

**bon-db/alg/ineq/Z9D631.json (ISI B.Stat. 2007 P5)**

**Problem.** Show that

$$-2 \leq \cos \theta \left( \sin \theta + \sqrt{\sin^2 \theta + 3} \right) \leq 2$$

for all values of  $\theta$ .

**Solution** by **mcrasher** (#2 on the thread).

*Solution.* From Cauchy–Schwarz inequality,

$$\begin{aligned} \left( \cos \theta \left( \sin \theta + \sqrt{\sin^2 \theta + 3} \right) \right)^2 &= \left( \cos \theta \sin \theta + \cos \theta \sqrt{\sin^2 \theta + 3} \right)^2 \\ &\leq (\sin^2 \theta + \cos^2 \theta) \cdot (\cos^2 \theta + \sin^2 \theta + 3) \\ &= 4 \end{aligned}$$

which implies  $-2 \leq \cos \theta \left( \sin \theta + \sqrt{\sin^2 \theta + 3} \right) \leq 2$ , as desired. ■