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№11.3.18

$1{,}07^{3{,}97}$

$f(x;y)=x^y$

$x=1{,}07 \qquad x_0=1 \qquad \Delta x=x-x_0=1{,}07-1=0{,}07$

$y=3{,}97 \qquad y_0=4 \qquad \Delta y=y-y_0=3{,}97-4=-0{,}03$

$f(x_0;y_0)=f(1;4)=1^4=1$

$y_0=3 \qquad \Delta y=y-y_0=3{,}97-3=0{,}97$

$|-0{,}03|<|0{,}97|$

лучше

вычисление без приближения

1) $f(x_0;y_0)=f(1;4)=1^4=1$

2) $f'_x(x_0;y_0)=(x^y)'_x\big|_{(x_0;y_0)}=y\cdot x^{y-1}\big|_{(x_0;y_0)}=y\cdot x^{y-1}\big|_{(1;4)}=4\cdot 1^{4-1}=4\cdot 1^3=4$

3) $f'_y(x_0;y_0)=(x^y)'_y\big|_{(x_0;y_0)}=x^y\cdot \ln x\big|_{(x_0;y_0)}=x^y\ln x\big|_{(1;4)}=1^4\cdot \underbrace{\ln 1}_{0}=$
$1\cdot 0=0$

4) $f(1{,}07;3{,}97)=[1{,}07^{3{,}97}]\approx 1+4\cdot 0{,}07+0\cdot(0{,}03)=$
$=1+0{,}28+0=1{,}28$ То есть $1{,}07^{3{,}97}\approx 1{,}28$

№11.3.19

$1{,}04^{2{,}03}$

$f(x;y)=x^y$

$x=1{,}04 \qquad x_0=1 \qquad \Delta x=x-x_0=0{,}04$

см. №11.3.18

$y=2{,}03 \qquad y_0=2 \qquad \Delta y=y-y_0=0{,}03$

№11.3.20

$\sqrt{(1{,}04)^2+(3{,}01)^2} \approx ?$

$f(x;y) = \sqrt{x^2+y^2}$

$x = 1{,}04 \qquad x_0 = 1 \qquad \Delta x = x - x_0 = 0{,}04$

$y = 3{,}01 \qquad y_0 = 3 \qquad \Delta y = y - y_0 = 0{,}01$

1) $f(x_0;y_0) = \sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10} \approx$

$f(x) = \sqrt{x} \qquad \overline{x} = 10 \qquad \overline{x_0} = 9 \qquad \overline{\Delta x} = 1$

$f'(x) = (\sqrt{x})' = \frac{1}{2\sqrt{x}} \qquad f(\overline{x}) \approx f(\overline{x_0}) + f'(\overline{x_0}) \cdot \overline{\Delta x} = \sqrt{9} + \frac{1}{2\sqrt{9}} \cdot 1 =$

$= 3 + \frac{1}{6} = \frac{19}{6} \approx 3{,}1(6) \approx 3{,}2$

№11.3.21

$\sin 28^\circ \cdot \cos 61^\circ \approx ?$

$f(x;y) = \sin(x) \cdot \cos(y)$

$x = (28^\circ)\, 28 \qquad x_0 = (30^\circ)\, 30 \qquad \Delta x = x - x_0 = (-2^\circ) -2$

$y = (61^\circ)\, 61 \qquad y_0 = (60^\circ)\, 60 \qquad \Delta y = y - y_0 = (1^\circ)\, 1$

№11.3.22

$\sqrt{(\sin^2 1{,}55 + 8 \cdot e^{0{,}015})^5}$

$f(x;y) = \sqrt{(\sin^2 x + 8 \cdot e^{y})^5} = (\sin^2 x + 8e^{y})^{\frac{5}{2}}$

1) $f'_x(x;y) = ((\sin^2 x + 8e^{y})^{\frac{5}{2}})'_x = \frac{5}{2} \cdot ((\sin^2 x + 8e^{y})^{\frac{5}{2}-1}) \cdot$

$\cdot (2 \sin x \cdot (\sin x)'_x + 0) = \frac{5}{2} \cdot 2 \sin x \cos x \, (\sin^2 x + 8e^{y})^{\frac{3}{2}} =$

$= \frac{5}{2}\sin(2x)\cdot(\sin^2 x+8e^y)^{\frac{3}{2}}$

2) $f_y'(x;y) = ((\sin^2 x+8e^y)^{5/2})_y' = \frac{5}{2}((\sin^2 x+8e^y)^{\frac{5}{2}-1})\cdot(0+8\cdot e^y) =$

$= \frac{5}{2}\cdot 8e^y\cdot(\sin^2 x+8e^y)^{3/2} = 20\cdot e^y\cdot(\sin^2 x+8e^y)^{3/2}$

3) $x = 1,55$ $\quad x_0 = 1,571 = \frac{\pi}{2}$ $\quad \Delta x = x - x_0 = 0,021$

$y = 0,015$ $\quad y_0 = 0$ $\quad \Delta y = y - y_0 = 0,015$

4) $f(x_0;y_0) = f(\frac{\pi}{2};0) = (\sin^2\frac{\pi}{2}+8e^0)^{\frac{5}{2}} = (1^2+8\cdot 1)^{5/2} = 9^{5/2} = \sqrt{9^5} =$

$= \sqrt{9^4\cdot 9} = \sqrt{(9^2)^2\cdot 9} = 9^2\cdot 3 = 81\cdot 3 = 243$

5) $f_x'(x_0;y_0) = (\frac{5}{2}\sin(2x)(\sin^2 x+8e^y)^{3/2})|_{(\frac{\pi}{2};0)} =$

$= \frac{5}{2}\cdot\sin(2\cdot\frac{\pi}{2})(\sin^2\frac{\pi}{2}+8e^0)^{\frac{3}{2}} = \frac{5}{2}\sin\pi(1^2+8\cdot 1)^{\frac{3}{2}} = \frac{5}{2}\cdot 0\cdot 9^{3/2} = 0$

6) $f_y'(x_0;y_0) = (20e^y\cdot(\sin^2 x+8e^y)^{\frac{3}{2}})|_{(\frac{\pi}{2};0)} =$

$= 20\cdot e^0(\sin^2\frac{\pi}{2}+8e^0)^{\frac{3}{2}} = 20\cdot 1\cdot(1^2+8\cdot 1)^{\frac{3}{2}} = 20\cdot 9^{3/2} = 20\sqrt{9^3} = 20\sqrt{9^2\cdot 9} =$

$= 20\cdot 9\cdot 3 = 180\cdot 3 = 540$

7) $f(x;y) \approx f(x_0;y_0) + f_x'(x_0;y_0)\cdot\Delta x + f_y'(x_0;y_0)\Delta y$

$f(1,55;0,015) \approx 243 + 0\cdot(-0,021) + 540\cdot 0,015 = 243 + 540\cdot 0,015 = 251,1$

То есть $\sqrt{(\sin^2 1,55+8,1^{0,015})^5} \approx 251,1$