Robert Frenken

Homework 10

1.

a) Given the 2nd rule that left and right subtrees can only differ by at most one, this would apply throughout the whole tree. Since the child of root with height h' only goes down 1 level, then at least one of the child must be $h'-1$, and the other can be $h'-1$ or $h'-2$ given our 2nd condition. This must be the case, otherwise it would violate our predetermined conditions

b) The shortest possible path of this tree is where the height of the child is $h-2$ of the parent. Given K levels, the path of the child is $h-2K=0$ where the height at the NIL nodes is zero. $K=\lceil h/2 \rceil$

so given K levels, the shortest path height is $\lceil h/2 \rceil$

c) The total nodes of height h on a tree is

$2^0+2^1+2^2+\dots+2^K = 2^{K+1}-1$ given our conditions

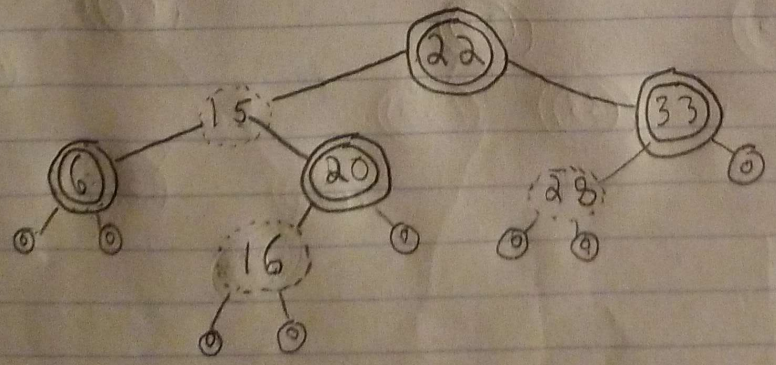
where $K=\lceil h/2 \rceil$ so the tree has at least $2^{\lceil h/2 \rceil+1}-1$ nodes

d) n = # nodes

$2^{\lceil h/2 \rceil}-1 \le n$ $\quad h \le 2\log_2(n+1)$

The height is at most $2\log_2(n+1)$ where n is # nodes

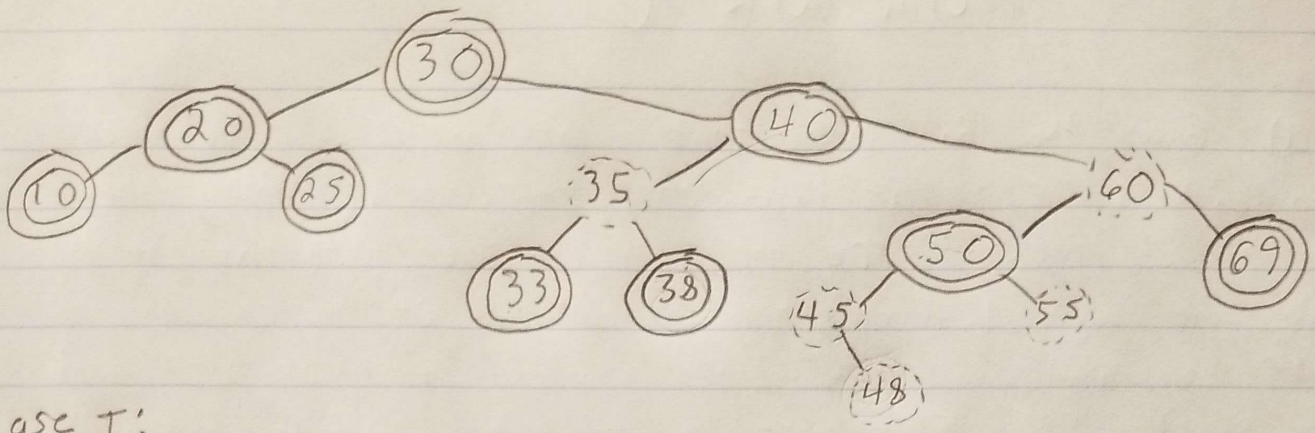
2.



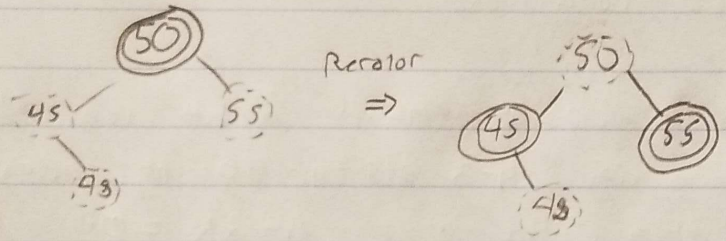
(0) = black

( ) = red

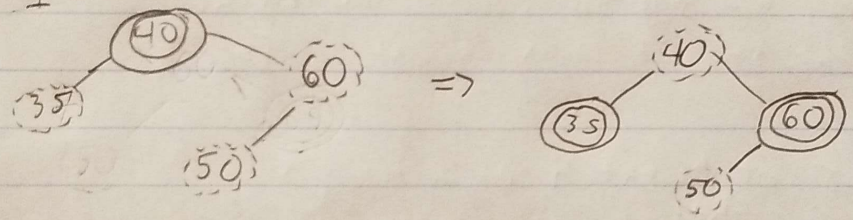
3.

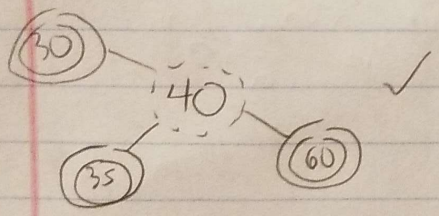


Case I:

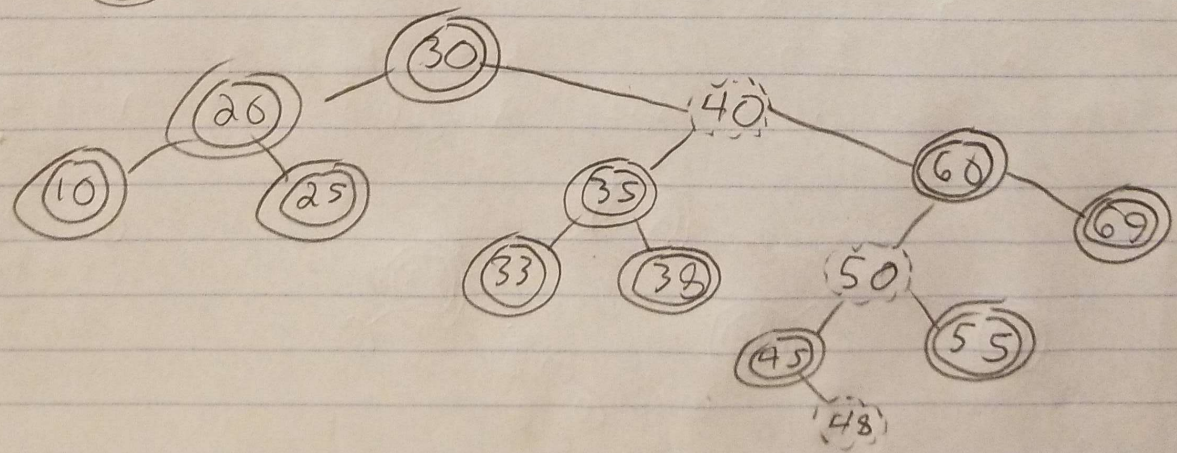


Case I:

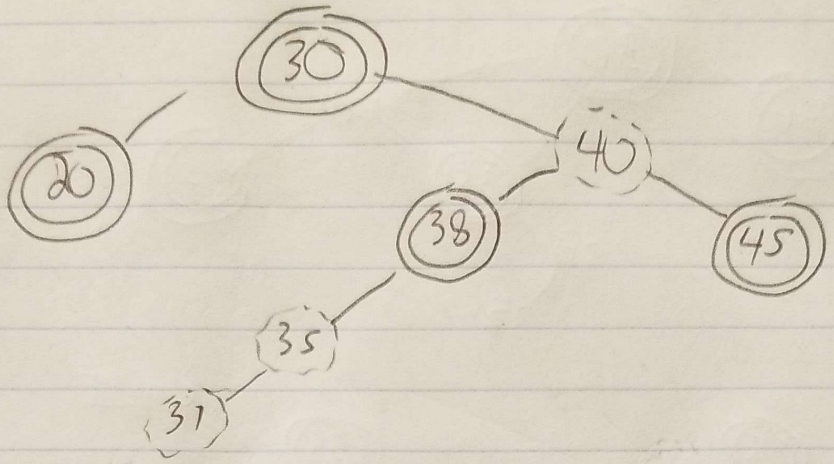




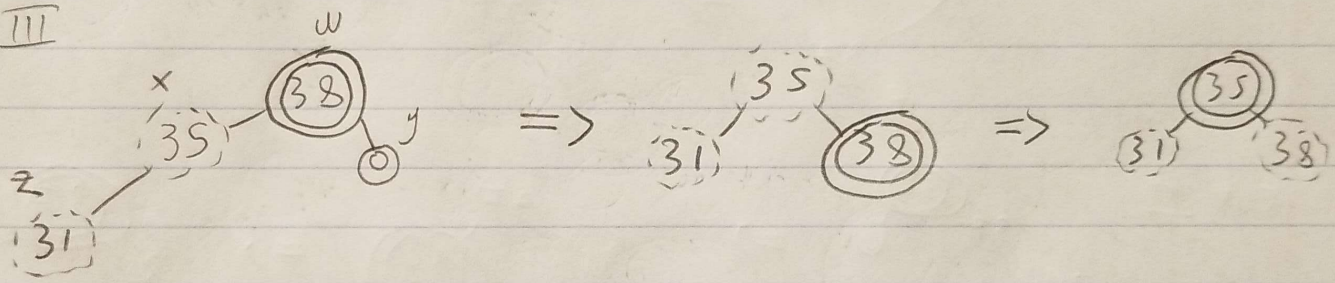
✓

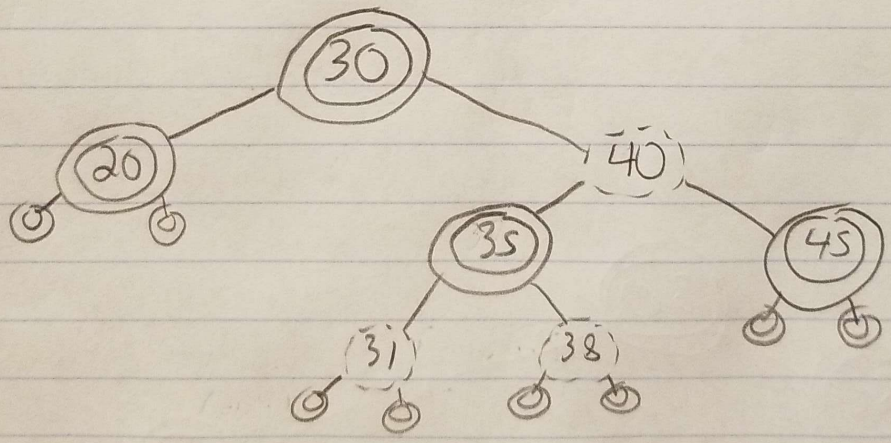


4.

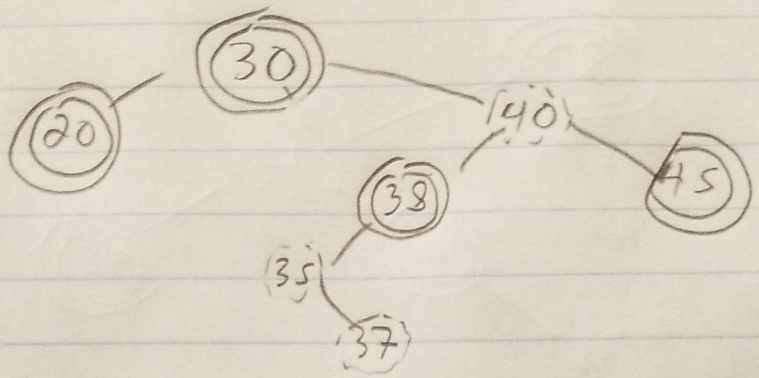

30
20
40
38
45
35
31

Case III

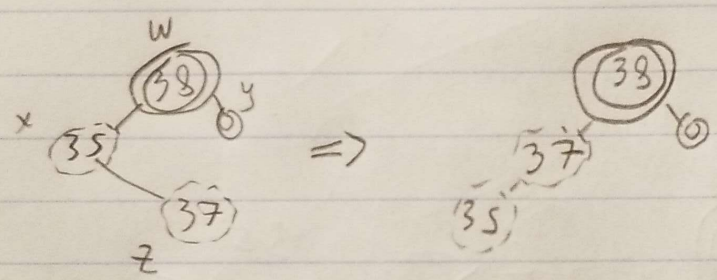

w
x
38
35
y
z
31
=>
35
31
38
=>
35
31
38


30
20
40
35
45
31
38

5.



Case II



Case III