№ 7.1.1

13.04.20

1) $y = 3x^2$

$y = f(x)$, $y' - ?$

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}; \Delta y = f(x_0 + \Delta x) - f(x_0)$$

$$\Delta y = 3 \cdot (x + \Delta x)^2 - 3x^2 = 3(x^2 + 2x \cdot \Delta x + \Delta x^2) - 3x^2 =$$

$$= 3x^2 + 6x\Delta x + \Delta x^2 - 3x^2 = 6x\Delta x + \Delta x^2 = 3\Delta x(2x + \Delta x)$$

$$f'(x) = \lim \frac{\Delta y}{\Delta x} = \lim \frac{3 \cdot \Delta x(2x + \Delta x)}{\Delta x} = \left[\frac{0}{0}\right] = \lim_{\Delta x \to 0} \cdot (2x + \Delta x) =$$

$$3\left(\lim_{\Delta x \to 0}(2x) + \lim_{\Delta x \to 0} \Delta x\right) = 3 \cdot (2x + 0) = 6x$$

2) $y = \sin x$

$$\Delta y = \sin(x + \Delta x) - \sin x = [\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha;$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha - \beta}{2} \cdot \cos\frac{\alpha + \beta}{2}] = 2 \cdot \sin\frac{x + \Delta x - x}{2} \cdot$$

$$\cdot \cos\frac{x + \Delta x + x}{2} = 2 \cdot \sin\frac{\Delta x}{2} \cdot \cos\frac{2x + \Delta x}{2} = 2\sin\frac{\Delta x}{2} \cdot \cos\left(x + \frac{\Delta x}{2}\right)$$

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\sin\frac{\Delta x}{2} \cdot \cos\left(x + \frac{\Delta x}{2}\right)}{\Delta x} = \left[\cos\left(x + \frac{\Delta x}{2}\right) \xrightarrow[\Delta x \to 0]{}\right.$$

$$\left.\cos(x + 0) = \cos x\right] = \lim_{\Delta x \to 0} (\cos(x + \Delta x)) \cdot \lim_{\Delta x \to 0} \frac{2 \cdot \sin\left(\frac{\Delta x}{2}\right)}{\Delta x} =$$

$$\left[\lim_{\Delta x \to 0} \frac{\sin x}{x} = 1; \frac{2 \cdot \sin\left(\frac{\Delta x}{2}\right)}{\Delta x} = \frac{\sin\left(\frac{\Delta x}{2}\right)}{\Delta x} \cdot 2 = \frac{\sin\left(\frac{\Delta x}{2}\right)}{\Delta x} \cdot \frac{1}{2} = \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}\right.$$

$\Delta x \to 0 \Rightarrow \frac{\Delta x}{2} \to 0$ Тогда $\left.\lim_{\frac{\Delta x}{2} \to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} = 1\right] = \cos x \cdot 1 = \cos x$

№ 7.1.6

$f'(x) - ?$ через основные правила

$$1)\ f(x) = \frac{9}{\sqrt[3]{x^2}} - 5^{x+1} = 9 \cdot x^{-\frac{2}{3}} - 5^{x+1} = [(x^\alpha)' = \alpha \cdot x^{\alpha - 1}; (a^x) = a^x \cdot \ln a;$$

$f'(\varphi(x)) = f'\varphi(x)\cdot\varphi'x;\ (u+v)' = u'+v'] = (9\cdot x^{-\frac{2}{3}})'_x -$

$(5^{x+1})'_x = 9\cdot(x^{-\frac{2}{3}})'_x - (5^{x+1})'_{x+1}\cdot(x+1)'_x = 9\cdot(-\frac{2}{3})\cdot x^{\frac{2}{3}-1} -$

$-5^{x+1}\cdot \ln 5\cdot(x'_x + 1'_x) = -6\cdot x^{-\frac{5}{3}} - 5^{x+1}\cdot\ln 5\cdot(1\cdot x^{1-1}+0) =$

$= -6x^{-\frac{5}{3}} - 5^{x+1}\cdot\ln 5\cdot 1 = -6x^{-\frac{5}{3}} - 5^{x+1}\cdot\ln 5$

2) $f(x) = (x^4 - x)(3tgx - 1)$

$f'(x) = ((x^4-x)(3tgx-1))' = (x^4-x)'\cdot(3tgx-1) + (x^4-x)$

$(3tgx-1)' = ((x^4)' - x')(3tgx-1) + (x^4-x)((3tgx)' - 1') =$

$= (4x^3-1)(3tgx-1) + (x^4-x)(3\cdot\frac{1}{\cos^2 x} - 0) =$

$= (4x^3-1)(3tgx-1) + (x^4-x)\cdot\frac{3}{\cos^2 x}$

№ 7.1.27

$f'(\varphi(x)) = f'\varphi(x)\cdot\varphi'(x)$

1) $y = \sin^2 x$

$y'x = (\sin^2 x)'_x = ((\sin x)^2)'_x = 2\cdot\sin x\cdot\cos x = \sin(2x)$

2) $y = \ln(arctg\,3x)$

$y' = (\ln(arctg(3x)))'_x = (\ln = \frac{1}{arctg\,3x}\cdot\frac{1}{1+(3x)^2}\cdot 3 =$

$= \frac{3}{(1+9x^2)\,arctg(3x)}$

№ 7.1.58

$(\ln y)'_x = \frac{y'}{y} \Rightarrow y' = y\cdot(\ln y)'_x$

1) $y = x^{\sin x}$ 1) логарифмируем

$\ln y = \ln x^{\sin x}$ 2) $\log_a b^c = c \cdot \log_a b$; используем св-во

$\ln y = \sin x \cdot \ln x$ 3) $(\ )'$

$(\ln y)' = (\sin x \cdot \ln x)'$

$\frac{y'}{y} = (\sin x)' \cdot \ln x + \sin x \cdot (\ln x)'$

$\frac{y'}{y} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x} \Rightarrow y' = y \cdot (\cos x \cdot \ln x + \frac{\sin}{x}) =$
$x^{\sin x} (\cos x \cdot \ln x + \frac{\sin x}{x})$

2) $y = \ln \frac{(x-1)^3 \sqrt{x+2}}{\sqrt[3]{(x+1)^2}}$

$\ln y = \ln \frac{(x-1)^3 \sqrt{x+2}}{\sqrt[3]{(x+1)^2}}$

$\ln y = \ln (x-1)^3 + \ln \sqrt{x+2} - \ln \sqrt[3]{(x+1)^2}$

$\ln y = 3 \cdot \ln (x-1) + \frac{1}{2} \ln (x+2) - \frac{2}{3} \ln (x+1)$

$(\ln y)' = (3 \ln (x-1) + \frac{1}{2} \ln (x+2) - \frac{2}{3} \ln (x+1))'$

$\frac{y'}{y} = 3 \cdot \frac{1}{x-1} \cdot (x-3)' + \frac{1}{2} \cdot \frac{1}{x+2} (x+2)' - \frac{2}{3} \cdot \frac{1}{x+1} \cdot (x+1)' =$
$\frac{3}{x-1} + \frac{1}{2(x+2)} - \frac{2}{3(x+1)} \Rightarrow y' = y \cdot (\frac{3}{x-1} + \frac{1}{2(x+2} - \frac{2}{3(x+1)}) =$
$\frac{(x-1)^3 \sqrt{x+2}}{\sqrt[3]{(x+1)^2}} \cdot \left(\frac{3}{x-1} + \frac{1}{2(x+2)} - \frac{2}{3(x+1)}\right)$