \quad m_2 = 1, \; N_2 = 16$$

$$\left.\Omega_3 = \frac{\pi}{2} = 2\pi\cdot\frac{m_3}{N_3}\right\} \quad m_3 = 1, \; N_3 = 4$$

Period

$$N = LCM(8, 16, 4) = 16$$

Ex $\mathcal{H}$ is given as

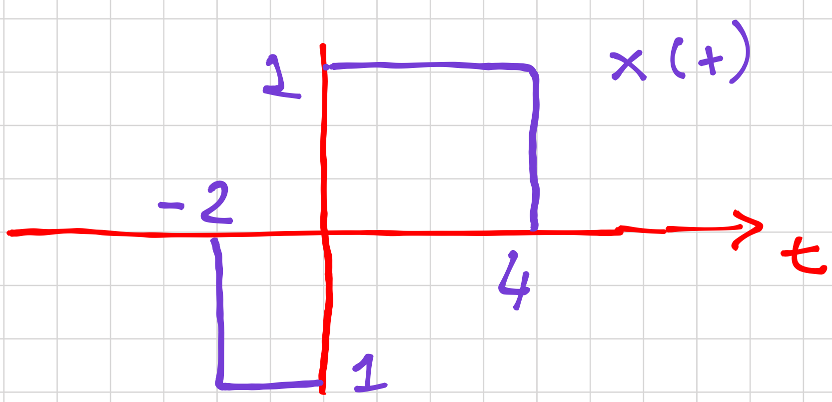
$$y(t) = \mathcal{H}\{x(t)\} = (t+1)^2\, x(t) \qquad \text{T.I?}$$

$$y_1(t) = \mathcal{H}\{x(t-t_0)\} = (t+1)^2\, x(t-t_0)$$

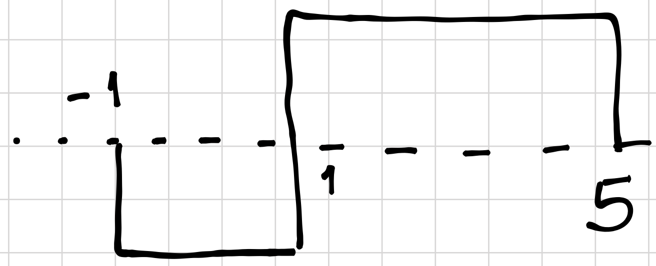
$$y_2(t) = y(t-t_0) = (t-t_0+1)^2\, x(t-t_0)$$

$$y_1(t) \neq y_2(t)$$ NOT TIME-INVARIANT

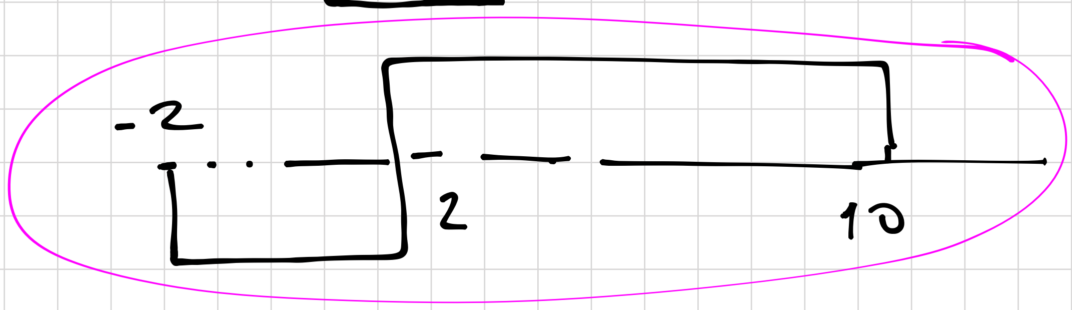
Ex $x(t)$ is given as



find and sketch $x\left(\frac{t}{2} - 1\right) + x(3t+1)$

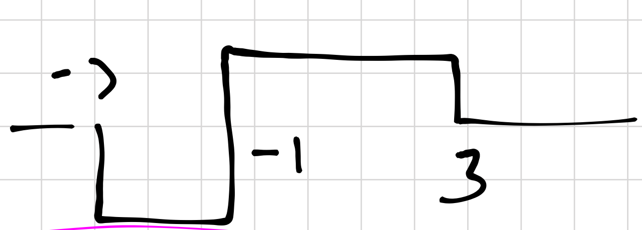


$x(t-1)$

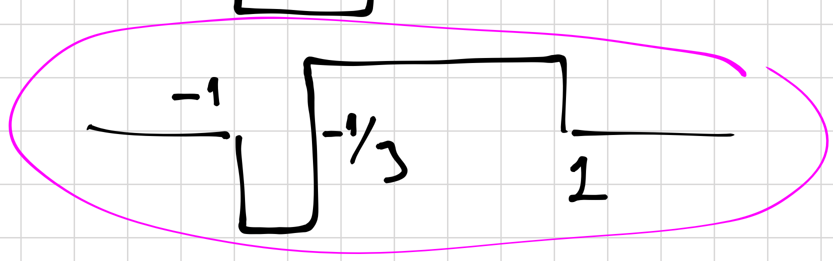


$x\left(\frac{t}{2}-1\right)$

Geçen sene yandık



$x(t+1)$



$x(3t+1)$