ENGR 3450
Project Scheduling

CRASHING
(TIME-COST TRADE-OFF)

AGENDA Today

- Crash Time and Cost
  - Computing crash data
- Minimum Cost schedule
- Minimum Time Schedule

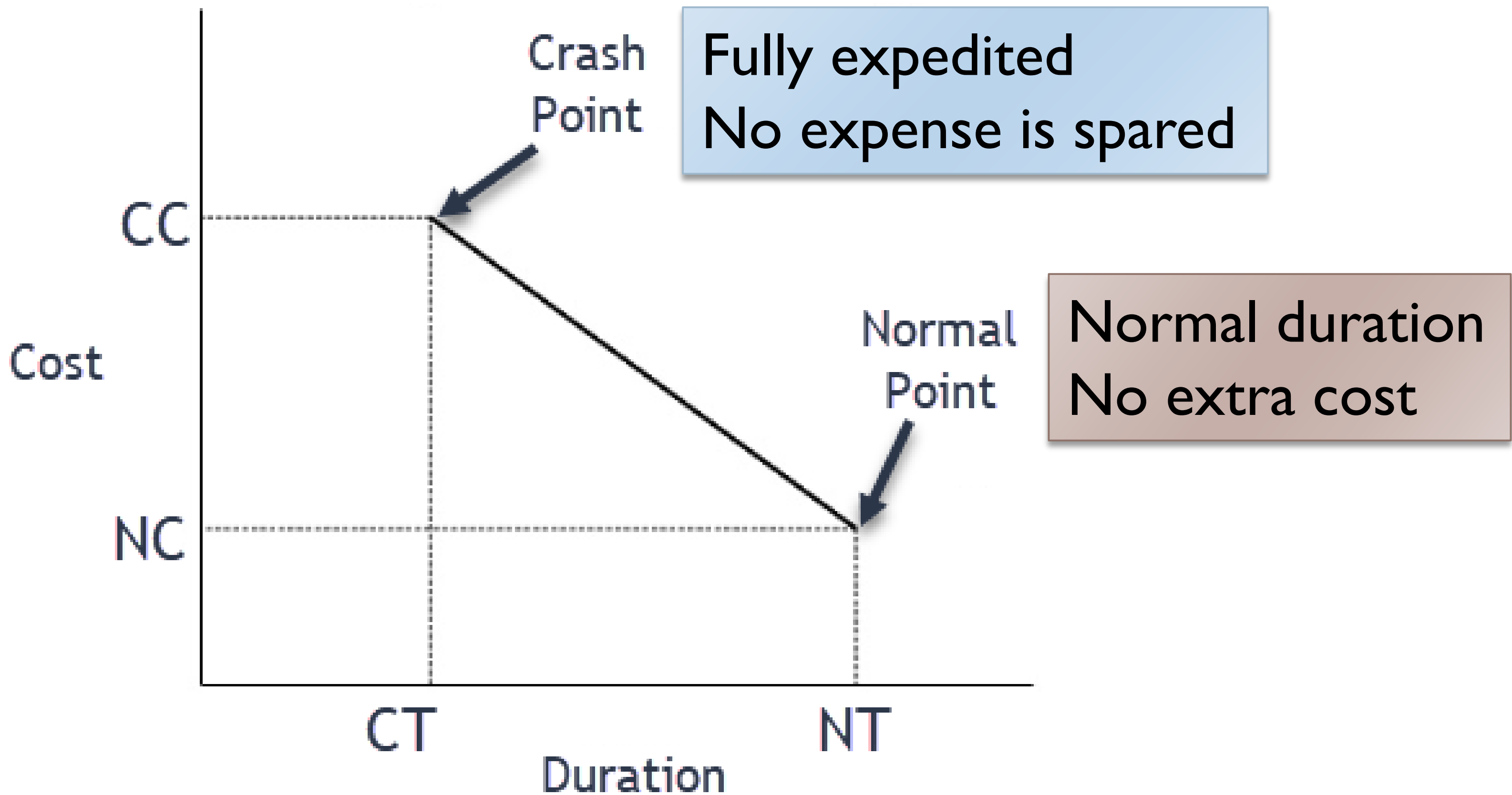
CRASHING

- When we say that an activity will take a certain number of days or weeks, what we really mean is: this activity takes **normally** that many days or weeks.

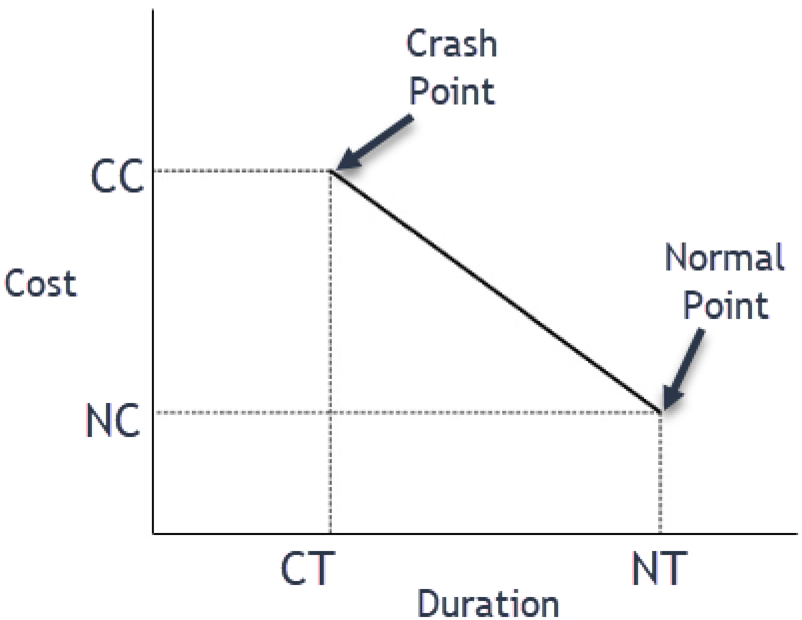
- We could make it take less time
  - but it would cost more money (resources).

- To spend more money so as to get something done more quickly is called "crashing" the activity.

Time – Cost trade offs for crashing activities



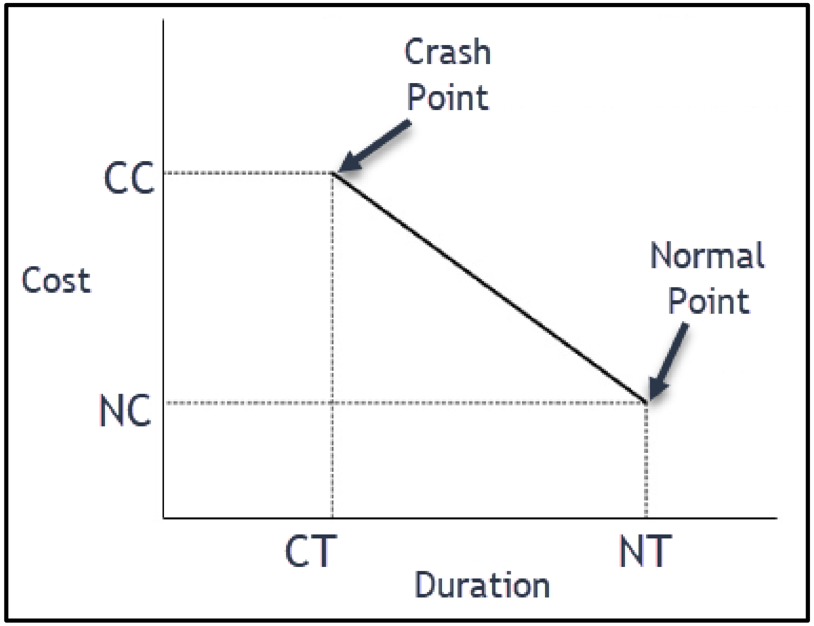
Time – Cost trade offs for crashing activities



- NT = normal time to complete an activity
- NC = normal cost to complete an activity
- CT = crash time to complete an activity, that is, the shortest possible time it could be completed in.
- CC = crash cost = the cost to complete the activity if it is performed in its shortest possible time (CT).

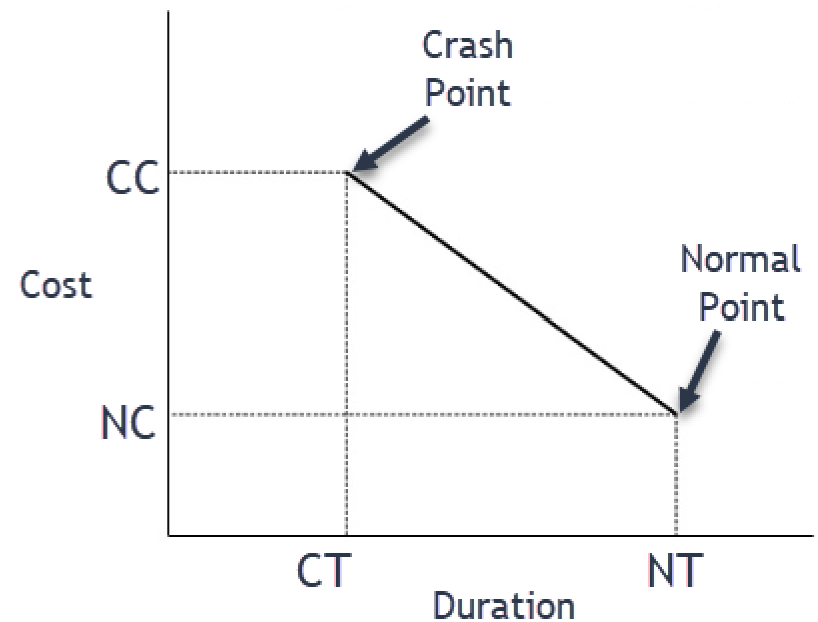
Parameters for crashing

- Maximum time reduction for an activity = NT – CT
- Cost to crash per period = $\frac{CC - NC}{NT - CT}$
- Note that the cost to crash per period assumes that the relationship between adding more money to the activity and reducing the time is **linear:**
- Spend half of the money, and get half the time reduction
- This is not always true in practice, but works alright for a rough planning technique.



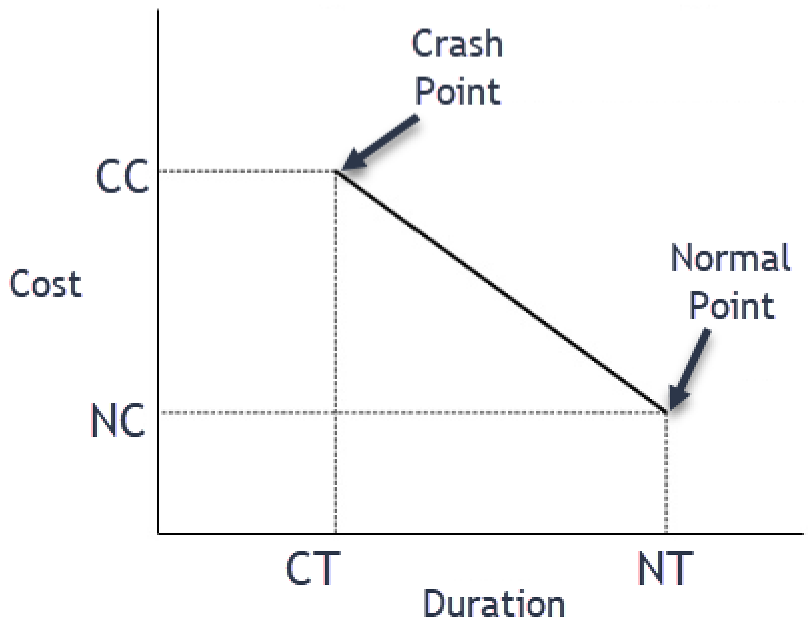
Computing crash data

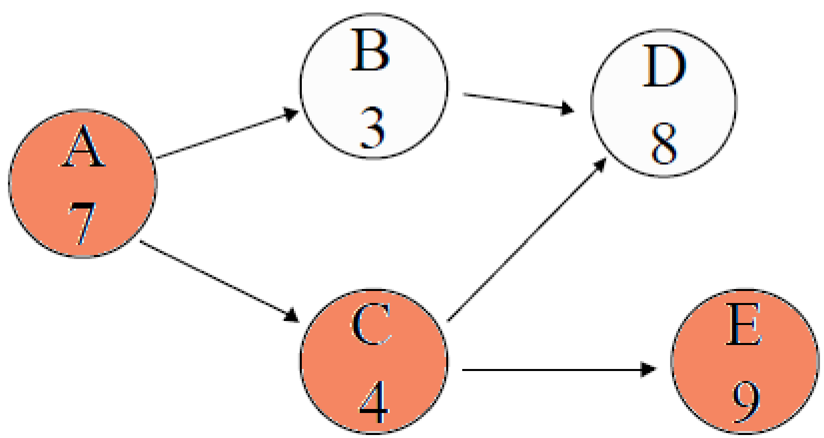
- Given:
  - activities
    - NT
    - NC
    - CT
    - CC
- Compute:
    - 1.maximum time reduction
    - 2.cost to crash per period



Example

Act.	NT	NC	CT	CC
A	7	3000	4	6000
B	3	4000	2	5500
C	4	15000	2	20000
D	8	10000	5	19000
E	9	7000	6	9100

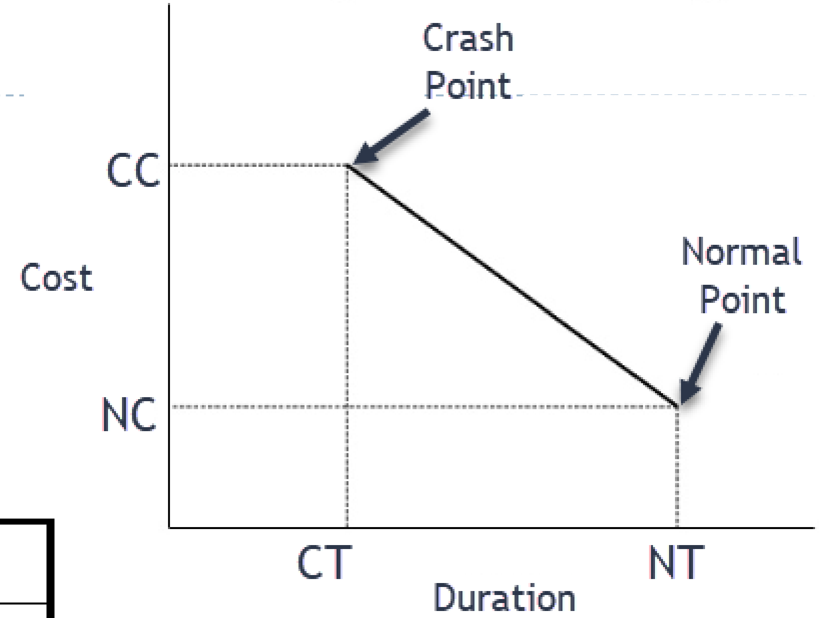




Critical Path: ACE

Example

1. Compute max. time reduction: NT-CT



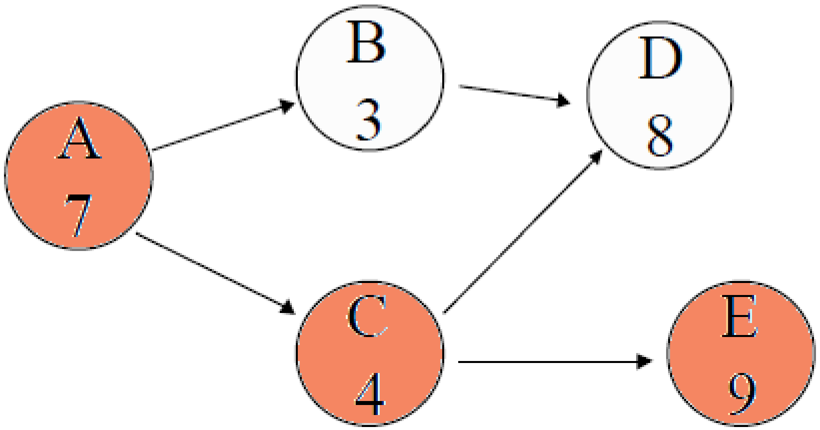
Act.	NT	NC	CT	CC	Max Red
A	7	3000	4	6000	7-4 = 3
B	3	4000	2	5500	3-2 = 1
C	4	15000	2	20000	4-2 = 2
D	8	10000	5	19000	8-5 = 3
E	9	7000	6	9100	9-6 = 3

Example

2. Compute cost to crash per period:

$$\frac{CC - NC}{NT - CT}$$

Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700

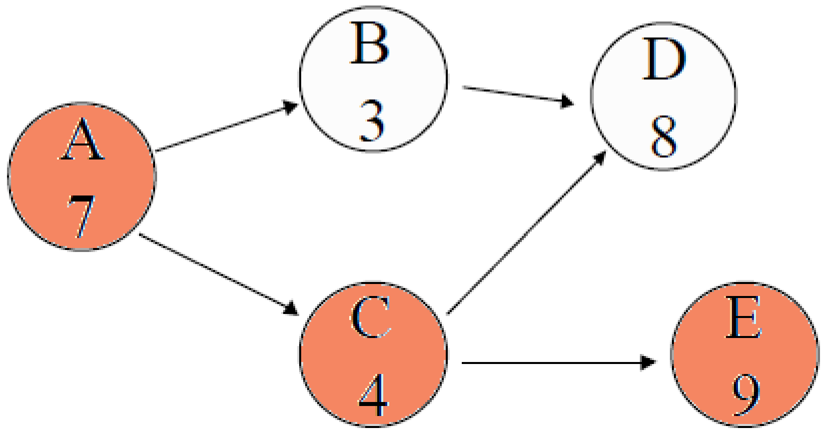


To Find the minimum cost schedule

- To shorten a project, crash only the activities that are critical.
- Crash from the least expensive to the most expensive.
- Each activity can be crashed until
  - it reaches its maximum time reduction
  - it causes another path to also become critical
  - it is more expensive to crash than not to crash
- Continue until no more activities can be crashed.

Example continued

- This project, under normal conditions takes 20 days. Suppose each day the project runs incurs an indirect project cost of **$1400** (overhead).

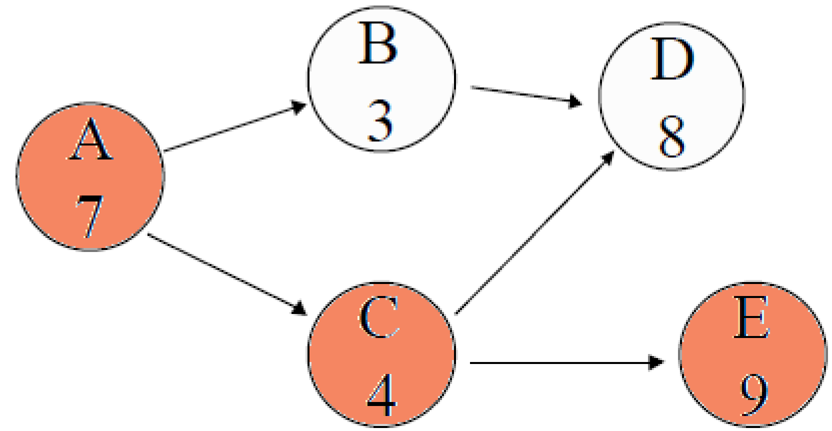


- Current (NT) Costs:
  - Sum of normal costs = 39000
  - Indirect costs = 20 days * 1400 = 28000
  - Total Costs: 67000

Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700

Example continued

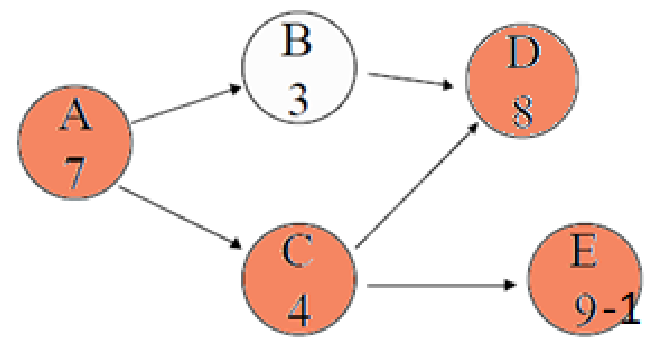
- Which activities should be crashed if any?
  - ABD 18
  - ACD 19
  - ACE 20 *
- Start by looking at activities on the critical path: A, C, and E.
- E is least expensive to crash.



Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700

Example continued

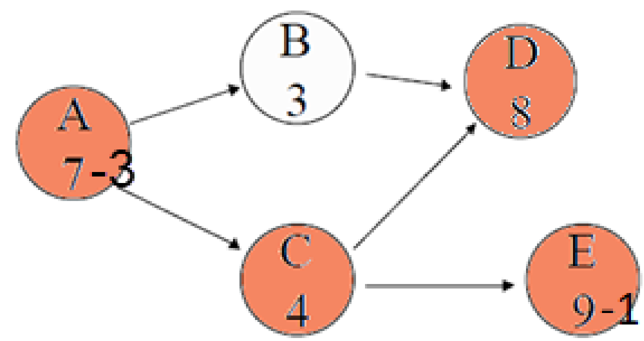
- How much to crash E ?
  - ABD 18
  - ACD 19
  - ACE 20 *
- E has maximum time reduction of 3, but if it is crashed by 1, then ACD also becomes a critical path.
- Also, we save **$1400** per day the project is shortened and would spend **$700** per day to crash E, so it is profitable to crash E.



Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700

Example continued

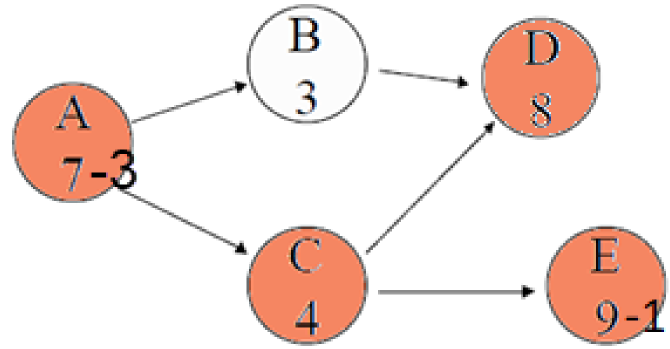
- Now there are two critical paths.
  - ABD : 18
  - ACD : 19 *
  - ACE : 19 *
- To finish the project earlier, we need to shorten both paths.
- Either Crash A or C (those activities are on both paths)
  - C : 2500
  - A : 1000
- Alternately, Crash both D and E together.
  - E-D : 3700
- **Crash A by 3 Since we gain 1400 for each project time gain.**



Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700

Stopping condition

- Now there are again two critical paths.
  - ABD : 15
  - ACD : 16 *
  - ACE : 16 *



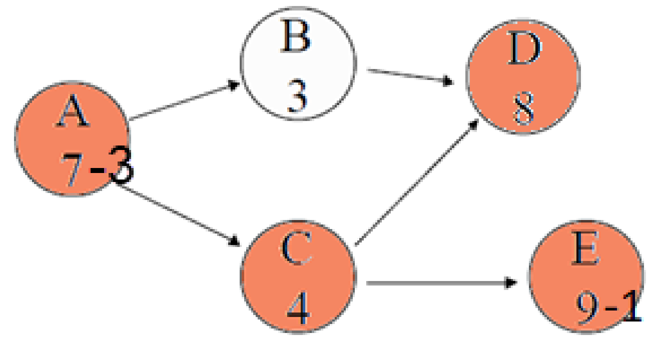
- To continue, we could crash C or both D and E. But in each case, the cost would be greater than the $1400 savings per day. So, we stop at this point.

Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700

- We can compute the cost to perform the project in 16 days.

Total Project Cost

- Sum of normal costs = 39,000
- Indirect costs = 16 days * 1400 = 22400
- Crashing cost
  - E by 1 = 700
  - A by 3 = 3000



= 39000 + 22400 + 3700

Min Project Cost = $65100

Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700
		39000				

Minimum time schedule

- Sometimes it is necessary to complete a project as short as possible (in min. time rather than min. cost)
- To find the shortest time possible, crash all activities completely and then find the times for all paths.
- The longest path is, of course, critical and tells us how long the project must take.

Minimum time schedule at minimum cost

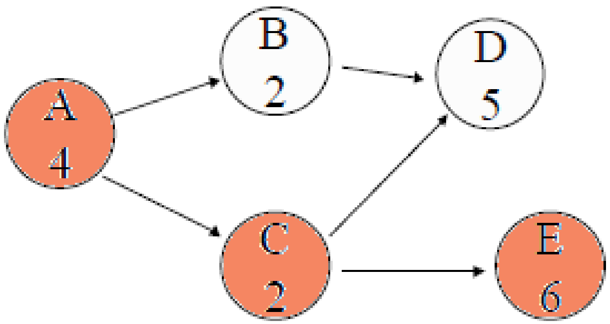
- Activities on non-critical paths may not need to be fully crashed in order for the project to be finished in the shortest possible time.

- These activities can be "**uncrashed**" one at a time, starting from the most expensive one to crash, till there is nothing left to uncrash.

Minimum time schedule at minimum cost

Same problem as earlier.

Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700



1. Set all activities to their crash (shortest) times

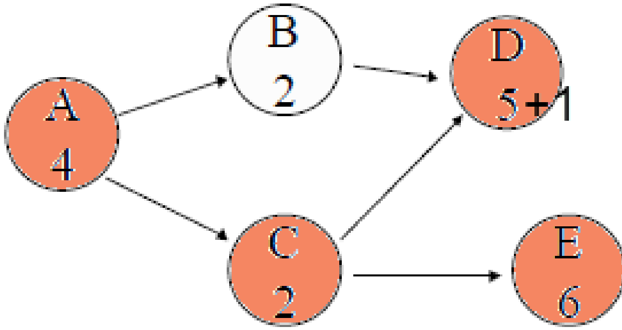
Three paths, but now with shorter times.

2. Critical activities are still A, C and E.
3. B and D are not critical and can be relaxed.

    B costs $1500 to crash.

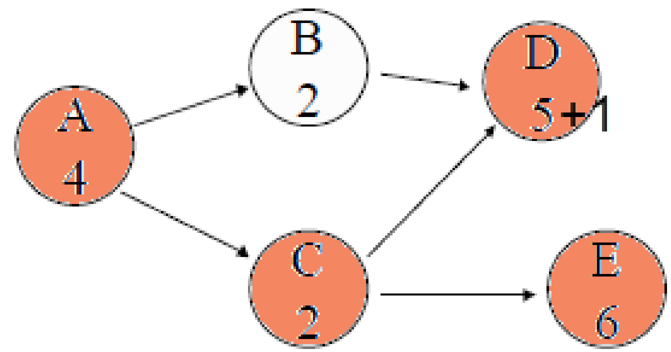
    D costs $3000 to crash.

To save some money, but still complete in 12 days **uncrush** D by 1. It means less cost and the same project time.



Total Cost

Act.	NT	NC	CT	CC	Max Red	Cost to crash per period
A	7	3000	4	6000	3	1000
B	3	4000	2	5500	1	1500
C	4	15000	2	20000	2	2500
D	8	10000	5	19000	3	3000
E	9	7000	6	9100	3	700
				59600		



Best Cost for the Least Project Time (12 days)
= 59600 + 12 * 1400 - 3000 = **$73,400**

Min Project Cost (16 days) = $65100