Вариант №42

№1

$f(x) = -49x^2 + 21x + 9$

$\left.\begin{array}{l} f'(x) = -98x + 21 \\ f'(x) = 0 \end{array}\right\} -98x + 21 = 0 \Rightarrow x = \frac{21}{98} = \frac{3}{14}$

$f\left(\frac{3}{14}\right) = -49 \cdot \left(\frac{3}{14}\right)^2 + 21 \cdot \frac{3}{14} + 9 = -49 \cdot \frac{9}{196} + \frac{63}{14} + 9 =$

$= -\left(\frac{7 \cdot 3}{14}\right)^2 + \frac{9}{2} + 9 = -\frac{9}{4} + \frac{9}{2} + 9 = -\frac{9}{4} + \frac{18}{4} + \frac{36}{4} =$

$= \frac{45}{4}$

$\left.\begin{array}{l} \text{Функция - парабола} \\ a < 0 \end{array}\right\} \text{Парабола направлена} \downarrow \Rightarrow y \leq \frac{45}{4}$

Ответ: $f(x) \leq \frac{45}{4}$

№2

$x_n = n^2 + \frac{n^3 - 3n^2 - 8}{(-1)^n \cdot n} - 5$

$x_1 = 1^2 + \frac{1^3 - 3 \cdot 1^2 - 8}{(-1)^1 \cdot 1} - 5 = 1 + \frac{1 - 3 - 8}{-1} - 5 = 6$

$x_2 = 2^2 + \frac{2^3 - 3 \cdot 2^2 - 8}{(-1)^2 \cdot 2} - 5 = 4 + \frac{8 - 12 - 8}{1 \cdot 2} - 5 = -7$

$x_3 = 3^2 + \frac{3^3 - 3 \cdot 3^2 - 8}{(-1)^3 \cdot 3} - 5 = 9 + \frac{27 - 27 - 8}{-1 \cdot 3} - 5 = 9 + \frac{8}{3} - 5 = \frac{20}{3}$

$x_4 = 4^2 + \frac{4^3 - 3 \cdot 4^2 - 8}{(-1)^4 \cdot 4} - 5 = 16 + \frac{64 - 48 - 8}{1 \cdot 4} - 5 = 13$

$x_5 = 5^2 + \frac{5^3 - 3 \cdot 5^2 - 8}{(-1)^5 \cdot 5} - 5 = 25 + \frac{125 - 75 - 8}{-1 \cdot 5} - 5 = 25 + \frac{42}{-5} - 5 = \frac{58}{5}$

Ответ: $6; -7; \frac{20}{3}; 13; \frac{58}{5}$

№3

1) $\lim\limits_{x\to-2}\frac{x^3+8}{-3x^3-15x^2-12x+12} = \frac{\lim\limits_{x\to-2}(x^3+8)}{\lim\limits_{x\to-2}(-3x^3-15x^2-12x+12)} =$ ~~[illegible]~~ $=$

$= \frac{-2^3+8}{-3(-2)^3-15(-2)^2-12(-2)+12} = \left[\frac{0}{0}\right] = \lim\limits_{x\to-2}\frac{(x+2)(x^2-2x+4)}{-3(x^3+5x^2+4x-4)} =$

$= \lim\limits_{x\to-2}\frac{(x+2)(x^2-2x+4)}{-3(x^3+2x^2+3x^2+6x-2x-4)} = \lim\limits_{x\to-2}\frac{(x+2)(x^2-2x+4)}{-3(x^2(x+2)+3x(x+2)-2(x+2))} =$

$\lim\limits_{x\to-2}\frac{\cancel{(x+2)}(x^2-2x+4)}{-3\cancel{(x+2)}(x^2+3x-2)} = \lim\limits_{x\to-2}\left(-\frac{x^2-2x+4}{3x^2+9x-6}\right) = -\frac{(-2)^2-2(-2)+4}{3(-2)^2+9(-2)-6} = 1$

Ответ: 1

2) $\lim\limits_{x\to1}\frac{7\sqrt{x+15}-28}{6-2\sqrt{10-x}} = \left[\frac{0}{0}\right] = \lim\limits_{x\to1}\frac{(7\sqrt{x+15}-28)'}{(6-2\sqrt{10-x})'} =$

$= \lim\limits_{x\to1}\frac{7/2\sqrt{x+15}}{1/\sqrt{10-x}} = \lim\limits_{x\to1}\frac{7\sqrt{10-x}}{2\sqrt{x+15}} = \frac{7\sqrt{10-1}}{2\sqrt{1+15}} = \frac{7\sqrt{9}}{2\sqrt{16}} = \frac{7\cdot3}{2\cdot4} = \frac{21}{8}$

Ответ: $\frac{21}{8}$

3) $\lim\limits_{x\to0}\frac{10\,tgx\cdot\sin x-10\sin^2x}{3x^3\sin x} = \left[\frac{0}{0}\right] =$ ~~$\lim\limits_{x\to0}\frac{(10\,tgx\cdot\sin x-10\sin^2x)'}{(3x^3\sin x)'}$~~ $=$

$\lim\limits_{x\to0}\frac{10\,tgx-10\sin x}{3x^3} = \lim\limits_{x\to0}\frac{10(tgx-\sin x)}{3x^3} = \lim\limits_{x\to0}\frac{(10(tgx-\sin x))'}{(3x^3)'} =$

$= \lim\limits_{x\to0}\frac{10(tg^2x-\cos x+1)}{9x^2} = \lim\limits_{x\to0}\frac{(10(tg^2x-\cos x+1))'}{(9x^2)'} =$

$= \lim\limits_{x\to0}\frac{5(\sin x+2tg^3x+2tgx)}{9x} = \lim\limits_{x\to0}\frac{(5(\sin x+2tg^3x+2tgx))'}{(9x)'} =$

$\lim\limits_{x\to0}\left(\frac{5\cos x}{9}+\frac{10tg^4x}{3}+\frac{40tg^2x}{9}+\frac{10}{9}\right) = \frac{5\cos0}{9}+\frac{10tg^40}{3}+\frac{40tg^20}{9}+\frac{10}{9} =$

$= \frac{5}{9}+\frac{0}{3}+\frac{0}{9}+\frac{10}{9} = \frac{15}{9} = \frac{5}{3}$

Ответ: $\frac{5}{3}$

4) $\lim\limits_{x\to\infty}\left(\frac{x-2}{x+25}\right)^{-3x} = \lim\limits_{x\to\infty}\left(\frac{(x+25)-27}{(x+25)}\right)^{-3x} = \lim\limits_{x\to\infty}\left(1-\frac{27}{x+25}\right)^{-3x}$ ~~[illegible]~~

~~$\frac{x+25}{-27}=n$ $= \lim\limits_{x\to\infty}\left(1+\frac{1}{n}\right)^{81n+75} = \lim\limits_{x\to\infty}\left(1+\frac{1}{n}\right)^{75}\left(1+\frac{1}{n}\right)^{81n} =$~~

~~$\lim\limits_{x\to\infty}\left(1+\frac{1}{n}\right)^{75}\cdot\lim\limits_{x\to\infty}\left(1+\frac{1}{n}\right)^{81n} = \lim\limits_{x\to\infty}\left(1+\frac{1}{n}\right)^{81n}$~~ $=$ По св-ву 2-го

замечат. предела: $a=-87$; $b=-3$ => $\lim\limits_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx} = e^{ab}$ =>

$\Rightarrow e^{-3\cdot -27} = e^{81}$

Ответ: e^{81}

№ 4

3) $\left(\sin(3xy-7y)+\dfrac{x^2+3xy}{y} = 2x+yx\right)' = 3y\cos(3xy-7y)+\dfrac{2x}{y}+3 = 2+y$