Unimetry: A Phase-Space Reformulation of Special Relativity

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Abstract

We propose a compact reformulation of special relativity in which spacetime measures (time and length) are treated as phase velocities, defined as directional derivatives of a single underlying parameter, the flow $\boldsymbol{\chi} \in \mathbb{H}$. In this framework, the observable Minkowski interval emerges as a conserved quantity under a change of parameter from the phase coordinate χ to the observer's proper time τ. Familiar relativistic effects, such as time dilation, the Lorentz factor, the Doppler shift, and relativistic velocity composition, arise as elementary projections and rotations within a Euclidean phase plane. Hyperbolic features of Lorentz kinematics reappear after a reparametrization of time, yielding the classical relations without altering empirical content. We provide closed-form derivations for the longitudinal and transverse Doppler effects, prove a lemma equating the total flow speed to the conserved Minkowski norm, and outline connections to gauge phases, rapidity, and a cosmological time gauge. Composition of non-collinear boosts (quaternionic d-rotations in $\mathbb{H}$) yields a Wigner rotation; in the continuous limit, this gives Thomas precession. Both effects emerge as purely kinematical consequences of the quaternionic phase formalism. The approach is advantageous for applications – especially in modeling non-collinear acceleration of particle beams – where it replaces matrix diagonalizations with algebraic rotor compositions and improves numerical stability.

Keywords: special relativity; phase; rapidity; Doppler shift; Lorentz factor; Wigner rotation; Thomas precession; phase parametrization.

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The tilt mechanism: changing frames as angle subtraction on the phase circle

Setup (recall). In observer A's split, let

$$\widehat{\mathbf{F}} = \frac{\boldsymbol{\chi}}{\|\boldsymbol{\chi}\|} = \cos\vartheta_{|A}\,\mathbf{e}_t^A + \sin\vartheta_{|A}\,\mathbf{u}_{|A}, \quad \mathbf{u}_{|A} \in S^A,\ \|\widehat{\mathbf{F}}\| = 1.$$

Here $\vartheta_{|A}$ is the *observer–object angle*. It is *not* your global phase angle Θ (except in the co–moving case when the time directions coincide).

Tilt operation (co–planar case). A *frame tilt by angle* α in the 2–plane spanned by $\{\mathbf{e}_t^A, \mathbf{u}_{|A}\}$ replaces A's basis by

$$\mathbf{e}_t^{A'} = \cos\alpha\,\mathbf{e}_t^A + \sin\alpha\,\mathbf{u}_{|A}, \qquad \mathbf{u}_{|A'} = \cos\alpha\,\mathbf{u}_{|A} - \sin\alpha\,\mathbf{e}_t^A.$$

Decomposing $\widehat{\mathbf{F}}$ in the new basis gives

$$\cos\vartheta_{|A'} = \widehat{\mathbf{F}}\cdot\mathbf{e}_t^{A'} = \cos(\vartheta_{|A} - \alpha), \quad \sin\vartheta_{|A'} = \widehat{\mathbf{F}}\cdot\mathbf{u}_{|A'} = \sin(\vartheta_{|A} - \alpha).$$

Mechanism in one line:

$$\boxed{\vartheta_{|A'} = \vartheta_{|A} - \alpha} \qquad \text{(tilt = angle subtraction on the phase circle).}$$

Minkowski interval from the tilt mechanism. Let $dt_{A'}$ be A''s lab time, $d\mathbf{x}_{|A'} \in S^{A'}$ the measured spatial shift, and $d\tau$ the object's proper time. By definition in each frame,

$$\frac{d\tau}{dt_{A'}} = \cos\vartheta_{|A'}, \qquad \frac{\|d\mathbf{x}_{|A'}\|}{c\,dt_{A'}} = \sin\vartheta_{|A'}.$$

Using $\cos^2 + \sin^2 = 1$,

$$\boxed{c^2\,d\tau^2 = c^2\,dt_{A'}^2 - \|d\mathbf{x}_{|A'}\|^2}$$

holds automatically. Thus the *tilt* changes what the observer reads (ϑ, dt, $d\mathbf{x}$), yet the Minkowski form stays invariant because the object's $d\tau$ is the same and the pair $(\cos\vartheta, \sin\vartheta)$ simply rotates.

Velocity and γ update (dictionary). In any frame X,

$$\beta_{|X} = \sin\vartheta_{|X}, \qquad \gamma_{|X} = \sec\vartheta_{|X}, \qquad \frac{d\tau}{dt_X} = \cos\vartheta_{|X}.$$

Under a tilt by α (co–planar case),

$$\beta_{|A'} = \sin(\vartheta_{|A} - \alpha), \quad \gamma_{|A'} = \sec(\vartheta_{|A} - \alpha).$$

Introduce half–angles $t = \tan(\vartheta_{|A}/2)$ and $s = \tan(\alpha/2)$. Then

$$\tan\frac{\vartheta_{|A'}}{2} = \frac{t-s}{1+ts}.$$

Reading back to β yields the Einstein collinear velocity addition in circular clothes:

$$\boxed{\beta_{|A'} = \frac{\beta_{|A} - \beta_{\text{tilt}}}{1 - \beta_{|A}\,\beta_{\text{tilt}}}}, \qquad \beta_{\text{tilt}} := \sin\alpha.$$

Equivalently, in "rapidity" η (with $\tanh\eta = \beta$) the tilt is just $\eta \mapsto \eta - \eta_{\text{tilt}}$, using the gudermann relation $\tan(\vartheta/2) = \tanh(\eta/2)$.

Two independent tilts (non–collinear) and the Wigner rotation. Tilting by α_1 along $\mathbf{u}_1$ and then by α_2 along $\mathbf{u}_2$ gives a net *tilt* plus a *spatial rotation* about $\hat{\mathbf{n}} \parallel \mathbf{u}_2 \times \mathbf{u}_1$ (the Wigner–Thomas rotation). Let $t_i = \tan(\alpha_i/2)$ and let χ be the angle between $\mathbf{u}_1$ and $\mathbf{u}_2$. The rotation angle satisfies the half–angle identity

$$\boxed{\tan\frac{\psi_W}{2} = \frac{\sin\chi\;t_1t_2}{1+\cos\chi\;t_1t_2}},$$

while the net tilt in the plane spanned by the resultant spatial direction obeys the same half–angle addition rule as above. (All of this follows from rotating the phase pair $\{\mathbf{e}_t, \mathbf{u}\}$ and then re-projecting.)

Relation to the global phase angle Θ. Your Θ is defined from the object's *self* split (its own "time" direction built from $\boldsymbol{\chi}_\perp$), whereas $\vartheta_{|A}$ is the object's angle *in the observer's split.* They coincide only when the observer's time axis aligns with the object's self time axis. The tilt mechanism above is exactly "how to move between splits."

Optical readout (if the medium changes). If time is read by EM propagation in a medium with $n(\omega) = \sec\zeta(\omega)$, use $dt_{\text{ph}} = n\,dt$; then

$$\frac{d\tau}{dt_{\text{ph}}} = \cos\vartheta_{|X}\,\cos\zeta(\omega),$$

so *tilt* controls kinematics (ϑ), while *refractivity* rescales the clock (ζ). They are distinct mechanisms and compose multiplicatively.