Unimetry: A Quaternionic Gravito–Electromagnetic Formulation

Timur Abizgeldin*

November 29, 2025

Abstract

Contents

*Email: `foo@example.org`. TODO: update contact/affiliation.

1 Introduction

2 Introduction

2.1 Context and motivation

Unimetry is a proposed phase–geometric framework in which physical systems are described in terms of stationary flows in an underlying Euclidean proto-space $\mathcal{E}$. Rather than postulating space–time as a primary arena, unimetry treats the observed space–time geometry, relativistic kinematics, and field interactions as effective structures derived from the orientation and coupling of such flows. A dimensionless scalar phase potential $\Phi : \mathcal{E} \to \mathbb{R}$ and its gradient define a normalized flow direction; the familiar Minkowski metric and Lorentzian phenomena then appear as particular projections of this underlying flow geometry.

In this sense, special relativity (SR) is not the endpoint, but the first benchmark for the framework: unimetry aims at a unified phase-based description of kinematics, gravity and gauge interactions, with SR recovered as a specific limit of the general construction. The present paper develops one important sector of this programme, namely a quaternionic gravito–electromagnetic (GEM) formulation built on top of the unimetrical flow picture.

At the classical level, gravito–electromagnetic analogies are well known: in the weak-field, slow-motion limit of general relativity, the Einstein equations can be cast into a Maxwell-like form, and moving masses generate a "gravitomagnetic" field. Quaternions and related algebras have also long been used to encode rotations and the Maxwell equations in a compact way. What unimetry adds to this landscape is a concrete phase-geometric interpretation: a single quaternionic object encodes both the temporal and spatial parts of a flow, and bilinear forms of such objects naturally split into scalar, symmetric vector, and axial (vorticity-like) channels. This suggests that gravity and electromagnetism might be viewed as different faces of the same bilinear structure acting on suitably dressed flow quaternions.

Our goal here is to make this statement precise. We construct a quaternionic GEM formalism in which gravitational and electromagnetic interactions originate from the *same* bilinear machinery applied to metrically dressed "body quaternions". In particular, we show that Newton and Coulomb potentials arise as two branches of a single scalar form, while the magnetic and gravitomagnetic sectors are associated with a vortical bilinear form whose physical calibration reveals a natural role for the constants ε_0, μ_0, G and c. The resulting description remains Euclidean at the level of the proto-space, yet reproduces relativistic kinematics and GEM fields in the observable three-space.

2.2 Relation to the base unimetry paper

This work is a direct sequel to the base unimetry paper, *"Unimetry: A Phase-Space Reformulation of Special Relativity"* (henceforth "Paper I"). Paper I develops the core phase/flow structure: the phase potential Φ, the phase 1-form $\alpha = d\Phi$, the normalized flow $\widehat{\boldsymbol{\chi}}$, and the calibration $\boldsymbol{\chi} = c\,\widehat{\boldsymbol{\chi}}$, together with the derivation of the Minkowski interval and standard SR effects from a Euclidean proto-space. It also introduces the unimetrical D-rotation, which encodes Lorentz boosts as Euclidean rotations in a suitable plane of $\mathcal{E}$.

From the unimetry viewpoint, however, these SR results are only the first consistency test of a broader phase-based paradigm. The present paper assumes familiarity with the conceptual setting of Paper I, but is written to be as self-contained as reasonably possible. We briefly recall the key definitions of the phase proto-space, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations. All constructions that are essential for the GEM sector are reproduced or adapted here; more detailed discussions of SR and cosmological applications remain in Paper I and are only referenced when needed.

2.3 Main results

The main technical contributions of this paper can be summarized as follows.

- We introduce *metrically dressed body quaternions* $\widetilde{\mathsf{q}}_i = L_{E,i}\,\hat{h} + L_{G,i}\,\widehat{\boldsymbol{n}}_i$, whose components have the dimension of length. The "electric" and "gravitational" lengths

  $$L_{E,i} = \sqrt{\frac{G}{4\pi\varepsilon_0 c^4}}\,Q_i, \qquad L_{G,i} = \frac{G}{c^2}\,m_i$$

  encode the charge Q_i and mass m_i of the body in a unified geometric fashion. The unit vector $\widehat{\boldsymbol{n}}_i$ represents the spatial flow direction associated with the body.

- We show that the scalar bilinear form

$$A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2) = L_{E,1} L_{E,2} - \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

  (with $\boldsymbol{S}_i = L_{G,i} \widehat{\boldsymbol{n}}_i$) yields, after a single global calibration by c^4/G and a geometric $1/r$ factor, the combined Newton–Coulomb potential:

$$U(r) = \frac{c^4}{G} \frac{A}{r} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r} - G \frac{m_1 m_2}{r}.$$

  Thus gravity and electrostatics arise as two channels of a single invariant scalar form.

- We identify two vector-valued bilinear forms, $\boldsymbol{B}(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2)$ and $\boldsymbol{C}(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2)$, corresponding to the symmetric and axial parts of the quaternion product. In the dressed setting these naturally describe current-like and vortical channels. In particular, the vortical form $\boldsymbol{C}$ reproduces the geometry of magnetic and gravitomagnetic fields generated by moving charges and masses.

- We construct a quaternionic GEM field $\mathcal{F}_{\mathrm{GEM}}(\boldsymbol{x})$ over the observable three-space by combining dressed source quaternions with purely imaginary distance quaternions. Its scalar channel reproduces the gravitational and electrostatic potentials, while its vortical channel yields a physically natural "phase-vortical" field C_{phys} with the same dimension as $\boldsymbol{E}$. The standard magnetic field $\boldsymbol{B}$ in SI units then appears as

$$\boldsymbol{B} = \frac{1}{c} C_{\mathrm{phys}},$$

  so that the familiar μ_0 and ε_0 can be interpreted in terms of linear and areal stiffness of the vacuum, combined into an effective volumetric stiffness proportional to $1/(\varepsilon_0 c^3)$.

- We analyze the action of unimetrical D-rotations and ordinary spatial rotors on dressed quaternions. Pure spatial rotations act in the usual way on the vector channels and leave the scalar form A invariant, while D-rotations mix the scalar channel and the longitudinal component of $\boldsymbol{B}$ in a two-dimensional "energy–current" plane. This provides a quaternionic encoding of relativistic kinematics in the GEM setting, with Lorentz-consistent transformation properties of the fields.

- Finally, we outline a Hamiltonian and Lagrangian formulation of the quaternionic GEM theory in terms of the self-form A and the norm-squares of $\boldsymbol{B}$ and $\boldsymbol{C}$, and discuss how the standard Maxwell Lagrangian and linearized GEM equations arise in appropriate limits.

2.4 Structure of the paper

The paper is organized as follows. In Section 3 we recall the basic quaternion algebra and introduce the bilinear forms A, $\boldsymbol{B}$, and $\boldsymbol{C}$ that arise from the quaternion product, together with their matrix representation and geometric interpretation. **??** provides a brief overview of the unimetrical phase proto-space, the phase potential, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations.

In **??** we introduce metrically dressed body quaternions and define the electric and gravitational lengths L_E and L_G. **??** shows how the scalar form A for dressed quaternions reproduces the static Newton and Coulomb potentials. In **??** we construct a quaternionic GEM field over the observable three-space and identify the scalar and vortical channels with gravitational, electric, and magnetic sectors.

?? analyzes the action of spatial rotors and D-rotors on dressed quaternions and on the GEM field, clarifying the relativistic transformation properties of the scalar, current-like, and vortical channels. **??** is devoted to the calibration of $\boldsymbol{E}$ and $\boldsymbol{B}$, to the definition of the phase-vortical field C_{phys}, and to the interpretation of ε_0, μ_0, and c in terms of vacuum stiffness.

In **??** we outline Hamiltonian and Lagrangian formulations of quaternionic GEM, and in **??** we compare the resulting equations with the standard Maxwell and linearized GEM formalisms. Finally, **??** discusses limitations and open questions, and sketches possible extensions towards non-Abelian interactions and cosmological applications.

3 Quaternion algebra and bilinear forms

3.1 Basic notation and conventions

We denote by $\mathbb{H}$ the real quaternion algebra, viewed as a four-dimensional real vector space

$$\mathbb{H} \simeq \mathbb{R}^4(\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}),$$

where $\hat{h}$ is a distinguished real ("temporal") basis element and $\hat{\imath}, \hat{\jmath}, \hat{k}$ are purely imaginary basis elements. A general quaternion is written as

$$q = T\,\hat{h} + \boldsymbol{S}, \qquad \boldsymbol{S} = S^1\hat{\imath} + S^2\hat{\jmath} + S^3\hat{k},$$

with $T \in \mathbb{R}$ and $\boldsymbol{S} \in \operatorname{Im}\mathbb{H} \simeq \mathbb{R}^3$. We call T the scalar part and $\boldsymbol{S}$ the vector part of q.

The imaginary basis satisfies the usual quaternion relations

$$\hat{\imath}^2 = \hat{\jmath}^2 = \hat{k}^2 = \hat{\imath}\hat{\jmath}\hat{k} = -\hat{h},$$

and the mixed products reproduce the three-dimensional cross product structure, e.g. $\hat{\imath}\hat{\jmath} = \hat{k}$, $\hat{\jmath}\hat{k} = \hat{\imath}$, $\hat{k}\hat{\imath} = \hat{\jmath}$, with antisymmetry under exchange of factors. We use the standard identification $\operatorname{Im}\mathbb{H} \simeq \mathbb{R}^3$ with its Euclidean inner product $\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$ and cross product $\boldsymbol{S}_1 \times \boldsymbol{S}_2$.

Quaternionic conjugation is defined by

$$\overline{q} := T\,\hat{h} - \boldsymbol{S},$$

and the norm is $\|q\|^2 = q\,\overline{q} = \overline{q}\,q = T^2 + \|\boldsymbol{S}\|^2$. The real and imaginary parts are

$$\operatorname{Re}(q) = T, \qquad \operatorname{Im}(q) = \boldsymbol{S}.$$

In what follows we will systematically write

$$q = (T, \boldsymbol{S})$$

when it is convenient to emphasize the split into scalar and vector parts. We will also denote vectors in $\operatorname{Im}\mathbb{H}$ in boldface (e.g. $\boldsymbol{S}$) and reserve $\hat{h}$ for the distinguished scalar basis element. This choice aligns with the unimetrical interpretation, where the scalar part will later be associated with the temporal channel of a flow, and the vector part with its spatial channel.

3.2 Quaternion product and decomposition into A, B, C forms

Let $\{e_\mu\}_{\mu=0}^3 = \{\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}\}$ be the standard quaternion basis, with

$$\hat{h}^2 = \hat{h}, \qquad \hat{\imath}^2 = \hat{\jmath}^2 = \hat{k}^2 = -\hat{h},$$

and the usual multiplication table $\hat{\imath}\hat{\jmath} = \hat{k}$, $\hat{\jmath}\hat{k} = \hat{\imath}$, $\hat{k}\hat{\imath} = \hat{\jmath}$, with antisymmetry under exchange of factors. Any quaternion can be written as

$$q = x^\mu e_\mu = x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k},$$

with real components $x^\mu \in \mathbb{R}$.

Let

$$\mathbf{q}_1 = x^\mu e_\mu, \qquad \mathbf{q}_2 = y^\nu e_\nu, \qquad \mathbf{q}_1, \mathbf{q}_2 \in \mathbb{H}.$$

Then their quaternion product can be expanded as

$$\begin{aligned} \mathbf{q}_1 \circ \mathbf{q}_2 &= (x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k})(y^0\hat{h} + y^1\hat{\imath} + y^2\hat{\jmath} + y^3\hat{k}) \\ &= (\mathbf{q}_1 * \mathbf{q}_2) + (\mathbf{q}_1 \diamond \mathbf{q}_2) + (\mathbf{q}_1 \times \mathbf{q}_2), \end{aligned} \tag{1}$$

where we have grouped terms according to three bilinear structures:

- a scalar ("Minkowski-like") channel $\mathbf{q}_1 * \mathbf{q}_2$,
- a symmetric vector channel $\mathbf{q}_1 \diamond \mathbf{q}_2$,
- an axial/vortical vector channel $\mathbf{q}_1 \times \mathbf{q}_2$.

Explicitly,

$$\begin{aligned} \mathbf{q}_1 \circ \mathbf{q}_2 = {} & (x^0y^0\hat{h}^2 + x^1y^1\hat{\imath}^2 + x^2y^2\hat{\jmath}^2 + x^3y^3\hat{k}^2) \\ & + (x^0y^1\hat{h}\hat{\imath} + x^0y^2\hat{h}\hat{\jmath} + x^0y^3\hat{h}\hat{k} + x^1y^0\hat{\imath}\hat{h} + x^2y^0\hat{\jmath}\hat{h} + x^3y^0\hat{k}\hat{h}) \\ & + (x^1y^2\hat{\imath}\hat{\jmath} + x^1y^3\hat{\imath}\hat{k} + x^2y^1\hat{\jmath}\hat{\imath} + x^2y^3\hat{\jmath}\hat{k} + x^3y^1\hat{k}\hat{\imath} + x^3y^2\hat{k}\hat{\jmath}). \end{aligned} \tag{2}$$

Using the multiplication rules, this can be written in a compact tensor-like form

$$\mathbf{q}_1 \circ \mathbf{q}_2 = \sum_{\mu,\nu=0}^{3} \left(A_{\mu\nu}\,\hat{h} + B_{\mu\nu} + C_{\mu\nu}\right) x^\mu y^\nu,$$

where the three 4×4 coefficient matrices are

$$A_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{3}$$

$$B_{\mu\nu} = \begin{pmatrix} 0 & \hat{\imath} & \hat{\jmath} & \hat{k} \\ \hat{\imath} & 0 & 0 & 0 \\ \hat{\jmath} & 0 & 0 & 0 \\ \hat{k} & 0 & 0 & 0 \end{pmatrix}, \tag{4}$$

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{k} & -\hat{\jmath} \\ 0 & -\hat{k} & 0 & \hat{\imath} \\ 0 & \hat{\jmath} & -\hat{\imath} & 0 \end{pmatrix}. \tag{5}$$

Equivalently, if we collect the scalar and vector parts explicitly, we recover the more familiar invariant decomposition

$$\begin{aligned} \mathbf{q}_1 \circ \mathbf{q}_2 = {} & \left(x^0y^0 - x^1y^1 - x^2y^2 - x^3y^3\right)\hat{h} \\ & + (x^0\boldsymbol{y} + y^0\boldsymbol{x}) + (\boldsymbol{x} \times \boldsymbol{y}), \end{aligned} \tag{6}$$

where $\boldsymbol{x} = (x^1, x^2, x^3)$, $\boldsymbol{y} = (y^1, y^2, y^3)$, and the colour coding is as in Eqs. (3) to (5):

- $A_{\mu\nu}$ is the Minkowski-like bilinear form $\mathrm{diag}(1, -1, -1, -1)$ acting on the coordinate vectors x^μ, y^ν;
- $B_{\mu\nu}$ collects the symmetric mixed products between the scalar and vector components;

- $C_{\mu\nu}$ collects the antisymmetric products between the spatial components, encoding the cross product $\boldsymbol{x} \times \boldsymbol{y}$.

This tensor-like presentation makes explicit that the quaternion product can be viewed as the contraction of a rank-$(0,2)$ object with two four-vectors:

$$\mathbf{q}_1 \circ \mathbf{q}_2 = \left(A + B + C\right)_{\mu\nu} x^\mu y^\nu,$$

with a scalar (red), a symmetric vector-valued (green), and an antisymmetric vector-valued (blue) block. In the unimetrical setting we will reinterpret the scalar block $A_{\mu\nu}$ as the basic energy-like invariant, the symmetric block $B_{\mu\nu}$ as a current-like coupling, and the antisymmetric block $C_{\mu\nu}$ as the vortical channel that underlies magnetic and gravitomagnetic fields.

3.3 Geometric interpretation of A, B, C

The decomposition (**??**) plays a central role in what follows:

- The scalar form A is a symmetric bilinear form of signature $(+,-,-,-)$ when restricted to suitable unimetrical subspaces and will be used as the basic energy-like invariant. In the dressed setting it will produce both Newtonian and Coulomb potentials from a single scalar channel.

- The symmetric vector form $\boldsymbol{B}$ behaves like a current-like quantity: it is linear in each argument and symmetric under exchange. When applied to dressed body quaternions, it will encode the coupling between scalar (temporal) and vector (spatial) parts, and will be directly related to mass and charge currents in the GEM field.

- The axial vector form $\boldsymbol{C}$ is purely imaginary and antisymmetric. It is built from the cross product of the spatial parts and therefore encodes vorticity-like structures. In the GEM interpretation it will be responsible for the magnetic and gravitomagnetic sectors.

Thus the elementary quaternion product already contains, in a rigid algebraic way, the three channels that we will later reinterpret as (i) an energy-like scalar invariant, (ii) a current-like symmetric vector channel, and (iii) a vortical (axial) channel. In the next section we recall how unimetry associates physical flows and space–time structure to quaternions, so that the forms A, $\boldsymbol{B}$, and $\boldsymbol{C}$ can be given a gravito–electromagnetic meaning.

4 Phase proto-space and flow: brief unimetry overview

4.1 Proto-space, phase potential and 1-form

4.2 Flow vector and normalization

4.3 Interval, proper time and correspondence with SR

4.4 Notation table

5 Flow calibrations and energy-like functionals

5.1 Phase (kinematic) calibration

5.2 Proper-time (energetic) calibration

5.3 Energy of a flow and the self-form A(q,q)

6 Metrically dressed body quaternions

6.1 Free unit imaginary vector for isotropic Newtonian bodies

In the metrically dressed setting we write the spatial part of a body quaternion as

$$\boldsymbol{S}_i = L_{G,i}\,\widehat{\boldsymbol{n}}_i,$$

where $\widehat{\boldsymbol{n}}_i$ is a unit spatial direction associated with body i. For charged bodies it is natural to interpret $\widehat{\boldsymbol{n}}_i$ as an intrinsic flow direction (e.g. an orientation of the underlying streamlet structure), which will in general produce non-trivial contributions in both the symmetric vector form $\boldsymbol{B}$ and the vortical form $\boldsymbol{C}$.

For purely Newtonian, isotropic mass distributions, however, we may and should distinguish between an intrinsic direction and the *interaction* direction. To reflect this, we introduce the notion of a *free unit imaginary vector* for the gravitational channel.

Definition 6.1 (Free unit imaginary vector)**.** A free unit imaginary vector is a unit spatial quaternion $\widehat{\boldsymbol{u}} \in \operatorname{Im}\mathbb{H}$ whose orientation is not fixed by the internal structure of the body, but is freely assigned at the level of the interaction. For an isotropic Newtonian body i we take the gravitational spatial part of its dressed quaternion to be

$$\boldsymbol{S}_i^{(G)}(\boldsymbol{x}) := L_{G,i}\,\widehat{\boldsymbol{u}}_i(\boldsymbol{x}),$$

where, for a field point $\boldsymbol{x}$, the free unit vector is chosen to be aligned with the radius vector from the body to that point,

$$\widehat{\boldsymbol{u}}_i(\boldsymbol{x}) := \widehat{\boldsymbol{r}}_i(\boldsymbol{x}) = \frac{\boldsymbol{x}-\boldsymbol{x}_i}{\|\boldsymbol{x}-\boldsymbol{x}_i\|}.$$

In other words, for an isotropic Newtonian source the gravitational channel of the dressed quaternion is always taken to be *radial* with respect to the field point, and carries no intrinsic "spin" information. This has an important structural consequence for the bilinear forms.

Consider two isotropic masses m_1, m_2 at positions $\boldsymbol{x}_1, \boldsymbol{x}_2$, and evaluate their gravitational spatial parts at a common field point $\boldsymbol{x}$. By construction,

$$\boldsymbol{S}_1^{(G)}(\boldsymbol{x}) \parallel \widehat{\boldsymbol{r}}_1(\boldsymbol{x}), \qquad \boldsymbol{S}_2^{(G)}(\boldsymbol{x}) \parallel \widehat{\boldsymbol{r}}_2(\boldsymbol{x}).$$

In the static two-body configuration the interaction is along the line joining the bodies, so that effectively

$$\boldsymbol{S}_1^{(G)} \parallel \boldsymbol{S}_2^{(G)},$$

and therefore their contribution to the vortical form

$$\boldsymbol{C}(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2) = \boldsymbol{S}_1 \times \boldsymbol{S}_2$$

vanishes in the purely gravitational, isotropic limit:

$$\boldsymbol{C}^{(G)} = \boldsymbol{S}_1^{(G)} \times \boldsymbol{S}_2^{(G)} = \mathbf{0}.$$

Thus, by assigning the gravitational spatial part of an isotropic body to a free unit imaginary vector that is always chosen radial, we ensure that:

- the gravitational interaction of isotropic masses is purely scalar and radial, as in Newtonian gravity;
- there is *no* spurious contribution of the gravitational channel to the vortical form $\boldsymbol{C}$ in the static limit;
- all non-trivial vortical contributions in $\boldsymbol{C}$ are genuinely associated with anisotropy and/or motion (currents), i.e. with the EM and gravitomagnetic sectors rather than with static isotropic gravity.

In contrast, for charged bodies we will keep an intrinsic unit direction $\widehat{\boldsymbol{n}}_i$ in the electric channel, which can contribute to both the symmetric form $\boldsymbol{B}$ and the vortical form $\boldsymbol{C}$. This separation between a free gravitational direction and an intrinsic electromagnetic direction will be important when we analyze the GEM field and its magnetic and gravitomagnetic components in **????**.