Unimetry: A Quaternionic Gravito–Electromagnetic Formulation

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Abstract

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2.1 Context and motivation

Unimetry is a proposed phase–geometric framework in which physical systems are described in terms of stationary flows in an underlying Euclidean proto-space $\mathcal{E}$. Rather than postulating space–time as a primary arena, unimetry treats the observed space–time geometry, relativistic kinematics, and field interactions as effective structures derived from the orientation and coupling of such flows. A dimensionless scalar phase potential $\Phi : \mathcal{E} \to \mathbb{R}$ and its gradient define a normalized flow direction; the familiar Minkowski metric and Lorentzian phenomena then appear as particular projections of this underlying flow geometry.

In this sense, special relativity (SR) is not the endpoint, but the first benchmark for the framework: unimetry aims at a unified phase-based description of kinematics, gravity and gauge interactions, with SR recovered as a specific limit of the general construction. The present paper develops one important sector of this programme, namely a quaternionic gravito–electromagnetic (GEM) formulation built on top of the unimetrical flow picture.

At the classical level, gravito–electromagnetic analogies are well known: in the weak-field, slow-motion limit of general relativity, the Einstein equations can be cast into a Maxwell-like form, and moving masses generate a "gravitomagnetic" field. Quaternions and related algebras have also long been used to encode rotations and the Maxwell equations in a compact way. What unimetry adds to this landscape is a concrete phase-geometric interpretation: a single quaternionic object encodes both the temporal and spatial parts of a flow, and bilinear forms of such objects naturally split into scalar, symmetric vector, and axial (vorticity-like) channels. This suggests that gravity and electromagnetism might be viewed as different faces of the same bilinear structure acting on suitably dressed flow quaternions.

Our goal here is to make this statement precise. We construct a quaternionic GEM formalism in which gravitational and electromagnetic interactions originate from the *same* bilinear machinery applied to metrically dressed "body quaternions". In particular, we show that Newton and Coulomb potentials arise as two branches of a single scalar form, while the magnetic and gravitomagnetic sectors are associated with a vortical bilinear form whose physical calibration reveals a natural role for the constants ε_0, μ_0, G and c. The resulting description remains Euclidean at the level of the proto-space, yet reproduces relativistic kinematics and GEM fields in the observable three-space.

2.2 Relation to the base unimetry paper

This work is a direct sequel to the base unimetry paper, *"Unimetry: A Phase-Space Reformulation of Special Relativity"* (henceforth "Paper I"). Paper I develops the core phase/flow structure: the phase potential Φ, the phase 1-form $\alpha = d\Phi$, the normalized flow $\widehat{\boldsymbol{\chi}}$, and the calibration $\boldsymbol{\chi} = c\,\widehat{\boldsymbol{\chi}}$, together with the derivation of the Minkowski interval and standard SR effects from a Euclidean proto-space. It also introduces the unimetrical D-rotation, which encodes Lorentz boosts as Euclidean rotations in a suitable plane of $\mathcal{E}$.

From the unimetry viewpoint, however, these SR results are only the first consistency test of a broader phase-based paradigm. The present paper assumes familiarity with the conceptual setting of Paper I, but is written to be as self-contained as reasonably possible. We briefly recall the key definitions of the phase proto-space, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations. All constructions that are essential for the GEM sector are reproduced or adapted here; more detailed discussions of SR and cosmological applications remain in Paper I and are only referenced when needed.

2.3 Main results

The main technical contributions of this paper can be summarized as follows.

- We introduce *metrically dressed body quaternions* $\widetilde{\mathsf{q}}_i = L_{E,i}\,\hat{h} + L_{G,i}\,\widehat{\boldsymbol{n}}_i$, whose components have the dimension of length. The "electric" and "gravitational" lengths

$$L_{E,i} = \sqrt{\frac{G}{4\pi\varepsilon_0 c^4}}\,Q_i, \qquad L_{G,i} = \frac{G}{c^2}\,m_i$$

encode the charge Q_i and mass m_i of the body in a unified geometric fashion. The unit vector $\widehat{\boldsymbol{n}}_i$ represents the spatial flow direction associated with the body.

- We show that the scalar bilinear form

$$A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = L_{E,1}L_{E,2} - \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

  (with $\boldsymbol{S}_i = L_{G,i}\widehat{\boldsymbol{n}}_i$) yields, after a single global calibration by c^4/G and a geometric $1/r$ factor, the combined Newton–Coulomb potential:

$$U(r) = \frac{c^4}{G}\,\frac{A}{r} = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_1Q_2}{r} - G\,\frac{m_1m_2}{r}.$$

  Thus gravity and electrostatics arise as two channels of a single invariant scalar form.

- We identify two vector-valued bilinear forms, $\boldsymbol{B}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$ and $\boldsymbol{C}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$, corresponding to the symmetric and axial parts of the quaternion product. In the dressed setting these naturally describe current-like and vortical channels. In particular, the vortical form $\boldsymbol{C}$ reproduces the geometry of magnetic and gravitomagnetic fields generated by moving charges and masses.

- We construct a quaternionic GEM field $\mathcal{F}_{\mathrm{GEM}}(\boldsymbol{x})$ over the observable three-space by combining dressed source quaternions with purely imaginary distance quaternions. Its scalar channel reproduces the gravitational and electrostatic potentials, while its vortical channel yields a physically natural "phase-vortical" field C_{phys} with the same dimension as $\boldsymbol{E}$. The standard magnetic field $\boldsymbol{B}$ in SI units then appears as

$$\boldsymbol{B} = \frac{1}{c}\,C_{\mathrm{phys}},$$

  so that the familiar μ_0 and ε_0 can be interpreted in terms of linear and areal stiffness of the vacuum, combined into an effective volumetric stiffness proportional to $1/(\varepsilon_0 c^3)$.

- We analyze the action of unimetrical D-rotations and ordinary spatial rotors on dressed quaternions. Pure spatial rotations act in the usual way on the vector channels and leave the scalar form A invariant, while D-rotations mix the scalar channel and the longitudinal component of $\boldsymbol{B}$ in a two-dimensional "energy–current" plane. This provides a quaternionic encoding of relativistic kinematics in the GEM setting, with Lorentz-consistent transformation properties of the fields.

- Finally, we outline a Hamiltonian and Lagrangian formulation of the quaternionic GEM theory in terms of the self-form A and the norm-squares of $\boldsymbol{B}$ and $\boldsymbol{C}$, and discuss how the standard Maxwell Lagrangian and linearized GEM equations arise in appropriate limits.

2.4 Structure of the paper

The paper is organized as follows. In Section 3 we recall the basic quaternion algebra and introduce the bilinear forms A, $\boldsymbol{B}$, and $\boldsymbol{C}$ that arise from the quaternion product, together with their matrix representation and geometric interpretation. Section 4 provides a brief overview of the unimetrical phase proto-space, the phase potential, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations.

In **??** we introduce metrically dressed body quaternions and define the electric and gravitational lengths L_E and L_G. **??** shows how the scalar form A for dressed quaternions reproduces the static Newton and Coulomb potentials. In **??** we construct a quaternionic GEM field over the observable three-space and identify the scalar and vortical channels with gravitational, electric, and magnetic sectors.

?? analyzes the action of spatial rotors and D-rotors on dressed quaternions and on the GEM field, clarifying the relativistic transformation properties of the scalar, current-like, and vortical channels. **??** is devoted to the calibration of $\boldsymbol{E}$ and $\boldsymbol{B}$, to the definition of the phase-vortical field C_{phys}, and to the interpretation of ε_0, μ_0, and c in terms of vacuum stiffness.

In **??** we outline Hamiltonian and Lagrangian formulations of quaternionic GEM, and in **??** we compare the resulting equations with the standard Maxwell and linearized GEM formalisms. Finally, **??** discusses limitations and open questions, and sketches possible extensions towards non-Abelian interactions and cosmological applications.

3 Quaternion algebra and bilinear forms

3.1 Basic notation and conventions

We denote by $\mathbb{H}$ the real quaternion algebra, viewed as a four-dimensional real vector space

$$\mathbb{H} \simeq \mathbb{R}^4(\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}),$$

where $\hat{h}$ is the distinguished scalar basis element and $\hat{\imath}, \hat{\jmath}, \hat{k}$ are purely imaginary basis elements. A general quaternion is written as

$$\mathbf{q} = x^\mu e_\mu = x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k},$$

with real components $x^\mu \in \mathbb{R}$ and basis $e_0 := \hat{h}$, $e_1 := \hat{\imath}$, $e_2 := \hat{\jmath}$, $e_3 := \hat{k}$.

The imaginary basis satisfies the usual quaternion relations

$$\hat{\imath}^2 = \hat{\jmath}^2 = \hat{k}^2 = -\hat{h}, \qquad \hat{\imath}\hat{\jmath} = \hat{k}, \quad \hat{\jmath}\hat{k} = \hat{\imath}, \quad \hat{k}\hat{\imath} = \hat{\jmath},$$

with antisymmetry under exchange of factors. We identify $\operatorname{Im}\mathbb{H} \simeq \mathbb{R}^3$ with its Euclidean inner product $\boldsymbol{x} \cdot \boldsymbol{y}$ and cross product $\boldsymbol{x} \times \boldsymbol{y}$, so that

$$\boldsymbol{x} = x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k} \quad \longleftrightarrow \quad (x^1, x^2, x^3) \in \mathbb{R}^3.$$

Quaternionic conjugation is defined by

$$\overline{\mathbf{q}} := x^0\hat{h} - x^1\hat{\imath} - x^2\hat{\jmath} - x^3\hat{k},$$

and the norm is $\|\mathbf{q}\|^2 = \mathbf{q}\,\overline{\mathbf{q}} = \overline{\mathbf{q}}\,\mathbf{q} = (x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$.

When convenient we will write a quaternion as $\mathbf{q} = (T, \boldsymbol{S})$ with

$$T := x^0, \qquad \boldsymbol{S} := x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k},$$

emphasizing the split into scalar and vector parts.

Universal imaginary unit. Besides the fixed imaginary basis $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$ it will be convenient to introduce a "universal" unit imaginary quaternion $\hat{\boldsymbol{\ell}}$, defined abstractly by

$$\hat{\boldsymbol{\ell}}^2 = -\hat{h}, \qquad \|\hat{\boldsymbol{\ell}}\| = 1. \tag{1}$$

Geometrically, $\hat{\boldsymbol{\ell}}$ should be understood as a *joker* direction: in any concrete configuration it is identified with the unit vector along the relevant interaction axis (for instance, the radial direction between two approximately isotropic Newtonian bodies). Algebraically, $\hat{\boldsymbol{\ell}}$ behaves as any other unit imaginary quaternion, and all scalar invariants such as $A(q, q)$ remain well defined when the spatial part of q is restricted to the one-dimensional subspace $\mathbb{R}\hat{\boldsymbol{\ell}}$.

When describing isotropic Newtonian bodies we will often use quaternions of the form

$$q = T\,\hat{h} + S\,\hat{\boldsymbol{\ell}}, \tag{2}$$

with $T, S \in \mathbb{R}$, so that all interaction channels reduce to the radial line spanned by $\hat{\boldsymbol{\ell}}$. This eliminates spurious transverse contributions in the vortical form $\boldsymbol{C}$ for purely radial gravito–electric configurations.

3.2 Quaternion product and decomposition into A, B, C forms

Let

$$\mathbf{q}_1 = x^\mu e_\mu = x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k},$$
$$\mathbf{q}_2 = y^\nu e_\nu = y^0\hat{h} + y^1\hat{\imath} + y^2\hat{\jmath} + y^3\hat{k},$$

with $\mathbf{q}_1, \mathbf{q}_2 \in \mathbb{H}$. Their quaternion product can be expanded term by term as

$$\begin{aligned}\mathbf{q}_1 \circ \mathbf{q}_2 &= (x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k})(y^0\hat{h} + y^1\hat{\imath} + y^2\hat{\jmath} + y^3\hat{k}) \\ &= (x^0y^0\hat{h}^2 + x^1y^1\hat{\imath}^2 + x^2y^2\hat{\jmath}^2 + x^3y^3\hat{k}^2) \\ &\quad + (x^0y^1\hat{h}\hat{\imath} + x^0y^2\hat{h}\hat{\jmath} + x^0y^3\hat{h}\hat{k} + x^1y^0\hat{\imath}\hat{h} + x^2y^0\hat{\jmath}\hat{h} + x^3y^0\hat{k}\hat{h}) \\ &\quad + (x^1y^2\hat{\imath}\hat{\jmath} + x^1y^3\hat{\imath}\hat{k} + x^2y^1\hat{\jmath}\hat{\imath} + x^2y^3\hat{\jmath}\hat{k} + x^3y^1\hat{k}\hat{\imath} + x^3y^2\hat{k}\hat{\jmath}).\end{aligned} \tag{3}$$

Using the multiplication rules, this can be organised into three bilinear contributions:

$$\mathbf{q}_1 \circ \mathbf{q}_2 = (\mathbf{q}_1 * \mathbf{q}_2) + (\mathbf{q}_1 \diamond \mathbf{q}_2) + (\mathbf{q}_1 \times \mathbf{q}_2), \tag{4}$$

where:

- $\mathbf{q}_1 * \mathbf{q}_2$ collects the purely scalar terms,
- $\mathbf{q}_1 \diamond \mathbf{q}_2$ collects the mixed scalar–vector terms,
- $\mathbf{q}_1 \times \mathbf{q}_2$ collects the purely vector–vector terms.

Explicitly, one finds the familiar invariant decomposition

$$\begin{aligned}\mathbf{q}_1 \circ \mathbf{q}_2 = &\left(x^0y^0 - x^1y^1 - x^2y^2 - x^3y^3\right)\hat{h} \\ &+ (x^0\boldsymbol{y} + y^0\boldsymbol{x}) + (\boldsymbol{x} \times \boldsymbol{y}),\end{aligned} \tag{5}$$

where

$$\boldsymbol{x} := x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k}, \qquad \boldsymbol{y} := y^1\hat{\imath} + y^2\hat{\jmath} + y^3\hat{k}.$$

This suggests three natural bilinear maps:

$$A(\mathbf{q}_1, \mathbf{q}_2) := x^0y^0 - x^1y^1 - x^2y^2 - x^3y^3, \tag{6}$$
$$\boldsymbol{B}(\mathbf{q}_1, \mathbf{q}_2) := x^0\boldsymbol{y} + y^0\boldsymbol{x}, \tag{7}$$
$$\boldsymbol{C}(\mathbf{q}_1, \mathbf{q}_2) := \boldsymbol{x} \times \boldsymbol{y}. \tag{8}$$

In terms of these,

$$\mathbf{q}_1 \circ \mathbf{q}_2 = (A(\mathbf{q}_1, \mathbf{q}_2))\,\hat{h} + \boldsymbol{B}(\mathbf{q}_1, \mathbf{q}_2) + \boldsymbol{C}(\mathbf{q}_1, \mathbf{q}_2). \tag{9}$$

It is often convenient to view (5) in a tensor-like form. We can write

$$\mathbf{q}_1 \circ \mathbf{q}_2 = \sum_{\mu,\nu=0}^{3} \left(A_{\mu\nu}\,\hat{h} + B_{\mu\nu} + C_{\mu\nu}\right) x^\mu y^\nu,$$

with three 4×4 coefficient matrices:

$$A_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{10}$$

$$B_{\mu\nu} = \begin{pmatrix} 0 & \hat{\imath} & \hat{\jmath} & \hat{k} \\ \hat{\imath} & 0 & 0 & 0 \\ \hat{\jmath} & 0 & 0 & 0 \\ \hat{k} & 0 & 0 & 0 \end{pmatrix}, \tag{11}$$

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{k} & -\hat{\jmath} \\ 0 & -\hat{k} & 0 & \hat{\imath} \\ 0 & \hat{\jmath} & -\hat{\imath} & 0 \end{pmatrix}. \tag{12}$$

Here $A_{\mu\nu}$ is the usual Minkowski-like bilinear form $\mathrm{diag}(1, -1, -1, -1)$ acting on the coordinate components, while $B_{\mu\nu}$ and $C_{\mu\nu}$ collect the symmetric and antisymmetric vector-valued pieces of the product.

3.3 Geometric interpretation and tensor structure of A, B, C

The decomposition (9) and the matrices (10)–(12) make explicit that the quaternion product can be regarded as the contraction of a rank-$(0, 2)$ object with two four-vectors:

$$\mathbf{q}_1 \circ \mathbf{q}_2 = (A + B + C)_{\mu\nu}\, x^\mu y^\nu,$$

with three structurally distinct blocks:

- $A_{\mu\nu}$ is a symmetric scalar bilinear form of signature $(+, -, -, -)$. In the unimetrical context it will play the role of an energy-like invariant and will generate both Newtonian and Coulomb potentials once we pass to dressed body quaternions.
- $B_{\mu\nu}$ is symmetric and vector-valued; it couples the scalar component to the spatial components. It will later be interpreted as a current-like channel, encoding the coupling between temporal and spatial parts of dressed flow quaternions.
- $C_{\mu\nu}$ is antisymmetric and vector-valued; it encodes the cross product $\boldsymbol{x} \times \boldsymbol{y}$ of the spatial parts and thus represents a vorticity (axial) channel. This will underlie the magnetic and gravitomagnetic sectors of the GEM field.

In summary, the elementary quaternion product already contains, in a rigid algebraic way, the three channels that we will later reinterpret as

(i) an energy-like scalar invariant A,

(ii) a current-like symmetric vector channel $\boldsymbol{B}$,

(iii) a vortical (axial) vector channel $\boldsymbol{C}$.

In the next section we recall how unimetry associates physical flows and an effective space–time structure to quaternions, so that these three forms can be given a gravito–electromagnetic meaning.

4 Phase proto-space and flow: brief unimetry overview

4.1 Proto-space, phase potential and phase 1-form

In unimetry the basic kinematical arena is a Euclidean (or, more generally, Hilbert) proto-space $(\mathcal{E}, \langle\cdot,\cdot\rangle)$. Points of $\mathcal{E}$ will be denoted by X, and the inner product $\langle\cdot,\cdot\rangle$ is used to identify tangent and cotangent spaces via the Riesz isomorphism. For the purposes of this paper one may think of $\mathcal{E}$ as a finite- or countable-dimensional real Hilbert space.

The fundamental scalar field of unimetry is a dimensionless *phase potential*

$$\Phi : \mathcal{E} \to \mathbb{R}.$$

From Φ we obtain the *phase 1-form*

$$\alpha := d\Phi,$$

which is a smooth 1-form on $\mathcal{E}$. At each point $X \in \mathcal{E}$, the value α_X is a linear functional on the tangent space $T_X\mathcal{E}$:

$$\alpha_X : T_X\mathcal{E} \to \mathbb{R}, \qquad \alpha_X(V) = d\Phi_X(V).$$

Using the inner product, we define the gradient $\nabla\Phi(X) \in T_X\mathcal{E}$ by the standard relation

$$\alpha_X(V) = d\Phi_X(V) = \langle \nabla\Phi(X), V\rangle, \qquad \forall\, V \in T_X\mathcal{E}.$$

Thus α and $\nabla\Phi$ carry the same information; the former is covariant, the latter contravariant.

Physically, the phase potential Φ encodes the global phase structure of the underlying flow, while the phase 1-form α and the gradient $\nabla\Phi$ encode local directions in which the phase changes most rapidly. The key idea of unimetry is to use this structure to define a canonical flow through $\mathcal{E}$.

4.2 Flow vector and normalization

Whenever $\nabla\Phi(X) \neq 0$, we define the *normalized flow direction* at X by

$$\widehat{\boldsymbol{\chi}}(X) := \frac{\nabla\Phi(X)}{\|\nabla\Phi(X)\|}, \qquad \|\nabla\Phi(X)\| := \sqrt{\langle \nabla\Phi(X), \nabla\Phi(X)\rangle}. \tag{13}$$

Thus $\widehat{\boldsymbol{\chi}}(X)$ is a unit vector in $T_X\mathcal{E}$ pointing along the steepest phase ascent. We then introduce the *physical flow vector* by a global calibration

$$\boldsymbol{\chi}(X) := c\,\widehat{\boldsymbol{\chi}}(X), \qquad \|\boldsymbol{\chi}(X)\| \equiv c, \tag{14}$$

where c is the speed of light. In other words, in unimetry the physical flow is a unit-speed curve in $\mathcal{E}$ with respect to the fixed scale c.

A flow line (or *stream*) is then a curve $\gamma : \lambda \mapsto X(\lambda) \in \mathcal{E}$ whose tangent vector is everywhere aligned with the physical flow:

$$\dot{X}(\lambda) := \frac{dX}{d\lambda} = \boldsymbol{\chi}(X(\lambda)), \qquad \|\dot{X}(\lambda)\| = c. \tag{15}$$

The parameter λ is a proto-space parameter, not yet identified with any observed time. The geometric content of (15) is simply that physical objects are represented by flows of constant Euclidean speed c in the proto-space.

4.3 Intrinsic angle, proper time and correspondence with SR

In unimetry a macroscopic body B is represented not by a single flow line, but by an ensemble of streamlets with weights w_a and tilt angles Θ_a relative to the body's self–time fibre.[1] On this ensemble one defines the temporal second moment and the spatial shape tensor as

$$T_B := \sum_a w_a \cos^2 \Theta_a, \qquad \mathbf{C}_B := \sum_a w_a \sin^2 \Theta_a \, \mathbf{u}_a \otimes \mathbf{u}_a, \tag{16}$$

where $0 < T_B \leq 1$, $\mathbf{C}_B$ is a symmetric positive semidefinite tensor on the body's three-surface, and $\mathbf{u}_a$ are unit spatial directions of the streamlets' projections. Operationally, T_B captures the aggregate fraction of flow carried in the orthogonal (self–time) fibre, while $\mathbf{C}_B$ encodes the anisotropic distribution of spatial projections across the body.

From these second moments one can define an *intrinsic angle* $\zeta \in [0, \frac{\pi}{2}]$ as an effective statistical parameter of the ensemble. Introducing

$$C := \sum_a w_a \cos 2\Theta_a, \qquad S := \sum_a w_a \sin 2\Theta_a,$$

there exists a unique ζ such that

$$(\cos 2\zeta,\ \sin 2\zeta) = (C, S) \quad \Longleftrightarrow \quad T_B = \tfrac{1}{2}(1 + C) = \cos^2\zeta, \qquad \operatorname{tr} \mathbf{C}_B = \tfrac{1}{2}(1 - C) = \sin^2\zeta. \tag{17}$$

We call ζ the *intrinsic angle* of the body. It aggregates the second–moment information $(T_B, \mathbf{C}_B)$ into a single scalar and should be thought of as a *statistical* internal parameter: it is *not* a geometric direction and is not attached to any particular flow line.

The intrinsic angle controls the rate at which the body's own proper time τ_B accumulates with respect to the phase parameter χ used to parametrize the flow in proto-space. In the calibrated gauge $\|\boldsymbol{\chi}\| = c$, one has

$$d\tau_B = \cos\zeta \, d\chi, \tag{18}$$

so that the temporal second moment T_B appears as $T_B = \cos^2\zeta = (d\tau_B/d\chi)^2$. The corresponding intrinsic metric of the body, as a quadratic form on $(d\chi, d\boldsymbol{\ell})$, reads

$$ds_B^2 := c^2 \, d\tau_B^2 - d\boldsymbol{\ell}^\top \mathbf{C}_B \, d\boldsymbol{\ell} = c^2 T_B \, d\chi^2 - d\boldsymbol{\ell}^\top \mathbf{C}_B \, d\boldsymbol{\ell}. \tag{19}$$

For an isotropic texture one has $\mathbf{C}_B = \frac{\sin^2\zeta}{3} \mathbf{I}_S$, and with the rest gauge $T_B \equiv 1$ this reduces to the familiar Minkowski form in the body's rest frame (up to the overall phase gauge $d\chi$).

In the full unimetrical construction the intrinsic angle ζ is combined with a kinematic angle ϑ (associated with the relative motion between bodies) and, when present, with a gravitational angle ϕ (associated with an external tilt field). The resulting time-rate factor factorises into intrinsic, kinematic, and gravitational contributions. For the purposes of the present GEM paper, we only need the following structural facts:

- The intrinsic angle ζ is a scalar *second-moment* parameter of a body, not a direction: it encodes how the flow budget is split between self–time and spatial channels in the ensemble of streamlets.
- The proper time τ_B along the body's worldline is related to the phase parameter χ by (18), and the body's intrinsic metric takes the Minkowski form (19) once the rest gauge is fixed.
- The relativistic kinematics of unimetry can therefore be formulated entirely in terms of phase flow and second-moment data, with the usual SR interval emerging as a derived object; we will reuse this structure when interpreting the scalar form A as an energy-like invariant for dressed quaternions.

[1] For the detailed construction see Paper I, §**??** there.

4.4 Notation table

For reference, we collect here the main unimetrical symbols used in the remainder of the paper. A more extensive table can be found in Paper I; the subset below is chosen to make the present text self-contained.

Symbol	Meaning
$\mathcal{E}$	Euclidean/Hilbert proto-space with inner product $\langle\cdot,\cdot\rangle$
$\Phi : \mathcal{E} \to \mathbb{R}$	dimensionless phase potential
$\alpha = d\Phi$	phase 1-form, $\alpha_X(V) = \langle\nabla\Phi(X), V\rangle$
$\nabla\Phi(X)$	gradient of Φ at X, defined via the inner product
$\widehat{\boldsymbol{\chi}}(X)$	normalized flow direction, $\widehat{\boldsymbol{\chi}} = \nabla\Phi/\|\nabla\Phi\|$
$\boldsymbol{\chi}(X)$	physical flow vector, $\boldsymbol{\chi} = c\,\widehat{\boldsymbol{\chi}}$, $\|\boldsymbol{\chi}\| = c$
$\gamma(\lambda)$	flow line in $\mathcal{E}$ with tangent $\dot{X} = \boldsymbol{\chi}$
$\widehat{\boldsymbol{u}}$	unit rest direction associated with an observer (local temporal axis)
$\widehat{\boldsymbol{n}}$	unit spatial direction orthogonal to $\widehat{\boldsymbol{u}}$
ζ	flow angle between $\boldsymbol{\chi}$ and $\widehat{\boldsymbol{u}}$, see (**??**)
δT	effective temporal increment for the observer, see (**??**)
$\delta\boldsymbol{x}$	effective spatial increment in the observer's rest space, see (**??**)
δs^2	effective interval, $\delta s^2 = c^2\delta T^2 - \|\delta\boldsymbol{x}\|^2$, see (**??**)

Table 1: Key unimetrical quantities used in the quaternionic GEM construction.

In the next section we introduce two calibrations of the flow — one kinematic and one energetic — which will allow us to interpret the scalar form A as an energy-like invariant and to define metrically dressed body quaternions suitable for the gravito–electromagnetic setting.

5 Flow calibrations and energy-like functionals

In the unimetrical picture, physical bodies are represented by flows of constant Euclidean speed c in the proto-space $\mathcal{E}$, while the phase potential Φ provides a dimensionless scalar structure along these flows. To connect this geometric description to observable kinematics and to energy-like quantities, we need to relate the flow parameter to time coordinates and to introduce a structural scale. This section introduces two complementary calibrations of the flow parameter and shows how they lead to a factorized expression for energy, which will later be recast in quaternionic terms.

5.1 Flow parameter and phase frequencies

Let $\gamma : \chi \mapsto X(\chi)$ be a flow line in $\mathcal{E}$. We take the flow parameter χ to have units of time,

$$[\chi] = \mathrm{s},$$

and we assume that along the worldline of a given body B the phase potential Φ depends smoothly on χ. We define the *phase frequency* per unit χ by

$$\omega_\chi(\chi) := \frac{d\Phi}{d\chi}, \qquad [\omega_\chi] = \mathrm{s}^{-1}. \tag{20}$$

In many situations it is convenient to consider a gauge in which ω_χ is constant along the worldline of a given body; in that case $\Phi(\chi) = \omega_\chi\,\chi$. The product $\omega_\chi\chi$ is always dimensionless.

The geometric flow vector $\boldsymbol{\chi}$ is normalized by $\|\boldsymbol{\chi}\| \equiv c$, cf. (14), so that the flow is a unit-speed (with respect to c) curve in proto-space. The different calibrations introduced below relate χ to observable time parameters and thus furnish a bridge between the proto-space flow and physical time measurements.

5.2 Cyclic origin of local time and proto-parameter speed

We now make explicit a key unimetrical assumption: local time arises from a cyclic action of the flow along a compactified temporal axis x_0. For simplicity we model the geometry of this axis by a circle of variable radius and show how the ratio between the proto-parameter χ and the proper time τ emerges as a frequency.

Consider two circles in the (χ, τ)-plane:

- a "phase circle" of radius R_1 in the χ-direction, representing one phase step $\Delta\chi = 2\pi R_1$;
- a "time circle" of radius R_2 in the τ-direction, along which the flow advances with a characteristic proto-space speed component $\widetilde{X}_0$ along the temporal axis.

One full turn along the time circle then corresponds to a proper-time increment

$$\Delta\tau = \frac{2\pi R_2}{\widetilde{X}_0}. \tag{21}$$

The frequency of phase ticks with respect to proper time is

$$\nu := \frac{\Delta\chi}{\Delta\tau} = \frac{2\pi R_1}{2\pi R_2/\widetilde{X}_0} = \underbrace{\frac{R_1}{R_2}}_{=:k} \widetilde{X}_0. \tag{22}$$

In the high-frequency limit $R_1 \to 0$ the discrete ratio becomes a derivative and we obtain

$$\nu \xrightarrow{R_1\to 0} \frac{d\chi}{d\tau} =: \dot{\chi} \quad \Rightarrow \quad \boxed{\dot{\chi} = k\,\widetilde{X}_0}, \tag{23}$$

with dimensionless rate $\dot{\chi}$ and dimensionless structural ratio $k = R_1/R_2$. This is the fundamental relation between the accumulation of the proto-parameter χ and the proper time τ: local time is the frequency of phase ticks generated by the cyclic flow along the compactified temporal axis.

The same geometric picture yields a conversion factor between proto-space velocities (measured per unit χ) and velocities measured per unit proper time. Let $\widetilde{\mathcal{H}}$ be the Euclidean norm of the proto-space flow per unit χ, with components $\widetilde{X}_\mu$. Using the chain rule we have

$$\dot{\mathcal{H}} := \frac{d\mathcal{H}}{d\tau} = \frac{d\mathcal{H}}{d\chi}\frac{d\chi}{d\tau} = \widetilde{\mathcal{H}}\,\dot{\chi}. \tag{24}$$

Imposing the unimetrical calibration in which the magnitude of the proto-space flow equals its temporal component,

$$\widetilde{\mathcal{H}} = \widetilde{X}_0, \tag{25}$$

and using (23) we obtain

$$\dot{\mathcal{H}} = k\,\widetilde{X}_0^2. \tag{26}$$

This quantity $\dot{\mathcal{H}}$ can be interpreted as the full flow magnitude in local time: it measures how fast the flow advances in proto-space per unit proper time.

Requiring that the full magnitude of the flow in local time be preserved under rotations of the proto-space velocity (i.e. under changes of the temporal projection $\widetilde{X}_0$) leads to the constraint

$$k = \widetilde{X}_0^{-2}, \tag{27}$$

so that $\dot{\mathcal{H}}$ becomes a universal constant. In physical units this constant is identified with the speed of light:

$$\dot{\mathcal{H}} = c \quad \Longleftrightarrow \quad \text{the speed of light is the invariant magnitude of the flow in local time.} \tag{28}$$

In this sense the constancy of c is reinterpreted as the constancy of the Euclidean flow speed in proto-space, while variations of its projection between temporal and spatial directions are captured by the kinematic angle ϑ.

The relation

$$d\chi = k\,\widetilde{X}_0\,d\tau \tag{29}$$

shows that the proto-parameter χ plays the role of a global invariant for transformations mixing temporal and spatial channels. Expressing the interval dS^2 in terms of $d\chi$ and the components $\widetilde{X}_\mu$ one recovers the Minkowski form

$$dS^2 = \dot{X}_0^2\,d\tau^2 - dx_1^2 - dx_2^2 - dx_3^2, \tag{30}$$

i.e. the "inverted" Euclidean norm of the flow once the temporal geometry is taken into account. For a body of fixed structural complexity (fixed k) the quantity $\widetilde{X}_0$ must compensate changes in the local rate of time, whereas for a fixed full magnitude $\dot{\mathcal{H}}$ the ratio k must adjust so that the cyclic construction of time remains compatible with the invariance of the interval.

5.3 Kinematic (phase) calibration

Let us choose an inertial laboratory frame with coordinate time t and three-position $\boldsymbol{x}$. In the *kinematic calibration* we identify the flow parameter χ with the lab time:

$$\chi = t, \qquad \frac{d\chi}{dt} = 1. \tag{31}$$

Along the worldline of a body B we may then write

$$\Phi(t) = \omega_\chi\,t,$$

with $\omega_\chi = d\Phi/dt$ in that frame.

The normalized flow direction can be decomposed with respect to the lab-frame temporal unit vector $\widehat{\boldsymbol{u}}$ and a spatial unit vector $\widehat{\boldsymbol{n}}$ in the rest space:

$$\widehat{\boldsymbol{\chi}} = \cos\vartheta\,\widehat{\boldsymbol{u}} + \sin\vartheta\,\widehat{\boldsymbol{n}}, \qquad \|\widehat{\boldsymbol{n}}\| = 1, \tag{32}$$

where $\vartheta \in [0, \pi/2]$ is a *kinematic angle*. For a body moving with constant lab three-velocity $\boldsymbol{v} = d\boldsymbol{x}/dt$ we impose

$$\sin\vartheta = \beta := \frac{\|\boldsymbol{v}\|}{c}, \qquad \cos\vartheta = \sqrt{1-\beta^2}. \tag{33}$$

This calibration ensures that the spatial projection of the flow has magnitude $\|\boldsymbol{v}\| = c\sin\vartheta$, while the temporal projection matches the usual factor $\sqrt{1-\beta^2}$ that appears in time dilation.

Using (18) specialized to the kinematic sector and (31), we obtain the standard relation between proper time τ_B and lab time:

$$\frac{d\tau_B}{dt} = \cos\vartheta = \sqrt{1-\beta^2},$$

with

$$\gamma(\vartheta) := \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\cos\vartheta} \tag{34}$$

the usual Lorentz factor. Thus the kinematic angle ϑ encodes the purely kinematic tilt of the flow relative to the chosen inertial frame.

5.4 Structural parameter and volumetric coefficient

Besides kinematic tilts, unimetry allows for an internal rescaling of the cyclic structure of the flow. A convenient way to visualize this is to consider a pair of circles in the (χ, τ)-plane:

- a "phase circle" of radius R_1 in the χ-direction, which measures the size of a single phase step $\Delta\chi$;
- a "time circle" of radius R_2 in the τ-direction, along which the flow executes a cyclic motion with some characteristic proper-time angular frequency ω_* (units s^{-1}).

One full turn on the phase circle corresponds to a phase increment $\Delta\chi = 2\pi R_1$. One full turn on the time circle corresponds to a proper-time increment

$$\Delta\tau = \frac{2\pi R_2}{\omega_*}, \tag{35}$$

since the circumference is $2\pi R_2$ and the "speed" along the time circle is set by ω_*.

In the high-frequency limit, when the phase step becomes small ($R_1 \to 0$) and many cycles are accumulated, the effective frequency of phase ticks with respect to proper time is

$$\nu := \frac{\Delta\chi}{\Delta\tau} = \frac{2\pi R_1}{2\pi R_2/\omega_*} = \underbrace{\frac{R_1}{R_2}}_{=:k}\,\omega_*. \tag{36}$$

Passing to the differential limit, this gives

$$\dot{\chi} := \frac{d\chi}{d\tau} = k\,\omega_*, \tag{37}$$

with a dimensionless structural parameter

$$k := \frac{R_1}{R_2}. \tag{38}$$

Equation (37) expresses the rate at which the flow parameter χ is accumulated per unit proper time τ as the product of a geometric ratio k and a characteristic internal proper-time frequency ω_*.

We model the cumulative effect of this structural rescaling on energy by a *volumetric structural coefficient* $\kappa(k)$. Since a rescaling of a characteristic length by a factor k changes a three-dimensional volume by k^3, we take

$$\kappa(k) = \kappa_* \left(\frac{k}{k_*}\right)^3, \tag{39}$$

where κ_* is a reference value at some fiducial structural state $k = k_*$. Dimensional analysis then dictates that κ has units

$$[\kappa] = \mathrm{kg\,s\,m^{-1}}, \tag{40}$$

so that the combination κc^3 has the dimension of energy:

$$[\kappa c^3] = \mathrm{kg\,m^2\,s^{-2}} = \mathrm{J}.$$

We therefore define the *structural rest energy* and associated rest mass by

$$E_0(k) := \kappa(k)\,c^3, \qquad m_0(k) := \frac{E_0(k)}{c^2} = \kappa(k)\,c. \tag{41}$$

For a fixed structural state k these reduce to constants $E_0 = \kappa(k)c^3$, $m_0 = \kappa(k)c$, while spatial variations of k would correspond to variations of the local rest energy and rest mass scale.

5.5 Proper-time calibration and energy factorization

In the *proper-time calibration* we parametrize the flow by the body's proper time τ_B. In the unimetrical framework the rate at which proper time accumulates with respect to the flow parameter χ is controlled by the body's intrinsic angle ζ_B, introduced in Section 4.3 as a characteristic of the second-moment structure of the streamlet ensemble:

$$d\tau_B = \cos\zeta_B \, d\chi, \qquad \frac{d\tau_B}{d\chi} = \cos\zeta_B. \tag{42}$$

Both χ and τ_B carry units of time, so $\cos\zeta_B$ is a dimensionless factor determined by the internal flow texture of the body.

A key assumption of unimetry in this paper is that, in the body's own rest frame and in the absence of dissipation or absorption/emission of radiation, the intrinsic angle ζ_B is *fixed*:

$$\zeta_B = \text{const} \quad \text{for a given body with fixed internal energy.} \tag{43}$$

In other words, purely kinematic changes of motion relative to an external observer do not alter ζ_B; they act "outside" the internal flow texture.

By contrast, the kinematic calibration of Section 5.3 relates proper time to the laboratory time t via the kinematic angle ϑ:

$$\frac{d\tau_B}{dt} = \cos\vartheta = \sqrt{1-\beta^2}, \qquad \gamma(\vartheta) = \frac{1}{\cos\vartheta}. \tag{44}$$

Here ϑ encodes the tilt of the macroscopic body flow relative to the lab frame and is meaningful only for an external observer. It is conceptually distinct from ζ_B: the intrinsic angle describes the internal streamlet ensemble in the body's own frame, whereas ϑ describes relative motion between frames.

Using the structural rest-energy scale

$$E_0(\zeta_B, k) := \kappa(k;\zeta_B)\, c^3, \qquad m_0(\zeta_B, k) := \kappa(k;\zeta_B)\, c, \tag{45}$$

we *define* the energy of the body in the state $(\vartheta; \zeta_B, k)$ as

$$E(\vartheta; \zeta_B, k) := \gamma(\vartheta)\, E_0(\zeta_B, k) = \frac{1}{\cos\vartheta}\, \kappa(k;\zeta_B)\, c^3, \tag{46}$$

or equivalently

$$E(\vartheta; \zeta_B, k) = \gamma(\vartheta)\, m_0(\zeta_B, k)\, c^2.$$

Thus the energy factorizes into a *purely kinematic* factor $\gamma(\vartheta)$, coming from the macroscopic tilt of the body's flow in the laboratory frame, and a *purely intrinsic-structural* factor $E_0(\zeta_B, k) = \kappa(k;\zeta_B)c^3$, which encodes the internal flow texture (via ζ_B) and the internal cyclic geometry (via k) in the body's own frame.

For small velocities $\beta \ll 1$, using $\cos\vartheta = \sqrt{1-\beta^2}$ and $\gamma(\vartheta) = 1/\cos\vartheta$ we recover the usual expansion

$$E(\vartheta; \zeta_B, k) = m_0(\zeta_B, k)\, c^2 + \tfrac{1}{2} m_0(\zeta_B, k)\, v^2 + O(\beta^4), \qquad v = c\sin\vartheta. \tag{47}$$

Hence the structurally generalized energy retains the standard non-relativistic limit, while allowing the rest energy scale to vary with the intrinsic flow texture (through ζ_B) and with the internal cyclic geometry (through k), without ever identifying ζ_B with the kinematic angle ϑ.

5.6 From flow vector to flow quaternion

So far the flow has been represented as a unit vector $\hat{\boldsymbol{\chi}} \in T_X\mathcal{E}$ with a split into temporal and spatial channels controlled by the kinematic angle ϑ, see (32). To connect this to the quaternion algebra of Section 3, we now introduce a simple representation map from flow directions to unit quaternions.

Fix a body B and a local observer comoving with B. At a point $X \in \mathcal{E}$ on the body's flow choose an orthonormal frame $\{\hat{\boldsymbol{u}}, \hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3\}$ in $T_X\mathcal{E}$, where $\hat{\boldsymbol{u}}$ is the observer's temporal direction and $\hat{\boldsymbol{e}}_i$ span the three-dimensional rest space. The normalized flow direction can then be written as

$$\hat{\boldsymbol{\chi}} = \cos\vartheta\, \hat{\boldsymbol{u}} + \sin\vartheta\, \hat{\boldsymbol{n}}, \qquad \hat{\boldsymbol{n}} := n^1\hat{\boldsymbol{e}}_1 + n^2\hat{\boldsymbol{e}}_2 + n^3\hat{\boldsymbol{e}}_3, \quad \|\hat{\boldsymbol{n}}\| = 1. \tag{48}$$

On the quaternion side we have the basis $\{\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}\}$ introduced in Section 3. We fix a local identification between the observer's frame and the quaternion basis by declaring

$$\hat{\boldsymbol{u}} \longleftrightarrow \hat{h}, \qquad \hat{\boldsymbol{e}}_i \longleftrightarrow \text{an orthonormal triple in } \{\hat{\imath}, \hat{\jmath}, \hat{k}\}. \tag{49}$$

In the simplest case one may align $\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3$ with $\hat{\imath}, \hat{\jmath}, \hat{k}$ respectively, so that the spatial unit vector $\hat{\boldsymbol{n}}$ is represented by a unit imaginary quaternion with the same components n^i.

Definition 5.1 (Flow quaternion)**.** Given a normalized flow direction $\hat{\boldsymbol{\chi}}$ with kinematic angle ϑ and spatial unit vector $\hat{\boldsymbol{n}}$ in the observer's rest space, the associated *flow quaternion* is the unit quaternion

$$\hat{\mathsf{q}}(\vartheta, \hat{\boldsymbol{n}}) := \cos\vartheta\, \hat{h} + \sin\vartheta\, \hat{\boldsymbol{n}}, \qquad \|\hat{\boldsymbol{n}}\| = 1. \tag{50}$$

By construction $\|\hat{\mathsf{q}}\| = 1$. The scalar part of $\hat{\mathsf{q}}$ encodes the temporal fraction of the flow, while the vector part encodes the spatial fraction in the chosen rest space. Different choices of the spatial orthonormal frame correspond to spatial rotations of the imaginary basis and do not affect scalar invariants such as $A(\hat{\mathsf{q}}, \hat{\mathsf{q}})$.

5.7 Scalar self-form $A(q, q)$ as kinematic factor

We can now relate the kinematic factor in (**??**) to the quaternionic self-form $A(q, q)$ introduced in Section 3. For brevity we suppress the explicit dependence on $\hat{\boldsymbol{n}}$ and write $\hat{\mathsf{q}}(\vartheta)$ when no confusion can arise:

$$\hat{\mathsf{q}}(\vartheta) = \cos\vartheta\, \hat{h} + \sin\vartheta\, \hat{\boldsymbol{n}}, \qquad \|\hat{\boldsymbol{n}}\| = 1.$$

The scalar self-form A applied to this unit quaternion gives

$$A(\hat{\mathsf{q}}(\vartheta), \hat{\mathsf{q}}(\vartheta)) = \cos^2\vartheta - \sin^2\vartheta = \cos 2\vartheta. \tag{51}$$

Solving for $\cos^2\vartheta$ we obtain

$$1 - \beta^2 = \cos^2\vartheta = \frac{1 + A(\hat{\mathsf{q}}, \hat{\mathsf{q}})}{2}, \tag{52}$$

and hence the Lorentz factor can be expressed as

$$\gamma(\vartheta) = \frac{1}{\sqrt{1-\beta^2}} = \sqrt{\frac{2}{1 + A(\hat{\mathsf{q}}, \hat{\mathsf{q}})}}. \tag{53}$$

Substituting (53) into the factorized energy (**??**), we find

$$E(\hat{\mathsf{q}}, k) = \gamma(\vartheta)\, E_0(k) = \sqrt{\frac{2}{1 + A(\hat{\mathsf{q}}, \hat{\mathsf{q}})}}\, \kappa(k)\, c^3. \tag{54}$$

Thus:

- the scalar self-form $A(\widehat{\mathsf{q}},\widehat{\mathsf{q}})$ is a *purely kinematic, dimensionless* invariant of the flow quaternion;
- the structural coefficient $\kappa(k)$ carries the absolute energy scale, via $E_0(k)=\kappa(k)c^3$;
- energy emerges as the product of a quaternionic kinematic factor (a function of A) and a structural scale.

In the gravito–electromagnetic construction below we will apply the same scalar form A not to unit flow quaternions, but to *metrically dressed* body quaternions $\widetilde{\mathsf{q}}_i$ that package mass and charge into effective lengths. For isotropic Newtonian bodies the spatial parts of these quaternions will be further restricted to multiples of the universal unit imaginary $\hat{\boldsymbol{\ell}}$ introduced in Section 3, so that all interaction channels reduce to a single radial direction. The vector forms $\boldsymbol{B}$ and $\boldsymbol{C}$ will then provide, respectively, the current-like and vortical channels of the gravito–electromagnetic field.

6 Metrically dressed body quaternions

6.1 Free unit imaginary vector for isotropic Newtonian bodies

In the metrically dressed setting we write the spatial part of a body quaternion as

$$\boldsymbol{S}_i = L_{G,i}\,\widehat{\boldsymbol{n}}_i,$$

where $\widehat{\boldsymbol{n}}_i$ is a unit spatial direction associated with body i. For charged bodies it is natural to interpret $\widehat{\boldsymbol{n}}_i$ as an intrinsic flow direction (e.g. an orientation of the underlying streamlet structure), which will in general produce non-trivial contributions in both the symmetric vector form $\boldsymbol{B}$ and the vortical form $\boldsymbol{C}$.

For purely Newtonian, isotropic mass distributions, however, we may and should distinguish between an intrinsic direction and the *interaction* direction. To reflect this, we introduce the notion of a *free unit imaginary vector* for the gravitational channel.

Definition 6.1 (Free unit imaginary vector)**.** A free unit imaginary vector is a unit spatial quaternion $\widehat{\boldsymbol{u}}\in\operatorname{Im}\mathbb{H}$ whose orientation is not fixed by the internal structure of the body, but is freely assigned at the level of the interaction. For an isotropic Newtonian body i we take the gravitational spatial part of its dressed quaternion to be

$$\boldsymbol{S}_i^{(G)}(\boldsymbol{x}) := L_{G,i}\,\widehat{\boldsymbol{u}}_i(\boldsymbol{x}),$$

where, for a field point $\boldsymbol{x}$, the free unit vector is chosen to be aligned with the radius vector from the body to that point,

$$\widehat{\boldsymbol{u}}_i(\boldsymbol{x}) := \widehat{\boldsymbol{r}}_i(\boldsymbol{x}) = \frac{\boldsymbol{x}-\boldsymbol{x}_i}{\|\boldsymbol{x}-\boldsymbol{x}_i\|}.$$

In other words, for an isotropic Newtonian source the gravitational channel of the dressed quaternion is always taken to be *radial* with respect to the field point, and carries no intrinsic "spin" information. This has an important structural consequence for the bilinear forms.

Consider two isotropic masses m_1, m_2 at positions $\boldsymbol{x}_1,\boldsymbol{x}_2$, and evaluate their gravitational spatial parts at a common field point $\boldsymbol{x}$. By construction,

$$\boldsymbol{S}_1^{(G)}(\boldsymbol{x}) \parallel \widehat{\boldsymbol{r}}_1(\boldsymbol{x}), \qquad \boldsymbol{S}_2^{(G)}(\boldsymbol{x}) \parallel \widehat{\boldsymbol{r}}_2(\boldsymbol{x}).$$

In the static two-body configuration the interaction is along the line joining the bodies, so that effectively

$$\boldsymbol{S}_1^{(G)} \parallel \boldsymbol{S}_2^{(G)},$$

and therefore their contribution to the vortical form

$$\boldsymbol{C}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = \boldsymbol{S}_1 \times \boldsymbol{S}_2$$

vanishes in the purely gravitational, isotropic limit:

$$\boldsymbol{C}^{(G)} = \boldsymbol{S}_1^{(G)} \times \boldsymbol{S}_2^{(G)} = \mathbf{0}.$$

Thus, by assigning the gravitational spatial part of an isotropic body to a free unit imaginary vector that is always chosen radial, we ensure that:

- the gravitational interaction of isotropic masses is purely scalar and radial, as in Newtonian gravity;
- there is *no* spurious contribution of the gravitational channel to the vortical form $\boldsymbol{C}$ in the static limit;
- all non-trivial vortical contributions in $\boldsymbol{C}$ are genuinely associated with anisotropy and/or motion (currents), i.e. with the EM and gravitomagnetic sectors rather than with static isotropic gravity.

In contrast, for charged bodies we will keep an intrinsic unit direction $\widehat{\boldsymbol{n}}_i$ in the electric channel, which can contribute to both the symmetric form $\boldsymbol{B}$ and the vortical form $\boldsymbol{C}$. This separation between a free gravitational direction and an intrinsic electromagnetic direction will be important when we analyze the GEM field and its magnetic and gravitomagnetic components in **????**.