Question 1 (1 mark)

Which of the following relations (over $\mathbb{N}$) are functions? (Select all that apply)

(a) ☐	The = relation
(b) ☐	The | relation
(c) ☐	The ≤ relation
(d) ☐	The relation $\{(n,m,n+m) : n,m \in \mathbb{N}\}$
(e) ☐	The relation $\{(n-1, n) : n \in \mathbb{N}_{>0}\}$
(f) ☐	The relation {}

Question 2 (1 mark)

True or false:

Over $\mathbb{Z}$, the relation defined by $(\leq \oplus \geq)$ is the same as the relation ≠ ?

(a) ○	True
(b) ○	False

Question 3 (1 mark)

Let $\Sigma = \{0,1\}$ and consider the relation on Σ^* given by $R = \{(w,v) : \text{length}(w) \geq 2 \cdot \text{length}(v)\}$

Which of the following properties does R satisfy? Select all that apply

(a) ☐	Reflexivity (R)
(b) ☐	Antireflexivity (AR)
(c) ☐	Symmetry (S)
(d) ☐	Antisymmetry (AS)
(e) ☐	Transitivity (T)

Question 4 (1 mark)

Consider the relation $R = \{(m,n) \in \mathbb{Z} \times \mathbb{Z} : m^2 =_{(5)} n^2 \}$.

Which of the following properties does R satisfy? Select all that apply

(a) ☐	Reflexivity (R)
(b) ☐	Antireflexivity (AR)
(c) ☐	Symmetry (S)
(d) ☐	Antisymmetry (AS)
(e) ☐	Transitivity (T)

Question 5 (1 mark)

Suppose R is a symmetric relation.

True or false: $R = R^{\leftarrow}$

(a) ○	True
(b) ○	False

Question 6 (1 mark)

Let $X = \{\lambda, a, aa\}$ and $Y = \{aab, ab, b\}$.

What is $|XY|$?

Enter integer

Question 7 (1 mark)

Let $F = \mathbb{N}^{\mathbb{N}}$ denote the set of functions from $\mathbb{N}$ to $\mathbb{N}$. Define the relation R on F×F as follows:

$(f,g) \in R$ if $f(n) \neq g(n)$ for only finitely many $n \in \mathbb{N}$

Which of the following properties does R have? Select all that apply

(a) ○	Reflexivity (R)
(b) ○	Antireflexivity (AR)
(c) ○	Symmetry (S)
(d) ○	Antisymmetry (AS)
(e) ○	Transitivity (T)

Question 8 (1 mark)

Let F = $\mathbb{N}^{\mathbb{N}}$ denote the set of functions from $\mathbb{N}$ to $\mathbb{N}$. Define the relation R on F×F as follows:

(f,g) ∈ R if f(n) ≤ g(n) for infinitely many n∈$\mathbb{N}$

Which of the following properties does R have? Select all that apply

(a) ○	Reflexivity (R)
(b) ○	Antireflexivity (AR)
(c) ○	Symmetry (S)
(d) ○	Antisymmetry (AS)
(e) ○	Transitivity (T)

Question 9 (1 mark)

Let Σ = {0,1}

True or false:

For all languages X⊆Σ*: $(X^2)^* = (X^*)^2$

(a) ○	True
(b) ○	False

Question 10 (1 mark)

Let X = {do, re, mi} and Y = {re, mi, fa, so}

Which of the following words are in X*⊕Y*? Select all that apply.

(a) ☐	λ
(b) ☐	dodo
(c) ☐	mire
(d) ☐	sore
(e) ☐	doso
(f) ☐	dood

Q1

A function, $f: S \to T$ is a binary relation

for all $s \in S$, there is exactly one $t \in T$.

a) $\{(0,0), (1,1), (2,2) \cdots\}$ T

b) $\{(1,0), (1,1), (1,2) \cdots\}$ F

c) $\{(0,0), (0,1), (0,2) \cdots\}$ F

d) $\{(0,0,0), (0,1,1), (1,1,2) \cdots\}$. F

e) $\{(0,1), (1,2), (2,3) \cdots\}$. T.

f) no such y for $\{0, y\}$

with relation $\{\}$. However $\{\} \times \mathbb{N}$ is a

function (over domain $\{\}$).

Q_2

True

Let (x, y) be arbitary pair over $\mathbb{Z}$.

with Relation $(\geq \oplus \leq) \Rightarrow$ we exclude

all equal cases. Hence (x, y) must be

satisfied.

Q_3

a) not R since $w = 010$, $\langle w, w \rangle : 3 < 6$.

b) not (AR) since $w = \lambda$, $\langle w, w \rangle : 0 \geq 0$.

c) not (S) since $w = 010$, $v = 0$.

$\langle w, v \rangle : 3 > 2$ $\quad \langle v, w \rangle : 1 < 6$.

d) (AS) only possible case is λ.

e) (T) $\langle w, v \rangle : w \geq 2v$.

$\langle v, x \rangle : v \geq 2x$

$\Rightarrow \quad w \geq 2v \geq 2(2x) = 4x$

$\Rightarrow \langle w, x \rangle$.

Q4

a) (R) since $5 \mid m^2 - m^2$

b) not (AR)

c) (S) $m^2 \equiv n^2 \pmod 5 \iff n^2 \equiv m^2 \pmod 5$.

d) not (AS) consider $m = -n$

e) (T) $a^2 \equiv b^2 \pmod 5$ $b^2 \equiv c^2 \pmod 5$

$$\Downarrow$$

$$a^2 \equiv c^2 \pmod 5$$

Q5 True

suppose R is symmetric.

then $(x, y) \in R$, exist $(y, x) \in R$

so $(x, y) \in R^{\leftarrow}$

for $(x, y) \in R^{\leftarrow}$ means that

exist $(y, x) \in R$, however R is symmetric.

$(x, y) \in R$.

$\boxed{\#}$

Hence $R = R^{\leftarrow}$

Q6

$XY = \{aab,\ ab,\ b,$

$aaab,\ \cancel{aab},\ \cancel{ab},$

$daaab,\ \cancel{adab},\ \cancel{dab}\}$

$|XY| = 5.$

Q7

a) (R) since $f(n) = f(n) \in R$ (since 0 in finite)

b) Not (AR) since (R)

c) (S) since for all $(f(n), g(n))$

$(g(n), f(n))$ have same finite

number such that $g(n) \neq f(n)$.

Hence $(g(n), f(n)) \in R$

d) not (AS) Let $f(n) = 1$, $g(n) = \begin{cases} 0 & n=0 \\ 1 & n \neq 0 \end{cases}$.

then $(f(n), g(n))$, $(g(n), f(n)) \in R$.

but $f(n) \neq g(n)$.

e1. (T).

Suppose $f(n) \ne g(n)$, $g(n) \ne h(n)$.

then let $X = \{n : f(n) \ne g(n)\}$.

$Y = \{n : g(n) \ne h(n)\}$. By defn,

X, Y are both finite.

Then, if $f(n) \ne h(n)$, ~~the~~ the only

possible n's must in $X \cup Y$.

in other words. $n \in X \cup Y$, since X, Y

are both finite, n for $f(n) \ne g(n)$ is finite.

Hence (T). $\blacksquare$

Q8.

a) (R) since $f(n) = f(n) \in R$.

b) not (AR)

c) not (S) since let $f(n)=1$, $g(n)=2$.

but $f(n)=2$, $g(n)=1$ not in R.

d) not (AS) let $f(n) = 1$ $g(n) = \begin{cases} 2 & \text{if odd} \\ 0 & \text{if even.} \end{cases}$

$\left.\begin{array}{l} f(n) \geq g(n) \text{ when even.} \\ f(n) \leq g(n) \text{ when odd.} \end{array}\right\}$ infinite.

but $f(n) \neq g(n)$.

e). not (T). let $f(n)=1$, $g(n) = \begin{cases} 2 & \text{odd} \\ 0 & \text{even} \end{cases}$ $h(n) = 0$

$f(n) \leq g(n)$ when odd. $g(n) \leq h(n)$ when even.

but $f(n) > h(n)$. Hence not (T).

Q9.

let $X = \{0, 1\}$.

and consider $(X^2)^*$ possiblely take λ.

$(X^*)^2$ have length at least 2.

Hence not the same.

Q10

a) $\lambda \in X^*$ and $\lambda \in Y^*$ F

b) dodo $\in X^*$ and dodo $\notin Y^*$ T

c) mire $\in X^*$ and mire $\in Y^*$ F

d) sore $\notin X^*$ and sore $\in Y^*$ T

e) doso $\notin X^*$ and doso $\notin Y^*$ F

f) dood $\notin X^*$ and dood $\notin Y^*$ F