

**bon-db/calculus/diff/Z889BD.json (Putnam 2015 B1)**

**Problem.** Let  $f$  be a three times differentiable function (defined on  $\mathbb{R}$  and real-valued) such that  $f$  has at least five distinct real zeros. Prove that  $f + 6f' + 12f'' + 8f'''$  has at least two distinct real zeros.

**Solution** by [ZETA\\_in\\_olympiad](#) (#30 on the thread).

*Solution.* Let  $g(x) = e^{x/2}f(x)$ . Then  $g$  has at least five distinct zeroes. By Rolle's Theorem,  $g'$ ,  $g''$ , and  $g'''$  have at least four, three, and two distinct real zeroes, respectively. Since

$$g'''(x) = \frac{1}{8}e^{x/2}(f(x) + 6f'(x) + 12f''(x) + 8f'''(x))$$

and  $e^{x/2}$  is never zero, we're done. ■