

3. A string is tightly stretched and its ends are fastened at two points $x = 0$ and $x = l$. The mid point of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string at any time during the subsequent motion.
4. A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The point where $x = \frac{1}{3}$ is drawn aside a small distance h , the displacement $y(x, t)$ satisfies $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$. Determine $y(x, t)$ at any time t .
5. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position find the displacement y at any distance x from one end at any time t .
6. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in its equilibrium position. If it is set vibrating giving each point a velocity $y = 3x(l - x)$, find the displacement.
7. Find the solution to the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the conditions

$$(i) u(0, t) = 0$$

$$(ii) u(l, t) = 0 \text{ for } t > 0$$

$$(iii) u(x, 0) = \begin{cases} x, & 0 < x < \frac{l}{2} \\ l - x, & \frac{l}{2} < x < l \end{cases}$$

8. A bar 10cm long, with insulated sides, has its ends A and B kept at $20^\circ C$ and $40^\circ C$ respectively until steady state conditions prevail. The temperature at A is then suddenly raised $50^\circ C$ and at the same instant that at B is lowered to $10^\circ C$. Find the subsequent temperature at any point of the bar at any time.
9. A square plate is bounded by the lines $x = 0, y = 0, x = l$ and $y = l$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, l) = x(l - x)$ when $0 < x < l$ while the other three edges are kept at $0^\circ C$. Find the steady state temperature in the plate.
10. Find the steady state temperature at any point of a square plate whose two adjacent edges are kept at $0^\circ C$ and the other two edges are kept at the constant temperature $100^\circ C$.
11. Find the steady temperature distribution at points in a rectangular plate with insulated faces the edges of the plate being the lines $x = 0, y = 0, x = a$ and $y = b$, when three of the edges are kept at temperature zero and the fourth at a fixed temperature α .
12. A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The

temperature at short edges $y=0$ is given by $u = \begin{cases} 20x & 0 \leq x \leq 5 \\ 20(10 - x) & 5 \leq x \leq 10 \end{cases}$ and all the other three edges are kept at $0^\circ C$. Find the steady state temperature at any point in the plate.