Будагян Артем Игоревич, 1 курс ИВТ, 1 подгруппа

№11.3.1

$z = xy^2 - \frac{x}{y}$ $\quad \Delta_x z, \Delta_y z, \Delta z$ - ?

$M_0(3;-2)$; $\Delta x = 0,1$; $\Delta y = -0,05$

1) $M_0(3;-2)$

]$x_0 = 3, y_0 = -2$

Тогда: $x = x_0 + \Delta x = 3 + 0,1 = 3,1$ $\quad y = y_0 + \Delta y = -2 + (-0,05) = -2,05$

Тогда: $M_1(3,1; -2,05)$

2) $z(M_0) = z(3;-2) = \left[z = xy^2 - \frac{x}{y}\right] = 3\cdot(-2)^2 - \frac{3}{-2} = 3\cdot 4 + \frac{3}{2} = 13,5$

$z(x_0 + \Delta x; y_0) = z(3,1;-2) = 3,1\cdot(-2)^2 - \frac{3,1}{-2} = 3,1\cdot 4 + \frac{3,1}{2} = 13,95$

$z(x_0; y_0+\Delta y) = z(3; -2{,}05) = 3\cdot(-2{,}05)^2 - \frac{3}{-2{,}05} \approx 3\cdot4{,}2025 + \frac{3}{2{,}05} = 14{,}0409$

$z(M_1) = z(x; y) = z(3{,}1; -2{,}05) = 3{,}1\cdot(-2{,}05)^2 - \frac{3{,}1}{-2{,}05} = 3{,}1\cdot4{,}2025 + \frac{3{,}1}{2{,}05} \approx 14{,}54$

3) $\Delta_x z = z(x_0+\Delta x; y_0) - z(x_0; y_0) = 13{,}95 - 13{,}5 = 0{,}45$

$\Delta_y z = z(x_0; y_0+\Delta y) - z(x_0; y_0) = 14{,}04 - 13{,}5 = 0{,}54$

4) $\Delta z = z(x_0+\Delta x; y_0+\Delta y) - z(x_0; y_0) = 14{,}54 - 13{,}5 = 1{,}04$

№11.3.2

$z = x^2 y \quad M_0(1;2) \quad \Delta x = 0{,}1 \quad \Delta y = -0{,}2$

1) $M_0(1;2) \Rightarrow x_0 = 1,\ y_0 = 2 \Rightarrow$

$x = x_0 + \Delta x = 1 + 0{,}1 = 1{,}1$

$y = y_0 + \Delta y = 2 + (-0{,}2) = 1{,}8$

2) $z(x_0; y_0) = z(1; 2) = 1^2\cdot 2 = 2$

$z(x_0+\Delta x; y_0) = z(1{,}1; 2) = (1{,}1)^2\cdot 2 = 1{,}21\cdot 2 = 2{,}42$

$z(x_0; y_0+\Delta y) = z(1; 1{,}8) = 1^2\cdot 1{,}8 = 1{,}8$

$z(x_0+\Delta x; y_0+\Delta y) = z(1{,}1; 1{,}8) = (1{,}1)^2\cdot 1{,}8 = 1{,}21\cdot 1{,}8 = 2{,}178$

3) $\Delta_x z = z(x_0+\Delta x; y_0) - z(x_0; y_0) = 2{,}42 - 2 = 0{,}42$

$\Delta_y z = z(x_0; y_0+\Delta y) - z(x_0; y_0) = 1{,}8 - 2 = -0{,}2$

4) $\Delta z = z(x_0+\Delta x; y_0+\Delta y) - z(x_0; y_0) = 2{,}178 - 2 = 0{,}178$

Дифференциал функции

$$dz = \underbrace{z'_x dx}_{d_x z} + \underbrace{z'_y dy}_{d_y z}$$

полный дифференциал

частный дифференциал

$dx = \Delta x$ $\quad dy = \Delta y$

$z'_x = f'_x(x;y)$ $\quad f'_x(x_0;y_0)$

$z'_y = f'_y(x;y)$ $\quad f'_y(x_0;y_0)$

$$f(x_0+\Delta x; y_0+\Delta y) \approx f(x_0;y_0) + f'_x(x_0;y_0)\cdot\Delta x + f'_y(x_0;y_0)\cdot\Delta y$$

линеаризация функции $z=f(x;y)$ в окрестности точки $M_0(x_0;x_y)$

№1.3.9

$z = \frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}$ $\quad z'_x - ?$ $\quad z'_y - ?$

$z'_x = \left(\frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}\right)'_x = \frac{1}{y^3}\cdot(x)'_x + y\cdot(x^{-3})'_x - \frac{1}{6y}\cdot(x^{-2})'_x =$

$= \frac{1}{y^3}\cdot 1 + y\cdot(-3)\cdot x^{-3-1} - \frac{1}{6y}\cdot(-2)\cdot x^{-2-1} = \frac{1}{y^3} - \frac{3y}{x^4} + \frac{1}{3x^3y}$

$z'_y = \left(\frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}\right)'_y = x\cdot(y^{-3})'_y + \frac{1}{x^3}\cdot(y)'_y - \frac{1}{6x^2}\cdot(y^{-1})'_y = x\cdot(-3)\cdot y^{-3-1} +$

$+ \frac{1}{x^3}\cdot 1 - \frac{1}{6x^2}\cdot(-1)\cdot y^{-1-1} = -\frac{3x}{y^4} + \frac{1}{x^3} + \frac{1}{6x^2y^2}$

№1.3.10

$z = \frac{x^2-2xy}{y^2+2xy+1}$ $\quad z'_x, z'_y - ?$

$z'_x = \left(\frac{x^2-2xy}{y^2+2xy+1}\right)'_x = \frac{(x^2-2xy)'_x\cdot(y^2+2xy+1) - (x^2-2xy)(y^2+2xy+1)'_x}{(y^2+2xy+1)^2} =$

$= \frac{(2x-2y)(y^2+2xy+1) - (x^2-2xy)\cdot 2y}{(y^2+2xy+1)^2}$

$z'_y = \left(\frac{x^2-2xy}{y^2+2xy+1}\right)'_y = \frac{(x^2-2xy)'_y(y^2+2xy+1) - (x^2-2xy)(y^2+2xy+1)'_y}{(y^2+2xy+1)^2} =$

$= \frac{-2x(y^2+2xy+1) - (x^2-2xy)(2y+2x)}{(y^2+2xy+1)^2}$

$$z=\cos\frac{x^2+y^2}{x^3+y^3}\qquad z'_x,\ z'_y,\ d_xz,\ d_yz,\ dz-?$$

$$1)\ z'_x=\left(\cos\frac{x^2+y^2}{x^3+y^3}\right)'_x=-\sin\frac{x^2+y^2}{x^3+y^3}\cdot\left(\frac{x^2+y^2}{x^3+y^3}\right)'_x=$$

$$=-\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{(x^2+y^2)'_x(x^3+y^3)-(x^2+y^2)(x^3+y^3)'_x}{(x^3+y^3)^2}=$$

$$=-\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{2x(x^3+y^3)-(x^2+y^2)\cdot 3x^2}{(x^3+y^3)^2}=$$

$$=\frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}$$

$$z'_y=\left(\cos\frac{x^2+y^2}{x^3+y^3}\right)'_y=\ldots=\frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}$$

$$2)\ d_xz=z'_xdx=\frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}\,dx$$

$$d_yz=z'_ydy=\frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\sin\frac{x^2+y^2}{x^3+y^3}\,dy$$

$$3)\ dz=d_xz+d_yz=\frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\sin\frac{x^2+y^2}{x^3+y^3}\,dx+$$

$$+\frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\sin\frac{x^2+y^2}{x^3+y^3}\,dy=\frac{1}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}\cdot$$

$$\cdot\left(\left(3x^2(x^2+y^2)-2x(x^3+y^3)\right)dx+\left(3y^2(x^2+y^2)-2y(x^3+y^3)\right)dy\right)$$

№11.3.17

$$u=\frac{x}{\sqrt{y^2+z^2}}\qquad du-?$$

$$du=u'_xdx+u'_ydy+u'_zdz$$

$$u'_x=\left(\frac{x}{\sqrt{y^2+z^2}}\right)'_x=\frac{1}{\sqrt{y^2+z^2}}\cdot(x)'_x=\frac{1}{\sqrt{y^2+z^2}}$$

$$u'_y=\left(\frac{x}{\sqrt{y^2+z^2}}\right)'_y=x\cdot\left((y^2+z^2)^{-\frac{1}{2}}\right)'_y=x\cdot\left(-\frac{1}{2}\right)(y^2+z^2)^{-\frac{1}{2}-1}\cdot(y^2+z^2)'_y=$$

$$=x\cdot\left(-\frac{1}{2}\right)\cdot\frac{1}{\sqrt{y^2+z^2}}\cdot 2y=\frac{-xy}{\sqrt{y^2+z^2}}$$

$$u'_z=\left(\frac{x}{\sqrt{y^2+z^2}}\right)'_z=\ldots=\frac{2xz}{\sqrt{y^2+z^2}}$$

$$du=\frac{1}{\sqrt{y^2+z^2}}dx+\frac{-xy}{\sqrt{(y^2+z^2)^3}}dy+\frac{-xz}{\sqrt{(y^2+z^2)^3}}dz=$$

$$=\frac{dx}{\sqrt{y^2+z^2}}-\frac{xy\,dy+xz\,dz}{\sqrt{(y^2+z^2)^3}}$$