11.05.20

№ 7.3.11

1) $\lim\limits_{x\to0} \frac{\ln \sin 3x}{\ln x} = \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to0}\frac{(\ln\sin 3x)'}{(\ln x)'} = \lim\limits_{x\to0}\frac{1/\sin 3x \cdot \cos 3x\cdot 3}{1/x} =$

$= \lim\limits_{x\to0}\frac{3x\cdot\cos 3x}{\sin 3x} = \left[\frac{0}{0}\right] = \underbrace{\lim\limits_{x\to0}\cos(3x)}_{1} \cdot \lim\limits_{x\to0}\frac{3x}{\sin(3x)} = 1\cdot\lim\limits_{x\to0}\left(\frac{1}{(\sin 3x)/3x}\right) =$

$= [\text{если } x\to0;\ \text{то } 3x\to0] = \lim\limits_{3x\to0}\left(\frac{1}{(\sin 3x)/3x}\right) = \left[\lim\limits_{x\to0}\frac{\sin x}{x}=1\right] = \frac{1}{1} = 1$

2) $\lim\limits_{x\to0}\frac{x^3}{x-\sin x} = \left[\frac{0}{0}\right] = \left[\lim\limits_{x\to0}\frac{f(x)}{g(x)} = \left[\frac{0}{0}\right] \text{ или } \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to0}\frac{f'(x)}{g'(x)}\right] =$

$= \lim\limits_{x\to0}\frac{(x^3)'}{(x-\sin x)'} = \lim\limits_{x\to0}\frac{3x^2}{1-\cos x} = \left[\frac{0}{0}\right] = \lim\limits_{x\to0}\frac{(3x^2)'}{(1-\cos x)'} = \lim\limits_{x\to0}\frac{6x}{0+\sin x} =$

$= \lim\limits_{x\to0}\frac{6x}{\sin x} = \left[\frac{0}{0}\right] \xrightarrow{\text{правило Л.}} \lim\limits_{x\to0}\frac{6}{\cos x} = \frac{6}{1} = 6$

замечат. предел

$\lim \frac{6}{\frac{\sin x}{x}} = \frac{6}{1} = 6$

№ 7.3.12

$\lim\limits_{x\to2}\frac{x^3+x-10}{x^3-3x-2} = \left[\frac{0}{0}\right] = \lim\limits_{x\to2}\frac{3x^2+1}{3x^2-3} = \frac{3\cdot4+1}{3\cdot4-3} = \frac{13}{9}$

№ 7.3.13

$\lim\limits_{x\to1}\frac{\ln x}{x-1} = \left[\frac{0}{0}\right] = \lim\limits_{x\to1}\frac{(\ln x)'}{(x-1)'} = \lim\limits_{x\to1}\frac{1/x}{1-0} = 1$

№ 7.3.14

$$\lim_{x\to 0}\frac{e^x-1}{\sin x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(e^x-1)'}{(\sin x)'}=\lim_{x\to 0}\frac{e^x-0}{\cos x}=\frac{1}{1}=1$$

№ 7.3.15

$$\lim_{x\to +\infty}\frac{\ln x}{x}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to +\infty}\frac{1/x}{1}=\lim_{x\to\infty}\frac{1}{x}=\frac{1}{\infty}=0$$

$$\left[y=\frac{1}{x}\to 0 \quad \lim_{y\to 0}\frac{\ln\frac{1}{y}}{1/y}\ \left[\frac{\infty}{\infty}\right]\ (\text{док-во})\right]$$

№ 7.3.16

$$\lim_{x\to +\infty}\frac{e^x}{x^3}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to +\infty}\frac{e^x}{3x^2}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{e^x}{6x}=\lim_{x\to +\infty}\frac{e^x}{6}=\frac{+\infty}{6}=+\infty\ (+\infty)$$

№ 7.3.17

$$\lim_{x\to 0}\frac{\ln x}{\operatorname{ctg} 2x}=\left[\frac{-\infty}{-\infty}\right]=\lim_{x\to 0}\frac{1/x}{\left(\frac{-1}{\sin^2 2x}\right)2}=\lim_{x\to 0}\frac{-\sin^2 2x}{2x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\left(\frac{2\cdot\sin(2x)\cdot\cos(2x)\cdot 2}{-2}\right)$$

$$=\lim_{x\to 0}(-\sin 4x)=0$$

$$x\to 0+0 \Rightarrow x>0$$

№ 7.3.18

1) $$\lim_{x\to 0+0}x\cdot\ln x=[0\cdot(-\infty)]=\lim_{x\to 0+0}\ln x:\frac{1}{x}=\lim_{x\to 0+0}\frac{\ln x}{1/x}=\left[\frac{-\infty}{+\infty}\right]=\lim\frac{1/x}{-x^{-2}}=$$

$$=\lim_{x\to 0+0}\frac{1}{-x^{-1}}=\lim_{x\to 0+0}-x=0$$

2) $$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=[\infty-\infty]=\lim_{x\to 1}\frac{x-1-\ln x}{(x-1)\ln x}=\left[\frac{0}{0}\right]=\lim_{x\to 1}\frac{1-1/x}{(x-1)'\cdot\ln x+(x-1)\ln x'}=$$

$$=\lim_{x\to 1}\frac{1-1/x}{\ln x+(x-1)\frac{1}{x}}=\left[\frac{0}{0}\right]=\lim_{x\to 1}\frac{-x^{-2}}{\frac{1}{x}+x^{-2}}=\frac{-1}{-(1+1)}=\frac{1}{2}=0{,}5$$