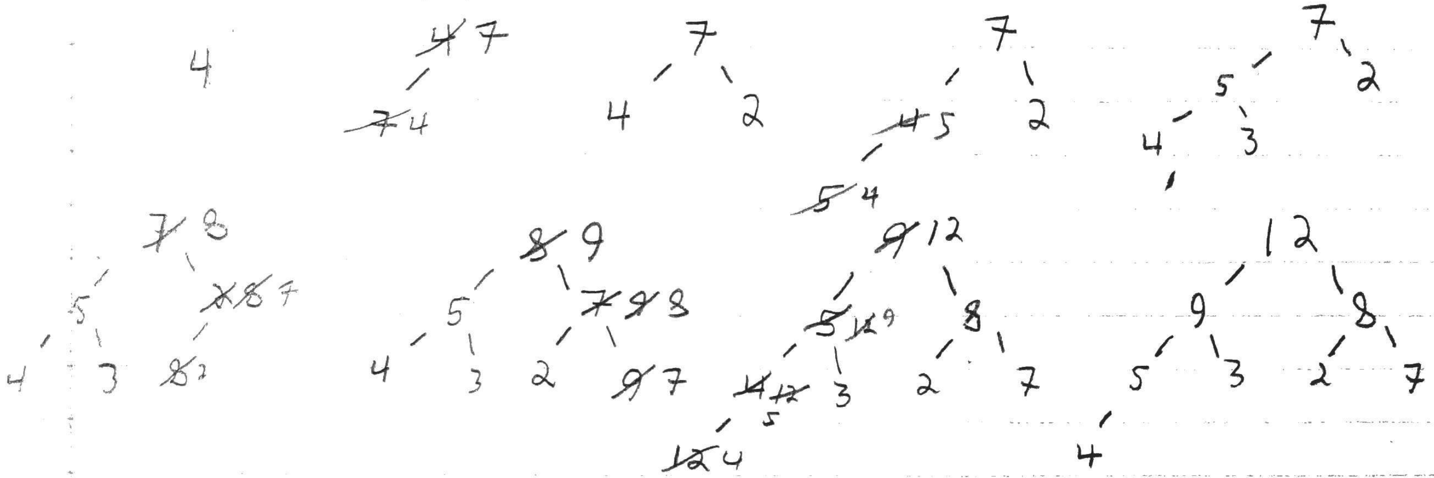
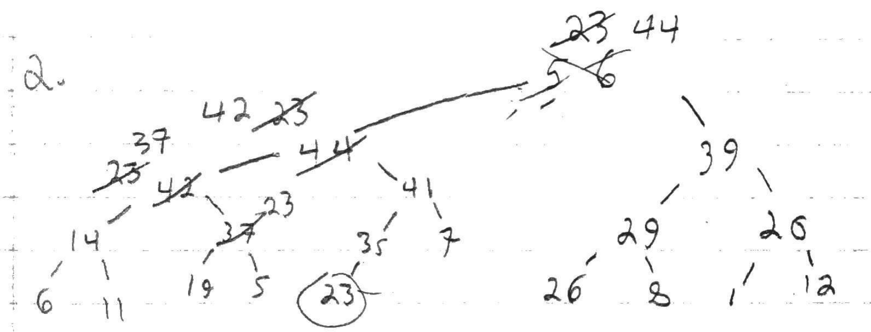
Robert Frenken
CSE 2331

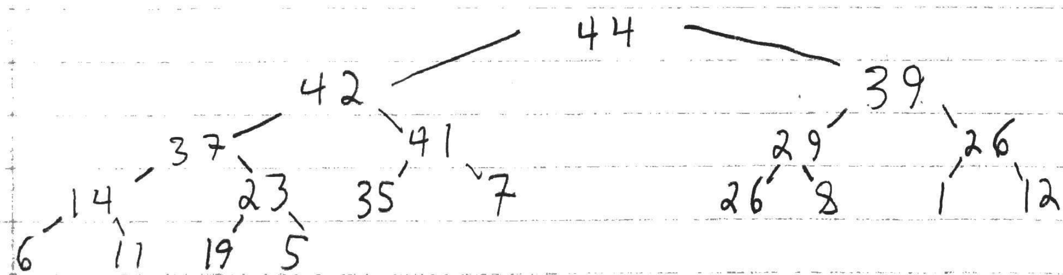
Homework 8

1. 4, 7, 2, 5, 3, 8, 9, 12

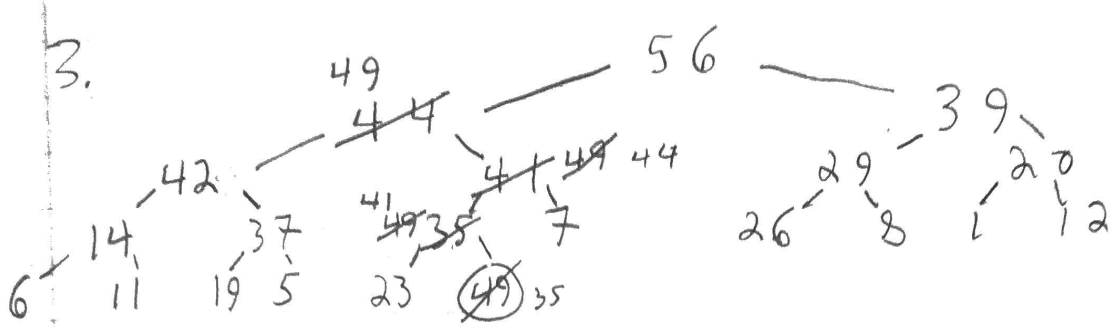


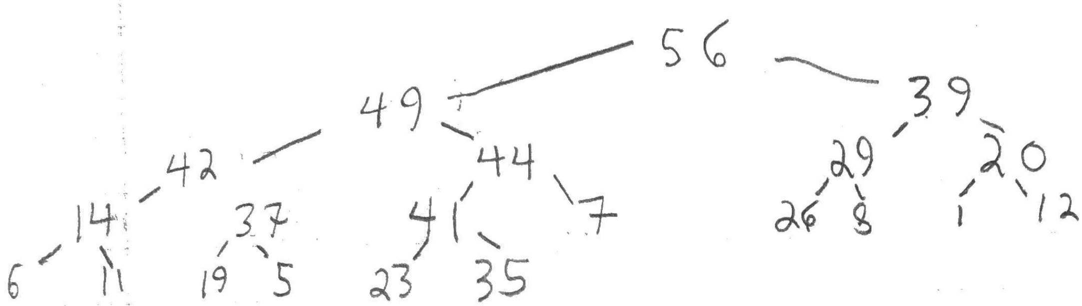
2.





3.





4. $P.\text{Init}() \in \Theta(1)$ $\quad$ $P.\text{ExtractMax}() \in \Theta(\log(P))$

$P.\text{Insert}(x) \in \Theta(\log(S))$ $\quad$ $P.\text{size} \in \Theta(1)$

$$T_{2-8}(n) = \sum_{i=1}^{\sqrt{n}} \sum_{j=1}^{n} P.\text{Insert} = \sum_{i=1}^{\sqrt{n}} \sum_{j=1}^{n} c\log(P) \qquad P\{\, P \in [1 \dots n^{3/2}]$$

$$= \sum_{i=1}^{n^{3/2}} c\log(P) \qquad UB: \leq \sum_{i=1}^{n^{3/2}} \tfrac{3}{2} c\log(n) = n^{3/2}\tfrac{3}{2} c\log(n).$$

$$LB: \geq \sum_{i=\frac{n^{3/2}}{2}}^{n^{3/2}} c\log\left(\tfrac{n^{3/2}}{2}\right) \leq \frac{n^{3/2}}{2} c\log\left(\tfrac{n^{3/2}}{2}\right)$$

$LB, UB \in \Theta(n^{3/2}\log(n))$

After line 6. $P.\text{size}() = n^{3/2}$

$P.\text{size}() = n^{3/2}$ $\quad$ after γ^{th} $P.\text{size}() = n^{3/2} - 1 \cdot K$

end $P.\text{size}() = 0$ $\quad$ $n^{3/2} - K = 0 \quad K = n^{3/2}$

$$UB: \leq \sum_{i=1}^{n^{3/2}} c\log(n^{3/2}) \leq c\, n^{3/2}\log(n^{3/2})$$

$$T_{7-10}(n) = \sum_{i=1}^{n^{3/2}} c\log(P)$$

$$LB. \geq \sum_{i=\frac{n^{3/2}}{2}}^{n^{3/2}} c\log\left(\tfrac{n^{3/2}}{2}\right) \geq c\,\frac{n^{3/2}}{2}\log\left(\tfrac{n^{3/2}}{2}\right)$$

$UB, LB \in \Theta(n^{3/2}\log n)$

$$T(n) = c\,n^{3/2}\log(n) + c\,n^{3/2}\log(n) \in \Theta(n^{3/2}\log(n))$$

Max-Heap $\quad s = \#$ elements

5. a) P. Insert $\log(s)$ $\quad$ P. ExtractMax $\log(s)$

$$T_{2\text{-}5}(n) = \sum_{j=1}^{n} P.\text{Insert}() = \sum_{j=1}^{n} c\log(s) \quad \begin{array}{l} U.B. \le cn\log(n) \\ L.B. \ge c\frac{n}{2}\log(\frac{n}{2}) \end{array} \in \Theta(n\log(n))$$

$$\sum_{i=1}^{\lfloor\sqrt{n}\rfloor} cn\log(n) + P.\text{ExtractMax}() = cn^{3/2}\log(n) + \sum_{i=1}^{\sqrt{n}} c\log(s)$$

$$\begin{array}{l} U.B.: \le cn^{3/2}\log(n) + c\sqrt{n}\log\sqrt{n} \\ L.B.: \ge cn^{3/2}\log(n) + c\frac{\sqrt{n}}{2}\log(\frac{\sqrt{n}}{2}) \end{array} \in \Theta(n^{3/2}\log(n))$$

$$T_{\text{Max-Heap}}(n) \in \Theta(n^{3/2}\log(n))$$

b) $P.\text{Insert}(s) = cs$ $\quad T_{EM}(s) = c$

$$T_{2\text{-}5}(n) = \sum_{j=1}^{n} P.\text{Insert}(s) = \sum_{j=1}^{n} cs = c\frac{n(n+1)}{2} \in \Theta(n^2)$$

$$T_{2\text{-}8}(n) = \sum_{i=1}^{\sqrt{n}} cn^2 + P.\text{ExtractMax} = cn^{2.5} + \sum_{i=1}^{\sqrt{n}} c = cn^{2.5} + c\sqrt{n}$$

$$T_{\text{Program 2}}(n) \in \Theta(n^{2.5})$$

c) $P.\text{Insert}() = c$ $\quad P.\text{ExtractMax}() = cs$

$$T_{2\text{-}5}(n) = \sum_{j=1}^{n} c = cn$$

$$T_{2\text{-}8}(n) = \sum_{i=1}^{\sqrt{n}} cn + P.\text{ExtractMax}() = cn^{3/2} + \sum_{i=1}^{n} cs$$

$$= cn^{3/2} + c\frac{n(n+1)}{2} \in \Theta(n^2)$$

$$T_{\text{Program3}}(n) \in \Theta(n^2)$$

$$T_{\text{program2}}(n) > T_{\text{program3}}(n) > T_{\text{program1}}(n)$$