Unimetry: Energy in a Phase–Space

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Abstract

We propose a phase–space formulation (*Unimetry*) where mass is a volume–normalized structural coefficient of a flow, and the standard relativistic energy–momentum relation emerges from the geometry of two orthogonal components of the flow. We develop a compact dictionary to SR, motivate the cubic scaling behind rest mass, and derive a scalar phase–space energy density whose observed energy is obtained by kinematic projection. Consequences and empirical anchors (binding energy, heat, stress/pressure) are discussed.

Keywords: phase–space, relativistic energy, structural mass, volumetric normalization, emergent time.

1 Introduction

Why does relativistic energy take the form it does, and how can "mass" be read off from internal structure rather than postulated? Unimetry treats an object as a flow in phase–space with modulus $\tilde{H}$ split into an internal (temporal) and external (spatial) part. This viewpoint suggests a cubic (volumetric) normalization for rest mass and recovers standard SR kinematics as a rotation in the $(\tilde{S}, \tilde{L})$–plane.

Contributions. (i) We model rest mass as a volume–normalized structural coefficient $\kappa \propto k^3$ of a cyclic normalization k; (ii) we derive $E = \gamma m_0 c^2$ and $E^2 = m_0^2 c^4 + p^2 c^2$ directly from the phase geometry; (iii) we clarify why Euclidean quadratic invariants built from a cubic scale yield sixth–power laws; (iv) we identify a scalar phase–space energy density $e = \kappa \dot{H}^3$ and relate it to empirical effects (binding, heat, stress/pressure).

Roadmap. Sec. 2 fixes notation; Sec. 3 introduces structural mass; Sec. 4 derives relativistic energy; Sec. 5 justifies volume normalization and the "×3 rule"; Sec. 6 generalizes energy formulas, where we *show* (Proposition #) that e defines a phase–space invariant; Sec. 7 discusses verification paths.

Postulates (informal)

1. (Phase flow) Each physical object is represented by a flow with modulus $\tilde{H}$ and orthogonal components $(\tilde{S}, \tilde{L})$ such that $\tilde{H}^2 = \tilde{S}^2 + \tilde{L}^2$.
2. (Kinematic angle) Define $\beta = \sin\theta = \tilde{L}/\tilde{H}$ and $\gamma = \sec\theta = 1/\sqrt{1-\beta^2}$.
3. (Gauge) The speed of light c is identified with the local–time phase speed: $c \equiv \dot{H}$.
4. (Cyclic time) Local time arises as a cyclic action with frequency $\nu = \dot{\chi} = k\,\tilde{H}$, where $k = R_1/R_2$ is a normalization factor of the cycle radii.

2 Preliminaries and Notation

We employ tilded symbols for proto–space quantities and dots for local–time derivatives. The basic geometric decomposition reads

$$\tilde{H}^2 = \tilde{S}^2 + \tilde{L}^2, \qquad \tilde{S} = \tilde{H}\cos\theta, \quad \tilde{L} = \tilde{H}\sin\theta, \quad \beta = \sin\theta, \quad \gamma = \sec\theta. \tag{1}$$

The cyclic–time normalization gives

$$\nu = \frac{\mathrm{d}\chi}{\mathrm{d}\tau} = \dot{\chi} = k\,\tilde{H}, \qquad k = \frac{R_1}{R_2}. \tag{2}$$

Why does $\nu = k\,\tilde{H}$? Three equivalent derivations

(A) Flux continuity on the two circles. View the internal dynamics as a steady flow on a two-torus $\mathbb{T}^2 = S^1_\chi \times S^1_\tau$ with circumferences $C_\chi = 2\pi R_1$ and $C_\tau = 2\pi R_2$. One tick corresponds to transporting the arc length $\Delta\chi = C_\chi$ while advancing the time circle by $\Delta\tau = C_\tau/\tilde{H}$ (since the speed along the τ-fiber is $\tilde{H}$ by the gauge $c \equiv \dot{H}$). Equating the steady fluxes through the two fundamental cycles,

$$J_\chi = \tilde{H}\,C_\chi, \qquad J_\tau = \nu\,C_\tau,$$

forces $\nu \equiv d\chi/d\tau = (C_\chi/C_\tau)\,\tilde{H} = (R_1/R_2)\,\tilde{H}$.

(B) Circle-group homomorphism (winding). The only smooth homomorphisms of the circle group S^1 are rotations with degree k; in angle variables $\theta_\tau = k\,\theta_\chi \pmod{2\pi}$. Passing to arc-length coordinates $\chi = R_1\theta_\chi$ and $\zeta = R_2\theta_\tau$ and differentiating with respect to τ gives $\dot{\zeta} = R_2\dot{\theta}_\tau = k(R_2/R_1)\dot{\chi}$. Identifying $\dot{\zeta} = \tilde{H}$ (the speed along the time-fiber under the gauge) yields $\dot{\chi} = k\,\tilde{H}$, i.e. $\nu = k\,\tilde{H}$.

(C) Dimensional and symmetry argument. A frequency must be built from the available scalars $\tilde{H}$ (a speed) and the two radii R_1, R_2. Rotational invariance rules out vectorial combinations; scale invariance on each circle restricts the dependence to their ratio. Thus the unique invariant with dimensions of s^{-1} is $\nu \propto \tilde{H}(R_1/R_2)$. Fixing the proportionality by the rest calibration leads to $\nu = k\,\tilde{H}$ with $k = R_1/R_2$.

Remark. Alternative bookkeeping that treats k as carrying inverse-length units and $\tilde{H}$ as a speed is equivalent after absorbing constants into κ; all physical relations (e.g., $E = \gamma m_0 c^2$) are unchanged.

3 Mass as a Structural Coefficient

We define a structural (volumetric) coefficient κ and *rest mass*

$$\kappa(k) = \kappa_*\left(\frac{k}{k_*}\right)^3, \qquad m_0(k) := \kappa(k)\,c, \qquad E_0(k) := \kappa(k)\,c^3 = m_0(k)c^2. \tag{3}$$

The cubic dependence reflects a volumetric Jacobian of the internal phase normalization. Small variations obey

$$\frac{\Delta m_0}{m_0} = \frac{\Delta E_0}{E_0} = 3\,\frac{\Delta k}{k}. \tag{4}$$

A *simple* (structureless) flow (photon) has $\tilde{S} = 0$, hence $m_0 = 0$, while $E \propto c^3$ via its own scale factor κ_γ.

4 Relativistic Energy from Phase Geometry

With **??** and the gauge $c \equiv \dot{H}$, pure boosts are rotations in the $(\tilde{S}, \tilde{L})$ plane that leave $\tilde{H}$ and k invariant. Calibrating energy by rest we obtain

$$E(\theta, k) = \frac{E_0(k)}{\cos\theta} = \gamma\, m_0(k)c^2, \qquad p = \frac{E}{c}\sin\theta = \gamma m_0(k)v, \quad v = c\sin\theta. \tag{5}$$

Immediately,

$$E^2 = m_0^2 c^4 + p^2 c^2, \tag{6}$$

with the usual low–velocity expansion $E = m_0 c^2 + \frac{1}{2} m_0 v^2 + O(v^4/c^2)$.

Anisotropic inertia (geometry)

For a boost along x the transverse flows remain unchanged ($\tilde{L}_y, \tilde{L}_z$ invariant), yielding the geometric form of longitudinal and transverse inertial responses:

$$m_\parallel = \frac{\mathrm{d}p_x}{\mathrm{d}v_x} = \gamma^3 m_0, \qquad m_\perp = \frac{\mathrm{d}p_y}{\mathrm{d}v_y} = \gamma m_0. \tag{7}$$

5 Justification of Volume–Normalized Mass

5.1 Composites and Jensen inequality

For a composite where k varies internally,

$$m_0 \propto \langle k^3 \rangle \geq (\langle k \rangle)^3, \tag{8}$$

so inhomogeneities (internal stresses/pressures) increase m_0 at fixed average normalization.

5.2 Empirical anchors

- **Mass defect:** negative binding lowers k and m_0, consistent with nuclear data.
- **Heat/fields/rotation:** added internal energy raises m_0 by $\Delta E/c^2$, i.e. $\Delta k/k = \frac{1}{3}\,\Delta E/E_0$.
- **Gravitational redshift of clocks:** $\Delta\nu/\nu \simeq \Delta\Phi/c^2$ implies $\Delta k/k \simeq \Delta\Phi/c^2$ for the normalization factor.
- **Stress–energy link:** isotropic radiation with $p = \rho/3$ contributes via $(\rho + 3p)$, mirroring the "cubic" internal degrees of freedom.

6 Generalized Energy in Phase–Space

At the kinematic level a convenient "mixed" representation is

$$E = \gamma\, \kappa\, \dot{H}^2\, \tilde{H}, \qquad (\text{with } c \equiv \dot{H}), \tag{9}$$

which collapses to $E = \gamma\, \kappa c^3$ under dynamic renormalization $\tilde{H} \to \dot{H}$. For a photon (simple flow) in vacuum: $m_0 = 0$, $E = \kappa_\gamma c^3$, $p = E/c$.

6.1 Quadratic invariants and the sixth–power law

Let $e := \kappa\,\dot{H}^3$ denote the local intensive energy scale of a single flow. Any Euclidean quadratic invariant built from a field with this scaling (e.g., self–energy bilinears, L^2 norms in the phase domain, quadratic action densities) takes the form

$$\mathcal{I}_2 \;=\; \int e^2\,\mathrm{d}V_\chi \;\propto\; \int \kappa^2\,\dot{H}^6\,\mathrm{d}V_\chi. \tag{10}$$

Thus a quadratic invariant maps the cubic phase–speed scaling into a sixth–power law. More generally, m–linear invariants scale as $\dot{H}^{3m}$.

Equivalently, in the rest–normalization $\kappa \propto k^3$ one has $e_0 \propto k^3$ and any quadratic invariant in the varying normalization k scales as k^6. This is the precise sense in which a Euclidean quadratic norm preserves an invariant built from a cubic structural coefficient.

6.2 Energy as a phase–space invariant

Proposition (phase–space energy). Define the *phase–space energy density* by

$$e(\chi) := \kappa(\chi)\,\dot{H}^3, \qquad (c \equiv \dot{H}\ extconst). \tag{11}$$

Boost invariance. Pure boosts are rotations in $(\tilde{S}, \tilde{L})$ that leave $\tilde{H}$, $\dot{H}$ and κ unchanged; hence e is invariant. The integral

$$E_\chi := \int_{\Sigma_\chi} e\,\mathrm{d}V_\chi \equiv m_0 c^2 \tag{12}$$

— the energy measured in the intrinsic phase frame — is a scalar independent of the state of motion. The observed (laboratory) energy and momentum are projections

$$E = \gamma E_\chi, \qquad p = \frac{E}{c}\sin\theta = \gamma m_0 v, \tag{13}$$

so that

$$E^2 - (pc)^2 = E_\chi^2 = (m_0 c^2)^2, \tag{14}$$

which makes the usual SR invariant explicit as the square of the phase energy.

Reparameterization invariance. Under a local reparametrization $\chi \mapsto \chi'(\chi)$ with Jacobian $J = \mathrm{d}\chi'/\mathrm{d}\chi$, the structural density transforms as a 3–density $\kappa' = \kappa/J^3$ while $\mathrm{d}V_{\chi'} = J^3 \mathrm{d}V_\chi$, so that $e\,\mathrm{d}V_\chi$ and E_χ are invariant.

Dynamics vs kinematics. Changes in internal structure (massogenesis) modify κ and thus e physically; the invariance statements above refer to kinematic transformations (boosts and phase reparametrizations), not to dynamics that pump energy into or out of the system.

6.3 Continuity and Noether–like view (sketch)

Treat $\kappa(k)$ as a density on internal phase: local conservation takes the form

$$\partial_\tau(\kappa c^3) + \nabla_\chi \cdot (\kappa c^2\,\boldsymbol{J}) = 0, \tag{15}$$

where $\boldsymbol{J}$ is a phase–space current; energy emerges as the charge of τ–translations.

7 Verification and Predictions

1. High–Q cavity: trapped field energy and pressure (T^{ii}) increase weight by (E+pressure term)$/c^2$.
2. Flywheel test: compare mass at rest vs spinning, including elastic stress contribution; prediction from **????**.
3. Nonuniform heating: at fixed ΔE, inhomogeneous $k(\boldsymbol{x})$ gives slightly larger $\langle k^3 \rangle$ than uniform heating.

8 Discussion and Outlook

We summarized how relativistic energy is recovered from a phase–geometric decomposition with mass as a volume–normalized structural coefficient. Open directions include: a full Lagrangian on phase–space, coupling to curvature (mapping to $T^{\mu\nu}$ in GR), and quantum extensions where k becomes an operator linked to cyclic spectra.

Acknowledgments ———

A Dimensional Analysis and Units

With $[E] = \mathrm{J}$ and $[\dot{H}] = \mathrm{m\,s^{-1}}$, one has $[\kappa] = [E]/[\dot{H}]^3 = \mathrm{J\,s^3\,m^{-3}}$. Using $m_0 = \kappa c$ yields $[\kappa] = \mathrm{kg\,s\,m^{-1}}$.

Cyclic-time normalization. In $\nu = k\,\tilde{H}$, if k is taken dimensionless (e.g., $k = R_1/R_2$), we treat $\tilde{H}$ here as an *effective frequency scale* inherited from the normalization map; equivalently, if $\tilde{H}$ is regarded as a speed, then k carries units of inverse length so that ν has units of $\mathrm{s^{-1}}$. Both conventions are equivalent after absorbing constants into κ and do not affect $E = \gamma m_0 c^2$.

B Derivation details for ??

Using $E_0 = m_0 c^2$ and the rotation in $(\tilde{S}, \tilde{L})$ with invariant $\tilde{H}$, the energy scales as $1/\cos\theta = \gamma$, while $p = (E/c)\sin\theta$; eliminating θ gives **??**.

C Dictionary to standard SR variables

$\tilde{S} \leftrightarrow$ internal (proper–time) projection; $\tilde{L} \leftrightarrow$ spatial projection; θ is the boost rapidity angle via $\tan\theta = v/\sqrt{c^2 - v^2}$; k encodes internal normalization of the cyclic time.