Quiz 1

Q1. Since 1~100 has 33 numbers that divisible by 3, and -1~-100 also has 33 numbers. Hence total number is (# of 1~100) + (# of -1~-100) + 0 = $\boxed{67}$

Q2.
$$\begin{cases} x+1 & \text{if } x \in \mathbb{Z} \\ \lceil x \rceil & \text{if } x \notin \mathbb{Z} \end{cases}$$

Ex. $x = 1.1$

$\Rightarrow$ $\lceil x+1 \rceil$	$\Longleftrightarrow$	$\lceil x \rceil + 1$		$=3$
$\lfloor x+1 \rfloor$	$\Longleftrightarrow$	$\lfloor x \rfloor + 1$	O	$=2$
$\lceil x \rceil + 1$				$=3$
$\lfloor x \rfloor + 1$			O	$=2$
$2 - \lceil 1-x \rceil$	$\Longleftrightarrow$	$2-1+\lfloor x \rfloor$	O	$=2$
$2 - \lfloor 1-x \rfloor$	$\Longleftrightarrow$	$2-1+\lceil x \rceil$		$=3$

Q3

$$6^6 \ \% \ 11 \iff 6^6 \equiv a \pmod{11}$$

$$\begin{aligned} 6^6 \pmod{11} &\equiv (6^2)^3 \pmod{11} \\ &\equiv (36)^3 \pmod{11} \\ &\equiv (3)^3 \pmod{11} \\ &\equiv 27 \pmod{11} \\ &\equiv 5. \pmod{11} \end{aligned}$$

Hence $a = 5 \iff 6^6 \% 11 = 5.$

Q4

$$\gcd(286, 396)$$
$$= \gcd(286, 396 \% 286)$$
$$= \gcd(286 \% 110, 110)$$
$$= \gcd(66, 110 \% 66)$$
$$= \gcd(66 \% 44, 44)$$
$$= \gcd(22, 44 \% 22)$$
$$= \gcd(22, 0) = \boxed{22}$$

Q5 $x \mid y$

① if $y \mid x$ then $x = y$ F

disproof: consider $x = 1$ and $y = -1$

② if $z \in \mathbb{Z}$ such that $y \mid z$, then $x \mid z$ T

proof: Since $x \mid y$ then $y = ax$, $a \in \mathbb{Z}$.

Since $y \mid z$ then $z = by$, $b \in \mathbb{Z}$.

then $z = by = bax = (ba)x$, $ba \in \mathbb{Z}$.

then $x \mid z$

③ if $z \in \mathbb{Z}$ and $x \mid z$, then $y \mid z$ F

disproof: $x = 2$ $y = 6$ $z = 4$.

$2 \mid 4$ but $6 \nmid 4$.

④. $\exists_z$ such that $x \mid z$ and $z \mid y$ T

proof: Let $z = x$, since $x \mid y$ then

$z \mid y$ with that $z \neq 0$,

also $x \mid z \Leftrightarrow x \mid x$

Hence exist such z.

or let $z = y$, since $y \mid y$ then $z \mid y$

also since $z = y$, $x \mid y$ then $x \mid z$.

$\blacksquare$

Q6

$17 \times 11 = (186)\ 1 \iff 17 \times 11 \equiv 1 \pmod{186}$

$17y \equiv 5 \pmod{186}$

$17 \times 11 \equiv 1 \pmod{186}$

$\iff 17 \times 11 \times 5 \equiv 1 \times 5 \pmod{186}$

$\iff 17 \times 55 \equiv 5 \pmod{186}$

$\iff y = 55.$

Q7 F

$a \equiv b \pmod{m}$ then $a \equiv b \pmod{mn}$

$a \equiv b \pmod{n}$

disproof : let $m = 4$, $n = 6$.

$a = 13$, $b = 1$

$a - b = 13 - 1 = 12$ $4 \mid 12$

$6 \mid 12$

but $24 \nmid 12$

Q8

① $\forall x \in \mathbb{Z}$, $\exists y$ such that $x \mid y$ T

proof: take $y = 0$.

② $\forall x \in \mathbb{Z}$, $\exists y$ such that $y \mid x$ T

proof: take $y = x$

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③ $\exists y$, such that $\forall x$, $x \mid y$

proof: take $y = 0$

④ $\exists y$, such that $\forall x$, $y \mid x$

proof: take $y = x$

very little expression difference

consider 1 case or n cases.

Q9

$$\begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$y = \begin{cases} x - |x| & F \\ x / \lceil x \rceil & F \\ x / 2|x| + \frac{1}{2} & T \\ |x+1| / 2x & F \\ x + |x| / 2x & T \end{cases}$$

$x = 1, \quad y = 0$

$x = -2, \quad y = 2$

$x = 2, \quad y = 3/4$

$$\frac{x}{2|x|} + \frac{1}{2} \quad \begin{cases} x > 0, & \frac{1\,\cancel{x}}{2\cancel{x}} + \frac{1}{2} = 1 \\ x < 0, & -\frac{x}{2x} + \frac{1}{2} = 0. \end{cases}$$

$$\frac{x + |x|}{2x} \quad \begin{cases} x > 0, & \frac{x+x}{2x} = 1 \\ x < 0, & \frac{x-x}{2x} = 0. \end{cases}$$

Q_{10}

$$\mathrm{lcm}(-40, 108) = \mathrm{lcm}(40, 108)$$

$$= \frac{40 \cdot 108}{\gcd(40, 108)} = \frac{40 \cdot 108}{4} = 1080.$$

$$\gcd(40, 108) = \gcd(40, 108 \% 40)$$

$$= \gcd(40 \% 28, 28)$$

$$= \gcd(12, 28 \% 12)$$

$$= \gcd(12 \% 4, 4)$$

$$= \gcd(0, 4) = 4.$$