Unit 3: Foundations for inference

2. Confidence intervals and hypothesis tests

Sta 101 - Spring 2015

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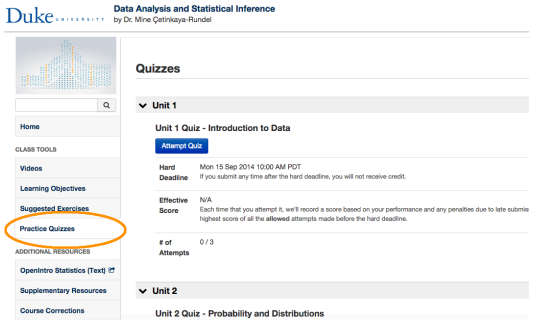
Slides posted at http://bitly.com/sta101sp15

Announcements

- No new PS this week, but review questions provided on new material that will be on the exam next week
- Sample midterm + review questions posted on course website
- Additional practice quizzes on Coursera



- PA3 will be posted on Monday after class and will be due that evening, so budget your time accordingly for that short turnaround!

1. Statistical inference methods based on the CLT depend on the same conditions as the CLT

Always check these in context of the data and the research question!

1. *Independence:* Sampled observations must be independent.

   This is difficult to verify, but is more likely if
   - random sampling/assignment is used, and,
   - if sampling without replacement, $n < 10\%$ of the population.

2. *Sample size/skew:* Either the population distribution is normal or
   $n > 30$ and the population distribution is not extremely skewed (the more skewed the distribution, the higher n necessary for the CLT to apply).

   This is also difficult to verify for the population, but we can check it using the sample data, and assume that the sample mirrors the population.

2. Use confidence intervals to estimate population parameters

$$CI : point\ estimate \pm margin\ of\ error$$

If the parameter of interest is the population mean, and the point estimate is the sample mean,

$$\bar{x} \pm Z^\star \frac{s}{\sqrt{n}}$$

Application exercise: 3.1 Confidence interval for a single mean

See course website for details.

3. Critical value depends on the confidence level

Clicker question

What is the critical value ($Z^\star$) for a confidence interval at the 91% confidence level?

(a) $Z^\star = 1.34$
(b) $Z^\star = 1.65$
(c) $Z^\star = 1.70$
(d) $Z^\star = 1.96$
(e) $Z^\star = 2.33$

Common misconceptions about confidence intervals

1. The confidence level of a confidence interval is the probability that a given interval contains the true population parameter.
   This is incorrect, CIs are part of the frequentist paradigm and as such the population parameter is fixed but unknown. Consequently, the probability any given CI contains the true value must be 0 or 1 (it does or does not).

2. A narrower confidence interval is always better.
   This is incorrect since the width is a function of both the confidence level and the standard error.

3. A wider interval means less confidence.
   This is incorrect since it is possible to make very precise statements with very little confidence.

4. Use hypothesis tests to make decisions about population parameters

Hypothesis testing framework:

1. Set the hypotheses.
2. Check assumptions and conditions.
3. Calculate a *test statistic* and a p-value.
4. Make a decision, and interpret it in context of the research question.

Hypothesis testing for a population mean

1. Set the hypotheses
   - $H_0 : \mu = \textit{null value}$
   - $H_A : \mu <$ or $>$ or $\neq \textit{null value}$
2. Check assumptions and conditions
   - Independence: random sample/assignment, 10% condition when sampling without replacement
   - Sample size / skew: $n \geq 30$ (or larger if sample is skewed), no extreme skew
3. Calculate a *test statistic* and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE}, \text{ where } SE = \frac{s}{\sqrt{n}}$$

4. Make a decision, and interpret it in context of the research question
   - If p-value $< \alpha$, reject H_0, data provide evidence for H_A
   - If p-value $> \alpha$, do not reject H_0, data do not provide evidence for H_A

4. Use hypothesis tests to make decisions about population parameters

Application exercise: 3.2 Hypothesis testing for a single mean

See course website for details.

Clicker question

Which of the following is the correct interpretation of the p-value?

(a) The probability that average GPA of Duke students has changed since 2001.

(b) The probability that average GPA of Duke students has not changed since 2001.

(c) The probability that average GPA of Duke students has not changed since 2001, if in fact a random sample of 63 Duke students this year have an average GPA of 3.58 or higher.

(d) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.

(e) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.

Common misconceptions about hypothesis testing

1. P-value is the probability that the null hypothesis is true
   This is incorrect, p-value is the probability of the observed or more extreme outcome if in fact the null hypothesis is correct. It is the conditional probability of the observed data (or something more extreme), conditioned on the null hypothesis being correct.

2. A high p-value confirms the null hypothesis.
   This is incorrect, a high p-value means the data do not provide convincing evidence for the alternative hypothesis and hence that the null hypothesis can't be rejected.

3. A low p-value confirms the alternative hypothesis.
   This is incorrect, a low p-value means the data provide convincing evidence for the alternative hypothesis, but not necessarily that it is confirmed.

1. Statistical inference methods based on the CLT depend on the same conditions as the CLT
2. Use confidence intervals to estimate population parameters
3. Critical value depends on the confidence level
4. Use hypothesis tests to make decisions about population parameters