

Алгебра, группа 1, н/з 1, Бугаев А. А.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} \quad \text{№ 1.1.36}$$

$$3A - 2B = \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 7 & 16 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 2 & 4 \\ -6 & 4 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 5 & 10 \\ -15 & 10 & 0 \end{pmatrix} \quad \text{№ 1.1.37}$$

$$2B - 5A = \begin{pmatrix} 0 & 10 & 20 \\ -30 & 20 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 10 & 20 \\ -30 & 20 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 3 \\ 3 & -2 \end{pmatrix} \quad \text{№ 1.1.38}$$

$$A - \lambda E = \begin{pmatrix} 2 & 3 \\ 3 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & -2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & 3 \\ 3 & -2-\lambda \end{pmatrix}$$

$$4A - 7B = 4 \begin{pmatrix} 1 & -2 & 5 & 3 \\ 2 & 0 & -3 & 1 \\ 5 & -1 & 0 & 4 \end{pmatrix} - 7 \begin{pmatrix} 0 & 2 & 7 & -5 \\ -8 & 1 & 3 & 0 \\ 4 & 2 & -2 & 5 \end{pmatrix} =$$

$$= \begin{pmatrix} 4 & -8 & 20 & 12 \\ 8 & 0 & -12 & 4 \\ 20 & -4 & 0 & 16 \end{pmatrix} - \begin{pmatrix} 0 & 14 & 49 & -35 \\ -56 & 7 & 21 & 0 \\ 28 & 14 & -14 & 35 \end{pmatrix} =$$

$$= \begin{pmatrix} 4 & -22 & -29 & 47 \\ 64 & -7 & -33 & 4 \\ -8 & -28 & 14 & -19 \end{pmatrix}$$

$$5A - 3B + 2C = 5 \begin{pmatrix} 1 & -2 & 0 \\ 3 & 5 & 1 \\ -1 & 2 & 4 \end{pmatrix} - 3 \begin{pmatrix} 5 & 1 & -2 \\ -3 & 2 & 7 \\ 4 & 0 & -1 \end{pmatrix} + 2 \begin{pmatrix} -5 & 3 & 1 \\ 2 & 0 & 5 \\ 6 & 4 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 5 & -10 & 0 \\ 15 & 25 & 5 \\ -5 & 10 & 20 \end{pmatrix} - \begin{pmatrix} 15 & 3 & -6 \\ -9 & 6 & 21 \\ 12 & 0 & -3 \end{pmatrix} + \begin{pmatrix} -10 & 6 & 2 \\ 4 & 0 & 10 \\ 12 & 8 & 4 \end{pmatrix} =$$

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$$= \begin{pmatrix} -20 & -7 & 8 \\ 28 & 19 & -6 \\ -5 & 18 & 27 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -3 & -4 \\ 7 & 10 \end{pmatrix}$$

$$AB = (1 \ -2 \ 3 \ 0) \cdot \begin{pmatrix} 5 \\ -3 \\ -4 \\ 1 \end{pmatrix} = (-1)$$

$$BA = \begin{pmatrix} 5 \\ -3 \\ -4 \\ 1 \end{pmatrix} \cdot (1 \ -2 \ 3 \ 0) = \begin{pmatrix} 5 & -10 & 15 & 0 \\ -3 & 6 & -9 & 0 \\ -4 & 8 & -12 & 0 \\ 1 & -2 & 3 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 0 & 3 \\ -1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

BA — не \exists , т.к. $1 \neq 2$

$$AB = \begin{pmatrix} 3 & 5 & -1 \\ 2 & -2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 \\ -3 & 0 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} -14 & 11 \\ 10 & 8 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & 0 \\ 5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 & -1 \\ 2 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 14 & 2 & -2 \\ -9 & -15 & 3 \\ 17 & 23 & -5 \end{pmatrix}$$

② $AB = \begin{pmatrix} -2 & 3 & 1 \\ 5 & 4 & 0 \\ 2 & -1 & -5 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 0 & -3 & 1 \\ 4 & -4 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -9 & 14 \\ 5 & -22 & -11 \\ -18 & 19 & -32 \end{pmatrix}$

$$BA = \begin{pmatrix} 1 & -2 & -3 \\ 0 & -3 & 1 \\ 4 & -4 & 5 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 & 1 \\ 5 & 4 & 0 \\ 2 & -1 & -5 \end{pmatrix} = \begin{pmatrix} -18 & -2 & 16 \\ -13 & -13 & -5 \\ -18 & -9 & -21 \end{pmatrix}$$

$$(AB) \cdot C = \left(\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix} \right) \cdot \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ -5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 13 & -8 \\ -13 & 8 \end{pmatrix}$$

$$A \cdot (BC) = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \left(\begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \right) = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 6 & -2 \\ -7 & 6 \end{pmatrix} = \begin{pmatrix} 13 & -8 \\ -13 & 8 \end{pmatrix}$$

$$(AB) \cdot C = \left(\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$$

$$A \cdot (BC) = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \cdot \left(\begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \right) = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 1 & -3 \end{pmatrix} \cdot \begin{pmatrix} -3 & 2 & 0 \\ -2 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 4 & -3 & 0 \\ 0 & 2 & 5 & -2 \\ 3 & -1 & 2 & 4 \end{pmatrix} =$$

$$= \begin{pmatrix} 3 & -13 & 3 \end{pmatrix} \cdot \begin{pmatrix} -2 & 4 & -3 & 0 \\ 0 & 2 & 5 & -2 \\ 3 & -1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -19 & -68 & 38 \end{pmatrix}$$

$$A \cdot (BC) = (1 \ -3) \cdot \left(\begin{pmatrix} -3 & 2 & 0 \\ -2 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 4 & -3 & 0 \\ 0 & 2 & 5 & -2 \\ 3 & -1 & 2 & 4 \end{pmatrix} \right) =$$

$$= (1 \ -3) \cdot \begin{pmatrix} 6 & -8 & 19 & -4 \\ 1 & 3 & 29 & -14 \end{pmatrix} = \begin{pmatrix} 3 & -17 & -68 & 38 \end{pmatrix}$$

$$(AB) \cdot C = \left(\begin{pmatrix} -5 & 0 \\ 4 & 1 \\ 2 & -3 \\ 1 & 5 \end{pmatrix} \cdot \begin{pmatrix} 3 & 0 \\ -2 & 1 \\ 4 & 3 \end{pmatrix} \right) \cdot \begin{pmatrix} -2 \end{pmatrix} =$$

$$= \begin{pmatrix} -3 & 9 \\ 6 & -2 \\ 20 & 3 \\ 5 & 14 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 33 \\ -18 \\ -31 \\ 32 \end{pmatrix}$$

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$$A \cdot (BC) = \begin{pmatrix} -5 & 0 & 3 \\ 4 & 1 & -1 \\ 2 & -3 & 2 \\ 1 & 5 & 3 \end{pmatrix} \cdot \left(\begin{pmatrix} 3 & 0 \\ -2 & 1 \\ 4 & 3 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} \right) =$$

$$= \begin{pmatrix} -5 & 0 & 3 \\ 4 & 1 & -1 \\ 2 & -3 & 2 \\ 1 & 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 7 \\ 1 \end{pmatrix} = \begin{pmatrix} 33 \\ -18 \\ -31 \\ 32 \end{pmatrix}$$

$$A^{n^2} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \quad \sim 1.1.50$$

$$A^{n^3} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

$$A^{n^4} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

$$\dots$$

$$A^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \sim 1.1.51$$

$$A^3 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\dots$$

$$A^n = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \sim 1.1.52$$

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$$A^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\dots$$

$$A^n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ кроме } A^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\sim 1.1.53$

$$f(x) = 2x^2 - 3x + 1 \quad A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$1) 2 \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$2) 3 \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & -3 \end{pmatrix}$$

$$3) 1 \cdot E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & -3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix}$$

$\sim 1.1.54$

$$f(x) = 3x^2 + 2x + 5 \quad A = \begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix}$$

$$1) 3 \cdot \begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix} = 3 \cdot \begin{pmatrix} 4 & -18 \\ 0 & 16 \end{pmatrix} = \begin{pmatrix} 12 & -54 \\ 0 & 48 \end{pmatrix}$$

$$2) 2 \cdot \begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 4 & -6 \\ 0 & 8 \end{pmatrix}$$

$$3) 5 \cdot E = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 12 & -54 \\ 0 & 48 \end{pmatrix} + \begin{pmatrix} 4 & -6 \\ 0 & 8 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 21 & -60 \\ 0 & 61 \end{pmatrix}$$

$\sim 1.1.55$

$$f(x) = 2x^3 - x^2 + 3$$

$$A = \begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix}$$

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$$1) 2 \cdot \begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix} = 2 \cdot \begin{pmatrix} 5 & -10 \\ 15 & -5 \end{pmatrix} = \begin{pmatrix} 10 & -20 \\ 30 & -10 \end{pmatrix}$$

$$2) \begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

$$3) 3 \cdot E = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 10 & -20 \\ 30 & -10 \end{pmatrix} - \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 18 & -20 \\ 30 & -2 \end{pmatrix}$$

~ 1.1.56

$$f(x) = 4x^3 - 2x^2 + 3x - 2 \quad A = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}$$

$$1) 4 \cdot \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} = 4 \cdot \begin{pmatrix} -20 & 21 \\ 7 & -6 \end{pmatrix} = \begin{pmatrix} -80 & 84 \\ 28 & -24 \end{pmatrix}$$

$$2) 2 \cdot \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} = 2 \cdot \begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 14 & -12 \\ -4 & 6 \end{pmatrix}$$

$$3) 3 \cdot \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -6 & 9 \\ 3 & 0 \end{pmatrix}$$

$$4) 2 \cdot E = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -80 & 84 \\ 28 & -24 \end{pmatrix} - \begin{pmatrix} 14 & -12 \\ -4 & 6 \end{pmatrix} + \begin{pmatrix} -6 & 9 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -102 & 105 \\ 35 & -32 \end{pmatrix}$$

~ 1.1.57

$$f(x) = x^2 - 3x + 2$$

$$A = \begin{pmatrix} 1 & -3 & 0 \\ 0 & 2 & 1 \\ 3 & -3 & 2 \end{pmatrix}$$

$$1) \begin{pmatrix} 1 & -3 & 0 \\ 0 & 2 & 1 \\ 3 & -3 & 2 \end{pmatrix} \begin{pmatrix} 1 & -3 & 0 \\ 0 & 2 & 1 \\ 3 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -9 & -3 \\ 3 & 1 & 4 \\ 9 & -21 & 1 \end{pmatrix}$$

$$2) 3 \cdot \begin{pmatrix} 1 & -3 & 0 \\ 0 & 2 & 1 \\ 3 & -3 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -9 & 0 \\ 0 & 6 & 3 \\ 9 & -9 & 6 \end{pmatrix}$$

$$3) 2 \cdot E = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -9 & -3 \\ 3 & 1 & 4 \\ 9 & -21 & 1 \end{pmatrix} - \begin{pmatrix} 3 & -9 & 0 \\ 0 & 6 & 3 \\ 9 & -9 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -3 \\ 3 & -3 & 1 \\ 0 & -12 & -3 \end{pmatrix}$$

~ 1.1.58

$$f(x) = 3x^2 + 5x - 2$$

$$A = \begin{pmatrix} 2 & 3 & -3 \\ 0 & 1 & 4 \\ 5 & -2 & 1 \end{pmatrix}$$

$$1) 3 \cdot \begin{pmatrix} 2 & 3 & -3 \\ 0 & 1 & 4 \\ 5 & -2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & -3 \\ 0 & 1 & 4 \\ 5 & -2 & 1 \end{pmatrix} = 3 \cdot \begin{pmatrix} -11 & 15 & 3 \\ 20 & -7 & 8 \\ 15 & 11 & -22 \end{pmatrix} =$$

$$\begin{pmatrix} -33 & 45 & 9 \\ 60 & -21 & 24 \\ 45 & 33 & -66 \end{pmatrix}$$

$$2) 5 \cdot \begin{pmatrix} 2 & 3 & -3 \\ 0 & 1 & 4 \\ 5 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 15 & -15 \\ 0 & 5 & 20 \\ 25 & -10 & 5 \end{pmatrix}$$

$$3) 2 \cdot E = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -33 & 45 & 9 \\ 60 & -21 & 24 \\ 45 & 33 & -66 \end{pmatrix} + \begin{pmatrix} 10 & 15 & -15 \\ 0 & 5 & 20 \\ 25 & -10 & 5 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -25 & 60 & -6 \\ 60 & -18 & 44 \\ 70 & 23 & -63 \end{pmatrix}$$

~ 1.1.59

$$f(x) = x^3 - x^2 + 5$$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$1) \begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 7 \\ 3 & -1 & 6 \\ 0 & 0 & 8 \end{pmatrix}$$

$$2) \begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{pmatrix}$$

$$3) 5 \cdot E = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 7 \\ 3 & -1 & 6 \\ 0 & 0 & 8 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 4 \\ 3 & 3 & 3 \\ 0 & 0 & 9 \end{pmatrix}$$

~ 1.1.60

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$$f(x) = 2x^3 - x^2 + 3x - 2$$

$$A = \begin{pmatrix} 2 & -3 & 4 \\ 0 & 5 & -1 \\ -2 & -1 & 3 \end{pmatrix}$$

$$1) 2 \cdot \begin{pmatrix} 2 & -3 & 4 \\ 0 & 5 & -1 \\ -2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -6 & 8 \\ 0 & 10 & -2 \\ -4 & -2 & 6 \end{pmatrix}$$

$$2) \begin{pmatrix} 2 & -3 & 4 \\ 0 & 5 & -1 \\ -2 & -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & -3 & 4 \\ 0 & 5 & -1 \\ -2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} -4 & -25 & 23 \\ 2 & 26 & -8 \\ -10 & -2 & 2 \end{pmatrix}$$

$$3) 3 \cdot \begin{pmatrix} 2 & -3 & 4 \\ 0 & 5 & -1 \\ -2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 6 & -9 & 12 \\ 0 & 15 & -3 \\ -6 & -3 & 9 \end{pmatrix}$$

$$4) 2 \cdot E = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -108 & -292 & 156 \\ 40 & 264 & -84 \\ -48 & 86 & -64 \end{pmatrix} - \begin{pmatrix} -4 & -25 & 23 \\ 2 & 26 & -8 \\ -10 & -2 & 2 \end{pmatrix} + \begin{pmatrix} 6 & -9 & 12 \\ 0 & 15 & -3 \\ -6 & -3 & 9 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} -100 & -256 & 145 \\ 38 & 251 & -79 \\ -44 & 35 & -59 \end{pmatrix}$$

$$AB = (1 \ 2 \ 3) \cdot \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = (32) \quad BA = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \cdot (1 \ 2 \ 3) = \begin{pmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{pmatrix}$$

$$AB \neq BA$$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -5 & 4 \end{pmatrix}$$

$$BA = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ -1 & -1 \end{pmatrix}$$

$$AB \neq BA$$

$$AB = \begin{pmatrix} 2 & -3 \\ 4 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -2 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 12 & -13 \\ 0 & -8 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & -2 \\ -4 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} -8 & 0 \\ 4 & 12 \end{pmatrix}$$

$$AB \neq BA$$

$$AB = \begin{pmatrix} 2 & -1 & 0 \\ 3 & 2 & 5 \\ 4 & -2 & 7 \end{pmatrix} \begin{pmatrix} -2 & 1 & 0 \\ -3 & -2 & 5 \\ -4 & 2 & -7 \end{pmatrix} = \begin{pmatrix} -1 & 4 & -5 \\ -32 & 9 & -25 \\ -30 & 22 & -59 \end{pmatrix}$$

$$BA = \begin{pmatrix} -2 & 1 & 0 \\ -3 & -2 & 5 \\ -4 & 2 & -7 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 3 & 2 & 5 \\ 4 & -2 & 7 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 5 \\ 8 & -11 & 25 \\ -30 & 22 & -39 \end{pmatrix}$$

$$AB \neq BA$$

$$AB = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & b^2 & 0 & 0 \\ 0 & 0 & c^2 & 0 \\ 0 & 0 & 0 & d^2 \end{pmatrix}$$

$$BA = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & b^2 & 0 & 0 \\ 0 & 0 & c^2 & 0 \\ 0 & 0 & 0 & d^2 \end{pmatrix}$$

$$AB = BA$$

$$AB = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} -1 & 2 & -3 \\ -4 & 5 & -6 \\ -7 & 8 & -9 \end{pmatrix} = \begin{pmatrix} -30 & 36 & -42 \\ -66 & 81 & -96 \\ -102 & 126 & -150 \end{pmatrix}$$

$$BA = \begin{pmatrix} -1 & 2 & -3 \\ -4 & 5 & -6 \\ -7 & 8 & -9 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} -14 & -16 & -18 \\ -26 & -31 & -36 \\ -38 & -46 & -54 \end{pmatrix}$$

$$AB \neq BA$$

(8)

(9)

$$AB = \begin{pmatrix} 1 & 2 & -3 & 4 \\ 5 & -6 & 7 & 8 \\ -9 & 0 & 1 & 2 \\ 3 & 4 & 5 & -6 \end{pmatrix} \cdot \begin{pmatrix} -6 & 5 & 4 & 3 \\ 2 & 1 & 0 & -9 \\ 8 & 7 & -6 & 5 \\ 4 & -3 & 2 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -10 & -26 & 30 & -26 \\ 46 & 44 & -6 & 112 \\ 70 & -44 & -38 & -20 \\ 6 & 72 & -30 & -8 \end{pmatrix}$$

$$BA = \begin{pmatrix} -6 & 5 & 4 & 3 \\ 2 & 1 & 0 & -9 \\ 8 & 7 & -6 & 5 \\ 4 & -3 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & -3 & 4 \\ 5 & -6 & 7 & 8 \\ -9 & 0 & 1 & 2 \\ 3 & 4 & 5 & -6 \end{pmatrix} =$$

$$= \begin{pmatrix} -8 & -30 & 72 & 6 \\ -20 & -38 & -44 & 20 \\ 112 & -6 & 44 & 46 \\ -26 & 30 & -26 & -10 \end{pmatrix}$$

$$AB \neq BA$$

$$AB = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 7 & -6 & 1 \\ -5 & 3 & 1 \\ 6 & -3 & -3 \end{pmatrix} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$BA = \begin{pmatrix} 7 & -6 & 1 \\ -5 & 3 & 1 \\ 6 & -3 & -3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$AB = BA$$

$$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

N.I. 69

$$A^T = \begin{pmatrix} 1 & 3 & -4 \\ -2 & 5 & 1 \\ 0 & -7 & 2 \end{pmatrix}$$

N.I. 70

$$A^T = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

N.I. 71

$$AA^T = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 11 \\ 11 & 13 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 14 \\ 14 & 20 \end{pmatrix}$$

$$AA^T = (1 \ 2 \ 3 \ 4) \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = (30)$$

$$A^T A = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \cdot (1 \ 2 \ 3 \ 4) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 5 & -7 \\ -4 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & -4 \\ -2 & 5 & 1 \\ 0 & -7 & 2 \end{pmatrix} = \begin{pmatrix} 5 & -7 & -6 \\ -7 & 83 & -21 \\ -6 & -21 & 21 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 3 & -4 \\ -2 & 5 & 1 \\ 0 & -7 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 0 \\ 3 & 5 & -7 \\ -4 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 26 & 9 & -29 \\ 9 & 30 & -33 \\ -29 & -33 & 53 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{pmatrix}$$

N.I. 76

$$AA^T = \begin{pmatrix} 0 & 0 & -3 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 5 \\ 0 & 2 & 0 \\ -3 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 25 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 2 & 0 \\ -3 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & -3 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 25 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 66 & 78 & 90 \\ 78 & 93 & 108 \\ 90 & 108 & 126 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \xrightarrow{\text{II} - 4\text{I}} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{pmatrix} \xrightarrow{\text{I.I. 78}}$$

$$A = \begin{pmatrix} 2 & -1 & 5 \\ 1 & 1 & 3 \\ 1 & -5 & 1 \end{pmatrix} \xrightarrow{\text{II} - 0.5\text{I}, \text{III} - 0.5\text{I}} \sim \begin{pmatrix} 2 & -1 & 5 \\ 0 & 1.5 & 0.5 \\ 0 & -4.5 & -4.5 \end{pmatrix} \xrightarrow{\text{III} + 3\text{II}} \sim \begin{pmatrix} 2 & -1 & 5 \\ 0 & 1.5 & 0.5 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -2 & 3 & 1 \\ 3 & 2 & -4 & 2 \\ 5 & -2 & 2 & 4 \end{pmatrix} \xrightarrow{\text{II} - 3\text{I}, \text{III} - 5\text{I}} \sim \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & 8 & -13 & -1 \\ 0 & 8 & -13 & -1 \end{pmatrix} \xrightarrow{\text{III} - \text{II}} \sim \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & 8 & -13 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

~ I.I. 81

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 & 8 \\ 2 & -1 & -4 & 3 & 1 \\ 4 & -7 & -8 & 11 & -13 \\ 3 & 1 & -1 & 2 & 9 \end{pmatrix} \xrightarrow{\text{II} - 2\text{I}, \text{III} - 4\text{I}, \text{IV} - 3\text{I}} \sim \begin{pmatrix} 1 & 2 & 3 & -1 & 8 \\ 0 & -5 & -10 & 5 & -15 \\ 0 & -15 & -30 & 15 & -45 \\ 0 & -5 & -10 & 5 & -15 \end{pmatrix} \xrightarrow{\text{III} - 3\text{II}, \text{IV} - \text{II}} \sim \begin{pmatrix} 1 & 2 & 3 & -1 & 8 \\ 0 & -5 & -10 & 5 & -15 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 5 & -3 & 4 \\ 1 & 2 & -7 & 0 & 7 \\ 1 & 0 & 1 & -2 & 5 \end{pmatrix} \xrightarrow{\text{II} - \text{I}, \text{III} - 2\text{I}, \text{IV} - \text{I}} \sim \begin{pmatrix} 1 & -1 & 5 & -3 & 4 \\ 0 & 3 & -12 & 3 & 3 \\ 0 & 1 & -8 & 9 & -19 \\ 0 & 1 & -4 & 1 & 1 \end{pmatrix} \xrightarrow{\text{III} - \frac{1}{3}\text{II}, \text{IV} - \frac{1}{3}\text{II}} \sim \begin{pmatrix} 1 & -1 & 5 & -3 & 4 \\ 0 & 3 & -12 & 3 & 3 \\ 0 & 0 & -4 & 8 & -20 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -1 & 0 & 4 \\ 3 & -1 & -7 & -4 & 7 \\ 1 & -1 & -3 & -2 & 5 \end{pmatrix} \xrightarrow{\text{II} - 3\text{I}, \text{III} - 7\text{I}, \text{IV} - \text{I}} \sim \begin{pmatrix} 1 & 1 & -1 & 0 & 4 \\ 0 & -4 & -4 & -4 & -5 \\ 0 & -8 & -8 & -8 & -39 \\ 0 & -2 & -2 & -2 & 1 \end{pmatrix} \xrightarrow{\text{III} - 2\text{II}, \text{IV} - 0.5\text{II}} \sim \begin{pmatrix} 1 & 1 & -1 & 0 & 4 \\ 0 & -4 & -4 & -4 & -5 \\ 0 & 0 & 0 & 0 & -39 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & -1 & 0 & 4 \\ 0 & -4 & -4 & -4 & -5 \\ 0 & 0 & 0 & 0 & -29 \\ 0 & 0 & 0 & 0 & 3,5 \end{pmatrix}$$

$$\begin{aligned} A &= \begin{pmatrix} 1 & 5 & 3 & -10 \\ 3 & -1 & 1 & 10 \\ 2 & 1 & -1 & 0 \\ 7 & 10 & 6 & -10 \end{pmatrix} \begin{array}{l} \text{II} - 3\text{I} \\ \text{III} - 2\text{I} \\ \text{IV} - 7\text{I} \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 5 & 3 & -10 \\ 0 & -16 & -8 & 40 \\ 0 & -9 & -7 & 20 \\ 0 & -25 & -15 & 60 \end{pmatrix} \begin{array}{l} \\ \\ \text{IV} - (\text{II} + \text{III}) \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 5 & 3 & -10 \\ 0 & -16 & -8 & 40 \\ 0 & -9 & -7 & 20 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \\ \text{III} - \frac{9}{16}\text{II} \\ \\ \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 5 & 3 & -10 \\ 0 & -16 & -8 & 40 \\ 0 & 0 & -2,5 & -2,5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A &= \begin{pmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{pmatrix} \begin{array}{l} \text{II} - 8\text{I} \\ \text{III} + 4\text{I} \end{array} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & -5 & 2 \\ 0 & 3 & -1 \end{pmatrix} \begin{array}{l} \\ \text{III} + \frac{3}{5}\text{II} \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & -5 & 2 \\ 0 & 0 & 0,2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A &= \begin{pmatrix} 1 & 2 & -1 & 0 \\ 3 & -1 & 2 & 2 \\ 2 & 5 & -1 & 0 \\ 1 & -1 & 0 & 2 \end{pmatrix} \begin{array}{l} \text{II} - 3\text{I} \\ \text{III} - 2\text{I} \\ \text{IV} - \text{I} \end{array} \sim \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & -4 & 5 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & -3 & 1 & 2 \end{pmatrix} \begin{array}{l} \\ \\ \text{II} \leftrightarrow \text{III} \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & 5 & 2 \\ 0 & -3 & 1 & 2 \end{pmatrix} \begin{array}{l} \\ \\ \text{III} + 4\text{II} \\ \text{IV} + 3\text{II} \end{array} \sim \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 9 & 2 \\ 0 & 0 & 4 & 2 \end{pmatrix} \begin{array}{l} \\ \\ \text{IV} + \frac{4}{9}\text{III} \end{array} \sim \end{aligned}$$

$$\sim \begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 9 & 2 \\ 0 & 0 & 4 & 2 \end{pmatrix}$$

$$\begin{aligned} A &= \begin{pmatrix} 1 & 0 & 2 & -1 \\ 3 & -2 & 0 & -4 \\ 8 & 2 & 10 & -1 \\ 2 & -2 & -4 & 5 \end{pmatrix} \begin{array}{l} \text{II} - 3\text{I} \\ \text{III} - 2\text{I} \\ \text{IV} - \text{I} \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & -2 & -6 & -1 \\ 0 & 2 & 6 & 1 \\ 0 & -2 & -6 & 6 \end{pmatrix} \begin{array}{l} \\ \\ \text{III} + \text{II} \\ \text{IV} + \text{II} \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & -2 & -6 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix} \begin{array}{l} \\ \\ \\ \text{IV} \leftrightarrow \text{III} \end{array} \sim \\ &\sim \begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & -2 & -6 & -1 \\ 0 & 0 & 7 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$