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Homework 11

1. $\sum_{i=1}^{n}\left[c + \sum_{j=1}^{\deg(V_i)} \sum_{k=1}^{\deg(V_i)} c\right] = cn + \sum_{i=1}^{2m} cn$

$= cn + 2cmn$

$\in \Theta(n + mn)$

2.

a) Lines 1-7 $\sum_{i=1}^{n}\left[c + \sum_{j=1}^{\deg(V_i)} c\log(s)\right]$ s = size of queue

$= cn + \sum_{i=1}^{2m} c\log(s)$ U.B. $\leq cn + \sum_{i=1}^{2m} c\log(2m)$

$\leq cn + c2m\log(2m) \in \Theta(n + m\log(m))$

L.B. $\geq cn + \sum_{i=m}^{2m} c\log(m) \geq cn + cm\log(m) \in \Theta(n + m\log(m))$

Lines 1-7 $\in \Theta(n + m\log(m))$

b) Lines 8-11

start $i = s$ after γ^{th}: $i = s - \gamma$ end $i = 0$

$\gamma = s$ • runs s times • size of queue $s = 2m$ from above (# insertions)

$\sum_{i=1}^{2m} Q.\text{Delete min}() = \sum_{i=1}^{2m} c\log(s)$

U.B. $\leq \sum_{i=1}^{2m} c\log(2m) \leq c2m\log(2m) \in \Theta(m\log(m))$

L.B. $\geq \sum_{i=m}^{2m} c\log(m) \geq cm\log(m) \in \Theta(m\log(m))$

Lines 8-11 $\in \Theta(m \log(m))$

2. c) $T_{proc2}(n,m) = n + m\log(m) + m\log(m)$

$\in \Theta(n + m\log(m))$

3.

a) Lines 1-6 $\sum_{i=1}^{n} \sum_{j=1}^{n} c s^{1/4}$

U.B. $\leq \sum_{i=1}^{n} \sum_{j=1}^{n} c n^{1/4} \leq \sum_{i=1}^{n} c n^{5/4} \leq c n^{9/4}$

L.B. $\geq \sum_{i=1}^{n} \sum_{j=n/2}^{n} c (\frac{n}{2})^{1/4} \geq \frac{c}{2^{1/4}} \sum_{i=1}^{n} n^{5/4}/2 \geq \frac{c}{2 \cdot 2^{1/4}} n^{9/4}$

$\in \Theta(n^{9/4})$

b) Lines 7-13 $\sum_{i=1}^{n} \sum_{j=1}^{deg(V_i)} c\sqrt{s} = \sum_{i=1}^{2m} c\sqrt{s}$

U.B. $\leq \sum_{i=1}^{2m} c\sqrt{2m} \leq c\,2m \cdot 2\sqrt{m} \in \Theta(m^{3/2})$

L.B. $\geq \sum_{i=m}^{2m} c\sqrt{m} \geq c\, m\sqrt{m} \in \Theta(m^{3/2})$

$\in \Theta(m^{3/2})$

c) $T_{proc3} = c n^{9/4} + c m^{3/2}$

$\in \Theta(\quad m^{3/2})$ as $m \geq n$