

bon-db/calculus/seq/Z14689.json (Putnam 1966 A3)

Problem. Let $0 < x_1 < 1$ and $x_{n+1} = x_n(1 - x_n)$, $n = 1, 2, 3, \dots$. Show that

$$\lim_{n \rightarrow \infty} nx_n = 1.$$

Solution by **Tintarn** (#2 on the thread).

Solution. First of all, it is clear that $x_n \in (0, 1)$ for all n and hence $x_{n+1} < x_n$ and hence x_n converges and hence $\lim_{n \rightarrow \infty} x_n = 0$. Next, by Stolz-Cesaro, using $\frac{1}{x_{n+1}} = \frac{1}{x_n} + \frac{1}{1-x_n}$, we have

$$\lim_{n \rightarrow \infty} \frac{1}{nx_n} = \lim_{n \rightarrow \infty} \left(\frac{1}{x_{n+1}} - \frac{1}{x_n} \right) = \lim_{n \rightarrow \infty} \frac{1}{1-x_n} = 1. \quad \blacksquare$$