№ 7.3.19

$$\lim_{x\to+\infty}(x^2\cdot e^{-x})=\left[\frac{\infty}{\infty}\right]=\lim_{x\to+\infty}((x^2)'\cdot(e^{-x})')=\lim_{x\to+\infty}-\frac{2x\,e^{-x}}{1}=\lim_{x\to+\infty}((-2x)'\cdot(e^{-x})')=$$

$$=\lim_{x\to+\infty}(-2\cdot(-1)\cdot e^{-x})=\lim_{x\to+\infty}2e^{-x}=\frac{2}{\infty}=0$$

№ 7.3.20

$$\lim_{x\to0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)=[\infty-\infty]=\lim_{x\to0}\left(\frac{\sin x-x}{x\sin x}\right)=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{(\sin x-x)'}{(x\sin x)'}=$$

$$\lim_{x\to0}\frac{\cos x-1}{\sin x+x\cos x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{(\cos x-1)'}{(\sin x+x\cos x)'}=\lim_{x\to0}\frac{-\sin x}{\cos x+\cos x+(-x\sin x)}$$

$$=-\lim_{x\to0}\frac{\sin x}{2\cos x-x\sin x}=0$$

№ 7.3.21

$$\lim_{x\to\infty}x(e^{\frac{1}{x}}-1)=[\infty\cdot0]=\lim_{x\to\infty}\frac{e^{\frac{1}{x}}-1}{1/x}=\left[\frac{0}{0}\right]=\lim_{x\to\infty}\frac{(e^{\frac{1}{x}}-1)'}{(1/x)'}=$$

$$\lim_{x\to\infty}\frac{e^{\frac{1}{x}}\cdot(-\frac{1}{x^2})-0}{-1/x^2}=\lim_{x\to\infty}e^{\frac{1}{x}}=e^0=1$$

№ 7.3.22

$$\lim_{x\to1}\left(\frac{1}{1-x^3}-\frac{1}{1-x^2}\right)=\lim_{x\to1}\frac{1-x^2-1+x^3}{(1-x^3)(1-x^2)}=\lim_{x\to1}\frac{x^3-x^2}{(1-x^3)(1-x^2)}=\left[\frac{0}{0}\right]=$$

$$\lim_{x\to1}\frac{(x^3-x^2)'}{((1-x^3)(1-x^2))'}=\lim_{x\to1}\frac{3x^2-2x}{(0-3x^2)(1-x^2)+(1-x^3)(0-2x)}=$$

$$=\lim_{x\to1}\frac{3x^2-2x}{-3x^2(1-x^2)+(-2x)(1-x^3)}=\lim_{x\to1}\frac{3x^2-2x}{-3x^2+3x^4-2x+2x^4}=$$

$$\lim_{x\to1}\frac{3x^2-2x}{5x^4-3x^2-2x}=\frac{3\cdot1^2-2\cdot1}{5\cdot1^4-3\cdot1^2-2\cdot1}=\frac{1}{0}=\infty$$