Unit 3: Foundations for inference

3. Decision errors, significance levels, sample size & power

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

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Slides posted at http://bitly.com/sta101sp15

1. Housekeeping

2. Main ideas
    1. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
    2. Results that are statistically significant are not necessarily practically significant
    3. Calculate the sample size a priori to achieve desired margin of error
    4. Hypothesis tests are prone to decision errors
    5. Power depends on the effect size, α, n, and s

3. Summary

- PA3 due tonight
- Office hours today: ask questions after class
- Midterm review: online, you should have received an email from WebEx with instructions
- Office hours tomorrow: online/on campus depending on weather

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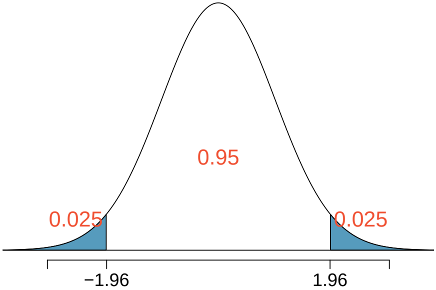
1. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

Two sided



95% confidence level
is equivalent to
two sided HT with $\alpha = 0.05$

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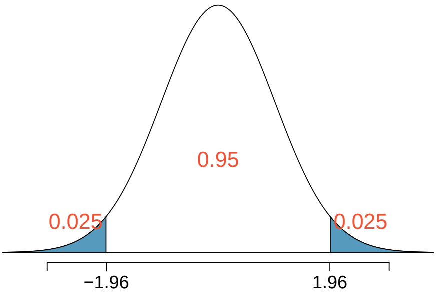

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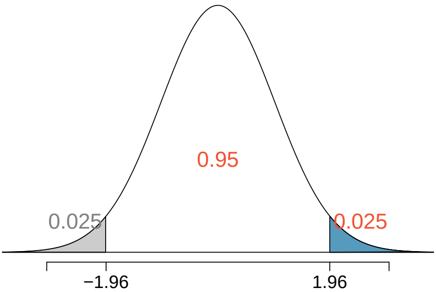
One sided



95% confidence level
is equivalent to
one sided HT with $\alpha = 0.025$

Clicker question

What is the significance level of a two-sided hypothesis test that is equivalent to a 90% confidence interval? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.001
(b) 0.01
(c) 0.025
(d) 0.05
(e) 0.10

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(c) 0.95
(d) 0.98
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Clicker question

A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is true?

(a) The hypothesis $H_A : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.

(b) The hypothesis $H_A : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.

(c) The hypothesis $H_A : \mu = 98$ would be rejected using a 90% confidence interval.

(d) The hypothesis $H_A : \mu = 98.2$ would not be rejected using a 99% confidence interval.

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(c) *The hypothesis $H_A : \mu = 98$ would be rejected using a 90% confidence interval.*

(d) The hypothesis $H_A : \mu = 98.2$ would not be rejected using a 99% confidence interval.

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2. Results that are statistically significant are not necessarily practically significant

Clicker question

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

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Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu \geq 4.5$.

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All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

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Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu \geq 4.5$.

$$Z_{n=100} = \frac{5 - 4.5}{\frac{2}{\sqrt{100}}}$$

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$$Z_{n=10000} = \frac{5 - 4.5}{\frac{2}{\sqrt{10000}}}$$

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Z_{n=10000} &= \frac{5-4.5}{\frac{2}{\sqrt{10000}}} = \frac{5-4.5}{\frac{2}{100}} = \frac{0.5}{0.02} = 25, \quad \textit{p-value} \approx 0
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As n *increases -* $SE \downarrow$, $Z \uparrow$, *p-value* $\downarrow$

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    4. Hypothesis tests are prone to decision errors
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3. Summary

3. Calculate the sample size *a priori* to achieve desired margin of error

Application exercise: 3.3 Sample size

See course website for details.

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		fail to reject H_0	reject H_0
Truth	H_0 true		
	H_A true		

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- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α

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	H_A true	*Type 2 Error*, β	*Power*, $1-\beta$

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  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true: β
- *Power* is the probability of correctly rejecting H_0, and hence the complement of the probability of a Type 2 Error: $1-\beta$

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- increasing the *effect size*

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