
Some known facts about financial data

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ABSTRACT

Many researchers are interesting in applying the neural networks methods to financial data. In fact these data are very complex, and classical methods do not always give satisfactory results. They need strong hypotheses which can be false, they have a strongly non-linear structures, and so on. But neural models must also be cautiously used. The black box aspect can be very dangerous. In this very simple paper, we try to indicate some specificity of financial data, to prevent some bad use of neural models.

KEYWORDS

Financial data, Neural networks, Empirical evidences

1. Introduction

The aim of this communication is to synthesize the main known facts on the analyze and the modeling of financial data. The econometric literature has developed an important stream of research on this topics and it seems to us absolutely necessary to be aware of the results accumulated since the last 30 years in this field in order to identify the potential applications of neural networks to financial data. The textbooks of Campbell et al. (1997) and Greene (2000) are

certainly useful starting points¹. Let us emphasize that we will certainly not pretend to be exhaustive in this survey. We only present a subjective selection of important points that should, at least we hope so, convince the reader to go ahead in this literature before trying his *magic box* on financial data. We will conclude our discussion by presenting a recent contribution of Wang (forthcoming) in the field of asset pricing, where the use of non parametric methods is central, opening interesting new potential applications of neural networks in finance research.

Potential application fields of neural networks are in fact numerous. Let us just quote Lo et al. (1997 – p. 467) on this subject: "Many aspects of economic behavior may not be linear. Experimental evidence and casual introspection suggest that investors' attitudes towards risk and expected return are nonlinear. The terms of many financial contracts such as options and other derivative securities are non linear.". A brief review of the books of Deboeck et al. (1998), Azoff (1994) and Refenes (1995) shows a large panel of them : non-linear arbitrage pricing theory, non-linear tests of the efficient market hypothesis, tracking of stock indexes, forecasting of prices, bond rating, bankruptcy prediction, prediction of corporate mergers, risk management, diversification, trading systems, interest rates simulations with application to Value-at-Risk, stock picking, ... In 1994, Hutchinson, Lo and Poggio present an application of neural networks (both MLP and RBF networks) to the problem of pricing and hedging of derivatives securities in the Journal of Finance. Since then (and even before but maybe not in the highest scientific journals in finance), numerous applications of neural networks have been proposed. All this shows clearly that the potential fields of application of neural networks in finance are the ones that financial research has always been interested in. Two remarks are yet worth to be done:

- the non linear nature of financial data remains, in the absence of theoretical models, mainly an hypothesis that have to be tested. Several propositions have been done in literature. The neural networks research community is used to use the notion of correlation dimension introduced by Grassberger et al. (1993) or the BDS statistics (Brock et al., 1987). This is not by far the only proposition and we find, for example, in Greene (2000), several more classical approaches, based on polynomial regression analysis.

- some scanning of the econometric literature quickly shows also that neural networks are not the only candidate to tackle the non linear nature of financial data. For time series data, polynomial models, proposed by Volterra (1959) (more than 40 years ago), piecewise-linear models, Markov-switching models, ... are potential competitors.

The main "producers" of financial data are the financial markets. They generate, at each moment millions and millions of prices of financial securities. Those are of different kinds. The most popular ones are securities that represent a piece of ownership of a firm (the classical share) and are exchanged on stock markets. But

¹ Lots of developments presented here are inspired from them.

bonds, currencies, futures, options, warrants, commodities, ... are all traded today on specific financial markets, with specific rules governing their quotation. All these data are mainly times series but the specificities of the security that they represent and of the environment in which they are formed are to be taken into account if we want to seriously analyze it. Moreover, not all financial data are prices (e.g., volumes, dividends, ratios, ...) and not all financial data are coming from financial markets (e.g., data coming from financial statements, forecasts of financial analysts, ...). It is clearly out of the scope of this contribution to cover all those kinds of financial data and we will concentrate our attention on stock market prices. This is a choice, partly justified by the very specific and historic interest that these markets have raised in the financial literature.

To put into light the main features of stock market data, we will try to respond to five questions: Must we work on prices or on returns? Are they Gaussian? Are they stationary? Is the variance constant through time? Is it possible to forecast them? In order to avoid a too classical review of literature, we will base our development on the analysis of the Dow Jones index² (there after, DJI) series on the period 1915 - 1990³. The figure 1 presents it. This represents 18840 days of quotation. The index starts at 55.4 on the 4/01/1915 to end at 2596.86 on the 20/2/1990. The inspection of figure 1, at least for eyes used to examine time series, reveals lots of things⁴. Let us discover them together.

² The Dow Jones Index is price weighted index composed by the 30 most important firms quoted on the New-York Stock Exchange. It is out of the scope of this paper to present it in full details but it is important to stress that the composition of the index has evolved through time and that this index is clearly biased toward blue chips (shares of big and profitable firms) of the market.

³ Those data are freely available on <http://www.nyse.com>.

⁴ It also gives a beautiful trace of the industrial and economic history of the 20th century.

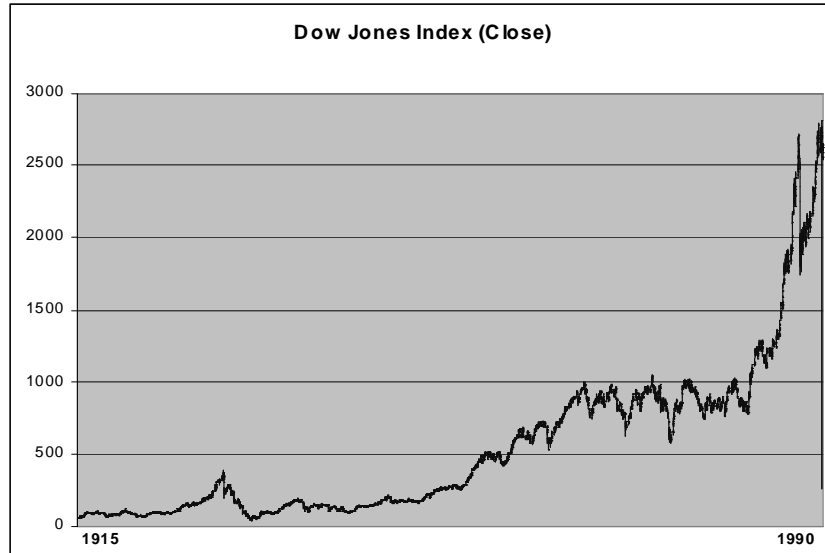


Figure 1. Dow Jones Index from 01/04/1915 to 02/20/1990

2. Must we work on prices or returns?

At first sight, it could be tempting to work directly on prices (on the case of the DJI, to work on the index level). Let us try, for example, to estimate a first order auto-regressive model on the DJI :

$$DJI_t = \alpha_0 + \alpha_1 DJI_{t-1} + \varepsilon_t,$$

where ε_t is a centered noise. An extract of the results⁵ is presented at table 1. The model seems very significant, with an impressive R-Squared⁶. A closer look to it will allow us to show that it is of no interest. Based on the estimated parameters, we have computed the number of times the model allows us to correctly estimated the sign of the variation of the index. The result is 9524 out of 18839 observations (the first one is lost to compute the initial variation), that is to say 50.5% of the time. Not really better than the result obtained by tossing a coin !

Std. error of regression = 26.1347

R-squared = .997617

⁵ All estimations presented in this paper have been realized with TSP 4.5.

⁶ Readers used to time series analysis will immediately suspect that the process is not stationary but we will defer this point up to section 4.

	Estimated	Standard		
Variable	Coefficient	Error	t-statistic	
Alpha_0	.581189	.271015	2.14449	**
Alpha_1	.999177	.355801E-03	2808.25	***

** is for significant at 5%, *** is for significant at 1%;

Table 1

The theoretical reasons of this result will be exposed later but, at this stage, this result is sufficient to put into light that any attempt to model or forecast stock market must be based on successive variations of price and not on the prices themselves. The classical measure of successive variations is the return, either calculated in discrete time as $r_t = \frac{P_{t+1} - P_t}{P_t}$ or as $r_t = \ln(\frac{P_{t+1}}{P_t})$. Figure 2 presents the returns of DJI. The very noisy nature of the data comes immediately to light.

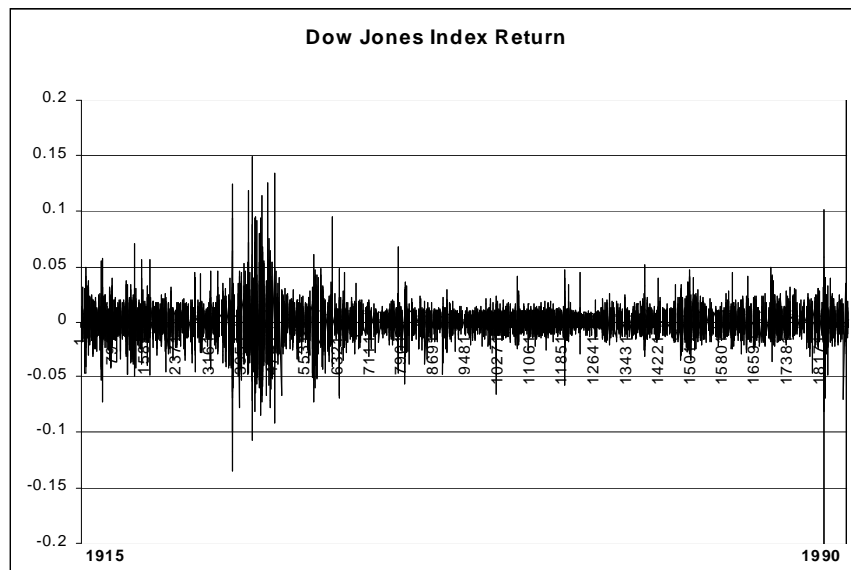


Figure 2. Dow Jones Index returns from 01/04/1915 to 02/20/1990

3. Are they Gaussian?

The Gaussian hypothesis on the distribution of returns is at the foundation of numerous theoretical developments in finance, such as the efficient frontier of Markovitz (1959) or the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965b), to quote only two of the most famous. It is needless to say that the Gaussian hypothesis makes considerably easier lots of mathematical and statistical developments. But is it a reasonable hypothesis? From a theoretical point of view, it is possible to build on the Central Limit Theorem to justify it, as Fama (1965) suggests it. Returns are indeed the result of the sum of the variations of prices from transaction to transaction on the market. If variations of prices from transaction to transaction can be considered as random outcome of probability distribution with finite mean and variance, then, returns should be Gaussian⁷. Which is the empirical evidence? We present, for the DJI returns series, at figures 3a, 3b and 3c, the distribution of returns calculated on a daily basis, weekly basis and monthly basis.

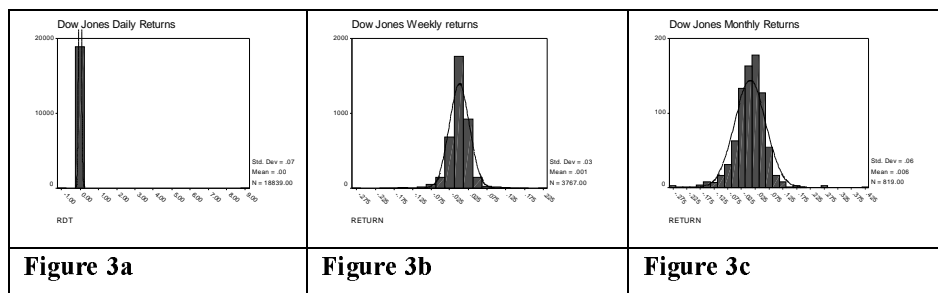


Figure 3. Dow Jones Index returns histograms at different frequencies

As awaited from the theoretical point of view (from the Central Limit Theorem), the more you aggregate returns, the more the distribution seems to be Gaussian. The same phenomenon can be observed concerning the aggregation of returns to form portfolios (the larger the number of securities in the portfolio, the closer are the returns to a Gaussian distribution). Empirical tests show that returns of individual securities are a lot less Gaussian than returns of portfolios of them.

4. Are they stationary?

A time process y_t is said weakly stationary (or covariance stationary) if $E(y_t)$ and $Var(y_t)$ are finite constants independent of time, and $Cov(y_t, y_s)$ is a finite function of the absolute value $|t-s|$. Does the returns of the DJI returns series respond to these conditions? The development of econometric test of stationarity has been a very

⁷ It should be noted that this assertion remains valid in the context of unequal variance between the distributions from which the variations of prices are coming.

active field and is closely related to the concept of random walk⁸. It is out of the scope of this paper to go into the details of this problematic. We will limit ourselves here to a presentation of the concept of random walk, to show that a random walk process is not stationary and, last but not least, to introduce the Dickey-Fuller test of the presence of a random walk. A random walk process is defined as

$$y_t = \mu + y_{t-1} + \varepsilon_t, \quad [1]$$

where ε_t is an i.i.d. sequence of random variables, with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \text{Cst}$.

Suppose that this process can be stationary, then we can reformulate the stationary solution of (1) as

$$y_t = \sum_{i=0}^{\infty} (\mu + \varepsilon_{t-i}).$$

But this solution cannot be a weakly stationary process since

- If μ is not null, y_t has an infinite expectation, (if μ is null, $E(y_t)=0$)
- Even if μ is null, the variance of y_t is infinite as an infinite summation of variance terms $Var(\varepsilon_t)$.

The usual test of the presence of a random walk is the Dickey-Fuller test. The test is constructed on the regression equation

$$y_t - y_{t-1} := \Delta y_t = \gamma y_{t-1} + \varepsilon_t \quad [2]$$

The presence of a random walk is revealed if γ is equal to 0. The classical test in this case is the Dickey-Fuller test and we have to use Dickey-Fuller critical values⁹.

We consider again the DJI returns series. As a previous study, we plot the auto-correlation function up to lag 30 (fig. 4). The auto-correlation function clearly indicates that it seems to have no significant auto-correlation in the series of returns of DJI, except for the first lag, which most probably is due to asynchronous trading, overall in the first part of the century¹⁰. This observation seems to indicate that the returns are not correlated and advocates for the presence of random walk (equation (2), with $\gamma = 0$).

⁸ In Econometry, this means that the process has an unit root.

⁹ The augmented version of the test allows to test for the presence of random walk in higher order auto-regressive processes.

¹⁰ Scholes and Williams (1977) analyze the consequence of non-synchronous data on the estimation of covariance of returns.

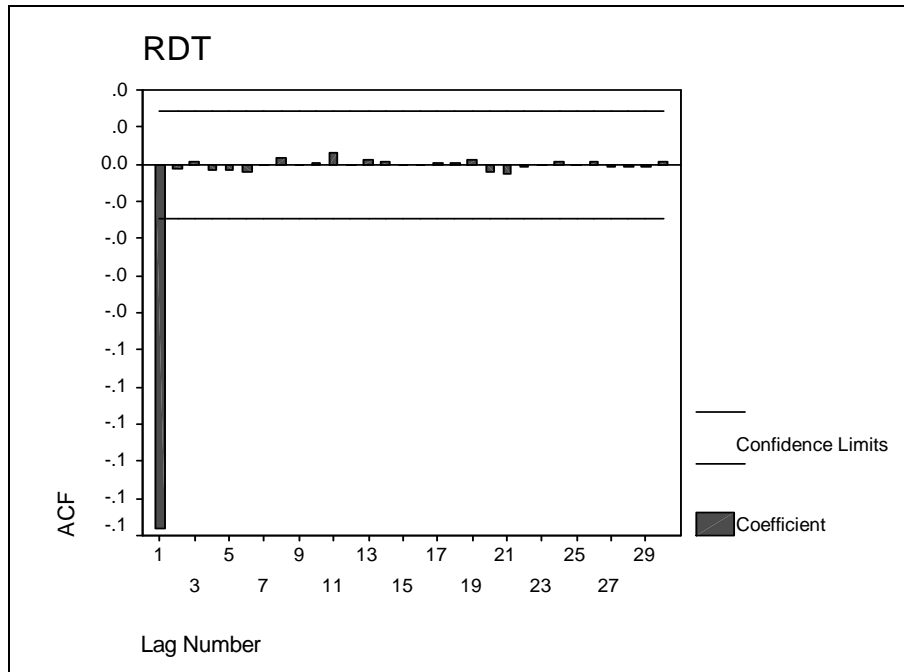


Figure 4. Autocorrelation function of the Dow Jones index returns.

However the hypothesis of random walk is rejected by the Dickey-Fuller test at a very high level of confidence ($> 99\%$). Those results conduct us to conclude, at this stage, that random walk model is not satisfactory for the DJI returns. Note that the absence of random walk do not allows to conclude to stationarity. Recent developments in this area have produced what is called "Long Memory Models", characterized by alternating positive and negative significant autocorrelations at very long lag. For example, Ding, Grander et *al.* (1993) find significant autocorrelations out to lags well over 2000 days in daily stock market returns. The main idea beyond these models is the notion of fractionally integrated processes, which are processes that lies somewhere between stationary and random walk.

5. Is the variance constant through time?

Engle (1982) put into light the notion of clustering of the variance in economic time series and introduces the ARCH model. The observation of fig. 2 clearly shows this feature of returns through time. The big depression, for example, between World War I and World War II seems clearly to be a period of high volatility. The ARCH(q) model is a model of the conditional variance of the noise ε_t :

$$\begin{cases} E[\varepsilon_t | F_{t-1}] = 0 \\ E[\varepsilon_t^2 | F_{t-1}] = h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \end{cases}$$

where $E(\cdot | F_{t-1})$ is the conditional expectation on the past of the process. Note that the $(\alpha_i)_{0 \leq i \leq q}$ are necessarily positive to ensure that the variance is a positive number.

A sufficient condition for the stationarity of the unconditional variance ($Var(\varepsilon_t) = E(\varepsilon_t^2)$) is :

$$\sum_{i=1}^q \alpha_i < 1.$$

For example, if we consider an ARCH(1) model, the necessary condition for the existence of a stationary solution of our model is $0 \leq \alpha_0$, $0 \leq \alpha_1 < 1$ and the unconditional variance of the noise will be

$$Var(\varepsilon_t) = \frac{\alpha_0}{1 - \alpha_1}.$$

Bollerslev (1986) introduces a generalization of the model in the form of a GARCH(p, q), which is defined as

$$\begin{cases} E[\varepsilon_t | F_{t-1}] = 0 \\ E[\varepsilon_t^2 | F_{t-1}] = h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j} \end{cases} \quad [3]$$

where $(\alpha_i)_{0 \leq i \leq q}$ and $(\beta_i)_{0 \leq i \leq p}$ are positive constants. As in the previous case, a sufficient condition for the existence of a stationary solution of (3) is

$$\sum_{i=1}^{\max(p, q)} (\alpha_i + \beta_i) < 1. \quad [4]$$

Note that this condition is not necessary since Nelson (1990) shows in the case of GARCH(1,1) model that for greater coefficients the stationarity can be ensured, but with a stationary solution which has an infinite variance.

With condition (4) for a GARCH(1,1), one has

$$Var(\varepsilon_t) = \frac{\alpha_0}{1 - (\alpha_1 + \beta_1)}$$

Table 2 presents results of a GARCH(1,1) estimate on the DJI returns series¹¹.

ALPHA0 = .762366	ALPHA1 = 1.000000	BETA1 = 0.
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Table 2.

The estimation is problematic. The inequality (4) is not verified. The persistence seems so high (the persistence is defined, in the framework of GARCH models, as the sum of α and β parameters) that we meet the limit of domain of validity of the model. In order to try to better understand the problem, we have decided to divide the period of analysis in 4 sub periods (1915-1928, from World War I to the Great Depression, 1929-1944, from the 1929 Crash to the end of World War II, 1945-1969, from the reconstruction up to the end the sixties and 1970-1990, from the petroleum crisis up to now). To identify each sub period, we have added three dummies in the variance equation (D_1 , D_2 and D_3), each one representing a shift in the unconditional variance. The model of the conditional variance is

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3 \quad [5]$$

and the results are presented at table 3.

ALPHA0 = .508330E-04	ALPHA1 = 1.000000	BETA1 = 0
GAMMA1 = .276468E-04	GAMMA2 = .110899E-03	GAMMA3 = 0

Table 3.

The same problem is met. Interestingly, the coefficients of the first and the second dummies (for which the estimation is realized) are highly significant. This corroborates the hypothesis of shifts in unconditional variances between those periods (with the unconditional variance being five times superior during the second period – the Great Depression than the first one). This result clearly contradicts the one of the previous section. If the unconditional variable changes through time, the DJI returns series is not weakly stationary on the observation period.

Numerous extensions of these models have been proposed in the econometric literature (see for example Hamilton (1994) for a review of them). The estimation is realized by the maximization of the log-likelihood function under the hypothesis of

¹¹ The GARCH(1,1) specification is the one that seems to best fit the behavior of stock market returns on most of published empirical works.

normality of the disturbance process¹². Let us emphasize at this point that GARCH models and their extensions are really non linear models of the process generating the data (y_t), allowing second order dependence in the variance. One of the interesting variants of the GARCH model is the one proposed by Glosten, et *al.* (1993). The variance equation is

$$h_t = \alpha_0 + (\alpha_1 + S_{t-1})\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad [6]$$

where S_{t-1} is a dummy variable which is equal to 1 if sign of ε_{t-1} is negative and 0 otherwise. That integrates an asymmetric effect of the bad news (which is captured by the sign of the previous disturbance) on the volatility.

To go one step ahead on the problem of high (too high) persistence, we present at table 4 the results obtained for GARCH(1,1) model with asymmetric effect (equation (6)). The estimation is presented on the 500 first observations (something like 2 years of data).

ALPHA0 = 45.2732	ALPHA1 = .281309	BETA1 = .425584
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Table 4.

This table shows a level of persistence far from one. The sum of α_i and β_i is 0.7 (interestingly, if we take the 1000 first observations, the sum is 0.9). It appears clearly that the longer is the estimation period, the higher the persistence. This raises in fact the question of structural breaks on the variance (the same kind of behavior is observed for ARMA models). Table 4 also shows that bad news have a lot more impact on the returns than good news (remember that S_t is equal to 1 when the disturbance is negative). This asymmetric effect of news is highly significant.

Two questions remains: why is there clustering in the variance of returns and is it possible to forecast future variance? Concerning the first point, several hypotheses have proposed and tested. It seems that the main reason is not clustering in the arrival of news on the market but well heterogeneity in the anticipations between investors (see for example Mordecai et *al.* (1999)). Can we forecast the variance? The title of the paper of Andersen et *al.* (1998), "Answering the Skeptics : Yes, Standard Volatility Models Do Provide Accurate Forecasts", seems to indicate that it is the case. Behind the title, the authors admit that, using only daily data and

¹² The case of non normality of disturbances is, under some restrictions, resolved by White (1982) and Gouriéroux, Montfort and Trognon (1982), who proposed the pseudo (or quasi) maximum likelihood approaches. Concerning the estimation procedure, several procedures have proposed in the literature to simplify the needed calculation (see for a review of them McCullough and Renfro (1999)).

being interested in ex-post observed variance, it is clearly not the case (at least by classical GARCH models). As shown for example by Blair *et al.* (1999) on the S&P100 index, the use of implied volatilities of options or the use of intra day data allows to get better results. These approaches impose however a stringent constraint on data availability.

6. Is it possible to forecast them?

Since the beginning of this century, the question of the predictability of financial series (at least of stock market prices) has been the subject of a highly controversial debate in finance. Fama (1965), in its seminal paper, recalls the meaning of the random walk hypothesis (first proposed by Bachelier (1900)) and presents different empirical tests of it. He concludes in those terms: "The main conclusion will be that the data seem to present consistent and strong support for the model. This implies, of course, that chart reading, though perhaps an interesting pastime, is of no real value to the stock market investor." The main theoretical argument of Fama is the notion of efficiency. On efficient market, all participants receive all information at any time. In such circumstances (very improbable in fact), the only reason for a stock price to move is the arrival of a new information, which is impossible to forecast by definition. With such an ideal representation of financial markets, what is surprising, it is the fact that most empirical works, mainly based on linear statistical tests, have conducted to the same conclusion in the years sixties and seventies, despite the heavy use of charts and technical indicators by the professional community.

However, as underlined by Campbell *et al.* (1997), "Recent econometric advances and empirical evidence seem to suggest that financial asset returns are predictable to some degree". Among those works, three of them have constituted main advances in this field. Brock *et al.* (1992) test two popular technical trading rules on the Dow Jones market index on the period going from 1897 to 1986. They use a bootstrap methodology to validate their results and conclude: "their results provide strong support for the technical strategies". Sullivan *et al.* (1999) propose new results on the same data set (extended with 10 new years of data). Their methodology, still relying on heavy use of bootstrap, allows avoiding (at least to some extent) the data-snooping bias (cfr *infra*) and is applied to a universe of 26 trading rules. They confirm that the results of Brock, Lakonishok and Le Baron stands up to inspection against data-snooping effects. The recent contribution of Lo, Mamaysky *et al.* (2000), using a new approach based on nonparametric kernel regression, confirms that "several technical indicators do provide incremental information and may have some practical value".

On the basis of all of those empirical evidences, we can consider that there is some interest in trying to predict the evolution of financial asset prices and that is

probably one of the potential field of application of neural networks¹³. This work must however be done very cautiously. The concept of data-snooping bias has been proposed by Lo *et al.* (1990) (at least in the financial literature). This bias in the evaluation of forecasting method appears as soon as a data set is used more than once for purposes of inference of model selection. It can be illustrated by the "Give me the data, I will give you the model" sentence. From our experience of the neural network modeling process, at our opinion, it is almost always present in our field. We do not present here an empirical results on the DJI index, partly by lack of place, partly because this index has been so much studied that any new result on past values of this series would be suspect from this point of view.

7. On the contribution of non parametric methods in asset pricing¹⁴.

Asset pricing has always been one of the key fields of research in finance. The most commonly quoted model is the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). Under a rather restrictive set of assumptions, the authors show that:

$$E(r_i) = E(r_m) \beta_i \quad [7]$$

In words, the expected excess return of a security ($E(r_i)$) is product the market risk premium ($E(r_m)$) times its β coefficient (note that we talk here about excess returns, this is to say returns minus the risk free rate). Applications of the model have been tremendous. For example, in corporate finance, $E(r_i)$, being the required rate of return of shareholders, allows us to determine the actualization necessary to evaluate the profitability of investment decisions.

Since its origination, the model has been subjected to numerous empirical tests. Evidences against this constant β model are well-known and largely documented. It has however been argued that conditional version of the model could hold even if the basic unconditional model does pass through empirical tests (see e.g. Dybbig *et al.*, 1985).

¹³ We have to stress also that a clear distinction must be made between the prediction of the evolution of a financial series and the possibility to win money using this prediction. This last point implies to build a trading strategy which, when we take into account all the transaction costs and the normal risk remuneration of the investor, is winning. This is a highly more complicated task than simply the prediction one, which must take into account the recent theoretical and empirical advances in finance on the subject of the time varying risk premia (see for example Chordia and Shivakumar (2000)).

¹⁴ This section is based on the recent contribution of Wang (forthcoming).

To introduce the conditional CAPM and the contribution of Wang (forthcoming), it is useful to adopt the Stochastic Discount Factor (SDF) framework¹⁵. Under it, the basic pricing equation of any asset is:

$$E_t(m_{t+1} p_{i,t+1}) = p_t \quad [8]$$

The price at time (p_t) is equal to the conditional expectation ($E_t()$) of the SDF (m_{t+1}) times the price at the next period ($p_{i,t+1}$). The conditional expectation is relative to all information available at time t . To obtain the relation in terms of returns, we simply both sides of eq. 8 by p_t :

$$E_t(m_{t+1} R_{i,t+1}) = 1 \quad [9]$$

where $R_{i,t+1}$ is the gross return ($p_{i,t+1}/p_t$). The conditional CAPM, in the SDF framework, amounts to stating that:

$$E_t(m_{t+1} r_{i,t+1}) = 0 \quad [10]$$

In words, the conditional expectation of the product of the SDF times the next period excess return is zero. Loosely said and in order to connect with the classical presentation of the CAPM, there is no intercept term (the Jensen alpha) in eq. 7. The connection between the classical β representation (eq. 7) and the SDF representation (eq. 10) is fully presented in Cochrane (2001).

The SDF is interesting because it allows us to focus on the determinants and the behavior of m_t , which is by nature time-varying. This is main focus of the Wang (forthcoming) contribution. The author directly tests eq. 10¹⁶, using a set of determinants to model the behavior of m_t through time. The choose variables are the dividend price ratio (DPR), the default premium (DEF), the treasure bill rate (TBR) and the market return (EWR). This choice is based on numerous previous researches in finance. While it is out of our scope to expose here fully the Wang methodology and results, the two key points, from our point of view, are the following:

- there are some strong theoretical arguments in Finance to state the relation between risk and return should be linear but, nothing is really known on the form of the relationship between SDF (or, almost equivalently, the time varying β) and its determinants. As the choice of a specific model (linear or not) would introduce the classical joint model problem (if the test reject the conditional CAPM, we do not know whether it is because the conditional CAPM is wrong or the chosen model is inadequate), Wang introduces the use a non parametric estimator to model the

¹⁵ Cocharne (2001) presents a in-depth investigation of it.

¹⁶ In fact, Wang presents the test of three models: CAPM, the Jagannathan and Wang (1996) model and the Fama and French (1993) model.

relation between the SDF and its determinants. The chosen estimation is the Rosenblatt-Parzen kernel density estimator. This is a key point because an alternative choice would have been the use neural networks such as Multi-Layer Perceptron, Radial Basis Function Network, ...

- the results presented by the author are particularly interesting. We only focus here on it the relation between the market factor β (the CAPM β) and its determinants. The result is presented at fig. 1 in the Wang paper. There are 4 panels. Each one presents the relation between the β (vertical axe) and one determinant (horizontal axe), at the mean level of the other determinants. The dashed lines are confidence region of estimations constructed by bootstrap. The results show that the relation when the time varying β and its determinants are clearly non linear.

These results give a clear and strong foundation to use of non linear modeling tools in asset pricing (and in all its applications) to capture the time varying behavior of expected returns.

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