Unit 2: Probability and distributions

4. Binomial distribution

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

February 4, 2015

1. Housekeeping

2. Main ideas

    1. Binomial distribution is used for calculating the probability of exact number of successes for a given number of trials

    2. Expected value and standard deviation of the binomial can be calculated using its parameters n and p

    3. Shape of the binomial distribution approaches normal when the S-F rule is met

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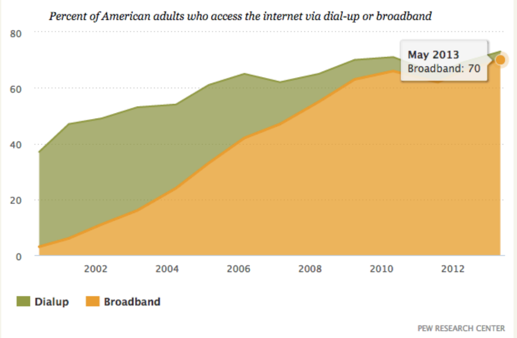
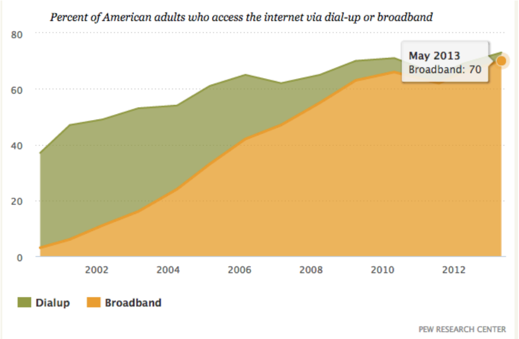
High-speed broadband connection at home in the US





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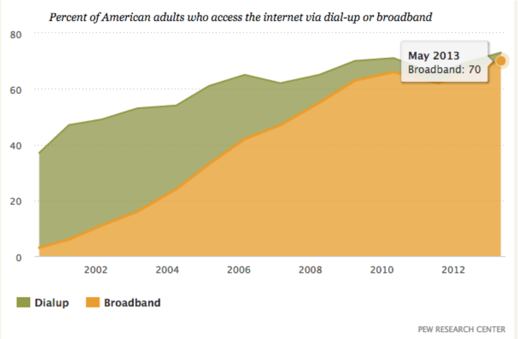

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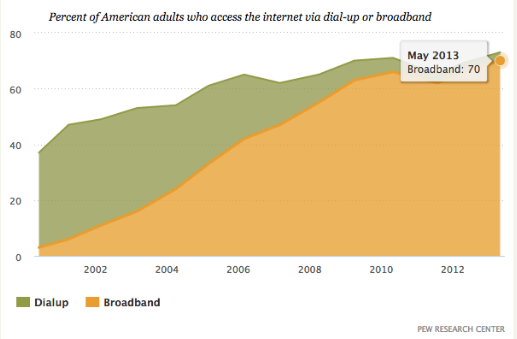

- Each person in the poll be thought of as a *trial*
- A person is labeled a *success* if s/he has high-speed broadband connection at home, *failure* if not
- Since 70% have high-speed broadband connection at home, *probability of success* is $p = 0.70$

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The probability of exactly one 1 of 3 people saying Yes is the sum of all of these probabilities.

$$0.063 + 0.063 + 0.063 = 3 \times 0.063 = 0.189$$

The question from the prior slide asked for the probability of given number of successes, k, in a given number of trials, n, ($\mathrm{k} = 1$ success in $\mathrm{n} = 3$ trials), and we calculated this probability as

$$\# \text{ of scenarios} \times \mathrm{P}(\text{single scenario})$$

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The *Binomial distribution* describes the probability of having exactly k successes in n independent trials (with only two possible outcomes) with probability of success p.

$$\mathrm{P}(\mathrm{k\ successes\ in\ n\ trials}) = \binom{\mathrm{n}}{\mathrm{k}}\, \mathrm{p}^{\mathrm{k}}\ (1 - \mathrm{p})^{(\mathrm{n-k})}$$

$$P(\text{k successes in n trials}) = \binom{n}{k}\, p^k\, (1-p)^{(n-k)}$$

Note: *You can also use R for the calculation of number of scenarios:*

```
> choose(5,3)
```

```
[1] 10
```

Clicker question

Which of the following is false?

(a) There are n ways of getting 1 success in n trials, $\binom{n}{1} = n$.

(b) There is only 1 way of getting n successes in n trials, $\binom{n}{n} = 1$.

(c) There is only 1 way of getting n failures in n trials, $\binom{n}{0} = 1$.

(d) There are $n-1$ ways of getting $n-1$ successes in n trials, $\binom{n}{n-1} = n-1$.

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(d) *There are $n - 1$ ways of getting $n - 1$ successes in n trials, $\binom{n}{n-1} = n - 1$.*

Clicker question

Which of the following is not a condition that needs to be met for the binomial distribution to be applicable?

(a) the trials must be independent

(b) the number of trials, n, must be fixed

(c) each trial outcome must be classified as a *success* or a *failure*

(d) the number of desired successes, k, must be greater than the number of trials

(e) the probability of success, p, must be the same for each trial

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Clicker question

According to the results of the Pew poll suggesting that 70% of Americans have high-speed broadband connection at home, is the probability of exactly 2 out of 15 randomly sampled Americans having such connection at home pretty high or pretty low?

(a) pretty high

(b) pretty low

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(a) pretty high

(b) *pretty low*

Clicker question

According to the results of the Pew poll 70% of Americans have high-speed broadband connection at home, what is the probability that exactly 2 out of 15 randomly sampled Americans have such connection at home?

(a) $0.70^2 \times 0.30^{13}$

(b) $\binom{2}{15} \times 0.70^2 \times 0.30^{13}$

(c) $\binom{15}{2} \times 0.70^2 \times 0.30^{13}$

(d) $\binom{15}{2} \times 0.70^{13} \times 0.30^2$

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(b) $\binom{2}{15} \times 0.70^2 \times 0.30^{13}$

(c) $\binom{15}{2} \times 0.70^2 \times 0.30^{13}$
$= \frac{15!}{13! \times 2!} \times 0.70^2 \times 0.30^{13} = 105 \times 0.70^2 \times 0.30^{13} = 8.2\mathrm{e} - 06$

(d) $\binom{15}{2} \times 0.70^{13} \times 0.30^2$

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    2. Expected value and standard deviation of the binomial can be calculated using its parameters n and p

    3. Shape of the binomial distribution approaches normal when the S-F rule is met

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  - $\sigma = \sqrt{\mathrm{np}(1-\mathrm{p})} = \sqrt{100 \times 0.70 \times 0.30} \approx 4.58$

Note: Mean and standard deviation of a binomial might not always be whole numbers, and that is alright, these values represent what we would expect to see on average.

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http://bitly.com/dist_calc

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S-F rule: The sample size is considered large enough if the expected number of successes and failures are both at least 10

$$np \geq 10 \qquad \text{and} \qquad n(1-p) \geq 10$$

Clicker question

Below are four pairs of Binomial distribution parameters. Which distribution's shape can be approximated by the normal distribution?

(a) $\mathrm{n} = 25, \mathrm{p} = 0.45$

(b) $\mathrm{n} = 100, \mathrm{p} = 0.95$

(c) $\mathrm{n} = 150, \mathrm{p} = 0.05$

(d) $\mathrm{n} = 500, \mathrm{p} = 0.015$

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Application exercise: 2.3 Binomial distribution

See course website for details.

Why do we care?

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