

Unit 3: Foundations for inference

3. Decision errors, significance levels, sample size & power

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

February 16, 2015

1. Housekeeping

2. Main ideas

1. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
2. Results that are statistically significant are not necessarily practically significant
3. Calculate the sample size a priori to achieve desired margin of error
4. Hypothesis tests are prone to decision errors
5. Power depends on the effect size, α , n , and s

3. Summary

- ▶ PA3 due tonight
- ▶ Office hours today: ask questions after class
- ▶ Midterm review: online, you should have received an email from WebEx with instructions
- ▶ Office hours tomorrow: online/on campus depending on weather

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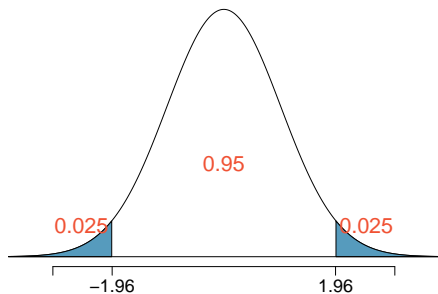
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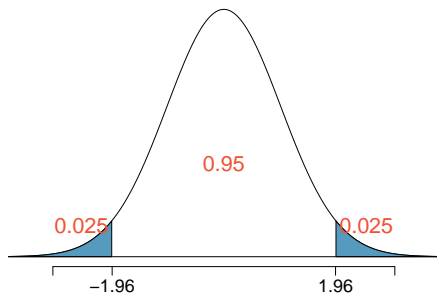
Two sided



95% confidence level
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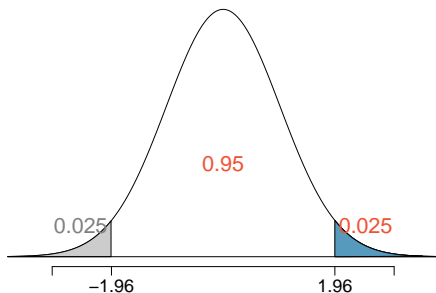
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One sided



95% confidence level
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one sided HT with $\alpha = 0.025$

Clicker question

What is the significance level of a two-sided hypothesis test that is equivalent to a 90% confidence interval? *Hint: Draw a picture and mark the confidence level in the center.*

- (a) 0.001
- (b) 0.01
- (c) 0.025
- (d) 0.05
- (e) 0.10

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What is the confidence level of a confidence interval that is equivalent to a two-sided hypothesis test with $\alpha = 0.01$. *Hint: Draw a picture and mark the confidence level in the center.*

- (a) 0.80
- (b) 0.90
- (c) 0.95
- (d) 0.98
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A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is true?

- (a) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.
- (b) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.
- (c) The hypothesis $H_0 : \mu = 98$ would be rejected using a 90% confidence interval.
- (d) The hypothesis $H_0 : \mu = 98.2$ would not be rejected using a 99% confidence interval.

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3. Summary

2. Results that are statistically significant are not necessarily practically significant

Clicker question

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

- (a) $n = 100$
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Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, and $H_A : \mu \geq 4.5$.

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Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, and $H_A : \mu \geq 4.5$.

$$Z_{n=100} = \frac{5 - 4.5}{\frac{2}{\sqrt{100}}}$$

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As n increases - $SE \downarrow$, $Z \uparrow$, $p\text{-value} \downarrow$

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3. Summary

3. Calculate the sample size *a priori* to achieve desired margin of error

Application exercise: 3.3 Sample size

See course website for details.

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3. Summary

4. Hypothesis tests are prone to decision errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true		
	H_A true		

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- ▶ A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
 - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
 - Increasing α increases the Type 1 error rate, hence we prefer to small values of α

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	Decision	
	fail to reject H_0	reject H_0
Truth	H_0 true	✓ <i>Type 1 Error, α</i>
	H_A true	<i>Type 2 Error, β</i>

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 - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
 - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- ▶ A *Type 2 Error* is failing to reject the null hypothesis when H_A is true: β

4. Hypothesis tests are prone to decision errors

Truth	Decision	
	fail to reject H_0	reject H_0
	H_0 true	H_A true
	✓	Type 1 Error, α
	Type 2 Error, β	Power, $1 - \beta$

- ▶ A **Type 1 Error** is rejecting the null hypothesis when H_0 is true: α
 - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
 - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- ▶ A **Type 2 Error** is failing to reject the null hypothesis when H_A is true: β
- ▶ **Power** is the probability of correctly rejecting H_0 , and hence the complement of the probability of a Type 2 Error: $1 - \beta$

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- ▶ increasing α
- ▶ increasing the *effect size*

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