Drell-Yan Lepton Angle Sampling

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The Drell-Yan angular distribution for lepton decays[1] states that the differential cross section for a back-to-back two-lepton decay at angle (θ, ϕ) in the rest frame of a Z is:

$$\frac{d\sigma}{d\Omega} = \sigma_0(1 + \cos^2\theta)$$

This means that the total cross section is:

$$\begin{aligned}
\sigma_{total} &= \int \frac{d\sigma}{d\Omega} d\Omega \\
&= \int_0^{2\pi} \int_0^{\pi} \sigma_0(1 + \cos^2\theta) \sin\theta d\theta d\phi \\
&= \frac{16}{3}\pi\sigma_0
\end{aligned}$$

Thus, the angular probability density for decaying within a given solid angle is:

$$\begin{aligned}
\rho(\theta, \phi) &= \frac{3}{16\pi}(1 + \cos^2\theta) \sin\theta \\
\rho(\theta) &= \frac{3}{8}(1 + \cos^2\theta) \sin\theta \\
\rho(\phi) &= \frac{1}{2\pi}
\end{aligned}$$

In order to generate angles (θ, ϕ) with these distributions, we first generate two numbers from a uniform distribution between 0 and 1, set the cumulative distribution functions for ϕ and θ equal to those numbers, then invert the equations.

[1] https://indico.cern.ch/event/777988/contributions/3236479/attachments/1762886/2860880/PHENO_18_Peng.pdf

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\begin{aligned}
u, v &\in [0,1] \\
\int_0^{\phi} \rho(\phi) d\phi &= \frac{\phi}{2\pi} = u \\
\int_0^{\theta} \rho(\theta) d\theta &= \frac{1}{2} - \frac{3}{8}\cos\theta - \frac{1}{8}\cos^3\theta = v \\
\phi &= 2\pi u \\
\theta &= \cos^{-1}\left(\frac{2^{\frac{1}{3}}(\sqrt{(4-8v)^2+4}+4-8v)^{\frac{2}{3}} - 2}{2^{\frac{2}{3}}(\sqrt{(4-8v)^2+4}+4-8v)^{\frac{1}{3}}} \right)
\end{aligned}
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