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**Number 34**

# Inverse Source Problems

**Victor Isakov**



**American Mathematical Society**

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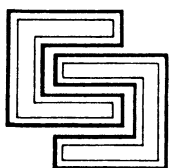
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MATHEMATICAL SURVEYS  
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# **INVERSE SOURCE PROBLEMS**

VICTOR ISAKOV

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**To my wife, Tatiana**

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## Preface

The principal theme of this book is uniqueness, stability, and existence of the solutions of inverse problems for partial differential equations when the right side is to be found. We describe the inverse problem of potential theory and some closely related inverse problems. These problems have important applications which we will not discuss in detail. We tell more about the content in the foreword.

This book has been written mostly in Akademgorodok, the U.S.S.R., in 1984–1986, when the author applied to emigrate from the Soviet Union and was dismissed from his teaching position at the Novosibirsk State University by Prof. M. M. Lavrent'ev and then from his research position at the Institute of Mathematics, Novosibirsk, by Prof. S. K. Godunov. Now this painful time is in the past, but it is difficult to forget it.

The final version was written while visiting the Courant Institute of NYU and at Wichita State University. The author would like to thank Alan Elcrat and Jeffrey Powell for their help.

Victor Isakov  
Wichita, October 1989

## Foreword

First we explain the concept of an inverse problem. It begins with the supposition that there is a direct problem, i.e., a well-posed problem of mathematical physics. In other words, we have a good mathematical description of a given “device”. If this device is in part unknown and supplementary information about a solution of the direct problem is given we obtain an inverse problem.

For example, for any mass distribution one can find its Newtonian potential. Moreover, this problem is well-posed although not very elementary. If a mass distribution is not known but its potential outside a certain ball is given and the aim is to determine this mass distribution, we find ourselves studying the inverse problem of potential theory. This inverse problem was formulated by Laplace 200 years ago. More generally, given a linear differential operator and boundary or initial data one can find a solution to a related well-posed boundary value problem. An inverse coefficients (identification) problem is to find coefficients or the right-hand side of this equation if additional boundary data are given.

Many inverse problems arise naturally and have important applications. As a rule, these problems are rather difficult to solve for two reasons: they are nonlinear and they are improperly posed. Probably, the second reason is more serious. Only in recent decades have we made certain progress in both the analytical and computational aspects.

Since our main goal is determining existing but unknown objects it is very important to be sure we have sufficient data. So uniqueness questions are of exceptional importance here.

Most direct problems can be reduced to finding values  $y = F(x)$  of an operator  $F$  acting from a topological space  $X$  into a topological space  $Y$ . Usually  $F$  is continuous and  $X, Y$  are Banach spaces with norms  $\| \cdot \|_X$  and  $\| \cdot \|_Y$ . The inverse problem is then connected with the inverse operator or with solving the equation

$$(0.1) \quad F(x) = y.$$

Many direct problems themselves are equivalent to such an equation. A related problem is said to be well-posed (with respect to the pair  $X, Y$ ) if

the operator  $F$  satisfies the following conditions:

(0.2) a solution  $x$  to Equation (0.1) is unique, i.e., if  $F(x_1) = F(x_2)$ ,  $x_j \in X$ , then  $x_1 = x_2$  (uniqueness of solution);

(0.3) a solution  $x$  to Equation (0.1) exists for any  $y \in Y$ , i.e., for any  $y \in Y$  there is  $x \in X$  satisfying Equation (0.1) (existence of solution);

(0.4) a solution to the equation under consideration is stable, i.e., if  $\tilde{y} \rightarrow y$ , then related solutions  $\tilde{x} \rightarrow x$  (stability of solution).

The problem is said to be well-posed in the sense of Hadamard if, in the definition given, both  $X$  and  $Y$  are spaces  $C^k$  or  $H^{k,p}$  or their subspaces of finite codimensions. Hadamard especially stressed meaning of the stability condition. In practice it is important because of inevitable errors when calculating or measuring something. Unfortunately, many important problems of mathematical physics, including basic inverse problems, are not well-posed according to Hadamard. Two examples are the Cauchy problem for the Laplace equation and the inverse problem of potential theory mentioned above.

A problem described by Equation (0.1) is said to be conditionally correct (correct according to Tikhonov) in a correctness class  $\mathbf{M}$  if the operator  $F$  satisfies the following conditions:

(0.2  $\mathbf{M}$ ) a solution  $x$  to Equation (0.1) is unique in  $\mathbf{M}$ , i.e., if  $F(x_1) = F(x_2)$ ,  $x_j \in \mathbf{M}$ , then  $x_1 = x_2$  (uniqueness of a solution in  $\mathbf{M}$ );

(0.4  $\mathbf{M}$ ) a solution is stable on  $\mathbf{M}$ , i.e.,  $\tilde{x} \rightarrow x$  in  $X$  if  $\tilde{x}, x \in \mathbf{M}$  and  $F(\tilde{x}) \rightarrow F(x)$  in  $Y$  (conditional stability).

So the requirements (0.2) and (0.4) are replaced by the less restrictive ones (0.2  $\mathbf{M}$ ) and (0.4  $\mathbf{M}$ ). There is no existence requirement at all. We remark that the convergence  $\tilde{x} \rightarrow x$  in  $X$  ( $\tilde{y} \rightarrow y$  in  $Y$ ) means that  $\|\tilde{x} - x\|_X \rightarrow 0$  ( $\|\tilde{y} - y\|_Y \rightarrow 0$ ). A theory of conditionally correct problems was created in the 1950s–1960s by Ivanov, John, Lavrent'ev, Pucci, and Tikhonov. It is described very briefly in Section 2.3. One can find detailed expositions in the books and papers of Ivanov, Vasin, Tanana [67], John [71], Lavrent'ev [87, 88], Nashed [103], and Tikhonov and Arsenin [163]. According to this theory, any conditionally correct problem can be solved numerically by means of regularization and the success of this solution process depends on the correctness class  $\mathbf{M}$ . Note that if  $\mathbf{M}$  is compact in  $X$  then the condition (0.4  $\mathbf{M}$ ) is a consequence of the condition (0.2  $\mathbf{M}$ ). Uniqueness questions are central in the theory of conditionally correct problems; nevertheless, existence theorems are of importance as well, since they guarantee that we do not use extra data.

The inverse problem of potential theory has been studied extensively, although many cardinal questions are waiting for answers. In 1943, analyzing stability of this problem (which is not well-posed in the sense of Hadamard), Tikhonov introduced certain important concepts of the theory of conditionally correct problems. In fact, we have to find the right-hand side (source) of the Laplace equation. This problem serves as a good pattern for other

inverse source problems. Often an identification problem can be reduced to an inverse source problem if we are interested only in uniqueness. For example, consider two differential equations of parabolic type:

$$(\partial/\partial t)u_j - \Delta u_j + a_j u_j = F, \quad j = 1, 2.$$

Letting  $u = u_2 - u_1$ ,  $f = a_1 - a_2$ , and  $\alpha = u_1$  we get

$$(\partial/\partial t)u - \Delta u + a_2 u = \alpha f.$$

Also we have additional information, for example, boundary data. Then, from a uniqueness theorem for the pair  $(u, f)$  (an inverse source problem), one can derive a uniqueness result for the inverse problem concerning identification of the coefficient  $a$ . This is why we are focusing on inverse source problems.

Now we just mention certain well-known and important inverse problems of mathematical physics referring to the books and expository papers given below: *the inverse seismic problem* (see the books of Lavrent'ev, Romanov, and Shishat'skii [90], and of Romanov [132, 133]); *integral geometry and tomography* (the books of Gel'fand, Graev, and Vilenkin [33] and of Helgason [38], the papers of Anikonov [6], Lavrent'ev and Buhgeim [89], Muhometov [101], Natterer [104], and of Smith, Solmon, and Wagner [145]); *the inverse spectral problem* (the books of Levitan [92] and of Pöschel and Trubowitz [117], the papers of Eskin, Ralston and Trubowitz [29], of Guillemin and Melrose [36], of M. Kac [73], of Prosser [125], and of Sleeman and Zayed [144]; and the *inverse scattering problem* (the books of Chadán and Sabatier [21], of Colton and Kress [24], of L. Faddeev [30], of Lax and Phillips, the papers of Angell, Kleinman, and Roach [4], of J. Keller [77], of Majda [98], of Nachman [102], and of R. Newton [106]). We emphasize that in the theory of non-well-posed problems for differential equations, *uniqueness in the Cauchy problem* is of great importance; the contemporary state of this problem is described by Hörmander [42, 45], Nirenberg [109], and Zuily [170].

The field of inverse problems is growing very rapidly. In 1979 there were two international conferences, in Newark, U.S.A., and in Halle, Germany [46], and in 1983 the conferences in New York [47] and in Samarkand, the U.S.S.R. [159], have been organized. New formulations and results were obtained. As a bright example we mention the inverse problem posed by Calderón in 1980 and investigated by R. Kohn and Vogelius [83]. Recently, very strong results including a complete solution of this problem in the three-dimensional case were obtained by Sylvester and Uhlmann [156].

Applications are growing very rapidly as well and now they include physics, geophysics, chemistry, medicine, and engineering. We refer to the books and expository articles of Baltes [10], Bolt [11], Buhgeim [16], J. Keller [78], R. Newton [105], Payne [114], Talenti [157], and Tarantola [158].

This book is devoted mainly to the inverse problem of potential theory and closely related questions such as coefficient identification problems. In

Chapter 1 we collect preliminary results concerning direct problems. We recommend to a reader who is mainly interested in applications first to read Chapter 2, where main results, including versions for parabolic and hyperbolic equations, are presented. Chapters 3, 4, and 5 contain a complete up-to-date exposition of the inverse problem of potential theory for elliptic equations, the first in the mathematical literature. Chapters 6 and 7 describe this problem as well as coefficient identification problems for parabolic and hyperbolic equations.

We hope that this book will be useful both for specialists and for students in partial differential equations, mathematical physics, and numerical analysis as well as in physics, geophysics, chemistry, and engineering.

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