

Task 2. Algorithms for unconstrained nonlinear optimization. Direct methods

Goal

The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss (coordinate descent), Nelder-Mead) in the tasks of unconstrained nonlinear optimization

Problems and methods

I. *Use the one-dimensional methods of exhaustive search, dichotomy and golden section search to find an approximate (with precision $\varepsilon = 0.001$) solution $x: f(x) \rightarrow \min$ for the following functions and domains:*

1. $f(x) = x^3, x \in [0, 1];$
2. $f(x) = |x - 0.2|, x \in [0, 1];$
3. $f(x) = x \sin \frac{1}{x}, x \in [0.01, 1].$

Calculate the number of f -calculations and the number of iterations performed in each method and analyze the results. Explain differences (if any) in the results obtained.

II. *Generate random numbers $\alpha \in (0,1)$ and $\beta \in (0,1)$. Furthermore, generate the noisy data $\{x_k, y_k\}$, where $k = 0, \dots, 100$, according to the following rule:*

$$y_k = \alpha x_k + \beta + \delta_k, \quad x_k = \frac{k}{100},$$

where $\delta_k \sim N(0,1)$ are values of a random variable with standard normal distribution. Approximate the data by the following linear and rational functions:

1. $F(x, a, b) = ax + b$ (linear approximant),
2. $F(x, a, b) = \frac{a}{1+bx}$ (rational approximant),

by means of least squares through the numerical minimization (with precision $\varepsilon = 0.001$) of the following function:

$$D(a, b) = \sum_{k=0}^{100} (F(x_k, a, b) - y_k)^2.$$

*To solve the minimization problem, use the methods of exhaustive search, Gauss and Nelder-Mead. If necessary, set the initial approximations and other parameters of the methods. Visualize the data and the approximants obtained in a plot separately for each type of **approximant** so that one can compare the results for the numerical methods used. Analyze the results obtained (in terms of number of iterations, precision, number of function evaluations, etc.).*