bon-db/combi/Z624F5.json (IMC 2023 D2 P1)

Problem. Ivan writes the matrix $\begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}$ on the board. Then he performs the following operation on the matrix several times:

1. he chooses a row or column of the matrix, and
2. he multiplies or divides the chosen row or column entry-wise by the other row or column, respectively.

Can Ivan end up with the matrix $\begin{pmatrix} 2 & 4 \\ 2 & 3 \end{pmatrix}$ after finitely many steps?

Solution by **Gryphos** (#2 on the thread).

Solution. Consider the entrywise logarithms of the matrix. In each step, we either add one row/column to the other or subtract one row/column from the other. These operations do not change the determinant of this log-matrix. Since

$$\det \begin{pmatrix} \log 2 & \log 3 \\ \log 2 & \log 4 \end{pmatrix} \neq \det \begin{pmatrix} \log 2 & \log 4 \\ \log 2 & \log 3 \end{pmatrix},$$

it is not possible to obtain the matrix $\begin{pmatrix} 2 & 4 \\ 2 & 3 \end{pmatrix}$. ■