Tech Tools in Functional Analysis

——A Note of My Functional Analysis Courese

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1 Introduction

This article is for recording my thoughts and notes in Functional Analysis Courese. Most of them are content that I personally think is important. Hope it's helpful for you:).

2 2 Important Inequalities-Schwartz and Minkowski

In 1st section, I want to introduce 2 important inequalities - also conclusion or tech tools - because they are common and useful in much proofs of propositions in my textbook. Besides, there are not unique method to prove this 2 inequalities, so I'll choose the most brief and elegant ones to show. Then I'll show you their contact with Bessel series of inequality and equality and Holder's inequality.

Theorem 2.1 (Basic)**.** ***Schwartz Inequality:*** *Let X be a inner product space, then for any $x, y \in X$, we have*

$$|\langle x, y\rangle| \leq \| x \| \| y \| \tag{1}$$

Minkowski Inequality: *Let X be a normed space, then for any $x, y \in X$, we have*

$$\| x + y \| \leq \| x \| + \| y \| \tag{2}$$

Theorem 2.2 (Tech)**.** ***Bessel Series of Inequality and Equality:***

$$\| f - \Sigma_{k=1}^{N} c_k \phi_k \|^2 \geq \| f - \Sigma_{k=1}^{N} \langle f, \phi_k \rangle \phi_k \|^2 = \| f \|^2 - \Sigma_{k=1}^{N} |\langle f, \phi_k \rangle|^2 \geq 0 \tag{3}$$

While $N \to \infty$, we have $\{\phi_k{}_{k=1}^{\infty}\}$ is complete orthornormal system, then we have

Theorem 2.3 (Tech)**.** ***Pasevarll Identity:*** $\forall f, f_1, f_2 \in L^2$

$$\begin{aligned} &1^{\circ}. \| f \|^2 = \Sigma_{k=1}^{\infty} |\langle f, \phi_k \rangle|^2 \\ &2^{\circ}. \| f_1, f_2 \| = \Sigma_{k=1}^{\infty} |\langle f_1, \phi_k \rangle \overline{\langle f_2, \phi_k \rangle}| \end{aligned} \tag{4}$$