# Unit 2: Probability and distributions

## 1. Probability and conditional probability

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

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Slides posted at http://bitly.com/sta101sp15

1. Housekeeping

2. Readiness assessment

3. Main ideas
    1. Disjoint and independent do mean the same thing
    2. Application of the addition rule depends on disjointness of events
    3. Bayes' theorem works for all types of events

4. Summary

- Please review the regrade policy on the course syllabus:
  *Regrade requests must be made within 3 days of when the assignment is returned, and must be submitted in writing. These will be honored if points were tallied incorrectly, or if you feel your answer is correct but it was marked wrong. No regrade will be made to alter the number of points deducted for a mistake. There will be no grade changes after the final exam.*
- Any regrade requests should be submitted to me, I will regrade the entire assignment.

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- 15 minutes individual – turn your clicker over when you're done
- 10 minutes team – put your team name on the front of the scratch off sheet + put **only** the names of the members who are present today on the back

- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But s/he might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $\mathrm{P}(\mathrm{A \text{ and } B}) = 0$

# 1. Disjoint and independent do not mean the same thing

- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But s/he might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $\mathrm{P(A \text{ and } B)} = 0$
- If A and B are *independent events*, having information on A does not tell us anything about B (and vice versa)
  - If A and B are independent:
    - $\mathrm{P(A \mid B) = P(A)}$
    - $\mathrm{P(A \text{ and } B) = P(A) \times P(B)}$

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## 2. Application of the addition rule depends on disjointness of events

- *General addition rule:* P(A or B) = P(A) + P(B) - P(A and B)
- A or B = either A or B or both

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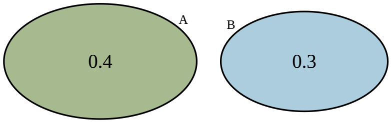
**disjoint events:**

P(A or B)
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0 = 0.7



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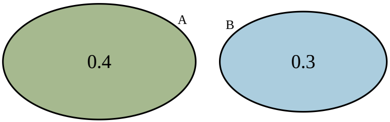
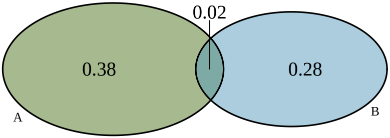
**non-disjoint events:**
P(A or B)
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0.02 = 0.68



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  = *P(A)* × P(B)

Application exercise: 2.1 Probability and conditional probability

See the course website for instructions.

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