Robert Frenken

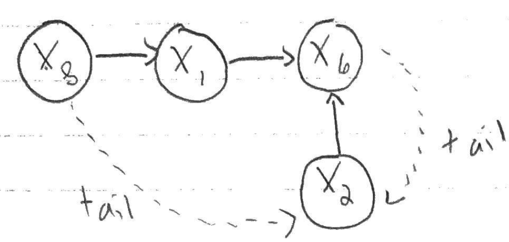
Homework 12

1. weightedUnion(x_1, x_6)

$x' \leftarrow x_2$ → $x' \leftarrow x_6$ x'.tail.next $\leftarrow x_2$

$y' \leftarrow x_6$ $y' \leftarrow x_2$ $w \leftarrow x_2$

w.head $\leftarrow x_6$ pointer



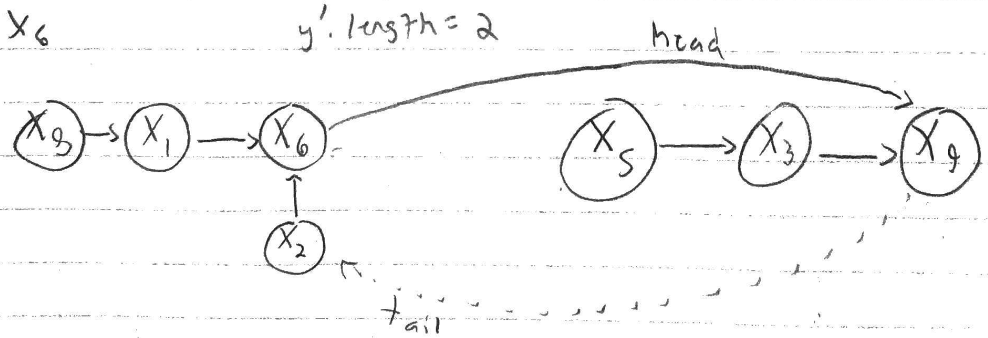
Findset(x_2) = x_6

Findset (x_9) = x_9

weightedUnion (x_3, x_6)

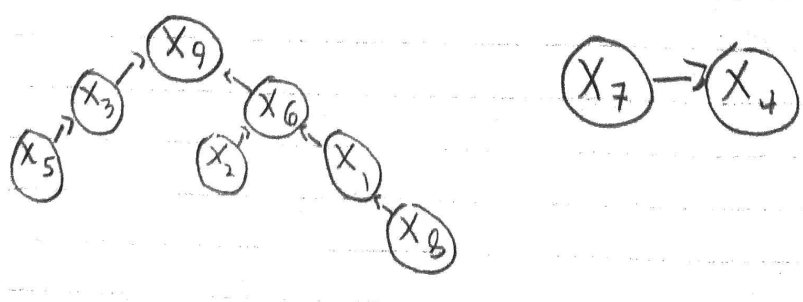
$x' \leftarrow x_3$ x'.length = 2

$y' \leftarrow x_6$ y'.length = 2



Findset(x_i) = x_9

Findset (x_9) = x_9

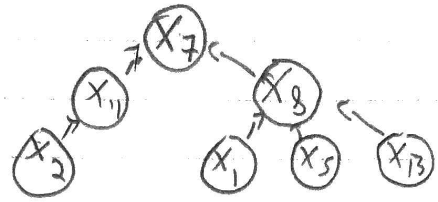


2. Union By Height(x_2, x_5)

$x' \leftarrow x_7$ x'.length = 2

$y' \leftarrow x_8$ y'.length = 1

y'.parent $\leftarrow x_7$



Findset(x_1) = x_7

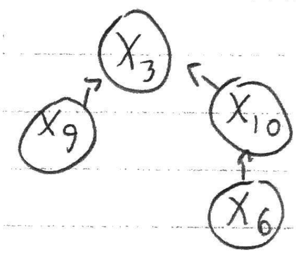
Findset(x_{11}) = x_7

Union By Height(x_9, x_6)

$x' \leftarrow x_3$ x'.length = 1

$y' \leftarrow x_{10}$ y'.length = 1

y'.parent $\leftarrow x_3$



Union By Height(x_4, x_6)

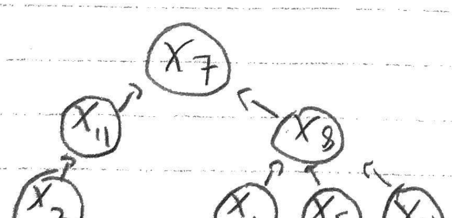
$x' \leftarrow x_{12}$ x'.length = 1

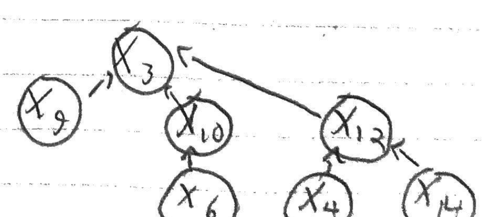
$y' \leftarrow x_3$ y'.length = 2

x'.parent $\leftarrow x_3$

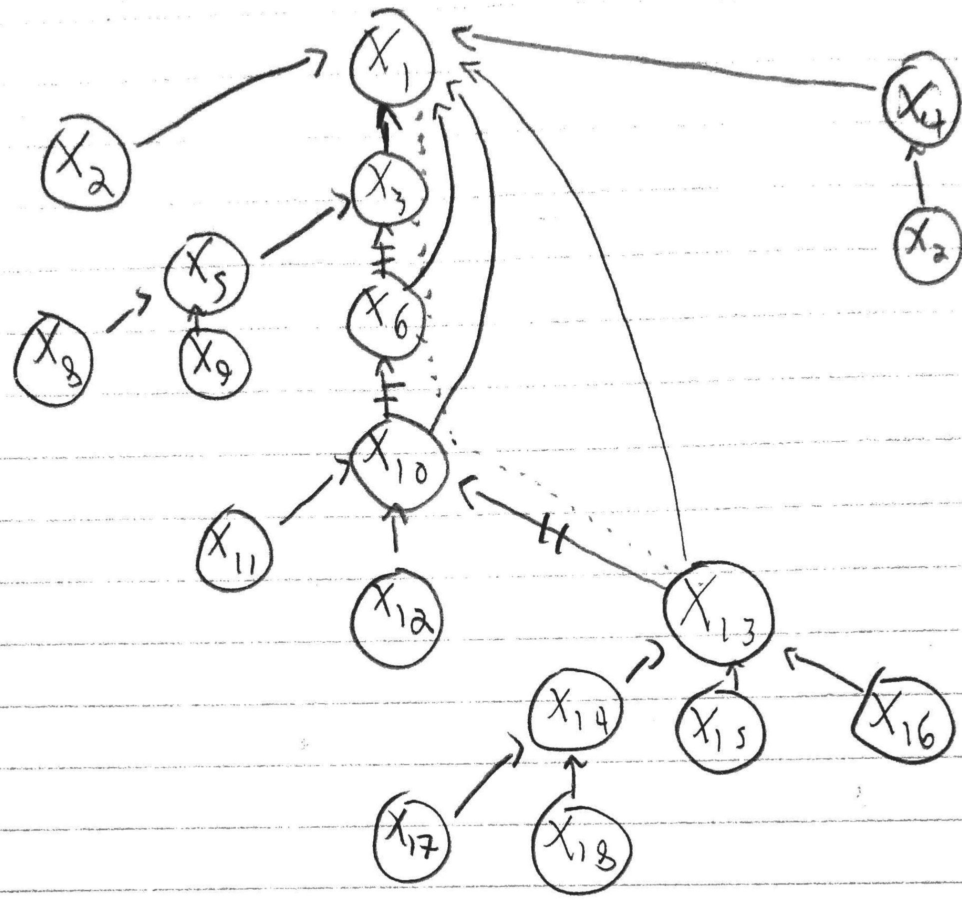
Findset(x_4) = x_3

Findset(x_9) = x_3





3.



Return: X_1

4.

a) Since the smaller or equal tree will become the child of the larger or equal tree, the max can either be the height of the first tree, in the case that it's larger than the 2nd tree, or it's the height of the 2nd tree + 1, if the heights of both trees were the same. The increment of 1 would be due to the equal sized 2nd tree having an additional parent node

By induction:

Case I: x.height > y.height r.height = x.height

Case II: x.height = y.height r.height = y.height + 1

b) Induction

Case I: $x'.height > y'.height$

$height(T) = r.height = x'.height$ part a

$size(T) = size(T_x) + size(T_y) \geq size(T_x) \geq 2^{height(T_x)}$

$= 2^{height(T)}$

Case II: $x'.height = y'.height$

$height(T) = r.height = x'.height + 1$ part a

$height(T_x) = height(T_y)$

$size(T) = size(T_x) + size(T_y) \geq 2^{height(T_x)} + 2^{height(y)}$

$= 2 \cdot 2^{height(T_x)} = 2^{height(T_x)+1} = 2^{height(T)}$

$size(T) \geq 2^{height(T)}$

c)

Since Tree T from part b was created by UnionBySize, it can be concluded that all trees follow the same constraints, with the only exception from part b if $y > x$.

$x'.height < y'.height$ same case as case I from part b

Therefore every tree follows $size(T) \geq 2^{height(T)}$

d) $size(T) \geq 2^{h}$ $height(T) = h$

$\log_2 size(T) \geq \log_2(2^h) = h$

$h \leq \log_2(size(T))$

$height(T) \leq \log_2(size(T))$