Question 1 (1 mark)

How many integers between -100 and 100 (inclusive) are divisible by 3?

Enter integer

Question 2 (1 mark)

Which of the following will give:

- x+1 if x is an integer, and
- The smallest integer greater than x if x is not an integer.

Select all that apply

(a) ☐	$\lceil x+1 \rceil$
(b) ☐	$\lfloor x+1 \rfloor$
(c) ☐	$\lceil x \rceil+1$
(d) ☐	$\lfloor x \rfloor+1$
(e) ☐	$2-\lceil 1-x \rceil$
(f) ☐	$2-\lfloor 1-x \rfloor$

Question 3 (1 mark)

What is 6^6 % 11?

Enter integer

Question 4 (1 mark)

What is gcd(286,396)?

Enter integer

Question 5 (1 mark)

Suppose x,y are arbitrary integers such that x|y.

Which of the following are always true (i.e. true for any such x and y)? Select all that apply

(a) ☐	If y|x then x=y
(b) ☐	If z is an integer such that y|z, then x|z
(c) ☐	If z is an integer such that x|z, then y|z
(d) ☐	There is an integer z such that x|z and z|y
(e) ☐	None of the above

Question 6 (1 mark)

Given that

$17 \times 11 =_{(186)} 1$

Find an integer y such that $17y =_{(186)} 5$ and $0 \leq y < 186$.

Enter integer

Question 7 (1 mark)

True or false:

For all positive integers m,n and all integers a,b: if $a =_{(m)} b$ and $a =_{(n)} b$ then $a =_{(mn)} b$

(a) ○	True
(b) ○	False

Question 8 (1 mark)

Which of the following are true. Select all that apply

(a) ☐	For every integer x, there exists an integer y such that x|y.
(b) ☐	For every integer x, there exists an integer y such that y|x.
(c) ☐	There exists an integer y, such that for every integer x, x|y.
(d) ☐	There exists an integer y, such that for every integer x, y|x.
(e) ☐	None of the above

Question 9 (1 mark)

Which of the following will give:

- 1 if x>0, and
- 0 if x<0

Select all that apply.

(a) ☐	x - |x|
(b) ☐	x / [x]
(c) ☐	x / (2|x|) + 1/2
(d) ☐	|x+1| / (2x)
(e) ☐	(x + |x|) / (2x)

Question 10 (1 mark)

What is lcm(-40,108)?

Enter number

Quiz 1

Q_1.

$$\left\lfloor \frac{100}{3} \right\rfloor - \left\lfloor \frac{-100-1}{3} \right\rfloor$$

$$= 33 - (-34)$$

$$= 67$$

Q_2.

$$\begin{cases} x+1 & \text{if } x \in \mathbb{Z} \\ \lceil x \rceil & \text{if } x \notin \mathbb{Z} \end{cases}$$

Ex. $x = 1.1$

$\Rightarrow$

$\lceil x+1 \rceil$	$\Leftrightarrow$	$\lceil x \rceil + 1$		$=3$
$\lfloor x+1 \rfloor$	$\Leftrightarrow$	$\lfloor x \rfloor + 1$	O	$=2$
$\lceil x \rceil + 1$				$=3$
$\lfloor x \rfloor + 1$			O	$=2$
$2 - \lceil 1-x \rceil$	$\Leftrightarrow$	$2-1+\lfloor x \rfloor$	O	$=2$
$2 - \lfloor 1-x \rfloor$	$\Leftrightarrow$	$2-1+\lceil x \rceil$		$=3$

Q3

$6^6 \% 11 \iff 6^6 \equiv a \pmod{11}$

$6^6 \pmod{11} \equiv (6^2)^3 \pmod{11}$

$\equiv (36)^3 \pmod{11}$

$\equiv (3)^3 \pmod{11}$

$\equiv 27 \pmod{11}$

$\equiv 5. \pmod{11}$

Hence $a = 5 \iff 6^6 \% 11 = 5.$

Q4

$$\gcd(286, 396)$$

$$= \gcd(286, 396 \% 286)$$

$$= \gcd(286 \% 110, 110)$$

$$= \gcd(66, 110 \% 66)$$

$$= \gcd(66 \% 44, 44)$$

$$= \gcd(22, 44 \% 22)$$

$$= \gcd(22, 0) = \boxed{22}$$

Q5 $x \mid y$

① if $y \mid x$ then $x = y$ F

disproof: consider $x = 1$ and $y = -1$

② if $z \in \mathbb{Z}$ such that $y \mid z$, then $x \mid z$ T

proof: Since $x \mid y$ then $y = ax$, $a \in \mathbb{Z}$.

since $y \mid z$ then $z = by$, $b \in \mathbb{Z}$.

then $z = by = bax = (ba)x$, $ba \in \mathbb{Z}$.

then $x \mid z$

③ if $z \in \mathbb{Z}$ and $x \mid z$, then $y \mid z$ F

disproof: $x = 2$ $y = 6$ $z = 4$.

$2 \mid 4$ but $6 \nmid 4$.

④. $\exists_z$ such that $x \mid z$ and $z \mid y$ T

proof: Let $z = x$, since $x \mid y$ then

$z \mid y$ with that $z \neq 0$,

also $x \mid z \iff x \mid x$

Hence exist such z.

or let $z = y$, since $y \mid y$ then $z \mid y$

also since $z = y$, $x \mid y$ then $x \mid z$.

▨

Q6

$17 \times 11 = (186) + 1 \iff 17 \times 11 \equiv 1 \pmod{186}$

$17y \equiv 5 \pmod{186}$

$17 \times 11 \equiv 1 \pmod{186}$

$\iff 17 \times 11 \times 5 \equiv 1 \times 5 \pmod{186}$

$\iff 17 \times 55 \equiv 5 \pmod{186}$

$\iff y = 55.$

Q7 F

$a \equiv b \pmod{m}$
$a \equiv b \pmod{n}$ then $a \equiv b \pmod{mn}$

disproof : let $m = 4$, $n = 6$.
$a = 13$, $b = 1$

$a - b = 13 - 1 = 12$

$4 \mid 12$
$6 \mid 12$
but $24 \nmid 12$

Q8

① $\forall x \in \mathbb{Z}$, $\exists y$ such that $x \mid y$ T

proof: take $y = 0$.

② $\forall x \in \mathbb{Z}$, $\exists y$ such that $y \mid x$ T

proof: take $y = x$

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③ $\exists y$, such that $\forall x$, $x \mid y$

proof: take $y = 0$

④ $\exists y$, such that $\forall x$, $y \mid x$

proof: take $y = x$

very little expression difference

consider 1 case or n cases.

Q9

$$\begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$y = \begin{cases} x - |x| \\ x / \lceil x \rceil \\ x/2|x| + \frac{1}{2} \\ |x+1|/2x \\ x + |x| / 2x \end{cases}$$

F	$x=1,\ y=0$
F	$x=-2,\ y=2$
T	
F	$x=2,\ y=3/4$
T	

$$\frac{x}{2|x|} + \frac{1}{2} \begin{cases} x>0, & \frac{|x|}{2x} + \frac{1}{2} = 1 \\ x<0, & -\frac{x}{2x} + \frac{1}{2} = 0. \end{cases}$$

$$\frac{x+|x|}{2x} \begin{cases} x>0, & \frac{x+x}{2x} = 1 \\ x<0, & \frac{x-x}{2x} = 0. \end{cases}$$

Q_{10}

$$\text{lcm}(-40, 108) = \text{lcm}(40, 108)$$

$$= \frac{40 \cdot 108}{\gcd(40, 108)} = \frac{40 \cdot 108}{4} = 1080.$$

$$\gcd(40, 108) = \gcd(40, 108 \% 40)$$

$$= \gcd(40 \% 28, 28)$$

$$= \gcd(12, 28 \% 12)$$

$$= \gcd(12 \% 4, 4)$$

$$= \gcd(0, 4) = 4.$$