

Unit 4: Inference for numerical data

1. Bootstrap intervals

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

February 23, 2015

1. Housekeeping

2. Bootstrapping

3. Randomization testing for a single numerical variable

4. Bootstrapping for categorical data



1. Housekeeping

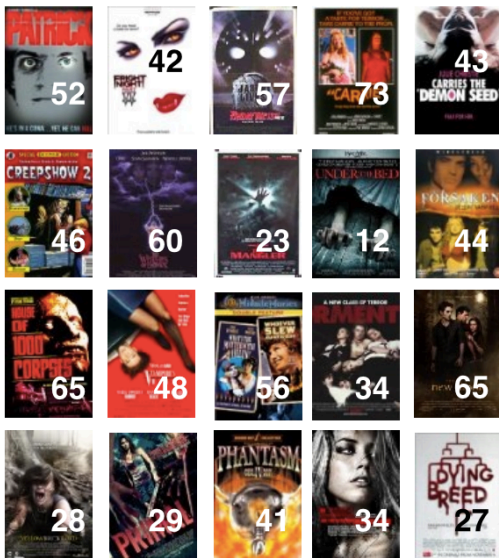
2. Bootstrapping

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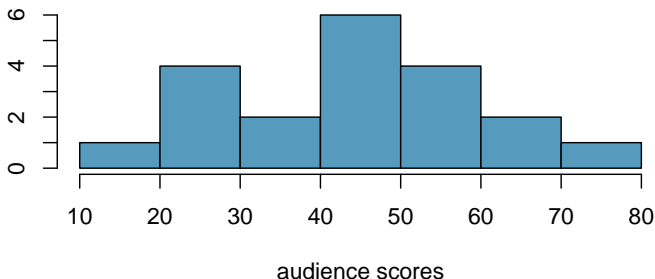


is a movie aggregator, where the audience is also able to review and score the movies. We want to estimate the average audience score of horror movies on RottenTomatoes.com. We start with a random sample of 20 horror movies.



	title	audience_score
1	Patrick	52
2	Demon Seed	43
3	Tormented	34
4	Under the Bed	12
5	Phantasm IV: Oblivion	41
6	Fright Night Part 2	42
7	House of 1000 Corpses	65
8	Creepshow 2	46
9	The Forsaken	44
10	All the Boys Love Mandy Lane	34
11	Jason Lives: Friday the 13th Part VI	57
12	Vampire's Kiss	48
13	The Witches of Eastwick	60
14	Yellowbrickroad	28
15	Dying Breed	27
16	Carrie	73
17	Whoever Slew Auntie Roo?	56
18	The Mangler	23
19	Primal	29
20	The Twilight Saga: New Moon	65

The dot plot below shows the distribution of the audience scores of these movies (ranging from 0 to 100). The median score in the sample is 43.5. Can we apply CLT based methods we have learned so far to construct a confidence interval for the median RottenTomatoes score of horror movies. Why or why not?



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- ▶ This term comes from the phrase "pulling oneself up by one's bootstraps", which is a metaphor for accomplishing an impossible task without any outside help.
- ▶ In this case the impossible task is estimating a population parameter, and we'll accomplish it using data from only the given sample.



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 - $\bar{x}_{boot} \pm z^* SE_{boot}$

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23, 27, 27, 29, 34, 41, 44, 46, 48, 48, 56, 56, 56, 57, 60, 65, 65, 65, 65, 65

median = $(48 + 56) / 2 = 52$

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(2) Calculate the median of the bootstrap sample:

23, 27, 27, 29, 34, 41, 44, 46, 48, 48, 56, 56, 56, 57, 60, 65, 65, 65, 65, 65

median = $(48 + 56) / 2 = 52$

(3) Record this value

(1) Take another bootstrap sample:

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	title	audience_score
1	Fright Night Part 2	42
2	Carrie	73
3	The Forsaken	44
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5	Primal	29
6	Patrick	52
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8	The Mangler	23
9	Vampire's Kiss	48
10	All the Boys Love Mandy Lane	34
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(2) Calculate the median of the bootstrap sample:

23, 23, 23, 28, 29, 34, 34, 34, 41, 42, 44, 48, 48, 52, 52, 57, 65, 65, 65, 73

median = $(42 + 44) / 2 = 43$

(1) Take another bootstrap sample:

	title	audience_score
1	Fright Night Part 2	42
2	Carrie	73
3	The Forsaken	44
4	The Mangler	23
5	Primal	29
6	Patrick	52
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(2) Calculate the median of the bootstrap sample:

23, 23, 23, 28, 29, 34, 34, 34, 41, 42, 44, 48, 48, 52, 52, 57, 65, 65, 65, 73

median = $(42 + 44) / 2 = 43$

(3) Record this value

(1) Take another bootstrap sample:

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	title	audience_score
1	Tormented	34
2	The Witches of Eastwick	60
3	The Witches of Eastwick	60
4	The Witches of Eastwick	60
5	The Mangler	23
6	The Witches of Eastwick	60
7	Patrick	52
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15	Fright Night Part 2	42
16	Whoever Slew Auntie Roo?	56
17	Fright Night Part 2	42
18	Fright Night Part 2	42
19	Under the Bed	12
20	Phantasm IV: Oblivion	41

(1) Take another bootstrap sample:

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1	Tormented	34
2	The Witches of Eastwick	60
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19	Under the Bed	12
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(2) Calculate the median of the bootstrap sample:

12, 23, 28, 28, 29, 34, 41, 41, 42, 42, 42, 42, 52, 56, 57, 57, 60, 60, 60, 60

median = $(42 + 42) / 2 = 42$

(1) Take another bootstrap sample:

	title	audience_score
1	Tormented	34
2	The Witches of Eastwick	60
3	The Witches of Eastwick	60
4	The Witches of Eastwick	60
5	The Mangler	23
6	The Witches of Eastwick	60
7	Patrick	52
8	Phantasm IV: Oblivion	41
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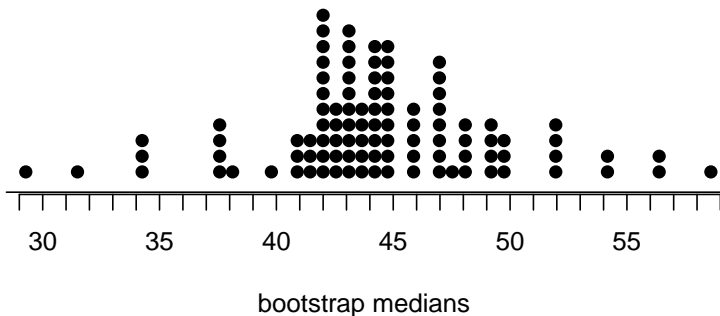
12, 23, 28, 28, 29, 34, 41, 41, 42, 42, 42, 42, 52, 56, 57, 57, 60, 60, 60, 60

median = $(42 + 42) / 2 = 42$

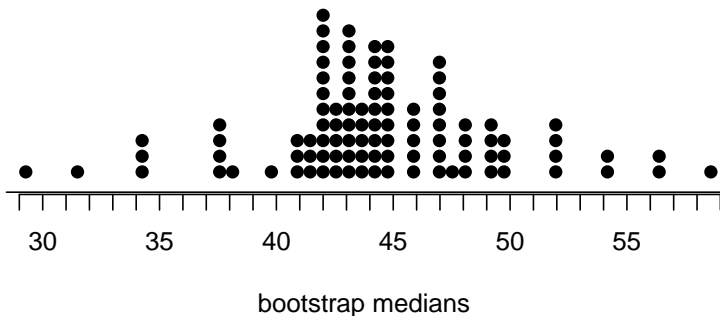
(3) Record this value

... repeat

The dot plot below is the bootstrap distribution of medians constructed using 100 simulations. What does each dot on the dot plot represent?

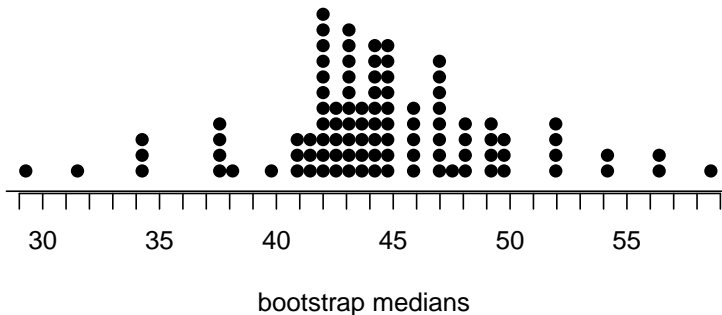


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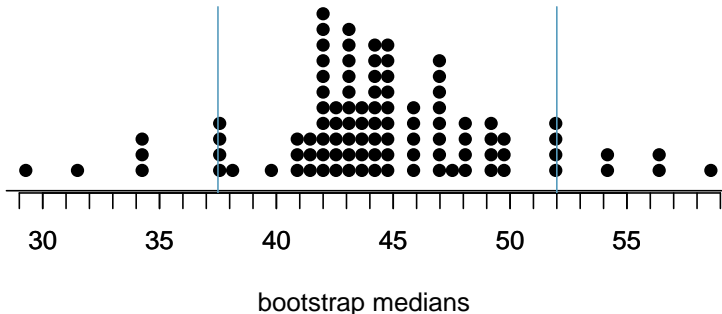


Each dot represents a bootstrap median from one simulation.

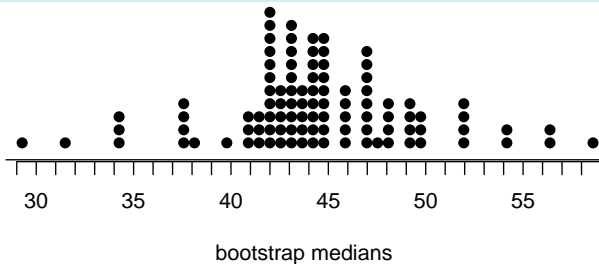
The dot plot below shows the distribution of medians of 100 bootstrap samples from the original sample. Estimate the 90% bootstrap confidence interval for the median RottenTomatoes score of horror movies.



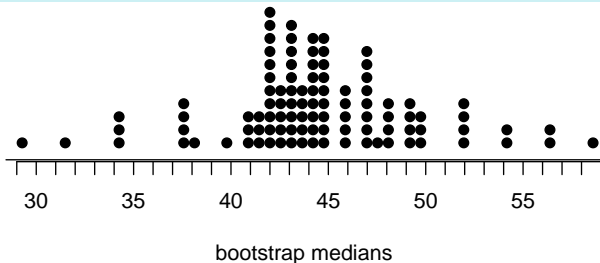
The dot plot below shows the distribution of medians of 100 bootstrap samples from the original sample. Estimate the 90% bootstrap confidence interval for the median RottenTomatoes score of horror movies.



The dot plot below shows the distribution of medians of 100 bootstrap samples from the original sample. Given that the mean of the bootstrap distribution is 44.39, and the bootstrap standard error is 4.88, estimate the 90% bootstrap confidence interval for the median RottenTomatoes score of horror movies.



The dot plot below shows the distribution of medians of 100 bootstrap samples from the original sample. Given that the mean of the bootstrap distribution is 44.39, and the bootstrap standard error is 4.88, estimate the 90% bootstrap confidence interval for the median RottenTomatoes score of horror movies.



$$44.39 \pm (1.65 \times 4.88) = (36.34, 52.44)$$

Application exercise: 4.1 Bootstrap intervals

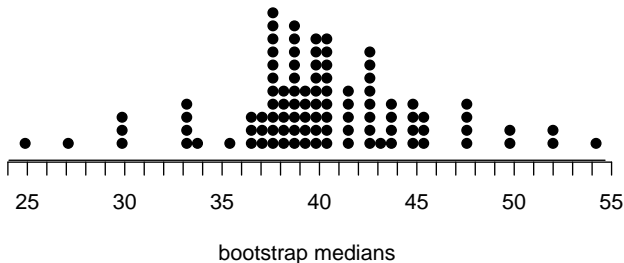
See the course webpage for details.

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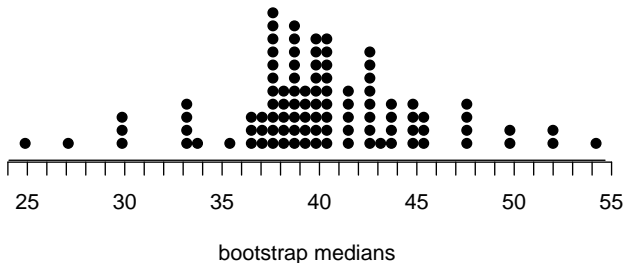
- ▶ This is very similar to bootstrapping, i.e. we randomly sample with replacement from the sample, but this time we shift the bootstrap distribution to be centered at the null value.

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- ▶ The p-value is then defined as the proportion of simulations that yield a sample statistic at least as favorable to the alternative hypothesis as the observed sample statistic.

Do these data provide convincing evidence that the median audience score of horror movies is greater than 40?



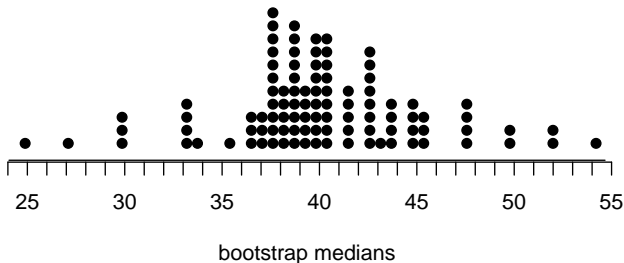
Do these data provide convincing evidence that the median audience score of horror movies is greater than 40?



H_0 : *median* = 40

H_A : *median* > 40

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H_0 : *median* = 40

H_A : *median* > 40

p-value: proportion of simulations where the simulated bootstrap sample median is at least as extreme as the one observed (43.5). $\rightarrow 20 / 100 = 0.20$

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Describe how you would construct a bootstrap interval for a proportion.