The derivative of $f(x) = a^x$

We use the definition of derivatives here

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} \\
&= \lim_{h \to 0} \frac{a^x \left(a^h - 1\right)}{h} \\
&= a^x \cdot \lim_{h \to 0} \frac{a^h - 1}{h} \\
&= a^x \cdot \lim_{h \to 0} \frac{e^{h \ln a} - 1}{h}
\end{aligned}
$$

Then, we apply L'Hôpital's rule to the second part

$$
\begin{aligned}
\lim_{h \to 0} \frac{e^{h \ln a} - 1}{h} &= \lim_{h \to 0} \frac{\left(e^{h \ln a} - 1\right)'}{h'} \\
&= \lim_{h \to 0} \frac{\ln a \cdot e^{h \ln a}}{1} \\
&= \lim_{h \to 0} \left(\ln a \cdot a^h\right) \\
&= \ln a
\end{aligned}
$$

So

$$
(a^x)' = a^x \cdot \ln a
$$

The derivative of $f(x) = \log_a x$

Since this function is the inverse function of $g(x) = a^x$, that is, $f^{-1}(x) = a^x$

$$
\begin{aligned}
f'(x) &= \frac{1}{(f^{-1})'(f(x))} \\
&= \frac{1}{(f^{-1})'(\log_a x)} \\
&= \frac{1}{a^{\log_a x} \cdot \ln a} \\
&= \frac{1}{x \cdot \ln a}
\end{aligned}
$$

So

$$
(\log_a x)' = \frac{1}{x \cdot \ln a}
$$

The derivative of $f(x) = \arcsin x$

$$
\begin{aligned}
(\arcsin x)' &= \frac{1}{\cos(\arcsin x)} \\
&= \frac{1}{\sqrt{1 - \sin^2(\arcsin x)}} \\
&= \frac{1}{\sqrt{1 - x^2}}, \quad |x| < 1
\end{aligned}
$$

The derivative of $f(x) = \operatorname{arcsec} x$

$$
\begin{aligned}
(\operatorname{arcsec} x)' &= \frac{1}{\sec(\operatorname{arcsec} x)\tan(\operatorname{arcsec} x)} \\
&= \frac{1}{|x|\sqrt{\sec(\operatorname{arcsec} x)^2 - 1}} \\
&= \frac{1}{|x|\sqrt{x^2 - 1}}, \quad |x| > 1
\end{aligned}
$$

The derivative of $f(x) = \arctan x$

$$
\begin{aligned}
(\arcsin x)' &= \frac{1}{\sec^2(\arctan x)} \\
&= \frac{1}{\tan^2(\arctan x) + 1} \\
&= \frac{1}{1 + x^2}
\end{aligned}
$$