Group Theory

Buce-Ithon

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Contents

Figure 1: Beautiful Algebra To My Elma

0 What is Group?-Introduction to Group

Accoding to Chiniese Wikipeida:

In mathematics, the **Group** is a special set equiped a binary operation, (the operation) which has associativity, identity and inverse element. core of Group

Since lots of mathematical structure are all groups (for instance: the integer system equiped with addition forms a Group), therefore, common results can be succinctly summarized from different mathematical structures, which makes groups a core concept in contemporary mathematics.

The unilateral (left) definition of Group (weak definition) is below:

If a given set G, and it's equiped binary operation $\circ$: $G \times G \to G$ satisfying (the result of operation $\circ(a,b)$ is simplified as $a \circ b$): simplest definition of the Group

Associativity	$\forall g_1, g_2, g_3 \in G$, we have $(g_1 \circ g_2) \circ g_3 = g_1 \circ (g_2 \circ g_3)$	
(L)Identity & (L)Inverse	$\exists e \in G, \forall g \in G$, we have	$e \circ g = g$ and exists $\gamma \in G$, s.t. $\gamma \circ g = e$

then we say $(G, \circ)$ is a **Group**. If $\circ$ is understood, $(G, \circ)$ is simplified as G.

The order of group operations is very important, i.e $a \cdot b = b \cdot a$(commutative) doesn't necessarily hold.
A group satisfies the commutative law is called Commutative Group (or Abelian Group named after Niels·Abel), those doesn't satisfy is called Non-commutative Group (Non-Abelian Group). (For instance: The Dihedra Group (appearing next chapter) is a Non-Abelian Group.)

Equivalance definition of Group: unilateral (right) definition of Group (strong definition) is below: The above definition part of identity and inverse can also be changed to: equivalance and full definition

Identity(R) & Inverse(R)	$\exists e \in G, \forall g \in G$, we have	$g \circ e = g$ and exists $\gamma \in G$, s.t. $g \circ \gamma = e$

Because no matter the original definition of lavender or the alternative definition of light-yellow, combined with the associativity, it's equivalent to the following definition:

Identity & Inverse	$\exists e \in G, \forall g \in G$, then we have	$e \circ g = g \circ e = g$ and exists $\gamma \in G$, s.t. $\gamma \circ g = g \circ \gamma = e$

Ohh! Please remenber the definitions above, because this is the most concise definition of Group. And it will be very helpful for us to understand Group Theory!