Cviko 1

$f: \mathbb{R}^n \to \mathbb{R}$ $f(x_1, \dots, x_n$

$\mathbb{R}^2 \to \mathbb{R}$ $f(x,y)$

$\mathbb{R}^3 \to \mathbb{R}$ $f(x,y,z)$

$\varphi: \mathbb{R} \to \mathbb{R}^2$ $\varphi(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$

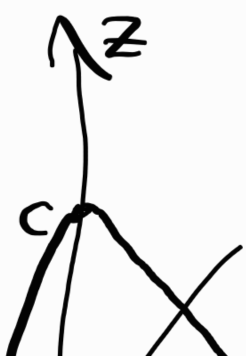
$\mathbb{R} \to \mathbb{R}^3$

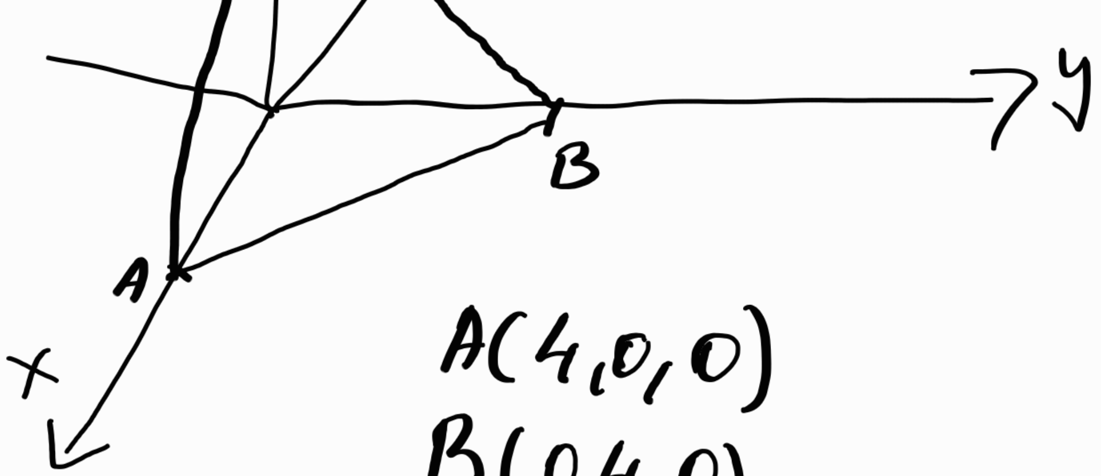
$= (x(t), y(t), z(t))$

$f(x,y) = 4 - x - y$ $D(f) = \mathbb{R}^2$

$H(f) = \mathbb{R}$

$f(4,0) = (4 - 4 - 0) = 0$






$A(4,0,0)$

$B(0,4,0)$

$C(0,0,4)$

$\vec{u} = \overrightarrow{AB} = B - A = (-4,4,0)$

$\vec{v} = \overrightarrow{AC} = C - A = (-4, 0, 4)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix}$$

$x = 4 - 4t - 4s$

$y = 4t$

$z = 4s$

$ax + by + cz + d = 0$

$\vec{n}(a, b, c)$

$x + y + z - 4 = 0$

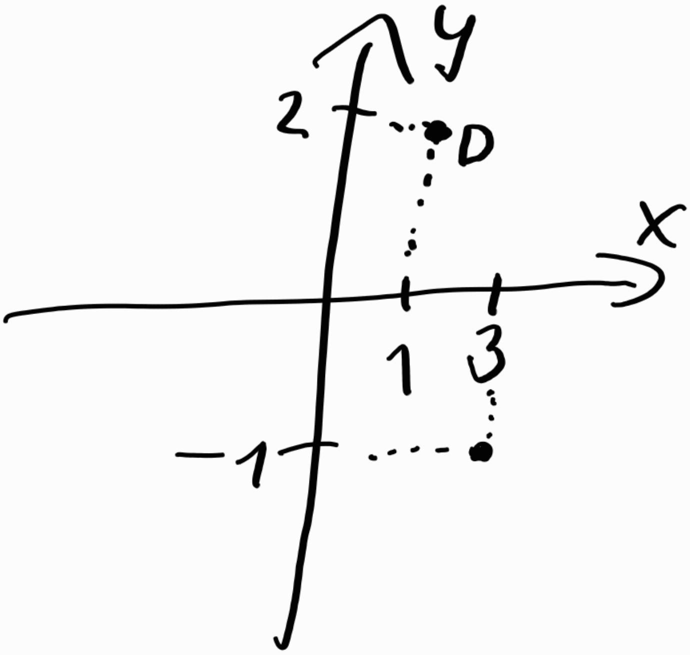
---

$$\varphi(t) = \begin{pmatrix} 1-2t \\ 2+3t \end{pmatrix}$$

$$\varphi(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 1-2t \\ 2+3t \end{pmatrix}$$

$$D(\varphi) = \mathbb{R}$$

$$t=0 \Rightarrow \varphi(0) = (x(0), y(0)) = (1,2)$$




Pr 3: $f(x,y) = \ln(-x-y)$

$$D(f) = \{(x,y) \in \mathbb{R}^2 : x+y<0\}$$

Pr 4

$$f(x,y) = \sqrt{4 - x^2 - y^2 + 2x - 4y}$$

$$4 - x^2 - y^2 + 2x - 4y \geq 0$$

$$D(f) = (0, \infty)$$

---

$$f(x,y,z) = \arccos(2x-1) + \sqrt{1-y^2} + \sqrt{y}$$
$$+ \ln(4 - z^2)$$

$$4 - z^2 > 0$$

$$-z^2 > -4$$

$$z^2 < 4$$

$$|z| < 2 \qquad\qquad y \geq 0 \qquad\qquad 1 - y^2 \geq 0$$

$$z \in (-2, 2) \qquad\qquad\qquad\qquad -y^2 \geq -1$$

$$y \leq \pm 1$$

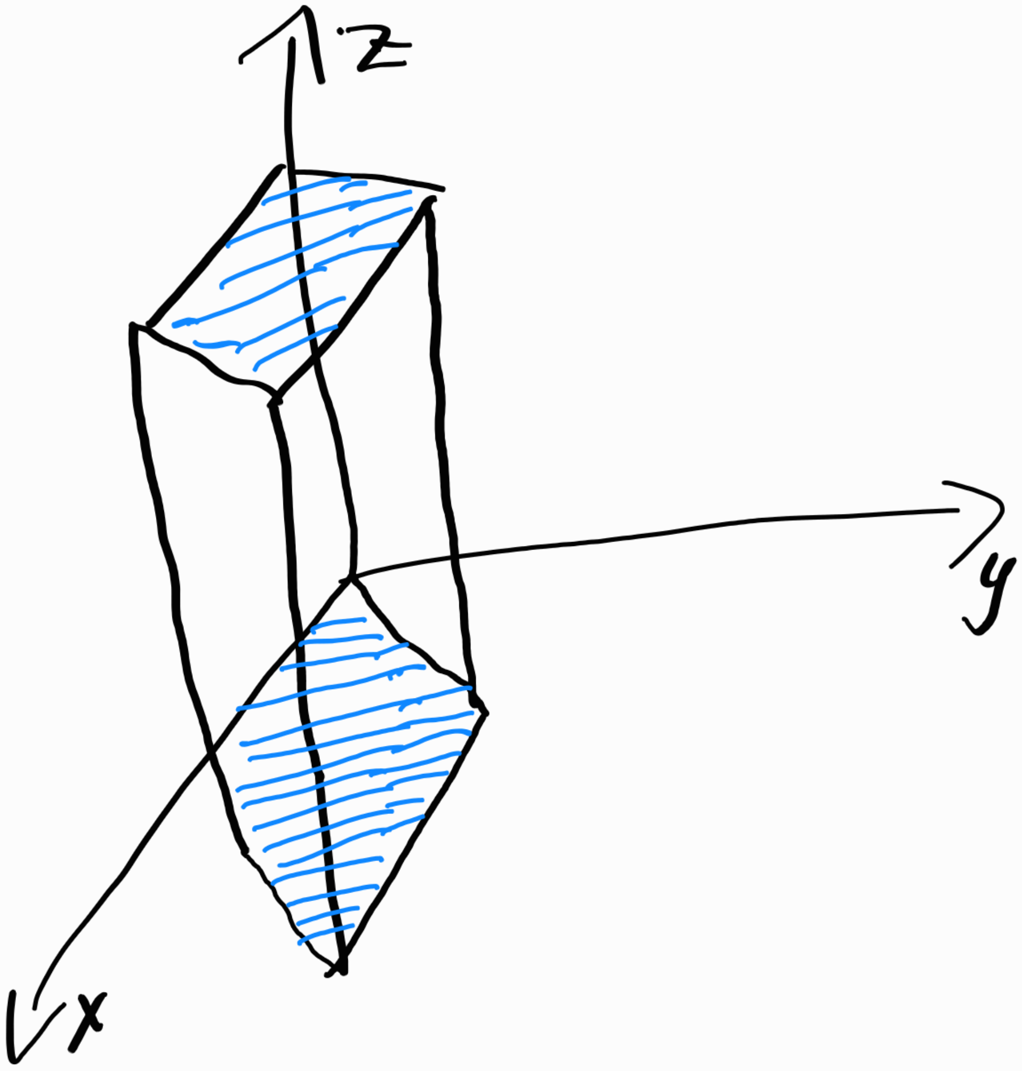
$$-1 \leq 2x - 1 \leq 1 \qquad\qquad\qquad 0 \leq y \leq 1$$

$$0 \leq 2x \leq 2 \quad :2$$

$0 \leq x \leq 1$

$x \leq 0$

$x \in (0, 1)$

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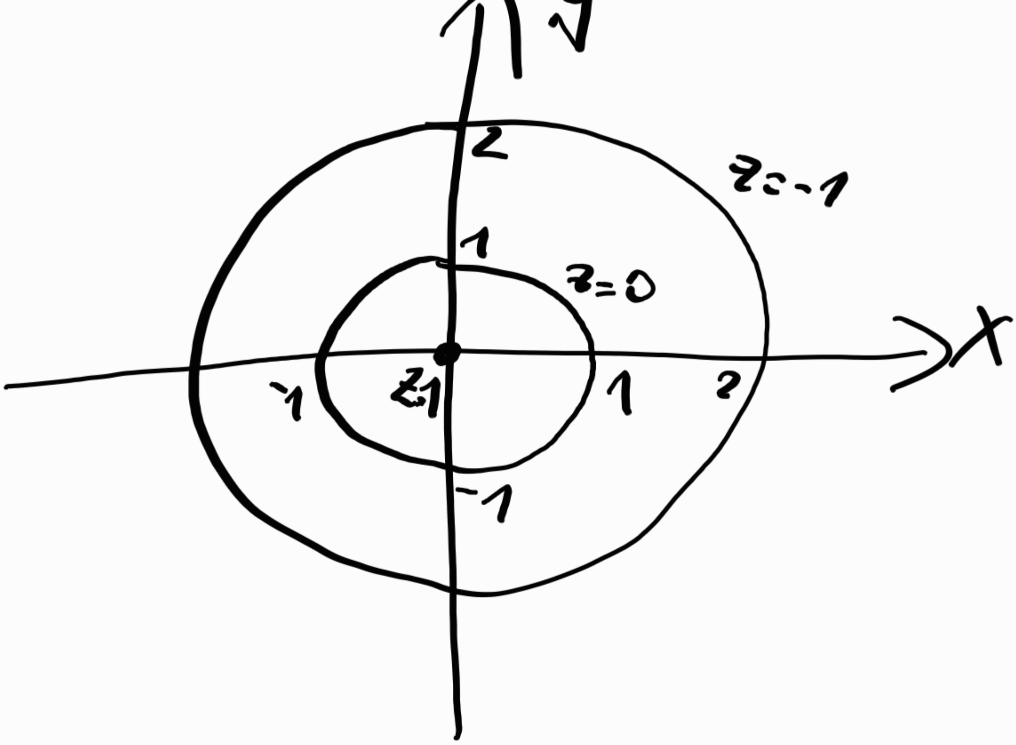
Pr 6 $f(x,y) = 1 - \sqrt{x^2+y^2}$

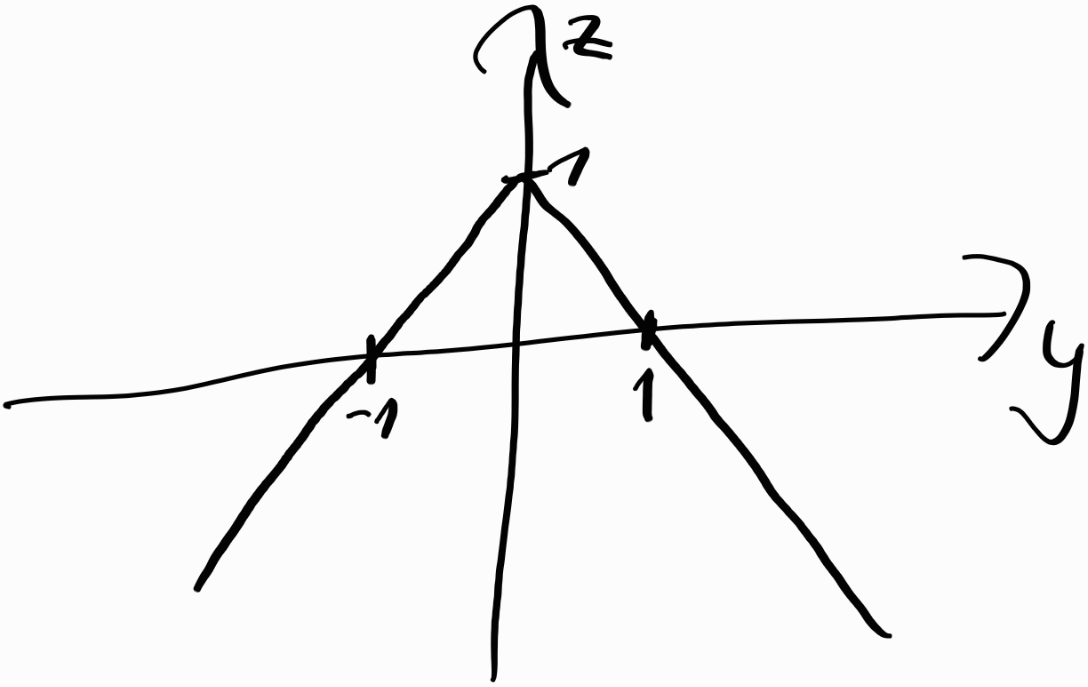
$D(f) = \mathbb{R}^2 \quad H(f) = (-\infty, 1\rangle$

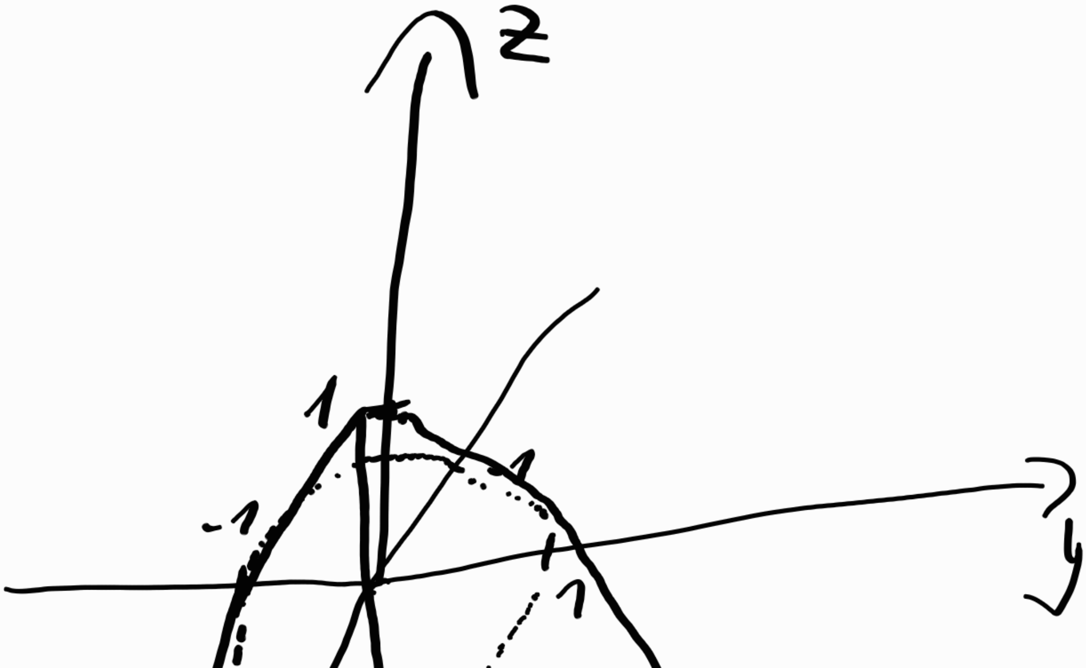
$z = f(x,y)$

$z = 0 \qquad 0 = 1 - \sqrt{x^2+y^2}$

$$\boxed{x^2+y^2 = 1}$$


y
2
z=-1
1
z=0
x
-1
1
2
-1


z
1
y
-1
1


z
1
1
-1
y
1


Pr. 7

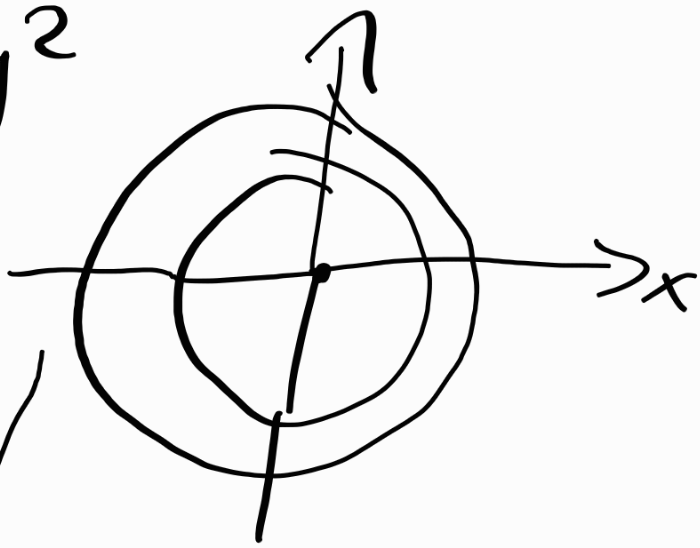
$$f(x,y) = x^2 + y^2$$

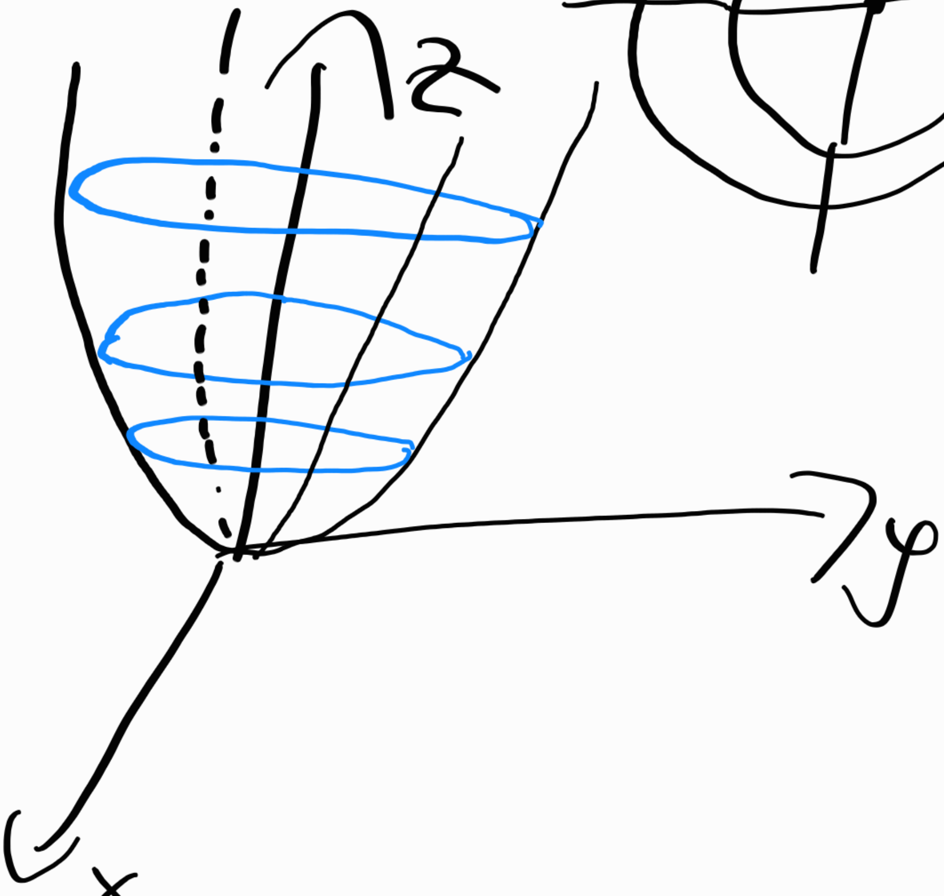
$D(f) = \mathbb{R}^2 \qquad H(f) = \langle 0, \infty)$

$z = f(x,y)$

$z = 0 \qquad 0 = x^2 + y^2$

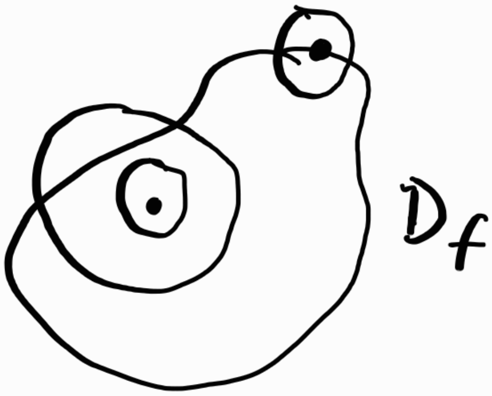
$z = 1$



Cviko 2 $\quad \mathbb{R}^2 \to \mathbb{R}$

$$\lim_{(x,y)\to(0,0)} f(x,y) = b$$



$$\lim_{(x,y)\to(2,3)} \frac{x}{x+y} = \frac{2}{5}$$

$$\lim_{(x,y)\to(0,0)} \frac{x}{x+y} \qquad D(f) = \{(x,y)\in\mathbb{R}^2 : y \neq -x\}$$

$$\varphi_1(t) = (x_1(t), y_1(t)) = (t,0)$$

$$\lim_{t\to 0} \frac{t}{t+0} = 1$$

$$\varphi_2(t) = (0,t) \qquad \text{limita neexistuje}$$

$$\lim \frac{0}{0+t} = 0$$

$t \to 0 \quad 0+t$

$y = kx \qquad k \neq -1$

$\varphi(t) = (t, kt)$

$$\lim_{t \to 0} \frac{t}{t+kt} = \lim_{t \to 0} \frac{t}{t(1+k)} = \frac{1}{1+k}$$

$$\lim_{(x,y) \to (0,0)} \frac{x+y}{x-y} \quad Df: x-y \neq 0$$

$x \neq y$

$\varphi_1(t) \quad (t, 0) \quad \frac{t+0}{t-0} = 1$

$\varphi_2(t) \quad (0, t) \quad \frac{0+t}{0-t} = -1$

---

$$\lim_{(x,y) \to (2,3)} \frac{y-3}{x+y-5}$$

$(2,3)$

$\varphi_1(t) = (t+2, t+3)$

$$\frac{t}{2t} = \frac{1}{2}$$

$$\varphi_2(t) = (2, t+3)$$

$$\lim_{t \to 0} \frac{t}{t} = 1$$

---

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$$

$$\varphi_1(t) = (0, t)$$

$$\varphi_2(t) = (t^2, t)$$

$$\varphi_1: \frac{0 \cdot t^2 = 0}{\underset{\neq 0}{0 + t^4}} = 0$$

$$\varphi_2(t) = \frac{t^2 \cdot t^2 \; \cancel{t^4}}{t^4 + t^4 \; \cancel{t^4} \cdot (2)}$$

$$\lim_{(x,y)\to(-2,1)} \frac{(2x+y)^2 - 9\ 3^2}{4xy + 2y^2 + 6y} = \frac{1}{2}$$

$$\frac{\cancel{(2x+y+3)} \cdot (2x+y-3)}{2y \cdot \cancel{(2x+y+3)}}$$

$$\frac{2\overset{-2}{x}+\overset{1}{y}-3}{2y} \quad \frac{-6}{2}=-3$$

---

$$\lim_{(x,y)\to(3,4)} \frac{4-\sqrt{x+3y+1}}{15-x-3y}$$

$D_f \quad x+3y\geq 1$

$x \geq -1-3y$

$\wedge \quad 15-x-3y\neq 0$

$-x \neq 3y-15$

$x\neq 15-3y$

$15-x-3y = -x-3y-1+16$

$-(x+3y+1)+16$

$$\frac{\left(4-\left(\sqrt{x+3y+1}\right)\right)\cdot\left(4+\left(\sqrt{x+3y+1}\right)\right)}{\left(16-(x+3y+1)\right)\cdot\left(4+\left(\sqrt{x+3y+1}\right)\right)}$$

$\cancel{16-}(x+3y+1)$

$$\frac{(16-(x+3y+1))}{(16-(x+3y+1))(4+\sqrt{x+3y+1})}$$

$$\frac{1}{4+\sqrt{x+3y+1}}$$

$$\frac{1}{4+\sqrt{\underbrace{3+3\cdot 4}_{12}+1}}$$

$$4+\overbrace{4}^{\sqrt{16}}=8$$

$$\lim_{(x,y)\to(0,6)} \frac{\sin(x\cdot y)}{x}\cdot\frac{y}{y}$$

$$\underbrace{y}_{\to 6}\cdot\underbrace{\frac{\sin(x,y)}{x\cdot y}}_{\to 1}$$

$$\lim_{w\to 0}\frac{\sin(w)}{w}=1$$

$$\lim_{(x,y)\to(4,0)} \frac{tg(3xy)}{y} = \frac{\frac{\sin(3xy)}{\cos(3xy)}}{\frac{y}{1}}$$

$$= \frac{\sin(3xy)\cdot 3x}{\cos(3xy)\cdot 3xy} = \frac{3x}{\cos(3xy)} = \frac{12}{1} = 12$$

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\cdot \sin\left(\frac{1}{xy}\right)$$

$0$ $\quad -1 \le \sin\left(\frac{1}{xy}\right) \le 1$

$$\lim_{(x,y)\to(0,0)} \frac{e^{-\frac{1}{x^2+y^2}}}{x^2+y^2} \qquad w = \frac{1}{x^2+y^2}$$

$$\lim_{w\to\infty} w\cdot e^{-w} = \lim_{w\to\infty} \frac{e^{-w}}{\frac{1}{w}} =$$

$\lim$ $\quad xy$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x-y}$$

$$\varphi_1(t) = (t,0)$$

$$\frac{t\cdot 0}{t-0} = \frac{0}{1} = 0$$

$$\varphi_1(t) = (0,t)$$

$$\frac{0}{-t} = 0$$

$$\varphi_3(t) = (t,-t)$$

$$\frac{t\cdot(-t)}{2t} = \frac{-t^2}{2t} = \frac{-t}{2}$$

$$= 0$$

$$\varphi_4(t) = (t^2, t)$$

$$\frac{t^2 \cdot t}{t^2 - t} = \frac{t^3}{t^2 - t} = \frac{t^2}{t - 1} = 0$$

$$\varphi_5(t) = (t, t + t^2) = -1$$

$$\cancel{\nexists}$$

---

Cviko 3

$$f(x,y) = \begin{cases} \dfrac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} & \text{pre } (x,y) \neq (0,0) \\ 2 & \text{pre } (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y) \to (0,0)} f(x,y) \stackrel{?}{=} f(0,0)$$

$$\lim_{(x,y) \to (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} \cdot \left( \frac{\sqrt{x^2 + y^2 + 1} + 1}{\sqrt{x^2 + y^2 + 1} + 1} \right)$$

$$= -\|- \frac{\cancel{(x^2+y^2)} \cdot \left(\sqrt{x^2+y^2+1}+1\right)}{\cancel{\left(x^2+y^2-1+1\right)}} = 2 = f(0,0)$$

$$f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2y^2+(x-y)^2}; & pre\ (x,y) \neq (0,0) \\ 0 & pre\ (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+(x-y)^2}$$

$$x^2 - 2xy + y^2$$

$$\frac{\cancel{x^2y^2}}{\cancel{x^2y^2} \cdot (1 - 2xy)} = \frac{1}{1-2xy}$$

$$\varphi(t) = (t, 0)$$

$$\lim_{(x,y)\to(0,0)} \frac{t^2 \cdot 0}{0 + t^2} = \frac{0}{t^2} = 0$$

Táto limita neexistuje

Nieje

$$\varphi_1(t) = (t, t)$$

$$t^2 \cdot t^2$$

$$\frac{}{t^2t^2+(\cancel{t}^{0}-t)^2} = 1$$ to spojité v Bode $(0,0)$

## Príklad 3

$f(x,y)=(y-x)^2-1$

$A=(1,0,0)$

$(x_0,y_0,f(x_0,y_0))$

$$f_x'(x_0,y_0)=\lim \frac{f(x,y_0)}{x-x_0}$$

$$f_x'(1,0)=\lim_{x\to 1}\frac{f(x,0)-f(1,0)}{x-1}$$

$g(x)=f(x,0)=x^2-1$

$g'(x)=2x \to g'(1)=2$

$$\lim_{x\to 1}\frac{x^2-1-0}{x-1} \quad x+1=2$$

$$f_y'(x_0,y_0)=\lim_{y\to y_0}\frac{f(x_0,y)-f(x_0,y_0)}{y-y_0}$$

• Lebo je $-x$

$f_x'(x,y)=2\cdot(y-x)\cdot(-1)$

$(y-x)\cdot(-1) = -2\cdot(y-x)$

$$f'_y(x,y) = 2\cdot(y-x)$$

---

$$z - 0 = 2\cdot(x-1) - 2\cdot(y-0)$$

$$z = 2x - 2y - 2$$

$$f(x,y) = x^4 + 2xy + y^3$$

$$T = (-1, 1, ?)$$

0

$$z_0 = f(-1,1)$$

$$f'_x(x,y) = 4x^3 + 2y$$

$$f'_y(x,y) = 2x + 3y^2$$

-4

-1 +

$$f'_x(-1,1) = 4\cdot(-1)^3 + 2 = -2$$

-2 3

$$f'_y(-1,1) = 2\cdot(-1) + 3 \ldots$$

$$\ldots) + 3\cdot(1)^2 = +1$$

$$z = -2\cdot(x+1) + 1\cdot(y-1)$$

$$f(x,y) = \sqrt{|xy|}$$

$$f'_x(0,0) = 0$$

$$f'_y(0,0) = 0$$

$$\lim_{(x,y)\to(0,0)} \sqrt{\frac{|x\,y|}{x^2+y^2}}$$

Df

$$\varphi(t) = (t, t)$$

$$\lim_{t\to 0^+} \sqrt{\frac{t^2}{t^2+t^2}} = \frac{1}{\sqrt{2}}$$

---

Cviko 4

$$f(x,y) = \sqrt{|xy|}$$

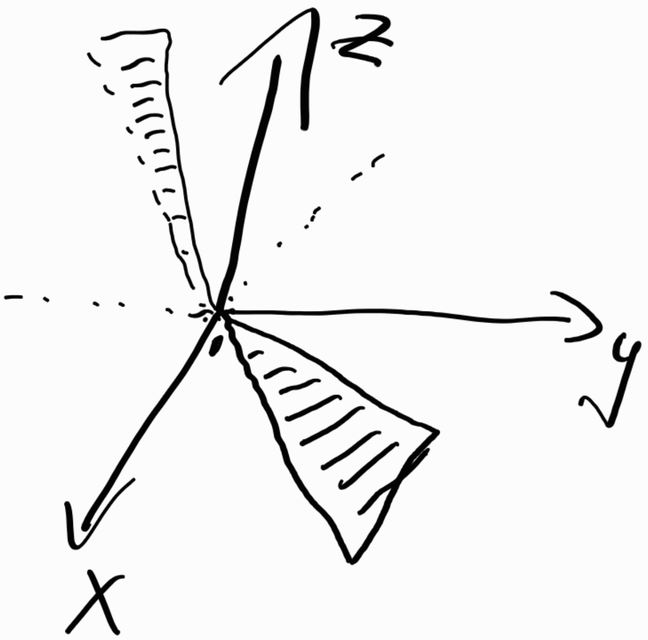
$f(x,y) = \sqrt{|xy|}$

$D(f) = \{(x,y) \in \mathbb{R}^2\}$

$D(f) = \mathbb{R}^2$

$$f'_x(0,0) = \lim_{x \to 0} \frac{0-0}{x-0} = 0$$

$f'_y(0,0) = 0$




$y = x$

$f(x,x)\sqrt{|x^2|} = \sqrt{x^2} =$

$|x|$

$y = 2x$

$f(x,2x) = \sqrt{2x^2} = \sqrt{2}|x|$

$f$ je diferencovatelná v $(x_0, y_0)$

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{|xy|}\ 0-0-0}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{\sqrt{|xy|}}{\sqrt{x^2+y^2}}$$

$\varphi(t) = (t,t)$

$$\lim_{t\to 0} \sqrt{\frac{t^2}{2t^2}} = \frac{1}{\sqrt{2}}$$

$$f(x,y) = \begin{cases} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} & \text{pre } (x,y) \neq (0,0) \\ 1 & \text{pre } (x,y) = (0,0) \end{cases}$$

$$f_x'(0,0) = \lim_{x\to x_0} \frac{\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} - 1}{x-0} = \frac{1}{0}$$

parcialna derivacia v bode x neexistuje

$$f(x) \begin{cases} \dfrac{xy^2}{x^2+y^4} & \text{pre } (x,y) \neq (0,0) \\ 0 & \text{pre } (x,y) = (0,0) \end{cases}$$

$$f_x'(0,0) = \lim_{x\to 0} \frac{0-0}{x-0} = 0$$

$$f_y'(0,0) = \lim_{y\to 0} \frac{0-0}{y-0} = 0$$

$$f(x,y) = \begin{cases} (x^2+y^2)\cdot \sin\left(\dfrac{1}{x^2+y^2}\right); & \text{pre } (x,y) \neq (0,0) \\ 0 & \text{pre } (x,y) = (0,0) \end{cases}$$

$(x,y) = (0,0)$

$$f'_x(0,0) = \lim_{x \to 0} \frac{f(x,0) - f(0,0)}{x-0} = \frac{x^2 \cdot \sin \frac{1}{x^2} - 0}{x - 0} = 0$$

$$f'_y(0,0) = \lim_{y \to 0} \frac{f(0,y) - f(0,0)}{y-0} = 0$$

$$f'_x(x,y) = (x^2+y^2)' \cdot \sin \frac{1}{x^2+y^2} + (x^2+y^2) \cdot \sin\left(\frac{1}{x^2+y^2}\right)' =$$

$$2x \cdot \sin\left(\frac{1}{x^2+y^2}\right) + (x^2+y^2) \cdot \cos\left(\frac{1}{x^2+y^2}\right) \cdot \frac{(-1) \cdot 2x}{(x^2+y^2)}$$

$$\lim_{(x,y) \to (0,0)} \frac{(x^2+y^2) \cdot \sin \frac{1}{x^2+y^2} - 0 - 0 - 0}{\sqrt{x^2+y^2}} =$$

$$\lim_{(x,y) \to (0,0)} \sqrt{x^2+y^2} \cdot \underbrace{\sin\left(\frac{1}{x^2+y^2}\right)}_{\text{ohr}} = 0 \Rightarrow \text{je diferencovateľná}$$

Príklad 6

$$f(x,y) = x^2 e^y \qquad D(f) = \mathbb{R}^2$$

$$f'_x(x,y) = 2x e^y \qquad (2,0) = 4$$

$$f'_y(x,y) = x^2 e^y \qquad (2,0) = 4$$

$$T = (2,0,4) \qquad \tau_T :$$

$x_0$ $y_0$ $z_0$

$$\boxed{z-4=4(x-2)+4y}$$

$\nabla f(x_0,y_0)=(f'_x(x_0,y_0).f'_y(x_0,y_0)$

$\vec{n}=(f'_x(x_0,y_0); f'_y(x_0,y_0); -1)$

$4x+4y-z-4=0$

$(1{,}94)^2 \cdot e^{0{,}12}$ $\quad f(1{,}94;\ 0{,}12)$

$z=4\cdot(x+y-1)$

$z(x,y)=4\cdot(1{,}94+0{,}12-1)=4{,}24$

$f(x,y)=\ln\left(\frac{x}{y}\right)$ $\quad \frac{x}{y}=\left(\frac{1}{y}\right)\cdot x$

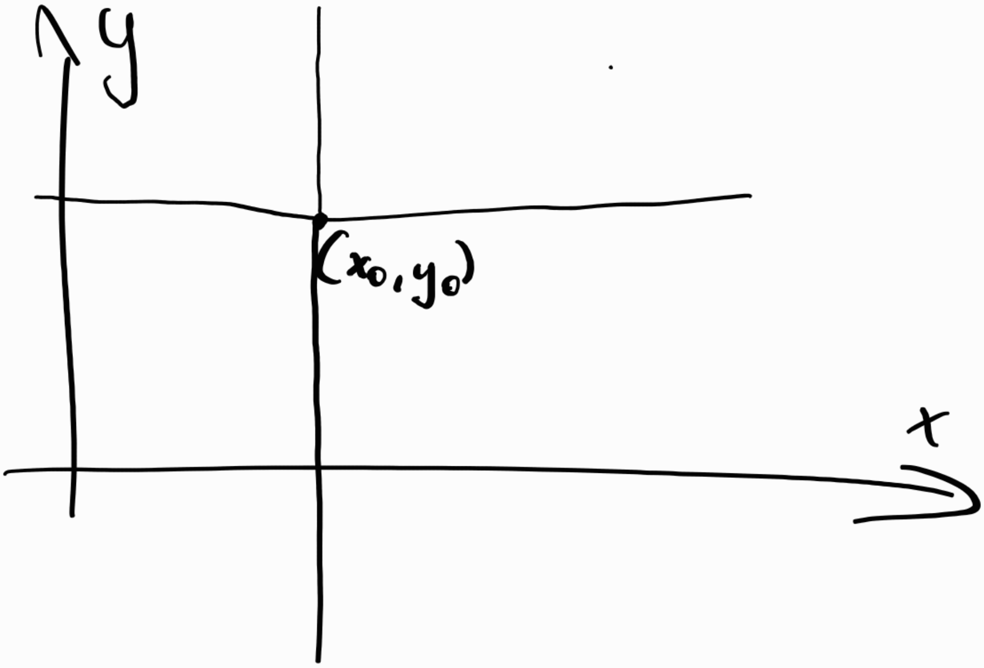
$f'_x(x,y)=\frac{1}{\frac{x}{y}}\cdot\frac{1}{y}=\frac{1}{x}$

$f'_y(x,y)=\frac{1}{\frac{x}{y}}\cdot\frac{x}{1}=$

$f(x,y)=\cos\cdot(x^2+2y)$

$$f(x,y) = \frac{\quad}{3x + y^2}$$

$$f_x' = \frac{-\sin(x^2+2y)\cdot(2x)\cdot(3x+y^2) - (\cos\cdot(x^2+2y)\cdot 3)}{(3x+y^2)^2}$$




$$f_x'(x_0,y_0) = \lim \frac{f(x,y_0)+f(x_0,y_0)}{x-0} = \lim_{t\to 0} \frac{f(x_0+t,y_0)-f(x_0,y_0)}{t}$$

$$(x_0+t, y_0) = (x_0,y_0)+t(1,0)$$

$$f_y'(x_0,y_0) = \lim_{t\to 0} \frac{f[(x_0,y_0)+t(0,1)]-f(x_0,y_0)}{t}$$

$$|\vec{e}| = \sqrt{e_1^2+e_2^2}$$

$$\frac{df}{d\vec{e}}(x_0,y_0) = \lim \frac{f[(x_0,y_0)+t(e_1,e_2)]-f(x_0,y_0)}{t}$$

$$f(x,y) = x^2 + y^2$$

$(-1,1)$

$$f'_x(x,y) = 2x \Rightarrow f'_x(-1,1) = -2$$

$$f'_y(x,y) = 2y \Rightarrow f'_y(-1,1) = 2$$

---

## $\boxed{\text{Cviko 5}}$

$$F(u,v) = F(u(x,y), v(x,y))$$

$$u(x,y) = x y^2$$

$$v(x,y) = x^2 + y^2$$

$$F(u,v) = 2 \ln u + v$$

$$\frac{\partial F}{\partial x}(x,y) = \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial F}{\partial v} \frac{\partial v}{\partial x}$$

$$\frac{\partial F}{\partial y}(x,y) = \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial F}{\partial v} \frac{\partial v}{\partial y}$$

$$\frac{\partial F}{\partial x}(x,y) = \frac{2}{u} y^2 + 2x = \frac{2}{x} + 2x$$

$$\frac{\partial f}{dy}(x,y) = \frac{2}{u}(2xy)+1 = \frac{4}{y}+1$$

$$f(x,y) = 2\ln(xy^2) + x^2 + y$$

$$f'_x = 2 \cdot \frac{1}{xy^2} \cdot y^2 + 2x = \frac{2}{x} + 2x$$

$$f'_y = \frac{2}{xy^2}(2xy)+1 = \frac{4}{y}+1$$

---

$$f(x,y) = \varphi \cdot (y e^{-x})$$

$\varphi$ - ľubovolná diferencovateľná funkcia

$$\boxed{f'_x + y \cdot f'_y = 0}$$

$$\varphi(u) = \sin u$$

$$\varphi(u) = 1 \quad \varphi(u) = u$$

$$f'_x = \varphi'(y e^{-x}) \cdot y \cdot e^{-x} (-1)$$

$$f'_y = \varphi'(y e^{-x}) \; e^{-x}$$

$y \quad (ye) \cdot e$

$$\varphi'\cancel{(ye^{-x})} \cdot (-y) \cdot e^{-x} = -y \cdot \cancel{\varphi(ye^{-x})} \cdot e^{-x}$$

$$-y \cdot e^{-x} = -y \cdot e^{-x}$$

---

$$f(x,y) = x^2 + xy + y^2$$

$$\frac{\partial f}{\partial x}(x,y) = 2x + y \qquad f'_y(x,y) = x + 2y$$

$$f''_{xx}(x,y) = 2 \qquad f''_{yx}(x,y) = 1$$

$$f''_{xy}(x,y) = 1 \qquad f''_{yy}(x,y) = 2$$

$$d^2 f\big((x_0,y_0);(x,y)\big) = f''_{xx}(x_0,y_0)(x-x_0)^2 + 2f''_{xy}(x_0,y_0)\cdot(x-x_0)\cdot(y-y_0) + f''_{yy}(x_0,y_0)(y-y_0)^2$$

$$d^2 f((x_0,y_0))(h,k) = 2h^2 + 2hk + 2k^2$$

$$\bar{a} = (0,0) \qquad \nabla f(\bar{a}) = (0,0)$$

$$T_2\big((x_0,y_0);(x,y)\big) = f((\ldots$$

$$\ldots f(x_0,y_0) + df((x_0,y_0),(x,y)) + \frac{1}{2}$$

$$d^2f((x_0,y_0),(x,y))$$

$$T_2(\bar{a}(x,y)) = f(0,0) + 0 + \frac{1}{2}(2x^2+2xy+2y^2) =$$

$$x^2+xy+y^2$$

každý bod $\to (x_0,y_0)$

$$T_2((x_0,y_0);(x,y)) = x_0^2 + x_0y_0 + y_0^2 + (2x_0+y_0)\cdot(x-x_0) +$$

$$(x_0+2y_0)\cdot(y-y_0) + (x-x_0)^2 + (x-x_0)\cdot(y-y_0) + (y-y_0)^2$$

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$$f(x,y) = x^3+y^3+3xy+2$$

$$f_x'(x,y) = 3x^2+3y \quad = 0$$

$$f_y'(x,y) = 3y^2+3x \quad = 0$$

$$\cancel{3}x^2 = -\cancel{3}y$$

$$x^2 = -y$$

$$y = -x^2$$

$$\cancel{3}(-x^2)^2 + \cancel{3}x$$

$$+x^4+x$$

$$x\cdot(x^3+1)$$

$$x_1 = 0 \quad y_1 = 0$$

$$x_2 = -1 \quad y_2 = -1$$

$$f''_{xx}(x,y) = 6x$$

$$f''_{yx}(x,y) = 3$$

$$f''_{xy}(x,y) = 3$$

$$f''_{yy}(x,y) = 6y$$

[0,0]

$$\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$$

[-1, -1]

$$\begin{pmatrix} -6 & 3 \\ \ldots \end{pmatrix}$$

$$\begin{pmatrix} 3 & -6 \end{pmatrix}$$

$$d^2 f((-1,-1))(h,k) = -6h^2 + 6hk - 6k^2$$

$$d^2 f((0,0))(h,k) = 6hk$$

$$-6\cdot(h^2 - hk + k^2) = -6\cdot\left(h^2 2h\frac{k}{2} + \frac{k^2}{4} + \frac{3}{4}k^2\right) -$$

$$-6\left(\left(h-\frac{k}{2}\right)^2 + \frac{3}{4}k^2\right) < 0$$

$$\overset{2}{f(0,0)} + 0 + \frac{1}{2}\cdot 6x_1y = 2+3xy$$

$$\overset{3}{f(-1,-1)} + 0 + \frac{1}{2}\cdot\left(-6\cdot(x+1)^2 + 6\cdot(x+1)\right.$$

$$\left.\cdot(y+1)\right] - 6\cdot(y+1)^2\Big) = 3 + \left(-3\cdot(x+1)^2 + 3(x+1)\cdot\right.$$

$$\left.(y+1) - 3(y+1)^2\right)$$