Unit 3: Foundations for inference

2. Confidence intervals and hypothesis tests

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

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Slides posted at http://bitly.com/sta101sp15

1. Housekeeping

2. Main ideas

    1. Statistical inference methods based on the CLT depend on the same conditions as the CLT

    2. Use confidence intervals to estimate population parameters

    3. Critical value depends on the confidence level

    4. Use hypothesis tests to make decisions about population parameters

3. Summary

Announcements

- No new PS this week, but review questions provided on new material that will be on the exam next week
- Sample midterm + review questions posted
- PA3 will be posted on Monday after class and will be due that evening, so budget your time accordingly for that short turnaround!

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Application exercise: 3.1 Confidence interval for a single mean

See course website for details.

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Clicker question

What is the critical value ($Z^\star$) for a confidence interval at the 91% confidence level?

(a) $Z^\star = 1.34$

(b) $Z^\star = 1.65$

(c) $Z^\star = 1.70$

(d) $Z^\star = 1.96$

(e) $Z^\star = 2.33$

Clicker question

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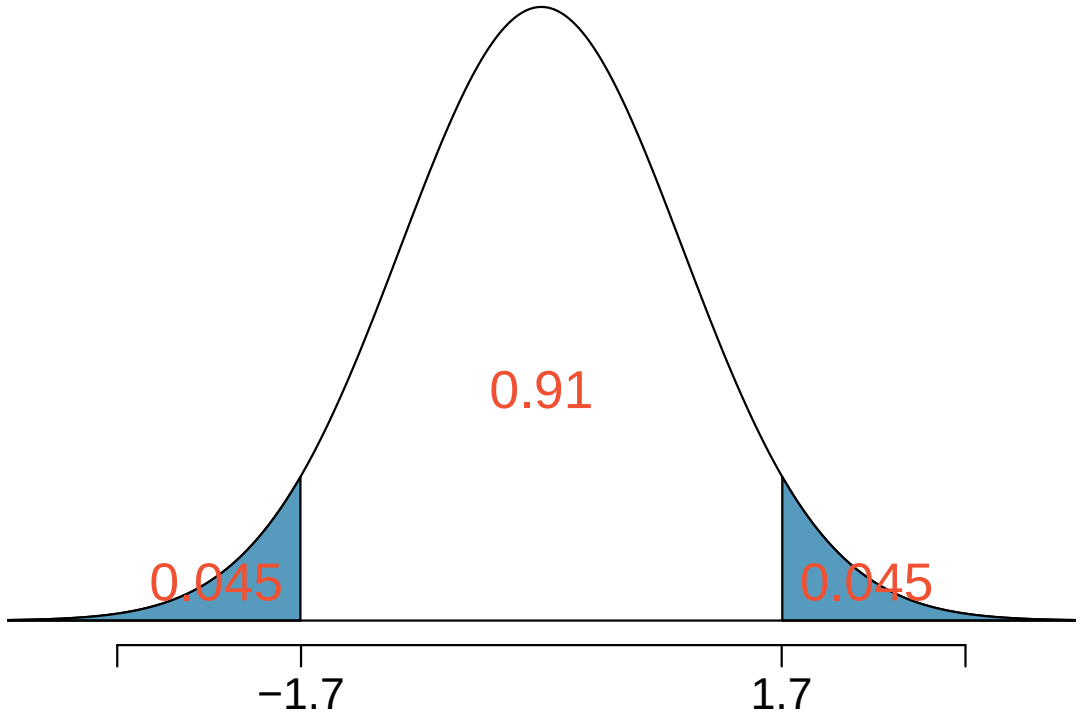

1. The confidence level of a confidence interval is the probability that a given interval contains the true population parameter.
   This is incorrect, CIs are part of the frequentist paradigm and as such the population parameter is fixed but unknown. Consequently, the probability any given CI contains the true value must be 0 or 1 (it does or does not).

Common misconceptions about confidence intervals

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3. A wider interval means less confidence.
   This is incorrect since it is possible to make very precise statements with very little confidence.

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Application exercise: 3.2 Hypothesis testing for a single mean

See course website for details.

Clicker question

Which of the following is the correct interpretation of the p-value?

(a) The probability that average GPA of Duke students has changed since 2001.
(b) The probability that average GPA of Duke students has not changed since 2001.
(c) The probability that average GPA of Duke students has not changed since 2001, if in fact a random sample of 63 Duke students this year have an average GPA of 3.58 or higher.
(d) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.
(e) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.

Clicker question

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(a) The probability that average GPA of Duke students has changed since 2001.
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(c) The probability that average GPA of Duke students has not changed since 2001, if in fact a random sample of 63 Duke students this year have an average GPA of 3.58 or higher.
(d) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.
(e) *The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.*

1. P-value is the probability that the null hypothesis is true
   This is incorrect, p-value is the probability of the observed or more extreme outcome if in fact the null hypothesis is correct. It is the conditional probability of the observed data (or something more extreme), conditioned on the null hypothesis being correct.

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   This is incorrect, a high p-value means the data do not provide convincing evidence for the alternative hypothesis and hence that the null hypothesis can't be rejected.

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2. A high p-value confirms the null hypothesis.
   This is incorrect, a high p-value means the data do not provide convincing evidence for the alternative hypothesis and hence that the null hypothesis can't be rejected.

3. A low p-value confirms the alternative hypothesis.
   This is incorrect, a low p-value means the data provide convincing evidence for the alternative hypothesis, but not necessarily that it is confirmed.

1. Set the hypotheses.
2. Check assumptions and conditions.
3. Calculate a *test statistic* and a p-value.
4. Make a decision, and interpret it in context of the research question.

1. Set the hypotheses
   - $H_0 : \mu = null\ value$
   - $H_A : \mu <$ or $>$ or $\neq null\ value$
2. Check assumptions and conditions
   - Independence: random sample/assignment, 10% condition when sampling without replacement
   - Normality: nearly normal population or $n \geq 30$, no extreme skew
3. Calculate a *test statistic* and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE}, \ where\ SE = \frac{s}{\sqrt{n}}$$

4. Make a decision, and interpret it in context of the research question
   - If p-value $< \alpha$, reject H_0, data provide evidence for H_A
   - If p-value $> \alpha$, do not reject H_0, data do not provide evidence for H_A

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