Robert [illegible]
CSE 2331

Homework 7

1.

$T(n)$ = cost of creating new table + cost of inserting elements into table

$T(n) = \sum_{i=1}^{n} c + \left(\frac{3}{2}cn + \left(\frac{3}{2}\right)^2 c + \left(\frac{3}{2}\right)^3 + \dots + (3/2)/2\, cn + \frac{3}{2}cn\right)$

$T(n) = cn + \frac{3}{2}cn\left(1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{n}\right)$

LB: $\geq cn \in \Theta(n)$

UB: $\leq \frac{3}{2}cn \cdot 2 + cn \in \Theta(n)$

2. $T(n) = cn + \left(10^2 c + 10^3 c + 10^4 c + \dots \frac{10cn}{10} + 10cn\right)$

$= cn + 10cn\left(1 + \frac{1}{10} + \frac{1}{10^2} + \dots + 10/n\right)$

UB: $\leq cn + \frac{1}{1-1/10} \cdot 10cn \in \Theta(n)$

LB: $\geq cn \in \Theta(n)$

3. j = table size n = # elements

$T(n) = cn + \left((2+1)^2 c + (3+1)^2 c + \dots (j-1+1)^2 c + (j+1)^2 c\right)$

$j = \sqrt{n}$

UB: $\leq cn + \left((\sqrt{n}+1)^2 c + (\sqrt{n}+1)^2 c + \dots + (\sqrt{n}+1)^2 n\right)$

$\leq cn + (\sqrt{n}+1)^2 c\,(\underbrace{1+1+1+\dots+1}_{\sqrt{n}})$

$\leq cn + (\sqrt{n}+\sqrt{n})^2 c\sqrt{n} \leq cn + (2\sqrt{n})^2 c \cdot \sqrt{n}$

$\leq 4n^{3/2}c + cn \in \Theta(n^{3/2})$

LB: $\geq \left((\sqrt{n}/2)^2 c + (\sqrt{n}/2)^2 c + \dots + (\sqrt{n}/2)^2 c\right)$ ($\sqrt{n}$ terms)

$\geq \frac{\sqrt{n}^2}{4} c\sqrt{n} \geq \frac{c}{4} n^{3/2} \in \Theta(n^{3/2})$

4. $S \to S - 200$ $\quad S = n$

$T(n) = cn + (\text{cost of decreasing})$

$= cn + (cn + c(n-200) + c(n-200\cdot 2) + c(n-200\cdot 3) + \dots 200c)$

UB: $\leq cn + (\underbrace{cn + cn + cn + cn + \dots}_{k} + cn)$ $\quad n - 200 \cdot k = 0$

$k = n/200$

$\leq cn + \frac{n}{200} \cdot cn \quad \in \Theta(n^2)$

LB: $\geq cn + (\underbrace{cn/2 + cn/2 + cn/2 + \dots + cn/2}_{k/2})$

$\geq cn + \frac{n}{400} \cdot cn/2 \quad \in \Theta(n^2)$