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CSE 2331

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Homework 1

1.

a) $f_a(n) = 9\log_7(n^4+2n^3)+5n^{0.2} \rightarrow \underline{f_a(n) \in \Theta(n^{0.2})}$

b) $f_b(n) = 3\log_{10}(4n-12) + 8\log_5(32n+100)$

$\lim_{n\to\infty} \frac{3\log_{10}(4n-12)}{8\log_5(32n+100)} = \frac{3\log_{10}(4)+3\log_{10}(n)}{8\log_5 32 + 8\log_5(n)} = \infty$ $\quad \underline{f_b(n) \in \Theta(\log_{10}(n))}$

* drop constants for limits

c) $f_c(n) = 13n + 2n\log_9(7n^{.5}+4n^{.8})$ $\quad 7n^{.5}+4n^{.8} \in \Theta(n^{.8})$

$\approx \log_9(n^{.8}) \in \Theta(\log_9(n))$ $\quad 2n \in \Theta(n)$

$\underline{f_c(n) = \Theta(n\log_9(n))}$

d) $f_d(n) = 5\log_{10}(7n^3-6n+9) + 9\log_2(5n^{4.5}+33n)$

$7n^3-6n+9 \in \Theta(n^3)$ $\quad (5n^{4.5}+33n) \in \Theta(n^{4.5})$

$\approx 5\log_{10}(n^3) \in \Theta(\log_{10}(n))$ $\quad \approx 9\log_2(n^{4.5}) \in \Theta(\log_2(n))$

$\lim_{n\to\infty} \frac{\log_{10}(n)}{\log_2(n)} = \infty$ $\quad \underline{f_d(n) \in \Theta(\log_{10}(n))}$

e) $f_e(n) = \log_6(13n+3)\cdot\log_4(9n+2) + \log_2(6n^5+5n^3+2n^2+5)$

$\in \Theta(\log_6(n)\cdot\log_4(n))$ $\quad \approx \in \Theta \log_2(n)$

$\log_4(n) = \log_6(n)/\log_6(4)$

$\approx \in \Theta\ (\log_6(n))^2$ $\quad \underline{f_e(n) \in \Theta(\log_6^2(n))}$

f) $f_f(n) = (5\log_3(3n^4+11) + 4\log_8(n+12))\cdot(6\sqrt{17n-5} + 3\log_{11}(3n^3+14))$

First part: $\approx \in \Theta(\log_8^2(n))$ $\quad$ 2nd part $\approx \in \Theta(n^{.5}\log_{11}(n))$

$(\log_8(n))^2 = \frac{(\log_{11}(n))^2}{(\log_{11}(8))^2}$

$\underline{f_f(n) \in \Theta(n^{.5}\log_{11}^3(n))}$

g) $f_g(n) = 3\sqrt{5n^3+8n-16}$ $\quad$ LB: $\geq 3\sqrt{5n^3-16} \geq 3\sqrt{5n^3-n^3}$ for $n\geq3$

$= 3\cdot2\,n^{1.5}$

UB: $\leq 3\sqrt{5n^3+8n^3} = 3\sqrt{13}\,n^{1.5}$ for $n\geq1$

$\underline{f_g(n) \in \Theta(n^{1.5})}$

1 2 3

h) $f_h(n) = 8\log_5(n) + 4n + 12\sqrt{6n-17}$

1. $\approx \in \Theta(\log_5(n))$ 2. $\approx \in \Theta(n)$ 3. UB: $12\sqrt{6}\, n^{.5}$

LB: $12\sqrt{6n-n}$ for $n \geq 17$

$12\sqrt{5}\, n^{.5} \in \Theta(n^{.5})$

$f_h(n) \in \Theta(n)$

i) $11^7 + 4^{13} \cdot 3\log_7(6^{23})$ all constants $\rightarrow$ $f_i(n) \in \Theta(1)$

j) $f_j(n) = \sqrt{3(\log_7(n))^3 + 5n^2}$

LB: $\geq \sqrt{5n^2} = \sqrt{5}\, n \in \Theta(n)$ UB: $\sqrt{5n^2+3n^2}$

$\sqrt{8}\, n \in \Theta(n)$

$f_j(n) \in \Theta(n)$

1. 2. 3. 4.

k) $(3n-23)\log_6(5n^3-15n^2-22n-24) + 9n + 85$

1. $\in \Theta(n)$ 2. $\in \Theta(\log_6(n))$ 3. $\in \Theta(n)$ 4. $\Theta(1)$

$f_k(n) \in \Theta(n\log_6(n))$

l) $f_l(n) = 9n^{.3} + 3n^{.4}$ UB: $9n^{.4}+3n^{.4} \in \Theta(n^{.4})$

LB: $3n^{.4} \in \Theta(n^{.4})$ $f_l(n) \in \Theta(n^{.4})$

m) $f_m(n) = \sqrt{13\log_5(7n) + 5n + 41} \geq$ LB: $\sqrt{5n} = \sqrt{5}\, n^{.5}$

UB: $\leq \sqrt{13n + 5n + 41n} = \sqrt{59}\, n^{.5}$

$f_m(n) \in \Theta(n^{.5})$

n) $f_n(n) = 4^n + 6^n + 8^n \geq$ LB: $8^n \in \Theta(8^n)$

UB: $\geq 8^n + 8^n + 8^n = 3\, 8^n \in \Theta(8^n)$ $f_n(n) \in \Theta(8^n)$

o) $f_o(n) = (4n^3+2n^2+1)\cdot(n^2+5n+13)\cdot(9n-6)$

LB: $\geq (4n^3)(n^2)(9n) = 36n^6 \in \Theta(n^6)$

UB: $(4n^3+2n^3+n^3)\cdot(n^2+5n^2+13n^2)\cdot(9n) = 1197n^6 \in \Theta(n^6)$

$f_o(n) \in \Theta(n^6)$

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p) $f_p(n) = 3 \cdot 7^{n+2} + 12 \cdot 7^{n+4} = 3 \cdot 7^2 \cdot 7^n + 12 \cdot 7^4 \cdot 7^n$

$(3 \cdot 7^2 + 12 \cdot 7^4)(7^n)^2 = (3 \cdot 7^2 + 12 \cdot 7^4)\, 7^{2n}$

$f_p(n) \in \Theta(7^{2n})$

q) $f_q(n) = 5^{7n} + 57^n = \Theta(5^{7n}) + \Theta(7^n) = \Theta(7^n)$

$f_q(n) \in \Theta(7^n)$

r) $f_r(n) = 7\log_6(5^n + n^5 + 5)$

LB: $\geq 7 \log_6(5^n) = 7n\log_6(5) = \Theta(n)$

UB: $\leq 7 \log_6(5^n + 5^n + 5^n)$ for $n \geq 1$ $= 7\log_6(3 \cdot 5^n)$

$= 7\log_6(3) + 7\log_6(5^n) = \Theta(1) + \Theta(n) = \Theta(n)$

$f_r(n) \in \Theta(n)$

s) $f_s(n) = 9n^5 + 5^{n+9} + 9^{n+5} = 9n^5 + 5^9 5^n + 9^5 9^n$

$= \Theta(n^5) + \Theta(5^n) + \Theta(9^n) = \Theta(9^n)$

$f_s(n) \in \Theta(9^n)$

t) $f_t(n) = 4 \cdot 7^{\log_7(2n^3+4n^2)} = 4(2n^3+4n^2) = 8n^3 + 16n^2$

LB: $\geq 8n^3 \in \Theta(n^3)$

UB: $\leq 8n^3 + 16n^3$ $n \geq 1$ $= 24n^3 \in \Theta(n^3)$

$f_t(n) \in \Theta(n^3)$

2. Example: $n^2 \log_2^2(n)$

$\lim_{n\to\infty} \frac{n^3}{n^2\log_2^2(n)} = \lim_{n\to\infty} \frac{n}{\log_2^2(n)} = \infty$ $n^2\log_2^2(n) \in O(n^3)$

$\lim_{n\to\infty} \frac{n^2\log_2(n)}{n^2\log_2^2(n)} = \lim_{n\to\infty} \frac{1}{\log_2(n)} = 0$ $n^2\log_2^2(n) \in \Omega(n^2\log_2(n))$

$\Theta(n^2\log_2(n)) < \Theta(n^2\log_2^2(n)) < \Theta(n^3)$

3. Example: $n^{.3}$

$$\lim_{n\to\infty} \frac{n^{.5}}{n^{.3}} = \lim_{n\to\infty} n^{.2} = \infty \qquad n^{.3} \in O(n^{.5})$$

$$\lim_{n\to\infty} \frac{n^{.1}}{n^{.3}} = \lim_{n\to\infty} \frac{1}{n^{.2}} = 0 \qquad n^{.3} \in \Omega(n^{.1})$$

$$\Theta(n^{.1}) < \Theta(n^{.3}) < \Theta(n^{.5})$$

4. $7\sqrt{4n^6+6n^3+23}$ LB: $\geq 7\sqrt{4n^6} = 14n^3$

UB: $\leq 7\sqrt{4n^6+6n^6+23n^6}$ for $n \geq 1$ $= 7\sqrt{33n^6} = 7\sqrt{33}\,n^3$

Thus, $14n^3 \leq 7\sqrt{4n^6+6n^3+23} \leq 7\sqrt{33}\,n^3$ and $7\sqrt{4n^6+6n^6+23n^6} \in \Theta(n^3)$

5. $$\lim_{n\to\infty} \frac{5n(\log_3(8n+12))^3}{7n\log_4(n^3+9)\cdot 3\log_{10}(n^2+2n)}$$

UB: $\leq \lim_{n\to\infty} \frac{5n\log_3^3(8n+12n)}{21n\log_4(n^3)\log_{10}(n^2)} = \lim_{n\to\infty} \frac{5n(\log_3(20)+\log_3(n))^3}{126n\log_4(n)\log_{10}(n)}$

$$= \lim_{n\to\infty} \frac{5}{126} \frac{(\log_3(20)+\log_3(n))^3}{\frac{1}{\log_{10}(4)}\log_{10}^2(n)} \leq \lim_{n\to\infty} \frac{5\cdot\log_4(10)}{126} \cdot \frac{(\log_3(n)+\log_3(n))^3}{\log_{10}^2(n)}$$

$$= \lim_{n\to\infty} \frac{\log_{10}(4)40}{126} \cdot \frac{\log_3^3(n)}{\log_{10}^2(n)} = \lim_{n\to\infty} \frac{\log_{10}(4)40}{126} \cdot \frac{\left(\frac{\log_{10} n}{\log_{10}(3)}\right)^3}{\log_{10}^2(n)}$$

$$= \lim_{n\to\infty} \frac{\log_{10}(4)40}{126\cdot\log_{10}^3(3)} \cdot \frac{\log_{10}^3(n)}{\log_{10}^2(n)} = \lim_{n\to\infty} \frac{\log_{10}(4)40}{126\cdot\log_{10}^3(3)} \cdot \log_{10}(n) = \infty$$

LB: $\geq \lim_{n\to\infty} \frac{5n(\log_3(8n))^3}{21n\cdot\log_4(10n^3)\cdot\log_{10}(3n^2)} \geq \lim_{n\to\infty} \frac{5\cdot\log_3^3(n)}{(21\log_{10}(10n^3)\log_{10}(3n^2))/\log_{10}(4)}$

$$= \lim_{n\to\infty} \frac{\log_{10}(4)5}{21\log_{10}^3(3)} \cdot \frac{\log_{10}^3(n)}{(\log_{10}(10)+3\log_{10}(n))\cdot(\log_{10}(3)+2\log_{10} n)}$$

$$\geq \lim_{n\to\infty} \frac{\log_{10}(4)5}{21\log_{10}^3(3)} \cdot \frac{\log_{10}^3(n)}{(\log_{10}(n)+3\log_{10}(n))\cdot(\log_{10}(n)+2\log_{10}(n))}$$

$$= \lim_{n\to\infty} \frac{\log_{10}(4)5}{21\log_{10}^3(3)} \cdot \frac{\log_{10}^3(n)}{12\log_{10}^2(n)} = \lim_{n\to\infty} \frac{\log_{10}(4)5}{21\cdot 12\cdot\log_{10}^3(3)} \cdot \log_{10}(n) = \infty$$

Since the limits for both bounds go to ∞
$f(n) \in \Omega(g(n))$