# bon-db/alg/ineq/Z9D631.json (ISI B.Stat. 2007 P5)

**Problem.** Show that

$$-2 \leq \cos\theta \left(\sin\theta + \sqrt{\sin^2\theta + 3}\right) \leq 2$$

for all values of $\theta$.

Solution by **mcrasher** (#2 on the thread).

*Solution.* From Cauchy–Schwarz inequality,

$$\begin{aligned}\left(\cos\theta \left(\sin\theta + \sqrt{\sin^2\theta + 3}\right)\right)^2 &= \left(\cos\theta\sin\theta + \cos\theta\sqrt{\sin^2\theta + 3}\right)^2 \\ &\leq (\sin^2\theta + \cos^2\theta)\cdot(\cos^2\theta + \sin^2\theta + 3) \\ &= 4\end{aligned}$$

which implies $-2 \leq \cos\theta \left(\sin\theta + \sqrt{\sin^2\theta + 3}\right) \leq 2$, as desired. ■