Probability Analysis Using R (Visualization of The Copoun Distribution)

Introduction

This is a demonstration of the analysis carried out in the development of a new distribution. Or in general, how plots, simulations, and maximum likelihood estimation can be analyzed using R programming languages.

I will demonstrate how the plots of the PDF and CDF of a new distribution known as the Copoun Distribution are analyzed using R codes that can be found here Copoun

First, let’s start by introducing the probability density function (pdf) and the cumulative distribution function (cdf) of the Copoun distribution and then illustrate the different shapes of the Copoun distribution.

Definition: A random variable X is said to have a Copoun distribution (CD) with parameters η and ϕ if its probability density function is given by

$$g(x;\eta,\phi)=\frac{\eta^2}{(\phi+\eta)}\left[1+\frac{\phi\eta^2 x^3}{6}\right] e^{-\eta x};\quad x>0,\eta>0,\phi>0 \qquad (1)$$

Remark 1: The pdf in equation 1 is a two-component density of an Exponential (η) and Gamma ($4,\eta$) distribution with mixing proportions $\pi_1= \eta/(\phi+\eta)$ and $\pi_2= \phi/(\phi+\eta)$ such that

$$g(x;\eta,\phi)=\pi_1 g_1 (x;\eta)+\pi_2 g_2 (x;\eta) \qquad (2)$$

The corresponding cdf of equation 1 is given by

$$G(x;\eta,\phi)=1-\left[1+\frac{\phi\eta^3 x^3+\phi\eta^2 x^3+\phi\eta x}{6(\phi+\eta)}\right] e^{-\eta x} \qquad (3)$$

Remark 2: It can be easily seen that the pdf in equation 1 is a proper pdf.

The graphical plots of the theoretical density and distribution function (for some selected but different real points of η and ϕ) of a Coupon distribution are shown in Figure 1 and Figure 2 below.

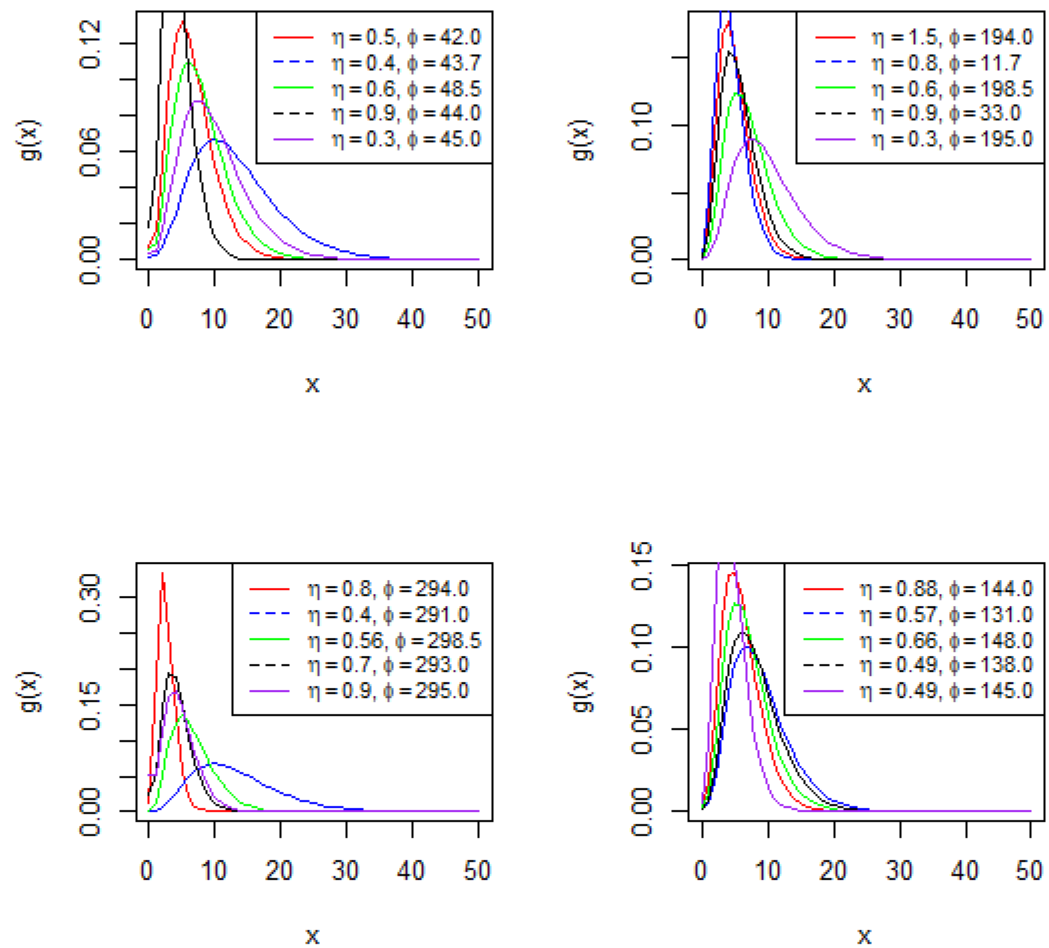

Figure 1: The graphical plots of the probability density function (for some selected but different real points of η and ϕ) of a Coupon distribution.

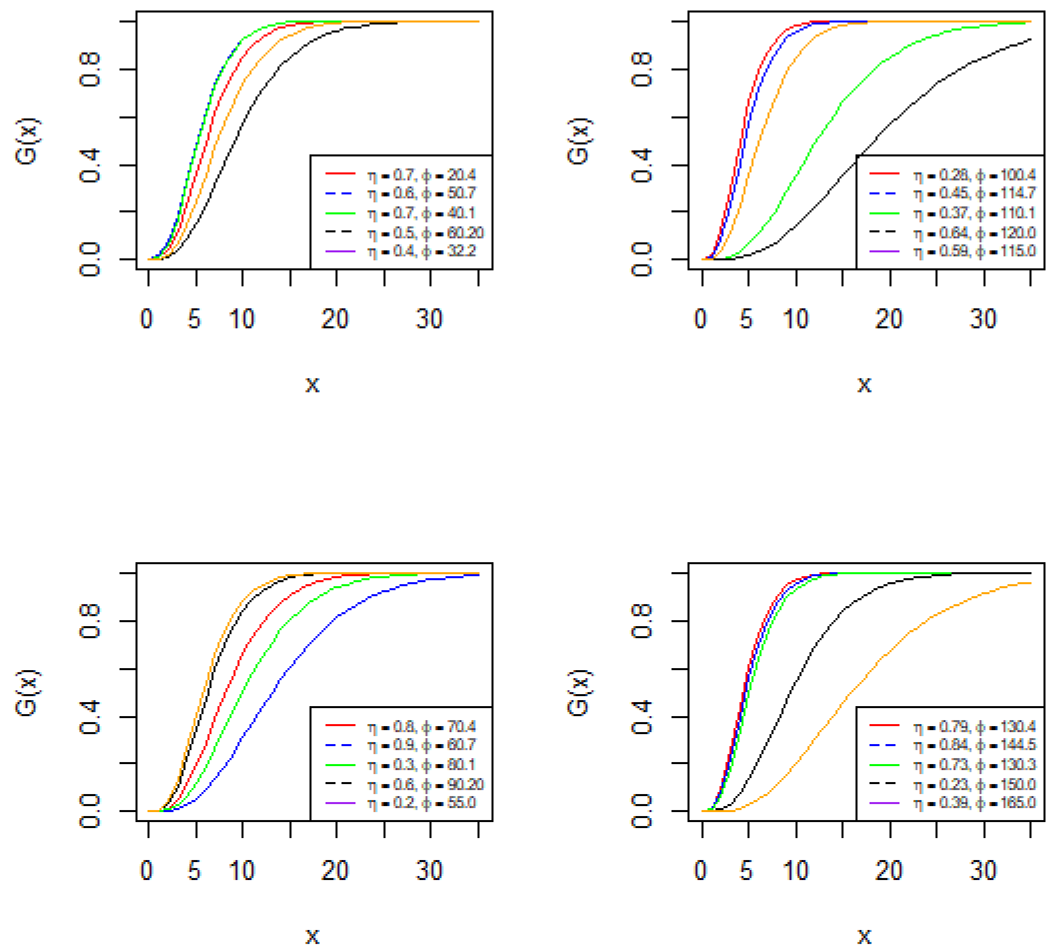

Figure 1: The graphical plots of the cumulative distribution function (for some selected but different real points of η and ϕ) of a Coupon distribution.

The curves displayed in Figure 1 are not bell-shaped but are positively skewed, unimodal, and right-tailed. In addition, the curve shows that increasing the value of ϕ leads to a considerable increase in the peak of the curve. In addition, the curves displayed in Figure 2 show that the cumulative distribution function converges to one.

Reliability Indices

Another property of the Copoun distribution is the reliability indices which I used R to plot the graphs.

So, for the reliability indices; Given any probability distribution, the reliability analysis is always considered based on the Survivorship or Existence Measurement Function, Risk Measurement Function, and Average Residual Measurement Life-Time Function.

Hence, for the Copoun distribution, the Survivorship or Existence Measurement Function, Risk Measurement Function, and Average Residual Measurement lifetime function are given below.

Survivorship or Existence Measurement Function

The survivorship or existence measurement function (also known as survival function) is defined as the probability that an item does not fail prior to some time t (Elechi et al., 2022; Epstein, 1958; Ronald et al., 2011; Shanker & Shukla, 2020).

The survivorship or existence measurement function of the Copoun distribution is given by

$$s(x)=1-G(x;\eta,\phi) \tag{3}$$

$$s(x)=1-\left[1-\left[1+\frac{{\eta^3 x}^3\phi+3{\eta^2 x}^2\phi+6\eta x\phi}{6(\phi+\eta)}\right]e^{-\eta x}\right] \tag{4}$$

$$\therefore s(x)=\left[1+\frac{{\eta^3 x}^3\phi+3{\eta^2 x}^2\phi+6\eta x\phi}{6(\phi+\eta)}\right]e^{-\eta x} \tag{5}$$

Risk Measurement Function

The risk measurement function (also known as hazard rate function) on the other hand can be seen as the conditional probability of failure, given it has survived to the time t (Elechi et al., 2022; Ronald et al., 2011; Shanker, 2016b; Umeh & Ibenegbu, 2019). It is obtained as

The risk measurement function of the Copoun distribution is given by

$$h(x)=\frac{g(x_k;\Phi)}{1-G(x_k;\Phi)} \tag{6}$$

$$h(x)=\frac{\frac{\eta^2}{(\phi+\eta)}\left[1+\frac{\phi\eta^2x^3}{6}\right]e^{-\eta x}}{\left[1-\left[1+\frac{{\eta^3 x}^3\phi+3{\eta^2 x}^2\phi+6\eta x\phi}{6(\phi+\eta)}\right]e^{-\eta x}\right]} \tag{7}$$

$$h(x)=\frac{6\eta^2+\phi\eta^4x^3}{{\eta^3 x}^3\phi+3{\eta^2 x}^2\phi+6\eta x\phi+6\eta+6\phi} \tag{8}$$

Average Residual Measurement Life-Time Function

The average residual measurement lifetime function of the new distribution is given by

$$m(x)=E[X-x \mid X>x]=\frac{1}{1-G(x)}\int_x^{\infty}[1-G(h)]\,dh \tag{9}$$

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m(x)=[( 〚η^3 x〛^3 ϕ+6 〚η^2 x〛^2 ϕ+18ηxϕ+24)+6η]/η[ 〚η^3 x〛^3 ϕ+3 〚η^2 x〛^2
ϕ+6xϕ+6(ϕ+η)]      (10)
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Remark 2: It can be easily observed that for x=0,

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S(0)=1, h(0)=η^2/(η+ϕ)=g(0), and m(0)=(6η+24)/η(6ϕ+6η).
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I now show the plots of the reliability indices in Figure 3 and Figure 4.

The figures show the graphical plots of h(x) and M(x) (for some selected but different real points of η and ϕ) of a Copoun distribution.

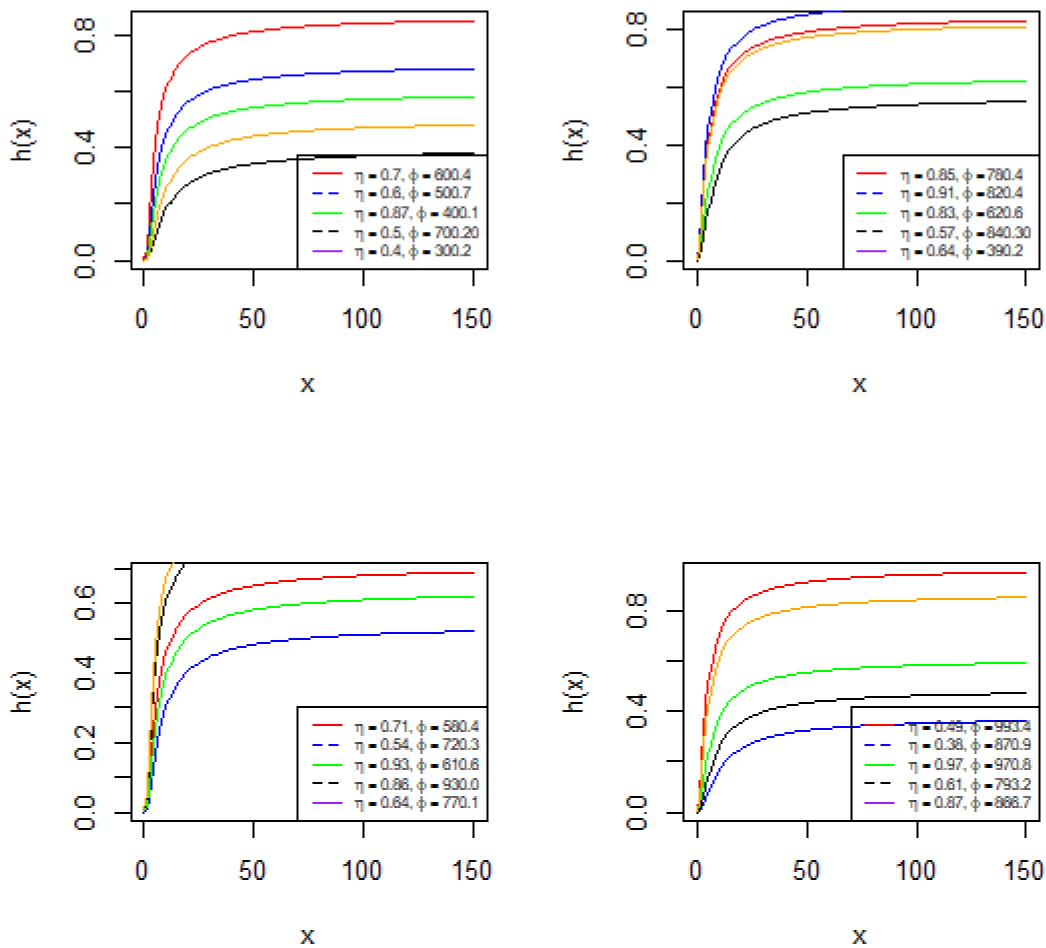

Figure 3 shows the graphical plots of h(x) (for some selected but different real points of η and ϕ) of a Copoun distribution.

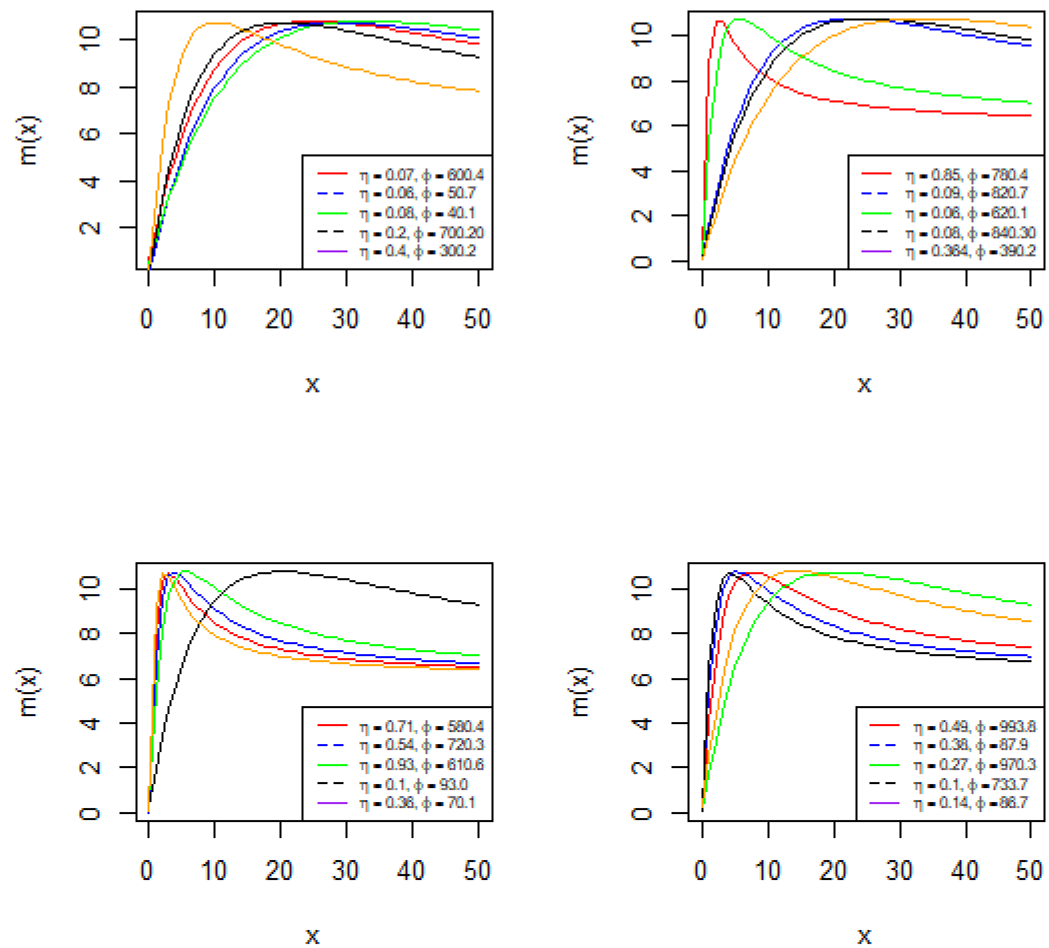

Figure 4 shows the graphical plots of m(x) (for some selected but different real points of η and ϕ) of a Copoun distribution.

The findings from Figure 3 show that the graphical plots of h(x) show that the hazard rate function, h(x) displays an increasing failure rate (IFR) function, increasing failure rate average (IFRA), new better than used (NBU), and new better than used in expectation (NBUE), respectively. (See (Barlow & Proschan, 1981)).

In addition, Figure 4 shows that the graphical plots of the mean residual function, M(x) show that the mean residual function M(x) displays a monotone non-decreasing function.

The implications of these visualizations for the new distribution are that the properties of the new Copoun distribution showed that Copoun distribution can be used to model lifetime datasets with unimodal, positively skewed, and right-tailed properties. Furthermore, the risk measurement function of the Copoun distribution can model datasets with increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU), and new better than used in expectation (NBUE) in survival analysis. In addition, the survivorship or existence measurement function and the average residual measurement lifetime function is a monotone non-increasing and monotone non-decreasing function.