Unit 3: Foundations for inference

3. Decision errors, significance levels, sample size & power

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

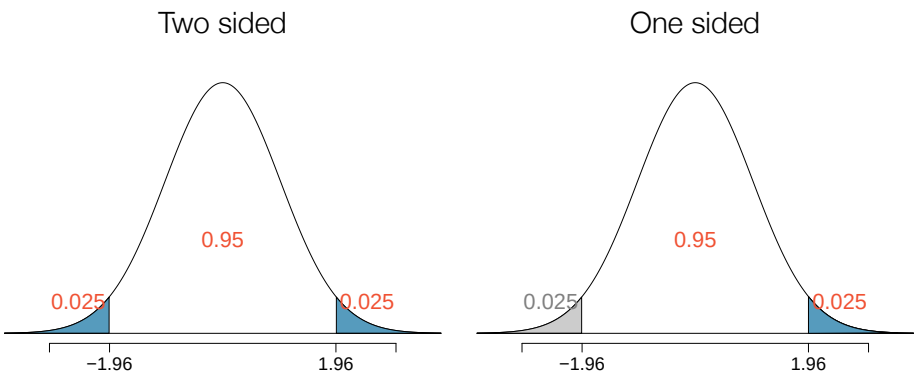
February 16, 2015

Dr. Çetinkaya-Rundel

Slides posted at http://bitly.com/sta101sp15

- PA3 due tonight
- Office hours today: ask questions after class
- Midterm review: online, you should have received an email from WebEx with instructions
- Office hours tomorrow: online/on campus depending on weather

1. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree



95% confidence level is equivalent to two sided HT with $\alpha = 0.05$

95% confidence level is equivalent to one sided HT with $\alpha = 0.025$

Clicker question

What is the significance level of a two-sided hypothesis test that is equivalent to a 90% confidence interval? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.001
(b) 0.01
(c) 0.025
(d) 0.05
(e) 0.10

Clicker question

What is the significance level of a one-sided hypothesis test that is equivalent to a 90% confidence interval? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.001
(b) 0.01
(c) 0.025
(d) 0.05
(e) 0.10

Clicker question

What is the confidence level of a confidence interval that is equivalent to a two-sided hypothesis test with $\alpha = 0.01$. *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.80
(b) 0.90
(c) 0.95
(d) 0.98
(e) 0.99

Clicker question

What is the confidence level of a confidence interval that is equivalent to a one-sided hypothesis test with $\alpha = 0.01$. *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.80
(b) 0.90
(c) 0.95
(d) 0.98
(e) 0.99

Clicker question

A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is true?

(a) The hypothesis $H_A : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.
(b) The hypothesis $H_A : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.
(c) The hypothesis $H_A : \mu = 98$ would be rejected using a 90% confidence interval.
(d) The hypothesis $H_A : \mu = 98.2$ would not be rejected using a 99% confidence interval.

2. Results that are statistically significant are not necessarily practically significant

Clicker question

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

3. Calculate the sample size *a priori* to achieve desired margin of error

Application exercise: 3.3 Sample size

See course website for details.

4. Hypothesis tests are prone to decision errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error,* α
	H_A true	*Type 2 Error,* β	*Power,* $1 - \beta$

- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true: β
- *Power* is the probability of correctly rejecting H_0, and hence the complement of the probability of a Type 2 Error: $1 - \beta$

5. Power depends on the n, a, α, effect size

Power can be increased (and hence Type 2 error rate can be decreased) by

- increasing the sample size
- decreasing the standard deviation of the sample (difficult to ensure but cautious measurement process and limiting the population so that it is more homogenous may help)
- increasing α
- increasing the *effect size*

1. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
2. Results that are statistically significant are not necessarily practically significant
3. Calculate the sample size a priori to achieve desired margin of error
4. Hypothesis tests are prone to decision errors
5. Power depends on the effect size, α, n, and s