

Robert Frenken

Homework 6

1. Chained hashing - array of lists

hash function

$$h(k) = 4 \cdot k \bmod 7$$

$$\text{Insert}(5, D_5) \rightarrow 20 \bmod 7 = 6$$

$$\text{Insert}(10, D_{10}) \rightarrow 40 \bmod 7 = 5$$

$$\text{Insert}(12, D_{12}) \rightarrow 48 \bmod 7 = 6$$

$$\text{Insert}(15, D_{15}) \rightarrow 60 \bmod 7 = 4$$

$$\text{Insert}(4, D_4) \rightarrow 16 \bmod 7 = 2$$

$$\text{Insert}(40, D_{40}) \rightarrow 200 \bmod 7 = 4$$

$$\text{Insert}(3, D_3) \rightarrow 12 \bmod 7 = 5$$

		(5, D ₅)		(15, D ₁₅), (40, D ₄₀)	(10, D ₁₀), (3, D ₃)	(5, D ₅), (12, D ₁₂)
--	--	----------------------	--	---	---	---

Index: 0 1 2 3 4 5 6

2.

Insert(3, D_e)

$$h(3, 0) = (4 \cdot 3 + 5 \cdot 0) \bmod 7 = 5 \rightarrow \text{coll.}$$

$$j++ \quad h(3, 1) = (4 \cdot 3 + 5 \cdot 1) \bmod 7 = 3 \rightarrow \text{coll.}$$

$$j++ \quad h(3, 2) = (4 \cdot 3 + 5 \cdot 2) \bmod 7 = 1 \rightarrow \text{Insert}$$

Insert(6, D_f)

$$h(6, 0) = (4 \cdot 6 + 5 \cdot 0) \bmod 7 = 3 \rightarrow \text{coll.}$$

$$j++ \quad h(6, 1) = (4 \cdot 6 + 5 \cdot 1) \bmod 7 = 1 \rightarrow \text{coll.}$$

$$j++ \quad h(6, 2) = (4 \cdot 6 + 5 \cdot 2) \bmod 7 = 6 \rightarrow \text{coll.}$$

$$j++ \quad h(6, 3) = (4 \cdot 6 + 5 \cdot 3) \bmod 7 = 4 \rightarrow \text{Insert}$$

0	1	2	3	4	5	6
D _a	D _e		D _b	D _f	D _c	D ₆

3. size = 77 $h(k) = 35 \cdot k \bmod 77$ Chained hashing
 h is not a good hash function, as since it's not a prime number, there could be a pattern that would cause many collisions, increasing the runtime. A better hash function would have a large prime number as the size, and implement a multiplicative or universal hashing instead of division hashing.

4. Prob 4 (Array<Integer> A, Array<Integer> B, Integer n)

a)

```

HashMap.Init()
for i ← 1 to n do
    num ← B[i]
    h(num) ← hashFunction
    HashMap.Insert(h(num), num)
count ← 0
for i ← 1 to n do
    numA ← A[i]
    h(numA) ← hashFunction
    if HashMap.Member(h(numA))
        count ++
return count

```

b) $ET(n) = \sum_{i=1}^n [ET_{insert}(s) + ET_{member}(s)] = \sum_{i=1}^n c = cn \in \Theta(n)$

Chained hashing: $T_{in}(s) = c$ $T_{mem}(s) = c's$ $ET_{in}(s) = c$ $ET_{mem}(s) = c'$

c)

$$T(n) = \sum_{i=1}^n [T_{in}(s) + T_{mem}(s)] = \sum_{i=1}^n [c + c'i] = cn + c \frac{n(n+1)}{2} \in \Theta(n^2)$$

5.

Prob 5 (Array A, Integer n)

Hash, Init()

for $i \leftarrow 1$ to n do

$h(A[i]) \leftarrow$ hashFunction

occurrence $\leftarrow 1$

if (Hash.member(A[i])

Hash.Retrieve(A[i], count)

count++

Hash.Insert(A[i], count)

else

Hash.Insert(A[i], occurrence)

for $i \leftarrow 1$ to Hash.size do

for $j \leftarrow 1$ to Hash.list.size do

print(Hash.list[j])

b)

$$T_{\text{insert}}(s) = c_s$$

$$T_{\text{mem}}(s) = c's^2$$

$$ET_{\text{in}}(s) = c$$

$$ET_{\text{mem}}(s) = c'$$

$$\sum_{i=1}^n [ET_{\text{in}}(s) + ET_{\text{mem}}(s)] = cn \in \Theta(n)$$

c)

$$\sum_{i=1}^n c + \sum_{i=1}^n \sum_{j=1}^i c = cn + cn^2 \in \Theta(n^2)$$