Question 1 (1 mark)

Let A be the set {b,a,n,a,n,a} and B be the set {p,i,n,e,a,p,p,l,e}.

Which of the following are subsets of $A \oplus B$?

Select all that apply.

(a) ☐	{b,p}
(b) ☐	{p,p,p,a,a,a,n,n,e,e}
(c) ☐	{b,p,i,e,p,p,l,e}
(d) ☐	{a,b,e,i,l,n,p}
(e) ☐	None of the above

Question 2 (1 mark)

True or false:

For all sets A,B, $A \cup B = A \cap B$ if and only if $A = B$

(a) ○	True
(b) ○	False

Question 3 (1 mark)

Which of the the following sets have exactly 6 elements? Select all that apply (intervals are over $\mathbb{N}$)

(a) ☐	$[0,1] \times (1,6)$
(b) ☐	$(0,1) \times [1,6]$
(c) ☐	$(0,2] \times [1,4)$
(d) ☐	$[0,2] \times (1,4)$
(e) ☐	None of the others

Question 4 (1 mark)

True or false:

For all sets A,B,C:

$A\times(B\cap C) = (A\times B) \cap (A\times C)$

(a) ○	True
(b) ○	False

Question 5 (1 mark)

Order the following sets by **increasing** cardinalities: (smallest at the top)

✥ $\{ n\in\mathbb{Z} : 6\|n \}$
✥ $\{ n\%6 : n\in\mathbb{Z}\}$
✥ $\{ 6\%n : n \in \mathbb{N}$ and $n>0 \}$
✥ $\{ n\in\mathbb{Z} : n\|6 \}$

Question 6 (1 mark)

Suppose $A = \{0,1,2\}$

For how many sets $B\subseteq\mathbb{N}$ is it the case that $A\times B = B\times A$?

(a) ○	0
(b) ○	1
(c) ○	2
(d) ○	3
(e) ○	Infinitely many
(f) ○	None of the above

Question 7 (1 mark)

Suppose A has 24 elements, B has 65 elements and $A \oplus B$ has 73 elements.

What can be said about the number of elements in $A \cup B$?

(a) ○	It can be any value in $[0,89] \cap \mathbb{N}$
(b) ○	It can be any value in $[73,89] \cap \mathbb{N}$
(c) ○	It is exactly 57
(d) ○	It is exactly 81
(e) ○	It can be any value in $[0,73] \cap \mathbb{N}$
(f) ○	None of the above

Question 8 (1 mark)

Which of the following statements are true for all sets A,B,C? Select all that apply.

(a) ☐	$(A \cup B) \cap A = A \cup (B \cap A)$
(b) ☐	$(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$
(c) ☐	$C \setminus (A \cup B) = (C \setminus A) \cup (C \setminus B)$
(d) ☐	$(C \setminus A)^C = (C^C) \setminus (A^C)$
(e) ☐	$(A \oplus B)^C = (A^C) \oplus (B^C)$

Question 9 (1 mark)

Let $\Sigma = \{c,u,p\}$ and $\Psi = \{s,a,u,c,e,r\}$

Which of the following words are in $\Sigma^* \setminus \Psi^*$? Select all that apply.

(a) ☐	λ
(b) ☐	cup
(c) ☐	saucer
(d) ☐	ppp
(e) ☐	eee

Question 10 (1 mark)

Let w be the word ab, and let v be the word babb. What is length(vww)?

Enter integer

Q1.

$A = \{b, a, n\}$ $B = \{p, i, n, e, a, l\}$

$A \oplus B = \{b, p, i, e, l\}$

Hence

a) is a subset since $\{b, p\} \subseteq A \oplus B$ T

b) is NOT since $\{a, n\} \nsubseteq A \oplus B$ F

c) is a subset since $\{b, p, i, e, l\} \subseteq A \oplus B$ T

d) is NOT since $\{a, n\} \nsubseteq A \oplus B$ F

Q_2 T

proof:

($\Rightarrow$) suppose for a contradiction that

$A \cup B = A \cap B$, but $A \neq B$.

Then WLOG, we say $\exists x \in A$ but

$x \notin B$. However, since $\exists x \in A$, $x \in A \cup B$.

and then $x \in A \cap B$ since $A \cup B = A \cap B$.

Hence $x \in B$, which is a contradiction.

($\Leftarrow$) Trivial.

since $A = B$, then $\forall x \in A$, $x \in B$.

Hence $x \in A \cap B$. ① also that $x \in A \cup B$. ②

Since all x satisfies ① and ②, trivial to see that $A \cap B \subseteq A \cup B$ and $A \cup B \subseteq A \cap B$. Hence $A \cap B = A \cup B$

Q_3

a) $|[0,1] \times (1,6)| = 2 \times 4 = 8$

b) $|(0,1) \times [1,6]| = 0 \times 6 = 0$

c) $|(0,2] \times [1,4)| = 2 \times 3 = 6$

d) $|[0,2] \times (1,4)| = 3 \times 2 = 6$

Q4

$(\Rightarrow)$ $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$

Note that $\forall (x,y) \in A \times (B \cap C)$, $x \in A$,

$y \in (B \cap C)$, since $x \in A$ and $y \in (B \cap C)$,

$(x,y) \subseteq A \times B$, similiarly $(x,y) \subseteq A \times C$.

Hence $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$

$(\Leftarrow)$ $A \times (B \cap C) \supseteq (A \times B) \cap (A \times C)$

Note that $\forall (x,y) \in (A \times B) \cap (A \times C)$

$x \in A$, $y \in B$ and $y \in C$

Hence by defn, all $(x,y) \in A \times (B \cap C)$

Hence $A \times (B \cap C) \supseteq (A \times B) \cap (A \times C)$. $\boxed{Q.E.D.}$

Q5

$\{6 \% n : n \in \mathbb{N}, n > 0\} = \{0, 2, 1, 6\}$ $\# = 4$

$\{n \% 6\} = \{0, 1, 2, 3, 4, 5\}$ $\# = 6$

$\{n \in \mathbb{Z} : n | 6\} = \{1, 2, 3, -1, -2, -3, 6, -6\}$ $\# = 8$

$\{n \in \mathbb{Z} : 6 | n\} = \{\cdots -6, 0, 6, 12, \cdots\}$ $\# = \infty$

Q6

$A = \{0, 1, 2\}$ $A \times B = B \times A$

$B = \{\}$ or $B = \{0, 1, 2\}$

For all other cases,

if $\exists x \in B$, $x \notin A$, then $(A, x) \neq (B, A)$

(eg: $B = \{0, 1, 2, 3\}$)

if $\exists x \in A$, $x \notin B$, then $(x, B) \neq (B, A)$

(eg: $B = \{0, 1\}$)

Q7

$|A| = 24, \quad |B| = 65.$

$|A \oplus B| = 73$

Note that by defn,

$|A \oplus B| = |A \setminus B \cup B \setminus A| = 73$

$|A| + |B| = 24 + 65 = 89$

$|A| + |B| - |A \oplus B| = 2 \times |A \cap B| = 89 - 73$

$\Rightarrow \quad |A \cap B| = 8$

Hence $A \cup B = |A \oplus B| + |A \cap B| = |A| + |B| - |A \cap B|$

$= 81.$

Q8

a) $(A \cup B) \cap A \Leftrightarrow x \in A \cup B$ and $x \in A$ T

$\Leftrightarrow x \in A$

$A \cup (B \cap A) \Leftrightarrow x \in A$ or $x \in (B \cap A)$

b) $(A \cup B) \setminus B \Leftrightarrow x \in A \cup B$ and $x \notin C$

T

$\Leftrightarrow (x \in A$ or $x \in B)$, $x \notin C$

$\Leftrightarrow x \in A, x \notin C$ or $x \in B, x \notin C$

$\Leftrightarrow (A \setminus C) \cup (B \setminus C)$

c) Let $x \in C \setminus A$ and $x \in B$ then

$x \in C \setminus A$ or $x \in C \setminus B$ (by defn) $\Leftrightarrow x \in (C \setminus A) \cup (C \setminus B)$

but $x \notin C \setminus (A \cup B)$ since $x \in B \subseteq A \cup B$.

d)

take $C = \phi$ A be a non-empty set

then $(C \setminus A)^c = (C \cap A^c)^c$

$$= (\phi)^c$$

$$= C^c = \mathcal{U}$$

$$C^c \setminus A^c = C^c \cap A^{cc}$$

$$= C^c \cap A$$

$$= A \neq \mathcal{U} \qquad \blacksquare$$

e)

$$(A \oplus B)^c = ((A \setminus B) \cup (B \setminus A))^c = ((\underset{\phi}{A} \cap B^c) \cup (\underset{\phi}{B} \cap A^c))^c \quad \mathcal{U}$$

$$A^c \oplus B^c = (A^c \setminus B^c \cup B^c \setminus A^c) = ((A^c \cap \underset{\phi}{B}) \cup (B^c \cap \underset{\phi}{A})) \quad \phi$$

take $A = \phi$, $B = \phi$, $(A \oplus B)^c = (\phi)^c = \mathcal{U}$

$$A^c \oplus B^c = \mathcal{U} \oplus \mathcal{U} = \phi$$

since $\forall x \in \mathcal{U}$, $x \in \mathcal{U}$

Can't find in φ^*

$$\Sigma^* = \{ \underset{0}{\lambda},\ \underset{1}{c},\ \underset{1}{u},\ p,\ \underset{2}{cu},\ \boxed{up},\ pc,\ cp,\ uc,\ pu \cdots \}$$

$$\varphi^* = \{ \underset{0}{\lambda},\ s,\ a,\ \underset{1}{u},\ \underset{1}{c},\ e,\ r,\ \underset{2}{\boxed{cu}}, \cdots \}$$

$$\Rightarrow \Sigma^* \setminus \varphi^* = \{ p,\ s,\ a,\ e,\ r,\ up,\ pc,\ cp,\ pu,\ sa,\ su,\ us \cdots \}$$

Q_9

a) $\lambda \in \Sigma^0$, $\lambda \in \Psi^0$ F

b) $cup \in \Sigma^3$, $cup \notin \Psi^*$ (no c's or p's) T

c) $saucer \notin \Sigma^*$, (no a's or c's ....) F

d) $ppp \in \Sigma^3$, $ppp \notin \Psi^*$ (no p's) T

e) $eee \notin \Sigma^*$ (no e's)

Q_{10}

$w = ab$, $v = babb$

$vww = babbabab$

length $= 4 + 2 + 2 = 8$