Name:
ID:
Date:
ITU, Computer Engineering Dept.
BLG527E, Machine Learning HW1
Due: March 28, 2024, 23:00 through Ninova.

NO LATE SUBMISSION WILL BE ACCEPTED. DO NOT SUBMIT THROUGH E-MAIL.

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Grading: You must complete the table below according to what you expect to get out of each question. At the beginning of your report you should give the following table.

		Q1	Q2	Q3	Q4	Q5	Total
Grade	Max	2	2	2	2	2	0 pts
	Expected						

Policy:
Please do your homeworks on your own. You are encouraged to discuss the questions with your class mates, but the code and the hw you submitted must be your own work. Cheating is highly discouraged for it could mean a zero or negative grade from the homework.
If a question is not clear, please let me know (via email or in class). Unless we indicate otherwise, do not use libraries for machine learning methods. When in doubt, email me.

There will be 3 homeworks this term. Each hw is worth 10 points and each question will be evaluated on a 0/1 basis.

Q1)
Make sure that you read Appendix A of the textbook and the resources on matrices, linear algebra and probability and statistics on Ninova.

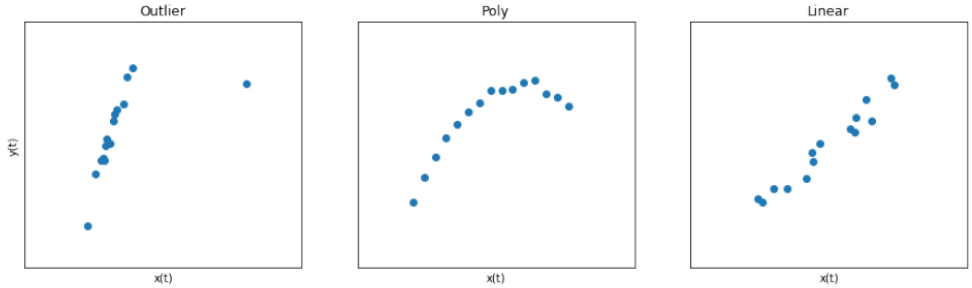
Given the table of joint probabilities between three discrete random variables X, Y, and Z.

P(X,Y,Z)	X	0			1		
	Y	-1	0	1	-1	0	1
Z	1	0.06	0.06	0.08	0.06	0.02	0.02
	2	0.09	0.09	0.12	0.24	0.08	0.08

a) Evaluate the joint probability mass function $P(Z|X=0, Y=0)$.

b) Are random variables Z and Y are independent? Show your solution. (**Hint:** marginalize X)

Q2) Do this exercise in python. Solve a) and b) using closed form solution



a) Given the following data points, compute the least-squares regression line and plot it with a red color on top of the data points for all three cases.

b) Compute the least-squares model for a polynomial of degree 4 and compare the expected absolute error of the linear and degree 4 polynomial models for all there cases.

c) Apply gradient descent and find the linear and degree 4 polynomial models for all three cases and compare your results with b.

Q3) A pharmaceutical company prepared a test amid the Coronavirus outbreak. The test is able to predict positive when applied to subjects with the virus with 90% accuracy while it predicts negative for healthy subjects again with 90% accuracy. We also know that only 2% of the population has the virus. What is the probability of actually being sick if the test predicts positive? Would you trust this test? A pharmaceutical company prepared a test amid the Coronavirus outbreak. The test is able to predict positive when applied to subjects with the virus with 90% accuracy while it predicts negative for healthy subjects again with 90% accuracy. We also know that only 2% of the population has the virus. What is the probability of actually being sick if the test predicts positive? Would you trust this test?

Q4) For a novel input x, a predictive model of the class c is given by p(c=1|x) = 0.7, p(c=2|x) = 0.2, p(c=3|x) = 0.1. The corresponding utility matrix U(ctrue,cpred) has elements:

$$\begin{bmatrix} 5 & 3 & 1 \\ 0 & 4 & -2 \\ -3 & 0 & 10 \end{bmatrix}$$

In terms of maximal expected utility, which is the best decision to take?
(Hint: Read 3.3 Losses and Risks from Ethem Alpaydın's book)

Q5) a) A continuous random variable x has the Laplace distribution

$$p(x) = \frac{1}{2}\exp(-|x|)$$

- Plot $p(x)$
- What is the probability that x > 2?

b) A discrete random variable has the binomial distribution

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

What is the E[x] = ? and E[x^2] = ? Show their derivations step by step.