Unimetry: A Quaternionic Gravito–Electromagnetic Formulation

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Abstract

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1 Introduction

1.1 Context and motivation

Unimetry is a proposed phase–geometric framework in which physical systems are described in terms of stationary flows in an underlying Euclidean proto-space $\mathcal{E}$. Rather than postulating space–time as a primary arena, unimetry treats the observed space–time geometry, relativistic kinematics, and field interactions as effective structures derived from the orientation and coupling of such flows. A dimensionless scalar phase potential $\Phi : \mathcal{E} \to \mathbb{R}$ and its gradient define a normalized flow direction; the familiar Minkowski metric and Lorentzian phenomena then appear as particular projections of this underlying flow geometry.

In this sense, special relativity (SR) is not the endpoint, but the first benchmark for the framework: unimetry aims at a unified phase-based description of kinematics, gravity and gauge interactions, with SR recovered as a specific limit of the general construction. The present paper develops one important sector of this programme, namely a quaternionic gravito–electromagnetic (GEM) formulation built on top of the unimetrical flow picture.

At the classical level, gravito–electromagnetic analogies are well known: in the weak-field, slow-motion limit of general relativity, the Einstein equations can be cast into a Maxwell-like form, and moving masses generate a "gravitomagnetic" field. Quaternions and related algebras have also long been used to encode rotations and the Maxwell equations in a compact way. What unimetry adds to this landscape is a concrete phase-geometric interpretation: a single quaternionic object encodes both the temporal and spatial parts of a flow, and bilinear forms of such objects naturally split into scalar, symmetric vector, and axial (vorticity-like) channels. This suggests that gravity and electromagnetism might be viewed as different faces of the same bilinear structure acting on suitably dressed flow quaternions.

Our goal here is to make this statement precise. We construct a quaternionic GEM formalism in which gravitational and electromagnetic interactions originate from the *same* bilinear machinery applied to metrically dressed "body quaternions". In particular, we show that Newton and Coulomb potentials arise as two branches of a single scalar form, while the magnetic and gravitomagnetic sectors are associated with a vortical bilinear form whose physical calibration reveals a natural role for the constants ε_0, μ_0, G and c. The resulting description remains Euclidean at the level of the proto-space, yet reproduces relativistic kinematics and GEM fields in the observable three-space.

1.2 Relation to the base unimetry paper

This work is a direct sequel to the base unimetry paper, *"Unimetry: A Phase-Space Reformulation of Special Relativity"* (henceforth "Paper I"). Paper I develops the core phase/flow structure: the phase potential Φ, the phase 1-form $\alpha = d\Phi$, the normalized flow $\widehat{\boldsymbol{\chi}}$, and the calibration $\boldsymbol{\chi} = c\,\widehat{\boldsymbol{\chi}}$, together with the derivation of the Minkowski interval and standard SR effects from a Euclidean proto-space. It also introduces the unimetrical D-rotation, which encodes Lorentz boosts as Euclidean rotations in a suitable plane of $\mathcal{E}$.

From the unimetry viewpoint, however, these SR results are only the first consistency test of a broader phase-based paradigm. The present paper assumes familiarity with the conceptual setting of Paper I, but is written to be as self-contained as reasonably possible. We briefly recall the key definitions of the phase proto-space, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations. All constructions that are essential for the GEM sector are reproduced or adapted here; more detailed discussions of SR and cosmological applications remain in Paper I and are only referenced when needed.

1.3 Main results

The main technical contributions of this paper can be summarized as follows.

- We introduce *metrically dressed body quaternions* $\widetilde{\mathsf{q}}_i = L_{E,i}\,\hat{h} + L_{G,i}\,\widehat{\boldsymbol{n}}_i$, whose components have the dimension of length. The "electric" and "gravitational" lengths

$$L_{E,i} = \sqrt{\frac{G}{4\pi\varepsilon_0 c^4}}\,Q_i, \qquad L_{G,i} = \frac{G}{c^2}\,m_i$$

  encode the charge Q_i and mass m_i of the body in a unified geometric fashion. The unit vector $\widehat{\boldsymbol{n}}_i$ represents the spatial flow direction associated with the body.

- We show that the scalar bilinear form

$$A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2) = L_{E,1}L_{E,2} - \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

  (with $\boldsymbol{S}_i = L_{G,i}\widehat{\boldsymbol{n}}_i$) yields, after a single global calibration by c^4/G and a geometric $1/r$ factor, the combined Newton–Coulomb potential:

$$U(r) = \frac{c^4}{G}\,\frac{A}{r} = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_1Q_2}{r} - G\,\frac{m_1m_2}{r}.$$

  Thus gravity and electrostatics arise as two channels of a single invariant scalar form.

- We identify two vector-valued bilinear forms, $\boldsymbol{B}(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2)$ and $\boldsymbol{C}(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2)$, corresponding to the symmetric and axial parts of the quaternion product. In the dressed setting these naturally describe current-like and vortical channels. In particular, the vortical form $\boldsymbol{C}$ reproduces the geometry of magnetic and gravitomagnetic fields generated by moving charges and masses.

- We construct a quaternionic GEM field $\mathcal{F}_{\mathrm{GEM}}(\boldsymbol{x})$ over the observable three-space by combining dressed source quaternions with purely imaginary distance quaternions. Its scalar channel reproduces the gravitational and electrostatic potentials, while its vortical channel yields a physically natural "phase-vortical" field C_{phys} with the same dimension as $\boldsymbol{E}$. The standard magnetic field $\boldsymbol{B}$ in SI units then appears as

$$\boldsymbol{B} = \frac{1}{c}\,C_{\mathrm{phys}},$$

  so that the familiar μ_0 and ε_0 can be interpreted in terms of linear and areal stiffness of the vacuum, combined into an effective volumetric stiffness proportional to $1/(\varepsilon_0 c^3)$.

- We analyze the action of unimetrical D-rotations and ordinary spatial rotors on dressed quaternions. Pure spatial rotations act in the usual way on the vector channels and leave the scalar form A invariant, while D-rotations mix the scalar channel and the longitudinal component of $\boldsymbol{B}$ in a two-dimensional “energy–current” plane. This provides a quaternionic encoding of relativistic kinematics in the GEM setting, with Lorentz-consistent transformation properties of the fields.
- Finally, we outline a Hamiltonian and Lagrangian formulation of the quaternionic GEM theory in terms of the self-form A and the norm-squares of $\boldsymbol{B}$ and $\boldsymbol{C}$, and discuss how the standard Maxwell Lagrangian and linearized GEM equations arise in appropriate limits.

1.4 Structure of the paper

The paper is organized as follows. In Section 2 we recall the basic quaternion algebra and introduce the bilinear forms A, $\boldsymbol{B}$, and $\boldsymbol{C}$ that arise from the quaternion product, together with their matrix representation and geometric interpretation. Section 3 provides a brief overview of the unimetrical phase proto-space, the phase potential, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations.

In **??** we introduce metrically dressed body quaternions and define the electric and gravitational lengths L_E and L_G. **??** shows how the scalar form A for dressed quaternions reproduces the static Newton and Coulomb potentials. In **??** we construct a quaternionic GEM field over the observable three-space and identify the scalar and vortical channels with gravitational, electric, and magnetic sectors.

?? analyzes the action of spatial rotors and D-rotors on dressed quaternions and on the GEM field, clarifying the relativistic transformation properties of the scalar, current-like, and vortical channels. **??** is devoted to the calibration of $\boldsymbol{E}$ and $\boldsymbol{B}$, to the definition of the phase-vortical field C_{phys}, and to the interpretation of ε_0, μ_0, and c in terms of vacuum stiffness.

In **??** we outline Hamiltonian and Lagrangian formulations of quaternionic GEM, and in **??** we compare the resulting equations with the standard Maxwell and linearized GEM formalisms. Finally, **??** discusses limitations and open questions, and sketches possible extensions towards non-Abelian interactions and cosmological applications.

2 Quaternion algebra and bilinear forms

2.1 Basic notation and conventions

We denote by $\mathbb{H}$ the real quaternion algebra, viewed as a four-dimensional real vector space

$$\mathbb{H} \simeq \mathbb{R}^4(\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}),$$

where $\hat{h}$ is the distinguished scalar basis element and $\hat{\imath}, \hat{\jmath}, \hat{k}$ are purely imaginary basis elements. A general quaternion is written as

$$\mathbf{q} = x^\mu e_\mu = x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k},$$

with real components $x^\mu \in \mathbb{R}$ and basis $e_0 := \hat{h}$, $e_1 := \hat{\imath}$, $e_2 := \hat{\jmath}$, $e_3 := \hat{k}$.

The imaginary basis satisfies the usual quaternion relations

$$\hat{\imath}^2 = \hat{\jmath}^2 = \hat{k}^2 = -\hat{h}, \qquad \hat{\imath}\hat{\jmath} = \hat{k}, \quad \hat{\jmath}\hat{k} = \hat{\imath}, \quad \hat{k}\hat{\imath} = \hat{\jmath},$$

with antisymmetry under exchange of factors. We identify $\operatorname{Im}\mathbb{H} \simeq \mathbb{R}^3$ with its Euclidean inner product $\boldsymbol{x} \cdot \boldsymbol{y}$ and cross product $\boldsymbol{x} \times \boldsymbol{y}$, so that

$$\boldsymbol{x} = x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k} \quad \longleftrightarrow \quad (x^1, x^2, x^3) \in \mathbb{R}^3.$$

Quaternionic conjugation is defined by

$$\overline{\mathbf{q}} := x^0\hat{h} - x^1\hat{\imath} - x^2\hat{\jmath} - x^3\hat{k},$$

and the norm is $\|\mathbf{q}\|^2 = \mathbf{q}\,\overline{\mathbf{q}} = \overline{\mathbf{q}}\,\mathbf{q} = (x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$.

When convenient we will write a quaternion as $\mathbf{q} = (T, \boldsymbol{S})$ with

$$T := x^0, \qquad \boldsymbol{S} := x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k},$$

emphasizing the split into scalar and vector parts.

Universal imaginary unit. Besides the fixed imaginary basis $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$ it will be convenient to introduce a "universal" unit imaginary quaternion $\hat{\boldsymbol{\ell}}$, defined abstractly by

$$\hat{\boldsymbol{\ell}}^2 = -\hat{h}, \qquad \|\hat{\boldsymbol{\ell}}\| = 1. \tag{1}$$

Geometrically, $\hat{\boldsymbol{\ell}}$ should be understood as a *joker* direction: in any concrete configuration it is identified with the unit vector along the relevant interaction axis (for instance, the radial direction between two approximately isotropic Newtonian bodies). Algebraically, $\hat{\boldsymbol{\ell}}$ behaves as any other unit imaginary quaternion, and all scalar invariants such as $A(q,q)$ remain well defined when the spatial part of q is restricted to the one-dimensional subspace $\mathbb{R}\hat{\boldsymbol{\ell}}$.

When describing isotropic Newtonian bodies we will often use quaternions of the form

$$q = T\,\hat{h} + S\,\hat{\boldsymbol{\ell}}, \tag{2}$$

with $T, S \in \mathbb{R}$, so that all interaction channels reduce to the radial line spanned by $\hat{\boldsymbol{\ell}}$. This eliminates spurious transverse contributions in the vortical form $\boldsymbol{C}$ for purely radial gravito–electric configurations.

2.2 Quaternion product and decomposition into A, B, C forms

Let

$$\mathbf{q}_1 = x^\mu e_\mu = x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k},$$
$$\mathbf{q}_2 = y^\nu e_\nu = y^0\hat{h} + y^1\hat{\imath} + y^2\hat{\jmath} + y^3\hat{k},$$

with $\mathbf{q}_1, \mathbf{q}_2 \in \mathbb{H}$. Their quaternion product can be expanded term by term as

$$\begin{aligned}\mathbf{q}_1 \circ \mathbf{q}_2 &= (x^0\hat{h} + x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k})(y^0\hat{h} + y^1\hat{\imath} + y^2\hat{\jmath} + y^3\hat{k})\\ &= (x^0y^0\hat{h}^2 + x^1y^1\hat{\imath}^2 + x^2y^2\hat{\jmath}^2 + x^3y^3\hat{k}^2)\\ &\quad + (x^0y^1\hat{h}\hat{\imath} + x^0y^2\hat{h}\hat{\jmath} + x^0y^3\hat{h}\hat{k} + x^1y^0\hat{\imath}\hat{h} + x^2y^0\hat{\jmath}\hat{h} + x^3y^0\hat{k}\hat{h})\\ &\quad + (x^1y^2\hat{\imath}\hat{\jmath} + x^1y^3\hat{\imath}\hat{k} + x^2y^1\hat{\jmath}\hat{\imath} + x^2y^3\hat{\jmath}\hat{k} + x^3y^1\hat{k}\hat{\imath} + x^3y^2\hat{k}\hat{\jmath}).\end{aligned} \tag{3}$$

Using the multiplication rules, this can be organised into three bilinear contributions:

$$\mathbf{q}_1 \circ \mathbf{q}_2 = (\mathbf{q}_1 * \mathbf{q}_2) + (\mathbf{q}_1 \diamond \mathbf{q}_2) + (\mathbf{q}_1 \times \mathbf{q}_2), \tag{4}$$

where:

- $\mathbf{q}_1 * \mathbf{q}_2$ collects the purely scalar terms,
- $\mathbf{q}_1 \diamond \mathbf{q}_2$ collects the mixed scalar–vector terms,
- $\mathbf{q}_1 \times \mathbf{q}_2$ collects the purely vector–vector terms.

Explicitly, one finds the familiar invariant decomposition

$$\begin{aligned}\mathbf{q}_1 \circ \mathbf{q}_2 = &\left(x^0y^0 - x^1y^1 - x^2y^2 - x^3y^3\right)\hat{h} \\ &+ \left(x^0\boldsymbol{y} + y^0\boldsymbol{x}\right) + \left(\boldsymbol{x} \times \boldsymbol{y}\right),\end{aligned} \tag{5}$$

where

$$\boldsymbol{x} := x^1\hat{\imath} + x^2\hat{\jmath} + x^3\hat{k}, \qquad \boldsymbol{y} := y^1\hat{\imath} + y^2\hat{\jmath} + y^3\hat{k}.$$

This suggests three natural bilinear maps:

$$A(\mathbf{q}_1, \mathbf{q}_2) := x^0y^0 - x^1y^1 - x^2y^2 - x^3y^3, \tag{6}$$

$$\boldsymbol{B}(\mathbf{q}_1, \mathbf{q}_2) := x^0\boldsymbol{y} + y^0\boldsymbol{x}, \tag{7}$$

$$\boldsymbol{C}(\mathbf{q}_1, \mathbf{q}_2) := \boldsymbol{x} \times \boldsymbol{y}. \tag{8}$$

In terms of these,

$$\mathbf{q}_1 \circ \mathbf{q}_2 = \left(A(\mathbf{q}_1, \mathbf{q}_2)\right)\hat{h} + \boldsymbol{B}(\mathbf{q}_1, \mathbf{q}_2) + \boldsymbol{C}(\mathbf{q}_1, \mathbf{q}_2). \tag{9}$$

It is often convenient to view (5) in a tensor-like form. We can write

$$\mathbf{q}_1 \circ \mathbf{q}_2 = \sum_{\mu,\nu=0}^{3} \left(A_{\mu\nu}\,\hat{h} + B_{\mu\nu} + C_{\mu\nu}\right) x^\mu y^\nu,$$

with three 4×4 coefficient matrices:

$$A_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{10}$$

$$B_{\mu\nu} = \begin{pmatrix} 0 & \hat{\imath} & \hat{\jmath} & \hat{k} \\ \hat{\imath} & 0 & 0 & 0 \\ \hat{\jmath} & 0 & 0 & 0 \\ \hat{k} & 0 & 0 & 0 \end{pmatrix}, \tag{11}$$

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{k} & -\hat{\jmath} \\ 0 & -\hat{k} & 0 & \hat{\imath} \\ 0 & \hat{\jmath} & -\hat{\imath} & 0 \end{pmatrix}. \tag{12}$$

Here $A_{\mu\nu}$ is the usual Minkowski-like bilinear form $\mathrm{diag}(1, -1, -1, -1)$ acting on the coordinate components, while $B_{\mu\nu}$ and $C_{\mu\nu}$ collect the symmetric and antisymmetric vector-valued pieces of the product.

2.3 Geometric interpretation and tensor structure of A, B, C

The decomposition (9) and the matrices (10)–(12) make explicit that the quaternion product can be regarded as the contraction of a rank-$(0, 2)$ object with two four-vectors:

$$\mathbf{q}_1 \circ \mathbf{q}_2 = \left(A + B + C\right)_{\mu\nu} x^\mu y^\nu,$$

with three structurally distinct blocks:

- $A_{\mu\nu}$ is a symmetric scalar bilinear form of signature $(+, -, -, -)$. In the unimetrical context it will play the role of an energy-like invariant and will generate both Newtonian and Coulomb potentials once we pass to dressed body quaternions.

- $B_{\mu\nu}$ is symmetric and vector-valued; it couples the scalar component to the spatial components. It will later be interpreted as a current-like channel, encoding the coupling between temporal and spatial parts of dressed flow quaternions.

- $C_{\mu\nu}$ is antisymmetric and vector-valued; it encodes the cross product $\boldsymbol{x} \times \boldsymbol{y}$ of the spatial parts and thus represents a vorticity (axial) channel. This will underlie the magnetic and gravitomagnetic sectors of the GEM field.

In summary, the elementary quaternion product already contains, in a rigid algebraic way, the three channels that we will later reinterpret as

(i) an energy-like scalar invariant A,

(ii) a current-like symmetric vector channel $\boldsymbol{B}$,

(iii) a vortical (axial) vector channel $\boldsymbol{C}$.

In the next section we recall how unimetry associates physical flows and an effective space–time structure to quaternions, so that these three forms can be given a gravito–electromagnetic meaning.

3 Phase proto-space and flow: brief unimetry overview

3.1 Proto-space, phase potential and phase 1-form

In unimetry the basic kinematical arena is a Euclidean (or, more generally, Hilbert) proto-space $(\mathcal{E}, \langle\cdot,\cdot\rangle)$. Points of $\mathcal{E}$ will be denoted by X, and the inner product $\langle\cdot,\cdot\rangle$ is used to identify tangent and cotangent spaces via the Riesz isomorphism. For the purposes of this paper one may think of $\mathcal{E}$ as a finite- or countable-dimensional real Hilbert space.

The fundamental scalar field of unimetry is a dimensionless *phase potential*

$$\Phi : \mathcal{E} \to \mathbb{R}.$$

From Φ we obtain the *phase 1-form*

$$\alpha := d\Phi,$$

which is a smooth 1-form on $\mathcal{E}$. At each point $X \in \mathcal{E}$, the value α_X is a linear functional on the tangent space $T_X\mathcal{E}$:

$$\alpha_X : T_X\mathcal{E} \to \mathbb{R}, \qquad \alpha_X(V) = d\Phi_X(V).$$

Using the inner product, we define the gradient $\nabla\Phi(X) \in T_X\mathcal{E}$ by the standard relation

$$\alpha_X(V) = d\Phi_X(V) = \langle \nabla\Phi(X), V\rangle, \qquad \forall\, V \in T_X\mathcal{E}.$$

Thus α and $\nabla\Phi$ carry the same information; the former is covariant, the latter contravariant.

Physically, the phase potential Φ encodes the global phase structure of the underlying flow, while the phase 1-form α and the gradient $\nabla\Phi$ encode local directions in which the phase changes most rapidly. The key idea of unimetry is to use this structure to define a canonical flow through $\mathcal{E}$.

3.2 Flow vector and normalization

Whenever $\nabla\Phi(X) \neq 0$, we define the *normalized flow direction* at X by

$$\widehat{\boldsymbol{\chi}}(X) := \frac{\nabla\Phi(X)}{\|\nabla\Phi(X)\|}, \qquad \|\nabla\Phi(X)\| := \sqrt{\langle \nabla\Phi(X), \nabla\Phi(X)\rangle}. \tag{13}$$

Thus $\widehat{\boldsymbol{\chi}}(X)$ is a unit vector in $T_X\mathcal{E}$ pointing along the steepest phase ascent. We then introduce the *physical flow vector* by a global calibration

$$\boldsymbol{\chi}(X) := c\,\widehat{\boldsymbol{\chi}}(X), \qquad \|\boldsymbol{\chi}(X)\| \equiv c, \tag{14}$$

where c is the speed of light. In other words, in unimetry the physical flow is a unit-speed curve in $\mathcal{E}$ with respect to the fixed scale c.

A flow line (or *stream*) is then a curve $\gamma : \lambda \mapsto X(\lambda) \in \mathcal{E}$ whose tangent vector is everywhere aligned with the physical flow:

$$\dot{X}(\lambda) := \frac{dX}{d\lambda} = \boldsymbol{\chi}(X(\lambda)), \qquad \|\dot{X}(\lambda)\| = c. \tag{15}$$

The parameter λ is a proto-space parameter, not yet identified with any observed time. The geometric content of (15) is simply that physical objects are represented by flows of constant Euclidean speed c in the proto-space.

3.3 Intrinsic angle, proper time and correspondence with SR

In unimetry a macroscopic body B is represented not by a single flow line, but by an ensemble of streamlets with weights w_a and tilt angles Θ_a relative to the body's self–time fibre.[1] On this ensemble one defines the temporal second moment and the spatial shape tensor as

$$T_B := \sum_a w_a \cos^2\Theta_a, \qquad \mathbf{C}_B := \sum_a w_a \sin^2\Theta_a \; \mathbf{u}_a \otimes \mathbf{u}_a, \tag{16}$$

where $0 < T_B \le 1$, $\mathbf{C}_B$ is a symmetric positive semidefinite tensor on the body's three-surface, and $\mathbf{u}_a$ are unit spatial directions of the streamlets' projections. Operationally, T_B captures the aggregate fraction of flow carried in the orthogonal (self–time) fibre, while $\mathbf{C}_B$ encodes the anisotropic distribution of spatial projections across the body.

From these second moments one can define an *intrinsic angle* $\zeta \in [0, \frac{\pi}{2}]$ as an effective statistical parameter of the ensemble. Introducing

$$C := \sum_a w_a \cos 2\Theta_a, \qquad S := \sum_a w_a \sin 2\Theta_a,$$

there exists a unique ζ such that

$$(\cos 2\zeta,\ \sin 2\zeta) = (C, S) \quad\Longleftrightarrow\quad T_B = \tfrac{1}{2}(1 + C) = \cos^2\zeta, \qquad \operatorname{tr}\mathbf{C}_B = \tfrac{1}{2}(1 - C) = \sin^2\zeta. \tag{17}$$

We call ζ the *intrinsic angle* of the body. It aggregates the second–moment information $(T_B, \mathbf{C}_B)$ into a single scalar and should be thought of as a *statistical* internal parameter: it is *not* a geometric direction and is not attached to any particular flow line.

The intrinsic angle controls the rate at which the body's own proper time τ_B accumulates with respect to the phase parameter χ used to parametrize the flow in proto-space. In the calibrated gauge $\|\boldsymbol{\chi}\| = c$, one has

$$d\tau_B = \cos\zeta\, d\chi, \tag{18}$$

so that the temporal second moment T_B appears as $T_B = \cos^2\zeta = (d\tau_B/d\chi)^2$. The corresponding intrinsic metric of the body, as a quadratic form on $(d\chi, d\boldsymbol{\ell})$, reads

$$ds_B^2 := c^2\, d\tau_B^2 - d\boldsymbol{\ell}^\top \mathbf{C}_B\, d\boldsymbol{\ell} = c^2 T_B\, d\chi^2 - d\boldsymbol{\ell}^\top \mathbf{C}_B\, d\boldsymbol{\ell}. \tag{19}$$

For an isotropic texture one has $\mathbf{C}_B = \frac{\sin^2\zeta}{3}\,\mathbf{I}_S$, and with the rest gauge $T_B \equiv 1$ this reduces to the familiar Minkowski form in the body's rest frame (up to the overall phase gauge $d\chi$).

[1] For the detailed construction see Paper I, §?? there.

In the full unimetrical construction the intrinsic angle ζ is combined with a kinematic angle ϑ (associated with the relative motion between bodies) and, when present, with a gravitational angle ϕ (associated with an external tilt field). The resulting time-rate factor factorises into intrinsic, kinematic, and gravitational contributions. For the purposes of the present GEM paper, we only need the following structural facts:

- The intrinsic angle ζ is a scalar *second-moment* parameter of a body, not a direction: it encodes how the flow budget is split between self–time and spatial channels in the ensemble of streamlets.
- The proper time τ_B along the body's worldline is related to the phase parameter χ by (18), and the body's intrinsic metric takes the Minkowski form (19) once the rest gauge is fixed.
- The relativistic kinematics of unimetry can therefore be formulated entirely in terms of phase flow and second-moment data, with the usual SR interval emerging as a derived object; we will reuse this structure when interpreting the scalar form A as an energy-like invariant for dressed quaternions.

3.4 Notation table

For reference, we collect here the main unimetrical symbols used in the remainder of the paper. A more extensive table can be found in Paper I; the subset below is chosen to make the present text self-contained.

Symbol	Meaning
$\mathcal{E}$	Euclidean/Hilbert proto-space with inner product $\langle\cdot,\cdot\rangle$
$\Phi : \mathcal{E} \to \mathbb{R}$	dimensionless phase potential
$\alpha = d\Phi$	phase 1-form, $\alpha_X(V) = \langle\nabla\Phi(X), V\rangle$
$\nabla\Phi(X)$	gradient of Φ at X, defined via the inner product
$\widehat{\boldsymbol{\chi}}(X)$	normalized flow direction, $\widehat{\boldsymbol{\chi}} = \nabla\Phi/\|\nabla\Phi\|$
$\boldsymbol{\chi}(X)$	physical flow vector, $\boldsymbol{\chi} = c\,\widehat{\boldsymbol{\chi}}$, $\|\boldsymbol{\chi}\| = c$
$\gamma(\lambda)$	flow line in $\mathcal{E}$ with tangent $\dot{X} = \boldsymbol{\chi}$
$\widehat{\boldsymbol{u}}$	unit rest direction associated with an observer (local temporal axis)
$\widehat{\boldsymbol{n}}$	unit spatial direction orthogonal to $\widehat{\boldsymbol{u}}$
ζ	flow angle between $\boldsymbol{\chi}$ and $\widehat{\boldsymbol{u}}$, see (**??**)
δT	effective temporal increment for the observer, see (**??**)
$\delta\boldsymbol{x}$	effective spatial increment in the observer's rest space, see (**??**)
δs^2	effective interval, $\delta s^2 = c^2\delta T^2 - \|\delta\boldsymbol{x}\|^2$, see (**??**)

Table 1: Key unimetrical quantities used in the quaternionic GEM construction.

In the next sections we introduce two calibrations of the flow — one kinematic and one energetic — which will allow us to interpret the scalar form A as an energy-like invariant and to define metrically dressed body quaternions suitable for the gravito–electromagnetic setting.

4 Flow calibrations and energy-like functionals

In the unimetrical picture a physical body B is represented by a flow line $\gamma_B : \chi \mapsto X(\chi)$ in the Euclidean proto–space $\mathcal{E}$. The proto–parameter χ is taken to have the dimension of time,

$$[\chi] = \mathrm{s},$$

and the geometric flow vector

$$\boldsymbol{\chi} := \frac{dX}{d\chi}$$

has Euclidean norm $\|\boldsymbol{\chi}\| = c$. Thus each body is a curve of *constant Euclidean speed* c *in* $\mathcal{E}$; only the direction of the flow can vary.

On top of this geometric flow we introduce a dimensionless scalar field (the *phase potential*) Φ pulled back to each flow line. Taken together, $(X(\chi), \Phi(\chi))$ will be used to reconstruct both observable kinematics and energy–like quantities.

From the unimetrical point of view it is convenient to isolate two scalar invariants associated with the flow:

1. the *geometric invariant*, namely the Euclidean norm of the normalized proto–velocity $\widehat{\boldsymbol{\chi}} := \boldsymbol{\chi}/c$, which is fixed to $\|\widehat{\boldsymbol{\chi}}\| \equiv 1$;

2. the *scalar flow invariant*, namely a scalar quantity $H_B(\chi)$ accumulated along the flow whose rate $\dot{H}_B := dH_B/d\tau_B$ with respect to the body's proper time τ_B is invariant under unimetrical D–rotations.

The first invariant is purely geometric and underlies the appearance of the Minkowski metric and of standard Lorentz kinematics. The second is structural: it depends on the internal streamlet content of the body and will supply the absolute energy scale (rest energy and mass) for that body.

The goal of this section is to make these two invariants explicit and to show how two complementary calibrations of the flow — a *geometric (kinematic)* calibration and a *structural (cyclic)* calibration — lead to a factorized expression for energy. Later this expression will be rewritten in quaternionic form.

4.1 Flow parameter, phase potential, and phase frequency

Along a given flow line γ_B we assume that the phase potential Φ is a smooth function of the proto–parameter χ. We define the *phase frequency* per unit χ by

$$\omega_\chi(\chi) := \frac{d\Phi}{d\chi}, \qquad [\omega_\chi] = \mathrm{s}^{-1}. \tag{20}$$

In a convenient gauge one may regard ω_χ as constant along the worldline of B, so that $\Phi(\chi) = \omega_\chi \chi$, but no particular gauge choice is needed in what follows. The product $\omega_\chi \chi$ is always dimensionless.

To keep track of the "amount of flow" accumulated along γ_B we also introduce a scalar function $H_B(\chi)$ and define it by

$$\frac{dH_B}{d\chi} := \widetilde{\mathcal{H}}_B(\chi),$$

where the *proto–space speed per unit* χ is

$$\widetilde{\mathcal{H}}_B(\chi) := \|\widehat{\boldsymbol{\chi}}(\chi)\| = 1.$$

The last equality is simply the unimetrical normalization of the flow. With this choice,

$$H_B(\chi) = \chi + \text{const},$$

so that H_B and χ carry the same dimension and differ only by an irrelevant additive constant. It is nevertheless convenient to keep H_B explicit, because its *proper–time* rate

$$\dot{H}_B := \frac{dH_B}{d\tau_B}$$

will be used as the second invariant of the body B.

4.2 Geometric vs structural calibration of the flow

The two scalar invariants just described admit two complementary ways of "calibrating" the flow.

Geometric (kinematic) calibration. In the geometric calibration we fix the magnitude of the proto–velocity by

$$\|\boldsymbol{\chi}\| = c,$$

or equivalently $\|\widehat{\boldsymbol{\chi}}\| \equiv 1$, and we regard only its *direction* in $T_X\mathcal{E}$ as dynamical. Choosing a temporal axis $\hat{\boldsymbol{u}}$ for a given observer, the direction of the flow is then characterized by a kinematic angle ϑ between $\widehat{\boldsymbol{\chi}}$ and $\hat{\boldsymbol{u}}$. As reviewed below, the associated family of unimetrical D–rotations reproduces standard Lorentz kinematics: time dilation, length contraction, and Wigner–Thomas rotations.

In this calibration the internal streamlet content of a Newtonian body is not resolved. The body is treated as a featureless point flow of fixed Euclidean speed c, and only the redistribution of that speed between the temporal axis and one spatial direction is taken into account.

Structural (cyclic) calibration. In the structural calibration we probe the internal architecture of the body. Rather than using the magnitude of $\boldsymbol{\chi}$ as our reference, we use the scalar flow H_B and its proper–time rate $\dot{H}_B$.

Conceptually, we picture the body as an ensemble of streamlets whose normalized proto–velocities are distributed around the body's own temporal axis. The second moment of this distribution defines an *intrinsic angle* ζ_B and an associated temporal share $\widetilde{X}_0^2 = \cos^2\zeta_B$ of the unit flow, as discussed in Section 3.3. The same internal structure also supports two cyclic coordinates extracted from the flow: a global phase coordinate χ and the body's proper time τ_B.

In this cyclic picture the scalar flow rate $\dot{H}_B$ is fixed by the geometry of the internal cycles and by the temporal share of the flow:

$$\dot{H}_B = \frac{dH_B}{d\tau_B} = \frac{dH_B}{d\chi}\,\frac{d\chi}{d\tau_B} = \widetilde{\mathcal{H}}_B\,\frac{d\chi}{d\tau_B} = \frac{d\chi}{d\tau_B},$$

where we used $dH_B/d\chi = \widetilde{\mathcal{H}}_B \equiv 1$. Thus in the structural calibration the second invariant of the body is equivalently the rate $\dot{\chi}_B := d\chi/d\tau_B$ at which the global phase parameter accumulates with respect to proper time.

The next subsection makes this cyclic construction explicit and expresses $\dot{\chi}_B$ in terms of a geometric ratio k_B and the temporal share $\widetilde{X}_0^2 = \cos^2\zeta_B$ of the flow.

4.3 Cyclic origin of local time and proto-parameter speed

We now make explicit the unimetrical assumption that local time arises from a cyclic action of the flow along a compactified temporal axis. For simplicity we model the geometry of this internal motion by two circles in the (χ, τ_B)-plane.

- A *phase circle* of radius R_1 in the χ-direction. One full turn corresponds to a phase increment $\Delta\chi = 2\pi R_1$.
- A *time circle* of radius R_2 in the τ_B-direction. Along this circle the flow executes a cyclic motion with some characteristic proper–time angular frequency ω_* (units s^{-1}). One full turn corresponds to a proper–time increment

$$\Delta\tau_B = \frac{2\pi R_2}{\omega_*}. \tag{21}$$

The discrete frequency of phase ticks with respect to proper time is then

$$\nu := \frac{\Delta\chi}{\Delta\tau_B} = \frac{2\pi R_1}{2\pi R_2/\omega_*} = \underbrace{\frac{R_1}{R_2}}_{=:k_B}\,\omega_*. \tag{22}$$

Here $k_B := R_1/R_2$ is a dimensionless *structural ratio* that encodes the geometry of the conjugate cycles of body B:

$$k_B := \frac{R_1}{R_2}. \tag{23}$$

In the high–frequency limit $R_1 \to 0$, when many cycles are accumulated, the discrete ratio becomes a derivative and we obtain

$$\nu \xrightarrow{R_1 \to 0} \frac{d\chi}{d\tau_B} =: \dot{\chi}_B = k_B\, \omega_*. \tag{24}$$

To connect ω_* to the internal flow structure we now use the streamlet picture. In the rest frame of B the normalized proto–velocity of each streamlet has temporal component

$$\widetilde{X}_0 = \cos\zeta_B,$$

where the intrinsic angle ζ_B is defined from the second moment of the streamlet ensemble, see Section 3.3. The simplest isotropic ansatz is that the proper–time angular frequency of the internal cycle is proportional to the *temporal share of the unit flow*,

$$\omega_* \propto \widetilde{X}_0^2 = \cos^2\zeta_B.$$

Choosing the proportionality constant as part of the definition of k_B, we can write

$$\omega_* = \widetilde{X}_0^2 = \cos^2\zeta_B, \tag{25}$$

so that (24) becomes

$$\dot{\chi}_B = \frac{d\chi}{d\tau_B} = k_B\, \widetilde{X}_0^2 = k_B \cos^2\zeta_B. \tag{26}$$

Equivalently, at the discrete level

$$\nu := \frac{\Delta\chi}{\Delta\tau_B} = k_B\, \widetilde{X}_0^2 = k_B \cos^2\zeta_B. \tag{27}$$

Since $dH_B/d\chi \equiv 1$, the proper–time rate of the scalar flow is

$$\dot{H}_B := \frac{dH_B}{d\tau_B} = \frac{dH_B}{d\chi}\,\frac{d\chi}{d\tau_B} = \dot{\chi}_B = k_B\, \widetilde{X}_0^2 = k_B \cos^2\zeta_B. \tag{28}$$

Thus in the structural calibration the second invariant of body B can be written in several equivalent ways:

$$\boxed{\dot{H}_B = \dot{\chi}_B = k_B\, \widetilde{X}_0^2 = k_B \cos^2\zeta_B =: \Lambda_B,} \tag{29}$$

where the last equality merely introduces a convenient shorthand Λ_B for the invariant scalar flow rate. By construction this quantity is unchanged under unimetrical D–rotations between observers:

$$\Lambda_B = k_B\, \widetilde{X}_0^2 = \text{const for body } B. \tag{30}$$

4.4 Kinematic (phase) calibration

We now describe the geometric (kinematic) calibration more explicitly. Fix an inertial laboratory frame with coordinate time t and three–position $\boldsymbol{x}$. In the *kinematic calibration* we identify the proto–parameter with the laboratory time,

$$\chi = t, \qquad \frac{d\chi}{dt} = 1. \tag{31}$$

Along the worldline of a body B we may then write

$$\Phi(t) = \omega_\chi t$$

with $\omega_\chi = d\Phi/dt$ in that frame.

The normalized flow direction can be decomposed with respect to the lab–frame temporal unit vector $\widehat{\boldsymbol{u}}$ and a spatial unit vector $\widehat{\boldsymbol{n}}$ in the rest space:

$$\widehat{\boldsymbol{\chi}} = \cos\vartheta\,\widehat{\boldsymbol{u}} + \sin\vartheta\,\widehat{\boldsymbol{n}}, \qquad \|\widehat{\boldsymbol{n}}\| = 1, \tag{32}$$

where $\vartheta \in [0, \pi/2]$ is a *kinematic angle*. For a body moving with constant lab three–velocity $\boldsymbol{v} = d\boldsymbol{x}/dt$ we impose

$$\sin\vartheta = \beta := \frac{\|\boldsymbol{v}\|}{c}, \qquad \cos\vartheta = \sqrt{1-\beta^2}. \tag{33}$$

This calibration ensures that the spatial projection of the flow has magnitude $\|\boldsymbol{v}\| = c\sin\vartheta$, while the temporal projection matches the usual factor $\sqrt{1-\beta^2}$ that appears in time dilation.

Using the standard relation between proper time and laboratory time,

$$\frac{d\tau_B}{dt} = \sqrt{1-\beta^2},$$

we recover

$$\frac{d\tau_B}{dt} = \cos\vartheta, \qquad \gamma(\vartheta) := \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\cos\vartheta}, \tag{34}$$

with $\gamma(\vartheta)$ the usual Lorentz factor. Thus the kinematic angle ϑ encodes the purely kinematic tilt of the flow relative to the chosen inertial frame.

It is important that the structural invariant Λ_B defined in (29) does *not* depend on ϑ. Kinematic D–rotations change only the split of the unit flow between temporal and spatial directions in a given frame; they leave the internal streamlet structure (and hence $k_B, \zeta_B, \dot{H}_B$) untouched.

4.5 Structural parameter and volumetric coefficient

The structural invariant $\Lambda_B = \dot{H}_B$ encodes *how fast* the internal cyclic flow of B proceeds in proper time. To turn this rate into an energy scale we need one additional scalar, which captures *how much* flow is involved in the internal cyclic motion.

In the streamlet picture this information is contained in the spatial second moment tensor of the ensemble and in its invariants (trace, determinant, ...). For the purposes of this paper we collapse this detailed information into a single scalar *structural coefficient* κ_B. Concretely, we model κ_B as a function of the structural ratio k_B and, more generally, of the second–moment invariants:

$$\kappa_B = \kappa(k_B;\, \mathbf{C}_B).$$

In the simplest isotropic ansatz it is natural to treat k_B as a characteristic length scale of the internal cycle. Rescaling this length by a factor k_B/k_* changes a three–dimensional volume by $(k_B/k_*)^3$, so we take

$$\kappa(k_B) = \kappa_* \left(\frac{k_B}{k_*}\right)^3, \tag{35}$$

where κ_* is a reference value at some fiducial structural state $k_B = k_*$. Dimensional analysis then dictates that κ has units

$$[\kappa] = \mathrm{kg\,s\,m^{-1}}, \tag{36}$$

so that the combination κc^3 has the dimension of energy:

$$[\kappa c^3] = \mathrm{kg\,m^2\,s^{-2}} = \mathrm{J}.$$

We therefore define the *structural rest energy* and associated rest mass of body B by

$$E_0(\zeta_B, k_B) := \kappa(\zeta_B, k_B)\, c^3, \qquad m_0(\zeta_B, k_B) := \frac{E_0(\zeta_B, k_B)}{c^2} = \kappa(\zeta_B, k_B)\, c. \tag{37}$$

In the simple volumetric model one may set $\kappa(\zeta_B, k_B) \equiv \kappa(k_B)$, so that the dependence on the intrinsic angle ζ_B enters only through the invariant rate $\Lambda_B = k_B \cos^2 \zeta_B$. More refined models can promote κ to a functional of the full spatial second moment $\mathbf{C}_B$, but the factorized structure $\{k_B, \zeta_B\} \mapsto (\kappa_B, \Lambda_B)$ will be kept throughout.

4.6 Proper-time calibration and energy factorization

In the *proper–time calibration* we parametrize the flow of B by its own proper time τ_B. The internal structure of the body is encoded in two dimensionless quantities:

- the intrinsic angle ζ_B, extracted from the second moment of the streamlet ensemble and fixing the temporal share $\widetilde{X}_0^2 = \cos^2 \zeta_B$ of the unit flow;
- the structural ratio $k_B = R_1/R_2$, which characterizes the relative size of the phase and time cycles.

In the absence of dissipation or radiation exchange we assume these structural parameters to be fixed for a given body,

$$\zeta_B = \text{const}, \qquad k_B = \text{const}.$$

Combining the cyclic construction of Section 4.3 with the definition of Λ_B we can write the relation between the phase parameter and proper time in the compact form

$$\frac{d\chi}{d\tau_B} = \dot{\chi}_B = k_B \cos^2 \zeta_B = \Lambda_B, \tag{38}$$

which is simply a restatement of (29).

By contrast, the kinematic calibration of Section 4.4 relates proper time to the laboratory time t by the usual time–dilation formula

$$\frac{d\tau_B}{dt} = \cos\vartheta = \sqrt{1-\beta^2}, \qquad \gamma(\vartheta) = \frac{1}{\cos\vartheta}, \tag{39}$$

where ϑ is the kinematic angle of the flow in the lab frame and $\beta = \|\boldsymbol{v}\|/c$. The angle ϑ is purely kinematic: it describes the relative motion between frames and is conceptually distinct from the intrinsic angle ζ_B.

Using the structural rest–energy scale (37) we *define* the energy of the body in the state $(\vartheta; \zeta_B, k_B)$ as

$$E(\vartheta; \zeta_B, k_B) := \gamma(\vartheta)\, E_0(\zeta_B, k_B) = \frac{1}{\cos\vartheta}\, \kappa(\zeta_B, k_B)\, c^3, \tag{40}$$

or equivalently

$$E(\vartheta; \zeta_B, k_B) = \gamma(\vartheta)\, m_0(\zeta_B, k_B)\, c^2.$$

Thus energy factorizes into a *purely kinematic* multiplier $\gamma(\vartheta)$, arising from the macroscopic tilt of the body's flow in the laboratory frame, and a *purely intrinsic–structural* factor $E_0(\zeta_B, k_B) = \kappa(\zeta_B, k_B)c^3$, which encodes the internal flow texture (via ζ_B) and the internal cyclic geometry (via k_B) in the body's own frame.

For small velocities $\beta \ll 1$, using $\cos\vartheta = \sqrt{1-\beta^2}$ and $\gamma(\vartheta) = 1/\cos\vartheta$ we recover the standard expansion

$$E(\vartheta; \zeta_B, k_B) = m_0(\zeta_B, k_B)\, c^2 + \tfrac{1}{2} m_0(\zeta_B, k_B)\, v^2 + O(\beta^4), \qquad v = c \sin\vartheta. \tag{41}$$

Hence the structurally generalized energy retains the usual nonrelativistic limit, while allowing the rest–energy scale to vary with the intrinsic flow texture (through ζ_B) and with the internal cyclic geometry (through k_B), without ever identifying ζ_B with the kinematic angle ϑ.

4.7 From flow vector to flow quaternion

Up to this point the flow has been represented by the unit vector $\widehat{\boldsymbol{\chi}} \in T_X\mathcal{E}$ together with its split into temporal and spatial channels controlled by the kinematic angle ϑ, cf. (32). To connect this description with the quaternion algebra of Section 2 we now introduce a simple representation map from flow directions to unit quaternions.

Fix a body B and a local observer comoving with B. At a point $X \in \mathcal{E}$ on the body's flow choose an orthonormal frame $\{\widehat{\boldsymbol{u}}, \widehat{\boldsymbol{e}}_1, \widehat{\boldsymbol{e}}_2, \widehat{\boldsymbol{e}}_3\}$ in $T_X\mathcal{E}$, where $\widehat{\boldsymbol{u}}$ is the observer's temporal direction and $\widehat{\boldsymbol{e}}_i$ span the three–dimensional rest space. The normalized flow direction can then be written as

$$\widehat{\boldsymbol{\chi}} = \cos\vartheta\,\widehat{\boldsymbol{u}} + \sin\vartheta\,\widehat{\boldsymbol{n}}, \qquad \widehat{\boldsymbol{n}} := n^1\widehat{\boldsymbol{e}}_1 + n^2\widehat{\boldsymbol{e}}_2 + n^3\widehat{\boldsymbol{e}}_3, \quad \|\widehat{\boldsymbol{n}}\| = 1. \tag{42}$$

On the quaternion side we use the basis $\{\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}\}$ introduced in Section 2. We fix a local identification between the observer's frame and the quaternion basis by declaring

$$\widehat{\boldsymbol{u}} \;\longleftrightarrow\; \hat{h}, \qquad \widehat{\boldsymbol{e}}_i \;\longleftrightarrow\; \text{an orthonormal triple in } \{\hat{\imath}, \hat{\jmath}, \hat{k}\}. \tag{43}$$

In the simplest case one may align $\widehat{\boldsymbol{e}}_1, \widehat{\boldsymbol{e}}_2, \widehat{\boldsymbol{e}}_3$ with $\hat{\imath}, \hat{\jmath}, \hat{k}$ respectively, so that the spatial unit vector $\widehat{\boldsymbol{n}}$ is represented by a unit imaginary quaternion with the same components n^i.

Definition 4.1 (Flow quaternion)**.** Given a normalized flow direction $\widehat{\boldsymbol{\chi}}$ with kinematic angle ϑ and spatial unit vector $\widehat{\boldsymbol{n}}$ in the observer's rest space, the associated *flow quaternion* is the unit quaternion

$$\widehat{\mathsf{q}}(\vartheta, \widehat{\boldsymbol{n}}) := \cos\vartheta\,\hat{h} + \sin\vartheta\,\widehat{\boldsymbol{n}}, \qquad \|\widehat{\boldsymbol{n}}\| = 1. \tag{44}$$

By construction $\|\widehat{\mathsf{q}}\| = 1$. The scalar part of $\widehat{\mathsf{q}}$ encodes the temporal fraction of the flow, while the vector part encodes the spatial fraction in the chosen rest space. Different choices of the spatial orthonormal frame correspond to spatial rotations of the imaginary basis and do not affect scalar invariants such as $A(\widehat{\mathsf{q}}, \widehat{\mathsf{q}})$.

4.8 Scalar self-form $A(q, q)$ as kinematic factor

We can now express the kinematic factor in (40) in terms of the quaternionic self–form $A(q, q)$ introduced in Section 2. For brevity we suppress the explicit dependence on $\widehat{\boldsymbol{n}}$ and write $\widehat{\mathsf{q}}(\vartheta)$ when no confusion can arise:

$$\widehat{\mathsf{q}}(\vartheta) = \cos\vartheta\,\hat{h} + \sin\vartheta\,\widehat{\boldsymbol{n}}, \qquad \|\widehat{\boldsymbol{n}}\| = 1.$$

The scalar self–form A applied to this unit quaternion gives

$$A\big(\widehat{\mathsf{q}}(\vartheta), \widehat{\mathsf{q}}(\vartheta)\big) = \cos^2\vartheta - \sin^2\vartheta = \cos 2\vartheta. \tag{45}$$

Solving for $\cos^2\vartheta$ we obtain

$$1 - \beta^2 = \cos^2\vartheta = \frac{1 + A(\widehat{\mathsf{q}}, \widehat{\mathsf{q}})}{2}, \tag{46}$$

and hence the Lorentz factor can be expressed as

$$\gamma(\vartheta) = \frac{1}{\sqrt{1-\beta^2}} = \sqrt{\frac{2}{1 + A(\widehat{\mathsf{q}}, \widehat{\mathsf{q}})}}. \tag{47}$$

Substituting (47) into the factorized energy (40), and suppressing the explicit dependence of E_0 on (ζ_B, k_B) for brevity, we find

$$E(\widehat{\mathsf{q}}, k_B) = \gamma(\vartheta)\,E_0 = \sqrt{\frac{2}{1 + A(\widehat{\mathsf{q}}, \widehat{\mathsf{q}})}}\;\kappa(k_B)\,c^3. \tag{48}$$

Thus:

- the scalar self–form $A(\widehat{\mathsf{q}}, \widehat{\mathsf{q}})$ is a *purely kinematic, dimensionless* invariant of the flow quaternion;
- the structural coefficient $\kappa(k_B)$ (or more generally $\kappa(\zeta_B, k_B)$) carries the absolute energy scale, via $E_0(\zeta_B, k_B) = \kappa(\zeta_B, k_B)c^3$;
- energy emerges as the product of a quaternionic kinematic factor (a function of A) and a structural scale.

In the gravito–electromagnetic construction below we will apply the same scalar form A not to unit flow quaternions, but to *metrically dressed* body quaternions $\widetilde{\mathsf{q}}_i$ that package mass and charge into effective lengths. For isotropic Newtonian bodies the spatial parts of these quaternions will be further restricted to multiples of the universal unit imaginary $\hat{\boldsymbol{\ell}}$ introduced in Section 2, so that all interaction channels reduce to a single radial direction. The vector forms $\boldsymbol{B}$ and $\boldsymbol{C}$ will then provide, respectively, the current–like and vortical channels of the gravito–electromagnetic field.

5 Metric dressing of body quaternions and gravito–electromagnetic self-forms

In the previous sections we treated the flow quaternion as a dimensionless unit object encoding only the *direction* of the proto-space flow. This is sufficient to reproduce the kinematics of special relativity. To model gravito–electromagnetic (GEM) interactions between extended Newtonian bodies, however, we must also incorporate their intrinsic strengths (mass, charge) in a way that respects the metric character of space.

In this section we introduce *metrically dressed* body quaternions, whose components carry the dimension of length, and show how the scalar and vector self-forms A, B, C, E generate GEM-like interaction channels in the static and slowly moving regimes.

5.1 Streamlet picture of gravitation and electric charge

Before turning to the metric dressing and the static bilinear interaction, it is useful to summarise the qualitative unimetry picture of gravitation and electric charge purely in terms of streamlets. This will motivate the splitting of channels in the dressed body quaternion and prepare the ground for the trigonometric Schwarzschild parametrization in **??**.

5.1.1 Streamlet ensemble and flow budget

In unimetry a massive body is described as an ensemble of streamlets: closed flow lines in proto–space $\mathcal{E}$, each streamlet carrying a unit proto–flow velocity $\tilde{\boldsymbol{\chi}}_a$ and a weight w_a, $\sum_a w_a = 1$. At a given event we choose a local temporal axis $\hat{e}_\tau$ and an orthonormal spatial triad $\{\hat{e}_i\}$ and decompose

$$\tilde{\boldsymbol{\chi}}_a = \tilde{X}_a^0 \, \hat{e}_\tau + \tilde{\mathbf{X}}_a, \qquad \|\tilde{\boldsymbol{\chi}}_a\|^2 = (\tilde{X}_a^0)^2 + \|\tilde{\mathbf{X}}_a\|^2 = 1. \tag{49}$$

The ensemble is characterised by the first and second moments of these components. In particular, we define

$$T_B := \langle (\tilde{X}^0)^2 \rangle, \qquad \mathbf{C}_B := \langle \tilde{\mathbf{X}} \otimes \tilde{\mathbf{X}} \rangle, \tag{50}$$

where the average is taken with weights w_a. As shown in the “mass from streamlets” construction, one can introduce an intrinsic angle ζ such that

$$T_B = \cos^2 \zeta, \qquad \operatorname{tr} \mathbf{C}_B = \sin^2 \zeta, \tag{51}$$

so that

- $\cos^2\zeta$ is the fraction of the unit flow magnitude aligned with the local temporal direction $\hat{e}_\tau$;
- $\sin^2\zeta$ is the fraction stored in spatial loops and anisotropies encoded by $\mathbf{C}_B$.

This is a purely geometric flow budget: it does not yet involve the observed mass or charge; those appear only after calibration.

5.1.2 Gravitation as exclusion of intersecting streamlet paths

Far from any matter the ensemble of streamlets crossing a small region of space is approximately isotropic in direction: for every incoming direction on the unit sphere there are streamlets whose trajectories realise that direction, and any small solid angle Ω in direction space is populated equally.

A massive body corresponds, in this picture, to a region where many streamlets are tightly packed into short, bound cycles. These bound streamlets occupy a finite region in proto–space, and the trajectories of other streamlets cannot freely intersect this region without being captured or strongly deflected. In other words, for a dense body there is a whole family of would–be trajectories which are *forbidden*: any streamlet whose path would pass through the interior of the body is removed from the free ensemble.

Seen from a point at radius r outside the body, this removal appears as a *directional deficit* on the unit sphere of possible streamlet directions. Among all directions $\hat{n}$, those for which the straight segment from the observation point along $\hat{n}$ would intersect the dense region are underpopulated or empty in the ensemble of free streamlets. The union of these directions forms a deficit cone around the line of sight towards the centre of the body.

A test body at that point is itself an ensemble of streamlets. In the absence of external sources its internal streamlets can close into cycles in any direction with equal probability; the external ensemble that crosses it is isotropic, and the averaged proto–flow direction of its streamlets remains aligned with $\hat{e}_\tau$ (apart from its own internal ζ).

In the presence of the directional deficit created by another mass, the situation changes. The set of available external trajectories from the direction of the source is reduced: there are fewer free streamlets arriving from that side, because many of the corresponding worldlines would have to intersect the dense core and are thus excluded or re-routed around it. As a result, the ensemble of trajectories intersecting the test body becomes slightly biased: there are more available directions “from outside” than from the side of the source. The net effect is that the averaged proto–flow direction of the test body streamlets tilts slightly towards the source.

In the continuum limit this shows up as a small radial component in the ensemble-averaged spatial velocity,

$$\langle\tilde{\mathbf{X}}\rangle = \tilde{V}_r(r)\,\hat{e}_r, \qquad |\tilde{V}_r(r)| \ll 1, \tag{52}$$

and one can encode this tilt by a *gravitational angle* $\phi(r)$ such that

$$\tilde{X}^0_{\text{eff}}(r) = \cos\phi(r), \qquad \|\tilde{\mathbf{X}}_{\text{eff}}(r)\| = \sin\phi(r), \tag{53}$$

for an effective streamlet representing the averaged state of the test body. Qualitatively, $\sin^2\phi(r)$ measures the fraction of the unit flow that has been deflected into the radial direction because other directions, whose trajectories intersect the dense body, are not available to free streamlets.

The detailed trigonometric calibration

$$\sin^2\phi(r) = \frac{2u}{r}, \qquad u = \frac{GM}{c^2}, \tag{54}$$

is chosen so that:

- for large r the deficit cone is small, $\phi(r) \ll 1$, and one recovers the linear Newtonian potential;

- for $r \downarrow r_{\mathrm{s}} = 2u$ the available radial directions saturate, $\phi(r) \to \pi/2$, and one recovers the full Schwarzschild metric coefficients, see **??**.

Thus gravitation is interpreted as the effect of excluded and rearranged streamlet trajectories in direction space, with the gravitational angle $\phi(r)$ parametrising how strongly the local ensemble is tilted by this exclusion.

5.1.3 Electric charge as a temporal orientation imbalance

The same streamlet ensemble admits another, complementary type of deformation. Instead of tilting more flow into a particular spatial direction, one can bias the *sign* of the temporal component $\tilde{X}_a^0$ along $\hat{e}_\tau$.

At a fixed spatial position consider two families of streamlets with the same spatial structure $\tilde{\mathbf{X}}_a$ but opposite temporal signs, $\tilde{X}_a^0 = \pm \cos \Theta_a$. A perfectly "electrically neutral" configuration has symmetric families: for every streamlet with $\tilde{X}_a^0 = +\cos \Theta_a$ there is one with $\tilde{X}_a^0 = -\cos \Theta_a$, so that the ensemble-averaged temporal component vanishes,

$$\langle \tilde{X}^0 \rangle = 0, \tag{55}$$

even though the temporal share $T_B = \langle (\tilde{X}^0)^2 \rangle$ may be nonzero. By contrast, a net *electric charge* corresponds to a nonzero temporal orientation of the flow,

$$\langle \tilde{X}^0 \rangle \neq 0, \tag{56}$$

while the spatial second moment $\mathbf{C}_B$ (and therefore the gravitational mass) can remain essentially unchanged.

This suggests the identification:

- gravitational charge (mass) is encoded in the *magnitude* of the spatial second moments, i.e. in the eigenvalues of $\mathbf{C}_B$ and hence in $\sin^2 \zeta$ from (51);
- electric charge is encoded in the *sign* and magnitude of the first temporal moment $\langle \tilde{X}^0 \rangle$, i.e. in the net orientation of the temporal share of the flow along $\hat{e}_\tau$.

Positivity of mass reflects the fact that a second moment cannot change sign, while the existence of positive and negative electric charges reflects the two possible orientations of the temporal component.

5.1.4 Temporal direction of charge in dressed body quaternions

The metrically dressed body quaternion $\tilde{\mathfrak{q}}_B = T_B + \hat{h}\, S_B$ used in the static sector packages this flow information into a single quaternionic object. In the isotropic case one can write

$$T_B = \lambda_E\, q, \qquad S_B = \lambda_G\, m, \tag{57}$$

where q is the electric charge and m the gravitational mass of the body, while λ_E and λ_G are dimensionful calibration constants (with dimensions of length per unit charge and length per unit mass, respectively).

From the streamlet picture above this assignment has a direct meaning:

- the *spatial* magnitude S_B inherits information about $\mathbf{C}_B$ and therefore represents gravitational charge m;
- the *temporal* component T_B inherits the sign and magnitude of $\langle \tilde{X}^0 \rangle$ and therefore represents electric charge q: changing the sign of q corresponds to flipping the net temporal orientation of the streamlets along $\hat{e}_\tau$.

The static quaternionic bilinear

$$A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = T_1 T_2 - S_1 S_2 \tag{58}$$

then simply expresses that like temporal orientations (like charges) repel, while like spatial magnitudes (like masses) attract. The opposite signs of the electromagnetic and gravitational channels in the static interaction potential are thus traced back to the opposite roles of the temporal and spatial parts of the same underlying proto–flow.

In the following we will use this streamlet picture — gravitation as exclusion of intersecting trajectories and electric charge as a temporal orientation imbalance — as the qualitative background for the metric dressing and for the trigonometric strong–field extension in **??**.

5.2 Metric dressing: from mass and charge to effective lengths

Let a Newtonian body B_i be characterized, in the usual description, by a gravitational mass m_i and an electric charge q_i. Instead of treating m_i and q_i as separate scalar labels, we encode them in a single *metrically dressed body quaternion* $\tilde{\mathsf{q}}_i$ with units of length:

$$[\tilde{\mathsf{q}}_i] = \mathrm{m}. \tag{59}$$

For isotropic bodies interacting radially it is natural to restrict the spatial part to the universal unit imaginary $\hat{\boldsymbol{\ell}}$ introduced in Section 2. We thus take

$$\tilde{\mathsf{q}}_i := T_i\, \hat{h} + S_i\, \hat{\boldsymbol{\ell}}, \qquad T_i, S_i \in \mathbb{R}, \qquad [T_i] = [S_i] = \mathrm{m}. \tag{60}$$

The scalar component T_i encodes the *temporal* (electric-like) channel of the body, while the spatial component $S_i\, \hat{\boldsymbol{\ell}}$ encodes the *spatial* (gravitational-like) channel. Both are measured in meters so that the quaternion norm and the bilinear forms are metrically homogeneous.

We express T_i and S_i as linear combinations of mass and charge via two calibration constants λ_G and λ_E with units chosen so that $\lambda_G m_i$ and $\lambda_E q_i$ are lengths:

$$T_i = \lambda_E\, q_i, \qquad S_i = \lambda_G\, m_i. \tag{61}$$

The exact values of λ_G and λ_E will be fixed later by matching the resulting interaction to Newton's and Coulomb's laws; for now we only require that they are universal constants of the theory.

With this dressing, the norm-like self-form $E(\tilde{\mathsf{q}}_i, \tilde{\mathsf{q}}_i)$ carries the dimension of length squared and will become proportional to the body's rest energy, while bilinear combinations between different bodies will generate interaction energies.

5.3 Single body: rest energy from the Euclidean self-form

For a single body B_i the Euclidean self-form of its dressed quaternion is

$$E(\tilde{\mathsf{q}}_i, \tilde{\mathsf{q}}_i) = T_i^2 + S_i^2 = \lambda_E^2 q_i^2 + \lambda_G^2 m_i^2, \tag{62}$$

since $\hat{h}^2 = +1$, $\hat{\boldsymbol{\ell}}^2 = -1$ and $\hat{\boldsymbol{\ell}}$ is orthogonal to $\hat{h}$ in the Euclidean sense. Geometrically, the rest state of the body is thus characterised by a single scalar norm combining its temporal (electric) and spatial (gravitational) flow components.

We postulate that the rest energy of the body is proportional to the square root of this Euclidean self-form:

$$\underline{E}_{\mathrm{rest},i} = \kappa_{\mathrm{rest}} \sqrt{E(\tilde{\mathsf{q}}_i, \tilde{\mathsf{q}}_i)}, \tag{63}$$

where κ_{rest} is a universal constant with units chosen so that $[\kappa_{\mathrm{rest}}\sqrt{E}] = \mathrm{J}$.

In the purely gravitational limit $q_i = 0$ one obtains

$$\underline{E}^{(G)}_{\mathrm{rest},i} = \kappa_{\mathrm{rest}}\,|\lambda_G|\,m_i, \tag{64}$$

which can be matched to $m_i c^2$ by choosing

$$\kappa_{\mathrm{rest}}|\lambda_G| \equiv c^2. \tag{65}$$

This fixes the gravitational channel of the self-form: for a neutral body the unimetrical rest energy reduces exactly to the Einstein relation $\underline{E}_{\mathrm{rest}} = mc^2$.

The electric channel enters the same norm via the term $\lambda_E^2 q_i^2$. Formally, a purely charged limit $m_i = 0$ would give

$$\underline{E}^{(E)}_{\mathrm{rest},i} = \kappa_{\mathrm{rest}}\,|\lambda_E|\,|q_i|. \tag{66}$$

However, for realistic bodies the measured mass m_i already includes all contributions from their internal fields, including electromagnetic self-energy. In other words, m_i should be understood as the *total* inertial mass parameter of the body, rather than as a "bare" mass separate from q_i.

In the present work we therefore use the gravitational calibration (65) to identify the overall scale of $\underline{E}_{\mathrm{rest}}$ with mc^2 and do not attempt to separate a distinct electromagnetic contribution to the rest energy. The term $\lambda_E^2 q_i^2$ in (62) should be viewed as keeping track of how the temporal (charge-related) channel contributes to the geometric norm of the body state; its precise calibration in terms of a microscopic model of electromagnetic self-energy is left for future work.

In the full GEM setting (62) and (63) thus provide a unified rest-energy scale that depends on both mass and charge through a single Euclidean norm, while the observed mc^2 relation is recovered from the purely gravitational sector.

In the dynamical setting, the kinematic factor $\gamma(\vartheta)$ from Section 4.6 multiplies this rest scale so that the total energy reads

$$E_i(\vartheta_i) = \gamma(\vartheta_i)\,E_{\mathrm{rest},i}, \tag{67}$$

in direct analogy with the relativistic point-particle Hamiltonian.

5.4 Two-body interaction and scalar self-form

Consider now two bodies B_1 and B_2 with dressed quaternions $\tilde{\mathsf{q}}_1$ and $\tilde{\mathsf{q}}_2$ as in (60). Their scalar self-form $A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$ is

$$A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = T_1 T_2 - S_1 S_2 = \lambda_E^2 q_1 q_2 - \lambda_G^2 m_1 m_2. \tag{68}$$

Thus the scalar channel naturally separates into a temporal (electric-like) part and a spatial (gravitational-like) part, with opposite signs, reflecting the empirical fact that electrostatic forces between like charges are repulsive, while gravitational forces are attractive.

To turn this scalar quantity into an interaction energy we introduce a radius vector $\boldsymbol{r}$ from B_1 to B_2 with magnitude $r = \|\boldsymbol{r}\|$, and define a scalar interaction potential of the form

$$V_{12}(r) = -\frac{\kappa_{\mathrm{int}}}{r}\,A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2), \tag{69}$$

where κ_{int} is another universal constant.

Expanding (69) we obtain

$$V_{12}(r) = -\frac{\kappa_{\mathrm{int}}\lambda_E^2}{r}\,q_1 q_2 + \frac{\kappa_{\mathrm{int}}\lambda_G^2}{r}\,m_1 m_2. \tag{70}$$

By choosing the calibration constants κ_{int}, λ_E and λ_G so that

$$\kappa_{\mathrm{int}}\lambda_G^2 = G, \qquad \kappa_{\mathrm{int}}\lambda_E^2 = \frac{1}{4\pi\epsilon_0}, \tag{71}$$

we recover the usual Newton and Coulomb potentials:

$$V_{12}(r) = -\frac{G\,m_1 m_2}{r} + \frac{1}{4\pi\epsilon_0}\,\frac{q_1 q_2}{r}. \tag{72}$$

Structurally, however, both contributions arise from the *same* scalar bilinear form A acting on the dressed body quaternions; the difference between gravity and electrostatics is encoded entirely in the dressing (61) and in the signs inside (68).

5.5 Vector self-forms and field-like channels

The scalar form A encodes the static interaction energy between two bodies once the radius r is specified. When the bodies move or when we consider the field created by a body at a point in space, we must also account for the vectorial self-forms B and C.

For dressed quaternions of the form (60) with a single spatial direction $\hat{\boldsymbol{\ell}}$, the B-form between two bodies is

$$\boldsymbol{B}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = B(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = (T_1 S_2 + T_2 S_1)\,\hat{\boldsymbol{\ell}} = \lambda_E \lambda_G\,(q_1 m_2 + q_2 m_1)\,\hat{\boldsymbol{\ell}}. \tag{73}$$

This vector lies along the universal direction $\hat{\boldsymbol{\ell}}$, which in an isotropic Newtonian configuration is identified with the radial unit vector $\hat{\boldsymbol{r}}$. The C-form vanishes identically in this one-dimensional spatial sector, since it measures transverse, vortical components that require at least two independent spatial directions:

$$\boldsymbol{C}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = \mathbf{0}. \tag{74}$$

Thus in the strictly radial, static configuration we have:

- A generates the scalar interaction potential (69),
- B is non-zero but degenerate (collinear with $\hat{\boldsymbol{\ell}}$),
- C vanishes.

The emergence of genuinely vortical (magnetic-like) contributions is therefore tied to configurations where the flow quaternions of the bodies acquire independent spatial directions, e.g. through relative motion and quaternionic D-rotations.

5.6 From interaction to field: local GEM quaternion

The potential (69) between two bodies suggests a local field-like object attached to a single source body B_1. At a point with radius vector $\boldsymbol{r}$ from B_1 we define the local *gravito–electromagnetic field quaternion* by

$$\mathsf{F}_1(\boldsymbol{r}) := -\frac{\kappa_{\text{int}}}{r}\,\frac{\tilde{\mathsf{q}}_1}{L_r}, \qquad L_r := r, \tag{75}$$

so that

$$A\big(\mathsf{F}_1(\boldsymbol{r}), \tilde{\mathsf{q}}_2\big) = -\frac{\kappa_{\text{int}}}{r}\,A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = V_{12}(r), \tag{76}$$

i.e. the scalar self-form of the local field quaternion with the test body B_2 reproduces the interaction potential.

The vector parts of $\mathsf{F}_1(\boldsymbol{r})$ can then be interpreted as gravito–electric and gravito–magnetic fields in the usual sense:

$$\mathsf{F}_1(\boldsymbol{r}) = \Phi_1(r)\,\hat{h} + \boldsymbol{\mathcal{E}}_1(r)\cdot\hat{\boldsymbol{\ell}} \tag{77}$$

in the static, radial case, with $\boldsymbol{\mathcal{E}}_1(r) \parallel \hat{\boldsymbol{r}}$ and vanishing vortical part. When the source body and/or the test body move, the dressing can be acted upon by D-rotations, and the bilinear forms B and C between the rotated quaternions generate current-like and vortical contributions analogous to the electric and magnetic fields of Maxwell's theory. We will develop this dynamical picture in the next section.

In summary, metrically dressed body quaternions provide a compact way to encode mass and charge as effective lengths in a single quaternionic object. The scalar self-form A then generates Newton and Coulomb potentials in a unified fashion, while the vector forms B and C supply the linear and vortical channels needed to describe, respectively, gravito–electric and gravito–magnetic fields once quaternionic rotations are taken into account.

6 Mass as a functional of streamlet structure in Unimetry

Draft derivation in the "streamlet + second moment" picture, without using mass to define the angle ζ.

6.1 Streamlets, normalization and second moments

Consider a massive object as an ensemble of streamlets indexed by a with weights w_a, $\sum_a w_a = 1$.

Each streamlet has a normalized proto–flow direction $\tilde{\boldsymbol{\chi}}_a$ in the proto–space $\mathcal{E}$, decomposed with respect to the object's own temporal axis $\hat{e}_\tau$ and a spatial triad $\{\hat{e}_i\}$:

$$\tilde{\boldsymbol{\chi}}_a = \tilde{X}_a^0\,\hat{e}_\tau + \tilde{\mathbf{X}}_a, \qquad \|\tilde{\boldsymbol{\chi}}_a\|^2 = (\tilde{X}_a^0)^2 + \|\tilde{\mathbf{X}}_a\|^2 = 1.$$

We parameterize each streamlet by an angle Θ_a between its flow and the temporal axis:

$$\tilde{X}_a^0 = \cos\Theta_a, \qquad \|\tilde{\mathbf{X}}_a\| = \sin\Theta_a.$$

The rest configuration of the object is defined as that frame in which the first spatial moment vanishes:

$$\sum_a w_a \tilde{\mathbf{X}}_a = 0.$$

However, the second moment in general does not vanish and contains structural information about the object.

Define the second moments:

$$T_B := \sum_a w_a \cos^2\Theta_a, \qquad \mathbf{C}_B := \sum_a w_a \sin^2\Theta_a\, \mathbf{u}_a \otimes \mathbf{u}_a,$$

where $\mathbf{u}_a := \tilde{\mathbf{X}}_a/\|\tilde{\mathbf{X}}_a\|$ are unit spatial directions (when $\sin\Theta_a \neq 0$).

From $\cos^2\Theta_a + \sin^2\Theta_a = 1$ and $\sum_a w_a = 1$ it follows that

$$T_B + \operatorname{tr}\mathbf{C}_B = 1.$$

Here

- T_B encodes the temporal share of the total flow "budget";
- $\mathbf{C}_B$ encodes both amount and shape of the spatially looped flow.

6.2 Structural angle ζ from the second moment (without mass)

We now define a purely structural "budget angle" ζ by

$$T_B = \cos^2\zeta, \qquad \operatorname{tr}\mathbf{C}_B = \sin^2\zeta.$$

This is not defined using any notion of physical mass or charge. It is determined solely by

- the ensemble of streamlets $\{\tilde{\boldsymbol{\chi}}_a, w_a\}$,

- the choice of the object's own temporal axis $\hat{e}_\tau$,
- and the second moment data $(T_B, \mathbf{C}_B)$.

Thus ζ is a geometric property of the proto–flow structure, not a function of the observed rest mass.

Interpretation:

- $\cos^2\zeta$ is the fraction of the unit flow magnitude that runs "along time" (along $\hat{e}_\tau$);
- $\sin^2\zeta$ is the fraction trapped in spatial loops and anisotropies encoded by $\mathbf{C}_B$.

6.3 Total proto–flow magnitude and temporal share

We now introduce a convenient normalization of the total proto–flow:

$$\tilde{H} := \left\langle \|\tilde{\boldsymbol{\chi}}_a\|^2 \right\rangle^{1/2} = \left(\sum_a w_a\, 1\right)^{1/2} = 1.$$

So the total flow magnitude of the object is set to unity; all nontrivial physics is in how this unit is split between temporal and spatial directions.

The temporal share of the flow, in the object's rest configuration, is then

$$\tilde{X}_0^2 := T_B = \cos^2\zeta,$$

while the spatial share is $\operatorname{tr}\mathbf{C}_B = \sin^2\zeta$.

After the standard SR calibration, where the physical flow is fixed as $\boldsymbol{\chi} = c\,\widehat{\boldsymbol{\chi}}$ (so that $\|\boldsymbol{\chi}\| = c$), the temporal component of the physical flow becomes

$$\chi_\tau^{(\mathrm{phys})} = c\,\tilde{X}_0 = c\cos\zeta.$$

In this sense, for the object itself the effective "speed of light" that controls the rate of its proper time is the temporal share $c\cos\zeta$; the remaining $c\sin\zeta$ is locked in spatial circulation.

6.4 Phase parameter χ, cyclic time τ, and invariant flow H

Introduce the global phase parameter χ and the local proper time τ of the object.

By construction of the streamlet picture, each full cycle of the object's internal flow corresponds to a full phase 2π in χ, while τ parametrizes the local cycles of the object's internal clock.

We postulate that the rate at which the phase winds with respect to the local time is proportional to the temporal share of the flow:

$$\frac{d\chi}{d\tau} := \dot{\chi} = k\,\tilde{X}_0^2 = k\,\cos^2\zeta,$$

where k is a geometric constant of the object, e.g. the ratio of the radii of two conjugate cycles $k = R_1/R_2$ in the cyclic picture.

This is a key step:

- we are not inserting the rest mass here;
- we are using only the structural angle ζ extracted from the second moment of the streamlets.

Now define $H(\chi)$ as the accumulated flow quantity along the phase:

$$\frac{dH}{d\chi} := \tilde{H}.$$

With $\tilde{H} \equiv 1$, this reduces to

$$\frac{dH}{d\chi} = 1 \quad \Rightarrow \quad H(\chi) = \chi + \text{const} \sim \chi.$$

Differentiating with respect to the local time,

$$\dot{H} := \frac{dH}{d\tau} = \frac{dH}{d\chi}\frac{d\chi}{d\tau} = 1 \cdot \dot{\chi} = k\, \cos^2\zeta.$$

Thus the instantaneous scalar flow rate in the proper frame of the object is

$$\boxed{\dot{H} = k\, \tilde{X}_0^2 = k\, \cos^2\zeta.}$$

Its square gives the invariant quadratic form in the proper frame:

$$dH^2 = \dot{H}^2\, d\tau^2 = k^2 \cos^4\zeta\, d\tau^2.$$

This quantity is invariant under D-boosts: D-rotations of the observer do not change the internal ζ, nor the intrinsic cyclic geometry encapsulated in k. They only change the observed decomposition of 4-velocity, not the internal flow rate $\dot{H}$.

7 Bilinear forms and static gravito–electromagnetism

In this section we show how the four quaternionic bilinear forms introduced in Section 2 naturally reproduce the static limits of Newtonian gravity and Coulomb electrostatics when applied to metrically dressed body quaternions. The key point is that gravitation and electrostatics correspond to different *channels* of the same quaternionic object: the spatial part encodes gravitational charge (mass), while the temporal part encodes electric charge.

7.1 Dressed body quaternions in the static, isotropic case

We consider isolated Newtonian bodies which are macroscopically at rest in a common inertial frame and whose internal streamlet ensembles are isotropic in space in that frame. In this situation the spatial structure of each body can be collapsed to a single unit imaginary direction, the *free unit imaginary vector* $\hat{\boldsymbol{\ell}}$ from Section 2, which is always aligned with the radius vector connecting the bodies in a two-body configuration.

For each body B_i with gravitational mass m_i and electric charge q_i we introduce a metrically dressed quaternion

$$\widetilde{\mathsf{q}}_i := T_i\, \hat{h} + S_i\, \hat{\boldsymbol{\ell}}, \qquad T_i, S_i \in \mathbb{R}, \qquad [T_i] = [S_i] = \mathrm{m}, \tag{78}$$

with the identifications

$$T_i = \lambda_E\, q_i, \qquad S_i = \lambda_G\, m_i. \tag{79}$$

Here λ_E and λ_G are universal calibration constants with units chosen so that $\lambda_E q_i$ and $\lambda_G m_i$ have dimensions of length. The scalar component T_i encodes the *temporal* (electromagnetic) channel, while the spatial component $S_i\hat{\boldsymbol{\ell}}$ encodes the *spatial* (gravitational) channel.

In this static, isotropic setting all three imaginary units $\hat{\imath}, \hat{\jmath}, \hat{k}$ collapse to the single direction $\hat{\boldsymbol{\ell}}$. As a consequence:

- the scalar forms A and E remain nontrivial and will encode, respectively, interaction strengths and total energies;
- the vector form B is collinear with $\hat{\boldsymbol{\ell}}$ and plays the role of a unified "radial field";
- the vortical form C vanishes identically, reflecting the absence of static vorticity (no gravito–magnetic or magnetic components in the purely static configuration).

7.2 Scalar bilinear form and Newton–Coulomb potentials

Let two bodies B_1 and B_2 be described by dressed quaternions $\widetilde{\mathsf{q}}_1$ and $\widetilde{\mathsf{q}}_2$ of the form (78). The scalar bilinear form A from Section 2 reduces in this two-channel case to

$$A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2) = T_1 T_2 - S_1 S_2 = \lambda_E^2 \, q_1 q_2 - \lambda_G^2 \, m_1 m_2. \tag{80}$$

The temporal part $T_1 T_2$ corresponds to the electromagnetic channel; the spatial part $S_1 S_2$ corresponds to the gravitational channel. The opposite signs reflect the empirical fact that like electric charges repel while like masses attract.

Let $\boldsymbol{r}$ be the radius vector from B_1 to B_2 in the common rest frame, with magnitude $r = \|\boldsymbol{r}\|$. We define the static interaction potential between the two bodies by

$$V_{12}(r) := \frac{\kappa_{\text{int}}}{r} \, A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2), \tag{81}$$

where κ_{int} is a universal constant with units chosen so that $[\kappa_{\text{int}} A/r] = \mathrm{J}$.

Expanding (81) using (80) we obtain

$$V_{12}(r) = \frac{\kappa_{\text{int}} \lambda_E^2}{r} \, q_1 q_2 - \frac{\kappa_{\text{int}} \lambda_G^2}{r} \, m_1 m_2. \tag{82}$$

By choosing the dressing constants such that

$$\kappa_{\text{int}} \lambda_E^2 = \frac{1}{4\pi\epsilon_0}, \qquad \kappa_{\text{int}} \lambda_G^2 = G, \tag{83}$$

we recover the familiar Newton and Coulomb potentials in a single expression:

$$V_{12}(r) = \frac{1}{4\pi\epsilon_0} \, \frac{q_1 q_2}{r} - \frac{G \, m_1 m_2}{r}. \tag{84}$$

Structurally, (84) shows that:

- the static electromagnetic interaction arises from the *temporal* channel T_i of the dressed quaternions;
- the static gravitational interaction arises from the *spatial* channel S_i of the same quaternions;
- both are captured by the *same* scalar bilinear form A, once mass and charge are encoded into effective lengths via (79).

In this sense the Newton and Coulomb laws appear as two contributions of one and the same quaternionic scalar interaction.

7.3 Euclidean self-form and decomposition of total energy

The Euclidean self-form E of a single dressed body quaternion $\widetilde{\mathsf{q}}_i$ reads

$$E(\widetilde{\mathsf{q}}_i, \widetilde{\mathsf{q}}_i) = T_i^2 + S_i^2 = \lambda_E^2 q_i^2 + \lambda_G^2 m_i^2. \tag{85}$$

As in Section 5.3, we associate to this a rest-energy scale

$$E_{\text{rest},i} = \kappa_{\text{rest}} \sqrt{E(\widetilde{\mathsf{q}}_i, \widetilde{\mathsf{q}}_i)}, \tag{86}$$

which reduces to $m_i c^2$ in the purely gravitational limit after calibration.

For a two-body system with total dressed quaternion

$$\widetilde{\mathsf{q}}_{\text{tot}} := \widetilde{\mathsf{q}}_1 + \widetilde{\mathsf{q}}_2, \tag{87}$$

the Euclidean self-form expands as

$$E(\tilde{\mathsf{q}}_{\mathrm{tot}}, \tilde{\mathsf{q}}_{\mathrm{tot}}) = E(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_1) + E(\tilde{\mathsf{q}}_2, \tilde{\mathsf{q}}_2) + 2\, A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2). \tag{88}$$

Thus the cross term in the Euclidean norm is *precisely* twice the scalar bilinear form A that generates the interaction potential. Up to a universal conversion factor this cross term can be identified with the static interaction energy (81), while the diagonal terms represent the self-energies of the bodies.

This decomposition mirrors the role of the Hamiltonian in classical mechanics:

$$E_{\mathrm{tot}} = E_{\mathrm{rest},1} + E_{\mathrm{rest},2} + V_{12}(r), \tag{89}$$

with $V_{12}(r)$ directly inherited from $A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$ via (81). In the static GEM sector the Euclidean self-form E therefore plays the role of a Hamiltonian channel (total energy), while the scalar form A encodes the interaction part.

7.4 Radial field from the vector form in the static regime

Although the static configuration has no vortical component, the vector bilinear form B already carries useful information about the direction and relative strength of the gravito–electromagnetic field.

For dressed quaternions of the form (78) we have

$$\boldsymbol{B}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = B(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = (T_1 S_2 + T_2 S_1)\, \hat{\boldsymbol{\ell}} = \lambda_E \lambda_G\, (q_1 m_2 + q_2 m_1)\, \hat{\boldsymbol{\ell}}. \tag{90}$$

In the static, isotropic case we align $\hat{\boldsymbol{\ell}}$ with the radial unit vector $\hat{\boldsymbol{r}} := \boldsymbol{r}/r$ connecting the bodies, so that $\boldsymbol{B} \parallel \hat{\boldsymbol{r}}$. The magnitude of $\boldsymbol{B}$ combines temporal and spatial charges in a symmetric way; it will play the role of a unified "radial flux" in the dynamical GEM formulation, where relative motion and quaternionic D-rotations create proper electric and magnetic fields.

In the strictly static limit considered here the vortical form $\boldsymbol{C}$ is identically zero, because all spatial directions collapse to $\hat{\boldsymbol{\ell}}$ and no transverse circulation is possible:

$$\boldsymbol{C}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = \boldsymbol{0}. \tag{91}$$

This matches the physical expectation that static mass and charge distributions generate only scalar and radial vector fields, with no magnetic-like components.

7.5 Summary of the static GEM sector

To summarize, in the static, isotropic gravito–electromagnetic sector:

- Each Newtonian body is represented by a metrically dressed quaternion $\tilde{\mathsf{q}}_i = \lambda_E q_i\, \hat{h} + \lambda_G m_i\, \hat{\boldsymbol{\ell}}$, where the temporal channel encodes electric charge and the spatial channel encodes mass.
- The scalar bilinear form $A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$ generates, after division by r, the unified Newton–Coulomb potential $V_{12}(r)$ in (84).
- The Euclidean self-form E of the sum $\tilde{\mathsf{q}}_1 + \tilde{\mathsf{q}}_2$ splits into two self-energy terms and an interaction term proportional to $A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$, and plays the role of a Hamiltonian channel.
- The vector form B reduces to a radial vector proportional to $\hat{\boldsymbol{r}}$ and combines mass and charge symmetrically, while the vortical form C vanishes, reflecting the absence of static gravito–magnetic and magnetic fields.

In the next section we will turn on relative motion and quaternionic D-rotations of the dressed body quaternions. This will lift the degeneracy of the vector forms: B and C will then separate into gravito–electric and gravito–magnetic parts, providing a quaternionic analogue of the Maxwell–Heaviside formulation of gravito–electromagnetism.

8 Quaternionic GEM field in the observable three–space

The previous section showed how the scalar and Euclidean bilinear forms A and E acting on metrically dressed body quaternions reproduce the static Newton and Coulomb potentials in a two–body configuration. We now pass from this two–body picture to a field picture in the observable three–space, and define a quaternion–valued gravito–electromagnetic (GEM) field whose bilinear couplings to a test body generate forces and energies.

8.1 Observable frame and 3+1 splitting

We fix an inertial "laboratory" frame with coordinate time t and Euclidean three–space $\mathbb{R}^3$ with orthonormal basis $\{\hat{e}_i\}_{i=1}^3$. The observer's temporal axis $\hat{e}_0$ defines a splitting of the proto–space flow into temporal and spatial parts, as in Section 4, and all observable quantities are projected onto this frame.

Each Newtonian body B_i is characterized by its worldline $\boldsymbol{x}_i(t) \in \mathbb{R}^3$, gravitational mass m_i, charge q_i and a metrically dressed quaternion $\widetilde{\mathsf{q}}_i$ of the form (78) in its own rest frame. In the laboratory frame, where B_i may move with some three–velocity $\boldsymbol{v}_i(t)$, the corresponding dressed quaternion is obtained by a quaternionic D–rotation,

$$\widetilde{\mathsf{q}}_i^{\mathrm{lab}}(t) := D_i(t)\,\widetilde{\mathsf{q}}_i\,D_i(t), \tag{92}$$

where $D_i(t)$ is the unimetrical rotor associated with the boost between the rest frame of B_i and the laboratory frame (see **??**).

In this section we concentrate on the quasi–static regime where the velocities are small and retardation can be neglected. The D–rotations will then mainly serve to generate the leading gravito–magnetic and magnetic corrections via the vector bilinear forms.

8.2 Local GEM field of a single static source

We first consider a single static source B_s at position $\boldsymbol{x}_s$ with dressed quaternion $\widetilde{\mathsf{q}}_s = T_s\hat{h} + S_s\hat{\boldsymbol{\ell}}$ as in (78), where $\hat{\boldsymbol{\ell}}$ is a free unit imaginary aligned with the radial direction from the source.

For a point $\boldsymbol{x} \in \mathbb{R}^3$ with radius vector $\boldsymbol{r} := \boldsymbol{x} - \boldsymbol{x}_s$ and $r := \|\boldsymbol{r}\|$, we define the local *static GEM field quaternion* of the source by

$$\mathsf{F}_s(\boldsymbol{x}) := \frac{\kappa_{\mathrm{f}}}{r^2}\,(T_s\,\hat{h} + S_s\,\widehat{\boldsymbol{r}}), \qquad \widehat{\boldsymbol{r}} := \frac{\boldsymbol{r}}{r}, \tag{93}$$

where κ_{f} is a universal field calibration constant.

The structure of (93) mirrors that of the standard Coulomb and Newton fields: the temporal component T_s/r^2 produces a radial electric–like field, while the spatial component $S_s\widehat{\boldsymbol{r}}/r^2$ produces a radial gravito–electric field. The use of the free unit imaginary $\widehat{\boldsymbol{r}}$ ensures that the field is always aligned with the radius vector, independently of the internal orientation of the source in proto–space.

A test body B_t at $\boldsymbol{x}$ with dressed quaternion $\widetilde{\mathsf{q}}_t = T_t\hat{h} + S_t\,\widehat{\boldsymbol{r}}$ then couples to $\mathsf{F}_s(\boldsymbol{x})$ through the scalar and vector bilinear forms:

$$A\big(\mathsf{F}_s(\boldsymbol{x}), \widetilde{\mathsf{q}}_t\big) = \frac{\kappa_{\mathrm{f}}}{r^2}(T_sT_t - S_sS_t), \tag{94}$$

$$\boldsymbol{B}\big(\mathsf{F}_s(\boldsymbol{x}), \widetilde{\mathsf{q}}_t\big) = \frac{\kappa_{\mathrm{f}}}{r^2}(T_sS_t + T_tS_s)\,\widehat{\boldsymbol{r}}, \tag{95}$$

$$\boldsymbol{C}\big(\mathsf{F}_s(\boldsymbol{x}), \widetilde{\mathsf{q}}_t\big) = \boldsymbol{0}, \tag{96}$$

where we used the one–dimensional spatial structure ($\hat{\boldsymbol{\ell}} \parallel \widehat{\boldsymbol{r}}$) to conclude that the vortical form $\boldsymbol{C}$ vanishes.

Up to universal calibration factors, the scalar channel $A(\mathsf{F}_s, \widetilde{\mathsf{q}}_t)$ reproduces the sum of the Newton and Coulomb field energies at the position of the test body, while the vector channel $\boldsymbol{B}(\mathsf{F}_s, \widetilde{\mathsf{q}}_t)$ yields a radial force–like quantity combining gravitational and electrostatic contributions. The absence of a nontrivial $\boldsymbol{C}$ in the static case reflects the absence of static magnetic or gravito–magnetic fields.

8.3 Superposition principle and total static field

For a collection of static sources B_i at positions $\boldsymbol{x}_i$ with dressed quaternions $\widetilde{\mathsf{q}}_i$, we define the total static GEM field at $\boldsymbol{x}$ by linear superposition,

$$\mathsf{F}(\boldsymbol{x}) := \sum_i \mathsf{F}_i(\boldsymbol{x}), \qquad \mathsf{F}_i(\boldsymbol{x}) := \frac{\kappa_{\mathrm{f}}}{r_i^2}\,(T_i\,\hat{h} + S_i\,\widehat{\boldsymbol{r}}_i), \tag{97}$$

with $\boldsymbol{r}_i := \boldsymbol{x} - \boldsymbol{x}_i$, $r_i := \|\boldsymbol{r}_i\|$, $\widehat{\boldsymbol{r}}_i := \boldsymbol{r}_i/r_i$.

A test body with dressed quaternion $\widetilde{\mathsf{q}}_t$ then experiences:

- a scalar field channel
$$A(\mathsf{F}(\boldsymbol{x}), \widetilde{\mathsf{q}}_t) = \sum_i A(\mathsf{F}_i(\boldsymbol{x}), \widetilde{\mathsf{q}}_t), \tag{98}$$
which, after appropriate calibration, coincides with the usual sum of Newton and Coulomb field energies;
- a radial vector field channel
$$\boldsymbol{B}(\mathsf{F}(\boldsymbol{x}), \widetilde{\mathsf{q}}_t) = \sum_i \boldsymbol{B}(\mathsf{F}_i(\boldsymbol{x}), \widetilde{\mathsf{q}}_t), \tag{99}$$
representing the static gravito–electric plus electrostatic force density at $\boldsymbol{x}$.

In this way the quaternionic field $\mathsf{F}(\boldsymbol{x})$ collects in a single object the information about the scalar potential and the radial field lines of the combined gravito–electromagnetic configuration in the observable three–space.

8.4 Turning on motion: D–rotations and vortical channel

When the sources and/or the test body move with respect to the laboratory frame, the corresponding dressed quaternions acquire nontrivial spatial orientations through the D–rotations in (92). As soon as two dressed quaternions with different spatial directions enter a bilinear form, the vortical channel $\boldsymbol{C}$ becomes nonzero.

To leading order in velocity, the static field expression (93) may still be used, but the dressed quaternions of the moving sources are boosted before entering the bilinear combinations. For a moving source B_s the field at $\boldsymbol{x}$ is

$$\mathsf{F}_s^{\mathrm{lab}}(\boldsymbol{x}, t) := \frac{\kappa_{\mathrm{f}}}{r^2}\,(T_s^{\mathrm{lab}}(t)\,\hat{h} + \mathbf{S}_s^{\mathrm{lab}}(t, \boldsymbol{x})), \tag{100}$$

where the spatial part $\mathbf{S}_s^{\mathrm{lab}}$ is no longer constrained to be parallel to $\widehat{\boldsymbol{r}}$. The bilinear forms with a test body then produce:

$$\boldsymbol{B}(\mathsf{F}_s^{\mathrm{lab}}, \widetilde{\mathsf{q}}_t^{\mathrm{lab}}) \sim \text{gravito–electric and electric fields}, \tag{101}$$
$$\boldsymbol{C}(\mathsf{F}_s^{\mathrm{lab}}, \widetilde{\mathsf{q}}_t^{\mathrm{lab}}) \sim \text{gravito–magnetic and magnetic fields}, \tag{102}$$

with the precise identification depending on the chosen calibration.

Dimensionally, the vortical channel $\boldsymbol{C}$ differs from the radial channel $\boldsymbol{B}$ by a factor of the flow speed. In the unimetrical calibration where the flow speed is fixed by the speed of light c, it is natural to define the *physical* magnetic–like field as

$$\boldsymbol{\mathcal{B}}_{\text{phys}} := c\,\boldsymbol{C}, \tag{103}$$

so that $\boldsymbol{\mathcal{B}}_{\text{phys}}$ has the same units as the gravito–electric and electric field obtained from $\boldsymbol{B}$ and couples to currents with the usual Lorentz–like structure. In this sense the unimetrical picture explains the appearance of the factor c in the relation between electric and magnetic fields as a consequence of the different geometric nature of the radial and vortical bilinear channels.

8.5 Quaternionic GEM field as a 3D observable

From the point of view of an observer living in the three–space $\mathbb{R}^3$, the quaternionic GEM field $\mathsf{F}(\boldsymbol{x}, t)$ provides a compact encoding of all static and quasi–static gravitational and electromagnetic effects:

- its scalar bilinear coupling $A(\mathsf{F}, \widetilde{\mathsf{q}}_t)$ yields potential energies;
- its vector bilinear coupling $\boldsymbol{B}(\mathsf{F}, \widetilde{\mathsf{q}}_t)$ yields gravito–electric and electric field strengths;
- its vortical bilinear coupling $\boldsymbol{C}(\mathsf{F}, \widetilde{\mathsf{q}}_t)$, rescaled by c, yields gravito–magnetic and magnetic field strengths.

All three channels arise from the same underlying quaternionic structure and the same dressed body quaternions; the distinction between gravitational and electromagnetic phenomena is traced to the temporal vs spatial channels of the dressing, not to different kinds of interaction at the algebraic level.

In the next section we will formulate dynamical equations for $\mathsf{F}(\boldsymbol{x}, t)$ and show how a Maxwell–Heaviside type system for the GEM field emerges from unimetrical conservation laws for the streamlet flow.

9 D-rotors and relativistic dynamics of GEM

In the static setting of Sections 7 and 8, the gravito–electromagnetic (GEM) interaction is encoded entirely in the scalar and vector bilinear forms A and $\boldsymbol{B}$ on metrically dressed body quaternions, with the vortical form $\boldsymbol{C}$ vanishing. Relativistic dynamics enters Unimetry through the *D-rotors* — special unit quaternions implementing real–imaginary rotations between the temporal and spatial channels of the flow. In this section we recall their structure and show how they generate the relativistic dynamics of the GEM field and its coupling to moving bodies.

9.1 Unimetrical D-rotors as real–imaginary rotations

Let $\{\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}\}$ be the orthonormal quaternion basis as in Section 2. A *D-rotation* is an orthogonal transformation of $\mathbb{H}$ which mixes the real unit $\hat{h}$ with a single imaginary unit $\hat{\boldsymbol{\ell}}$ and leaves the orthogonal imaginary directions unchanged. Geometrically, this is a rotation in the Euclidean plane spanned by the temporal axis $\hat{h}$ and the spatial direction $\hat{\boldsymbol{\ell}}$.

In matrix form, acting on the basis $(\hat{h}, \hat{\boldsymbol{\ell}}, \hat{\boldsymbol{e}}_\perp, \dots)$, the rotation by an angle ϑ is:

$$D(\vartheta) = \begin{pmatrix} \cos\vartheta & \sin\vartheta & 0 & 0 \\ -\sin\vartheta & \cos\vartheta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{104}$$

In the quaternionic algebra, this transformation is implemented by the "double-sided" product with a unit quaternion $\hat{\mathbf{d}}$. Crucially, to generate a rotation of angle ϑ in the $\hat{h}$-$\hat{\boldsymbol{\ell}}$ plane, the generator $\hat{\mathbf{d}}$ must carry the *half-angle*:

$$\mathbf{D}(\vartheta): \ \mathbf{q} \mapsto \hat{\mathbf{d}}\,\mathbf{q}\,\hat{\mathbf{d}}, \qquad \hat{\mathbf{d}} = \cos\frac{\vartheta}{2}\,\hat{h} + \sin\frac{\vartheta}{2}\,\hat{\boldsymbol{\ell}}. \tag{105}$$

Direct calculation shows that $\hat{\mathbf{d}}\,\hat{h}\,\hat{\mathbf{d}} = \hat{\mathbf{d}}^2 = \cos\vartheta\,\hat{h} + \sin\vartheta\,\hat{\boldsymbol{\ell}}$, recovering the matrix action (116).

The D-rotation preserves the Euclidean self-form E (the total flow magnitude):

$$E(\mathbf{D}(\mathbf{q}_1), \mathbf{D}(\mathbf{q}_2)) = E(\mathbf{q}_1, \mathbf{q}_2) \qquad \forall\, \mathbf{q}_1, \mathbf{q}_2 \in \mathbb{H}, \tag{106}$$

confirming it is an isometry of the Euclidean proto-space. However, the scalar form A (the interaction channel) is *not* preserved. Instead, it transforms as:

$$A(\mathbf{D}(\mathbf{q}_1), \mathbf{D}(\mathbf{q}_2)) = A(\mathbf{q}_1, \mathbf{D}^2(\mathbf{q}_2)), \tag{107}$$

where $\mathbf{D}^2$ corresponds to a rotation by 2ϑ in the active view. This modification of the scalar channel under boosts is precisely what generates the γ-factors and time dilation effects in the unimetrical projection.

We parametrize the D-rotation by the kinematic angle ϑ associated with the velocity v relative to the observer. Using the unimetrical calibration $\|\boldsymbol{\chi}\| = c$:

$$\beta := \frac{v}{c} = \sin\vartheta, \qquad \gamma := \frac{1}{\cos\vartheta}. \tag{108}$$

Note that γ appears here as a trigonometric secant of the Euclidean rotation angle, ensuring the formalism remains purely geometric.

9.2 Boosting dressed body quaternions

In Section 7 a static Newtonian body B_i is represented by

$$\widetilde{\mathsf{q}}_i = T_i\,\hat{h} + S_i\,\hat{\boldsymbol{\ell}}_i, \qquad T_i = \lambda_E q_i, \quad S_i = \lambda_G m_i. \tag{109}$$

In a laboratory frame where B_i moves with velocity $\boldsymbol{v}_i$, the dressed quaternion becomes

$$\widetilde{\mathsf{q}}_i^{\text{lab}} := \mathbf{D}_i\,\widetilde{\mathsf{q}}_i\,\mathbf{D}_i, \qquad \mathbf{D}_i = \mathbf{D}(\vartheta_i, \widehat{\boldsymbol{v}}_i), \tag{110}$$

where $\widehat{\boldsymbol{v}}_i$ provides the axis of the D-rotation. Physically:

- The *intrinsic* invariants (rest mass, charge) are protected by the invariance of the Euclidean form E.
- The *projected* components change: the temporal part T_i feeds into the spatial vector channel (creating currents), and the spatial part S_i feeds into the temporal channel.

9.3 Field quaternions under D-rotors

The GEM field quaternion $\mathsf{F}(\boldsymbol{x}, t)$ transforms under a change of frame via conjugation by the observer's D-rotor $\mathbf{D}_{\text{obs}}$:

$$\mathsf{F}'(\boldsymbol{x}', t') = \mathbf{D}_{\text{obs}}\,\mathsf{F}(\boldsymbol{x}, t)\,\mathbf{D}_{\text{obs}}. \tag{111}$$

Writing $\mathsf{F} = \Phi\,\hat{h} + \mathcal{E} + \mathcal{C}$ (where $\mathcal{E}$ is symmetric/radial and $\mathcal{C}$ is vortical), this transformation mixes $\mathcal{E}$ and $\mathcal{C}$ exactly as Lorentz boosts mix electric and magnetic fields. A pure radial field in one frame appears as a superposition of radial and vortical fields in a boosted frame.

9.4 Relativistic GEM force via bilinear couplings

For a test body B_t moving with rotor $\mathbf{D}_t$, the effective dressed quaternion is $\widetilde{\mathsf{q}}_t^{\mathrm{lab}} = \mathbf{D}_t \widetilde{\mathsf{q}}_t \mathbf{D}_t$. The force channels are defined by the bilinear forms with the local field F:

$$\mathcal{P} := A(\mathsf{F}, \widetilde{\mathsf{q}}_t^{\mathrm{lab}}) \quad \text{(Power channel)}, \tag{112}$$

$$\boldsymbol{\mathcal{F}}_B := \boldsymbol{B}(\mathsf{F}, \widetilde{\mathsf{q}}_t^{\mathrm{lab}}) \quad \text{(Radial/Symm. force)}, \tag{113}$$

$$\boldsymbol{\mathcal{F}}_C := \boldsymbol{C}(\mathsf{F}, \widetilde{\mathsf{q}}_t^{\mathrm{lab}}) \quad \text{(Vortical force)}. \tag{114}$$

The vortical channel $\boldsymbol{\mathcal{F}}_C$ is identically zero for static bodies but activates for moving ones. Dimensional analysis suggests the identification of the physical magnetic-like force density as:

$$\boldsymbol{\mathcal{F}}_{\mathrm{mag}} := c\,\boldsymbol{\mathcal{F}}_C. \tag{115}$$

In the low-velocity limit, $\boldsymbol{\mathcal{F}}_C$ reproduces the structure $q(\boldsymbol{v} \times \boldsymbol{B})$, confirming that the magnetic force is the manifestation of the antisymmetry of the quaternion product (the $\boldsymbol{C}$-form) acting on boosted flow components.

9.5 D-rotor fields and Unimetric equivalence principle

Gravity can be essentially described as a field of D-rotors $\mathbf{D}(\boldsymbol{x})$ acting on the local time direction. Free fall corresponds to a trajectory that follows the "straight" flow in proto-space, meaning the body's internal rotor is parallel to the local background rotor field. The equivalence principle arises because a spatially varying D-rotor field (gravity) is locally indistinguishable from the time-varying D-rotor of an accelerated observer.

10 D-rotors and relativistic dynamics of GEM

In the static setting of Sections 7 and 8 the gravito–electromagnetic (GEM) interaction is encoded entirely in the scalar and vector bilinear forms A and B on metrically dressed body quaternions, with the vortical form C vanishing. Relativistic dynamics enters Unimetry through the *D-rotors* — special unit quaternions implementing real–imaginary rotations between the temporal and spatial channels of the flow. In this section we recall their structure and show how they generate the relativistic dynamics of the GEM field and its coupling to moving bodies.

10.1 Unimetrical D-rotors as real–imaginary rotations

Let $\{\hat{h}, \hat{\imath}, \hat{\jmath}, \hat{k}\}$ be the orthonormal quaternion basis as in Section 2. A *D-rotation* is an orthogonal transformation of $\mathbb{H}$ which mixes the real unit $\hat{h}$ with a single imaginary unit $\hat{\boldsymbol{\ell}}$ and leaves the orthogonal imaginary directions unchanged. In matrix form its action on the basis can be written as

$$D(\psi) = \begin{pmatrix} \cos\psi & \sin\psi & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \det D(\psi) = 1, \tag{116}$$

where the first two rows/columns correspond to $\hat{h}$ and $\hat{\boldsymbol{\ell}}$, and the remaining ones to the transverse imaginary basis vectors.

Equivalently, one can represent the same transformation by a unit quaternion $\hat{\mathbf{d}}$ acting by conjugation:

$$\mathbf{D} : \ \mathbf{q} \mapsto \hat{\mathbf{d}}\,\mathbf{q}\,\hat{\mathbf{d}}, \qquad \|\hat{\mathbf{d}}\| = 1. \tag{117}$$

In this representation, $\hat{\mathbf{d}}$ implements a rotation in the $(\hat{h}, \hat{\boldsymbol{\ell}})$-plane, and the equality between (116) and (117) can be checked explicitly by applying $\mathbf{D}$ to the basis vectors.

The crucial property of the D-rotation is that it preserves the Euclidean self-form E:

$$E(\mathbf{D}(\mathbf{q}_1), \mathbf{D}(\mathbf{q}_2)) = E(\mathbf{q}_1, \mathbf{q}_2) \qquad \forall\, \mathbf{q}_1, \mathbf{q}_2 \in \mathbb{H}, \tag{118}$$

hence it is an orthogonal transformation in the sense of the Euclidean metric. In contrast, the scalar form A is not preserved; one finds

$$A(\mathbf{D}(\mathbf{q}_1), \mathbf{D}(\mathbf{q}_2)) = A(\mathbf{q}_1, \mathbf{D}^2(\mathbf{q}_2)) = A(\mathbf{D}^2(\mathbf{q}_1), \mathbf{q}_2), \tag{119}$$

so that the effect of a D-rotation on the scalar channel is equivalent to applying the *square* of the D-operator to one of the arguments. This “double” action will be responsible for the familiar Lorentz γ-factors in the kinematic sector.

As in the purely kinematic unimetrical formulation of special relativity, we parametrize the D-rotation angle ψ by the kinematic angle ϑ associated with the velocity v of the body with respect to a chosen observer. In the calibration where the Euclidean flow speed equals the speed of light ($\|\boldsymbol{\chi}\| = c$), the usual relations

$$\beta := \frac{v}{c} = \sin\vartheta, \qquad \gamma := \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\cos\vartheta} \tag{120}$$

allow us to identify the D-rotor angle with ϑ. The D-rotor is purely dimensionless; the factor c enters only through this calibration, not through the rotor itself.

10.2 Boosting dressed body quaternions

In Section 7 each Newtonian body B_i was represented in its rest frame by a metrically dressed quaternion

$$\widetilde{\mathsf{q}}_i = T_i\, \hat{h} + S_i\, \hat{\boldsymbol{\ell}}_i, \qquad T_i = \lambda_E q_i, \quad S_i = \lambda_G m_i, \tag{121}$$

where $\hat{\boldsymbol{\ell}}_i$ is the free unit imaginary aligned with the radial direction of interaction in the static configuration. In a laboratory frame where B_i moves with velocity $\boldsymbol{v}_i$, its dressed quaternion becomes

$$\widetilde{\mathsf{q}}_i^{\mathrm{lab}} := \mathbf{D}_i\, \widetilde{\mathsf{q}}_i\, \mathbf{D}_i, \qquad \mathbf{D}_i = \mathbf{D}(\vartheta_i, \widehat{\boldsymbol{v}}_i), \tag{122}$$

with ϑ_i related to $\boldsymbol{v}_i$ as above and $\widehat{\boldsymbol{v}}_i$ the direction of motion in the observer's three-space.

Because of (118), the Euclidean self-form — and hence the rest-energy scale $\sqrt{E(\widetilde{\mathsf{q}}_i, \widetilde{\mathsf{q}}_i)}$ — is invariant under the boost:

$$E(\widetilde{\mathsf{q}}_i^{\mathrm{lab}}, \widetilde{\mathsf{q}}_i^{\mathrm{lab}}) = E(\widetilde{\mathsf{q}}_i, \widetilde{\mathsf{q}}_i), \tag{123}$$

while the scalar form $A(\cdot,\cdot)$ and the vector forms B and C are modified according to (119) and the corresponding vectorial identities. Physically this means that:

- the internal structure of the body (encoded in the dressing) is unaffected by the boost;
- the decomposition of the flow between temporal and spatial channels changes, adjusting the effective temporal and spatial components seen by the laboratory observer;
- the cross-terms between bodies acquire γ- and β-dependent factors, reproducing relativistic time dilation and the emergence of magnetic- and gravito–magnetic contributions.

10.3 Field quaternions under D-rotors

Let $\mathsf{F}(\boldsymbol{x}, t)$ be the quaternionic GEM field in the laboratory frame as defined in Section 8. A change of inertial frame to an observer related by a D-rotation with rotor $\mathbf{D}$ acts on the field quaternion exactly as on any other quaternionic quantity:

$$\mathsf{F}'(\boldsymbol{x}', t') = \mathbf{D}\, \mathsf{F}(\boldsymbol{x}, t)\, \mathbf{D}, \tag{124}$$

with $(\boldsymbol{x}', t')$ related to $(\boldsymbol{x}, t)$ by the same unimetrical kinematics that underlie the SR reformulation.

Writing the field in the laboratory frame as

$$\mathsf{F} = \Phi\, \hat{h} + \mathcal{E} \cdot \hat{\boldsymbol{\ell}} + \mathcal{C}, \tag{125}$$

where:

- Φ is the scalar potential-like part;
- $\mathcal{E}$ is the radial gravito–electric plus electric field (associated mainly with the B-form);
- $\mathcal{C}$ is the vortical part (associated with the C-form),

the transformed field F' will in general have mixtures of these components, exactly as in the standard Lorentz transformation of electric and magnetic fields. In particular:

- a purely radial field in one frame acquires a vortical component in a boosted frame;
- the relative weights of the radial and vortical channels are governed by β and γ, coming from the D-rotor angle ϑ.

The key advantage of the quaternionic formulation is that both gravito–electric and electrostatic fields, as well as their magnetic counterparts, transform via the *same* conjugation rule (124).

10.4 Relativistic GEM force via bilinear couplings

Consider now a test body B_t with dressed quaternion $\widetilde{\mathsf{q}}_t$ and kinematic D-rotor $\mathbf{D}_t$ corresponding to its motion in the laboratory frame. The *effective* dressed quaternion in the lab frame is $\widetilde{\mathsf{q}}_t^{\mathrm{lab}} = \mathbf{D}_t\, \widetilde{\mathsf{q}}_t\, \mathbf{D}_t$. The local field felt by the test body is $\mathsf{F}(\boldsymbol{x}_t, t)$. We define the GEM force channels by the bilinear couplings:

$$\mathcal{P} := A\big(\mathsf{F}(\boldsymbol{x}_t, t), \widetilde{\mathsf{q}}_t^{\mathrm{lab}}\big), \tag{126}$$

$$\boldsymbol{\mathcal{F}}_B := \boldsymbol{B}\big(\mathsf{F}(\boldsymbol{x}_t, t), \widetilde{\mathsf{q}}_t^{\mathrm{lab}}\big), \tag{127}$$

$$\boldsymbol{\mathcal{F}}_C := \boldsymbol{C}\big(\mathsf{F}(\boldsymbol{x}_t, t), \widetilde{\mathsf{q}}_t^{\mathrm{lab}}\big). \tag{128}$$

Here:

- $\mathcal{P}$ is a scalar channel which, after calibration, can be identified with the instantaneous power input to the test body;
- $\boldsymbol{\mathcal{F}}_B$ is a radial force channel arising from the B-form and reproducing, in the nonrelativistic limit, the sum of the Newton and Coulomb forces;
- $\boldsymbol{\mathcal{F}}_C$ is a vortical force channel arising from the C-form and reproducing magnetic- and gravito–magnetic forces.

Dimensionally, $\boldsymbol{\mathcal{F}}_C$ differs from $\boldsymbol{\mathcal{F}}_B$ by a factor of the flow speed. In the unimetrical calibration where the Euclidean flow speed is fixed at c, it is natural to define the physical magnetic-like force density by

$$\boldsymbol{\mathcal{F}}_{\mathrm{mag}} := c\,\boldsymbol{\mathcal{F}}_C. \tag{129}$$

In the low-velocity limit ($\beta \ll 1$), the radial channel then yields

$$\boldsymbol{\mathcal{F}}_B \;\longrightarrow\; -\frac{G\,m_s m_t}{r^2}\,\widehat{\boldsymbol{r}} + \frac{1}{4\pi\epsilon_0}\,\frac{q_s q_t}{r^2}\,\widehat{\boldsymbol{r}}, \tag{130}$$

while the vortical channel reduces to the familiar Lorentz-like form

$$\boldsymbol{\mathcal{F}}_{\mathrm{mag}} \;\longrightarrow\; q_t\,\boldsymbol{v}_t \times \boldsymbol{B} \quad \text{(and its gravito–magnetic analogue)}, \tag{131}$$

with $\boldsymbol{B}$ extracted from $\boldsymbol{C}$ via the identification $\boldsymbol{B}_{\mathrm{phys}} = c\,\boldsymbol{C}$.

Thus the quaternionic bilinear couplings to the GEM field reproduce the structure of the relativistic Lorentz force in both electromagnetic and gravito–electromagnetic sectors, while keeping the underlying algebraic machinery unified.

10.5 D-rotor fields and Unimetric equivalence principle

Finally, we briefly comment on the role of spatially varying D-rotors. In Unimetry, a purely gravitational field can be viewed as a field of D-rotors $\mathbf{D}(\boldsymbol{x})$ acting on the temporal direction of the observer and on the body quaternions. In this picture:

- a uniform gravitational field corresponds to a linear variation of the D-rotor angle with position, mimicking a constant acceleration;
- free fall is represented by a trajectory along which the local D-rotor remains constant in the comoving frame — the object follows the locally "straight" flow in proto-space;
- the equivalence between acceleration and gravity appears as the equivalence between a time-dependent D-rotor in a flat GEM field and a spatially varying D-rotor in a static field.

The GEM field quaternion $\mathsf{F}(\boldsymbol{x}, t)$ and the D-rotor field $\mathbf{D}(\boldsymbol{x}, t)$ are thus two complementary descriptions of the same underlying unimetrical dynamics: F encodes the local gravito–electromagnetic stresses in the observable three-space, while $\mathbf{D}$ encodes how the local notion of time and space is rotated with respect to a global proto-space flow.

A full dynamical formulation would express the evolution of $\mathsf{F}(\boldsymbol{x}, t)$ and $\mathbf{D}(\boldsymbol{x}, t)$ through unimetrical conservation laws for streamlets, leading to a Maxwell–Heaviside–type system for GEM in the three-space. Here we have restricted ourselves to outlining how relativistic kinematics enters the GEM sector via D-rotors and how the Lorentz structure of forces emerges from the quaternionic bilinear forms.

11 Quaternionic GEM field equations in three–space

In the preceding sections we introduced metrically dressed body quaternions, the bilinear forms A, B, C, E, the static gravito–electromagnetic (GEM) interaction, and the relativistic kinematics of bodies and fields via D-rotors. We now outline how a Maxwell–Heaviside–type system of field equations for the quaternionic GEM field $\mathbf{F}(\boldsymbol{x}, t)$ can be formulated in the observable three–space, starting from conservation of streamlet flow.

Our goal here is structural rather than exhaustive: we emphasize how the usual Maxwell and gravito–electromagnetic equations emerge as different projections of a single quaternionic differential relation.

11.1 Quaternionic differential operator and source current

We work in a fixed laboratory frame with coordinate time t and Euclidean three–space $\mathbb{R}^3$ with orthonormal basis $\{\hat{e}_i\}_{i=1}^3$. Following the spirit of Sections 2 and 8, we introduce a quaternionic differential operator

$$\mathcal{D} := \hat{h}\,\frac{\partial}{\partial t} + \sum_{i=1}^{3} \hat{e}_i\,\frac{\partial}{\partial x^i}, \tag{132}$$

which acts on quaternion-valued fields by left multiplication and ordinary differentiation of components.

The total source current is represented by a quaternionic field $\mathbf{J}(\boldsymbol{x}, t)$ that combines mass and charge densities and their currents:

$$\mathbf{J} := \rho_E\,\hat{h} + \mathbf{j}_E + \rho_G\,\hat{\mathbf{u}} + \mathbf{j}_G, \tag{133}$$

where

- $\rho_E(\boldsymbol{x}, t)$ is the electric charge density;
- $\mathbf{j}_E(\boldsymbol{x}, t)$ is the electric current density vector (expressed as a pure imaginary quaternion);
- $\rho_G(\boldsymbol{x}, t)$ is the mass density ("gravitational charge");
- $\mathbf{j}_G(\boldsymbol{x}, t)$ is the mass current density (momentum density), again as a pure imaginary quaternion;
- $\hat{\mathbf{u}}$ is the free unit imaginary introduced earlier, which in isotropic configurations aligns with radial directions of interaction.

The conservation of total streamlet flow in the laboratory frame is then encoded in the quaternionic continuity equation

$$A(\mathcal{D}, \mathbf{J}) = 0, \tag{134}$$

whose expansion yields the usual continuity equations for mass and charge:

$$\frac{\partial \rho_E}{\partial t} + \nabla \cdot \mathbf{j}_E = 0, \tag{135}$$

$$\frac{\partial \rho_G}{\partial t} + \nabla \cdot \mathbf{j}_G = 0. \tag{136}$$

Here ∇ is the standard gradient in $\mathbb{R}^3$.

11.2 Quaternionic GEM field equation

We now postulate that the quaternionic GEM field $\mathbf{F}(\boldsymbol{x}, t)$ satisfies a first-order differential equation driven by the source current:

$$\mathcal{D} \circ \mathbf{F} = \mathbf{J}, \tag{137}$$

where $\circ$ denotes quaternion multiplication as in Section 2. This is the unimetrical analogue of the inhomogeneous Maxwell equations, combining both gravito–electric and electro–magnetic sectors in a single quaternionic relation.

A second equation of "Bianchi type" is obtained by demanding that the GEM field derives from an underlying quaternionic potential $\mathbf{A}$:

$$\mathbf{F} = \mathcal{D} \circ \mathbf{A}, \tag{138}$$

which implies the homogeneous constraint

$$\mathcal{D} \circ \mathbf{F} = \mathcal{D} \circ \mathcal{D} \circ \mathbf{A} \equiv 0 \qquad \Rightarrow \qquad \mathcal{D} \circ \mathbf{F} = \mathbf{J} \implies \mathcal{D} \circ \mathbf{J} = 0. \tag{139}$$

Thus the continuity equation (134) is automatically satisfied if $\mathbf{F}$ is derived from a potential $\mathbf{A}$.

The pair (137)–(138) is the quaternionic GEM analogue of Maxwell's system $dF = 0$, $d{\star}F = J$ in the language of differential forms.

11.3 3+1 decomposition: gravito–electric and magnetic sectors

To connect (137) with familiar field equations we decompose $\mathbf{F}$ into temporal, radial and vortical parts as in Section 8:

$$\mathbf{F} = \Phi\,\hat{h} + \mathcal{E}_E + \mathcal{E}_G + \mathcal{C}, \tag{140}$$

where, schematically,

- Φ is a scalar potential-like component;
- $\mathcal{E}_E$ is the electric field (temporal channel, sourced by $\rho_E, \mathbf{j}_E$);
- $\mathcal{E}_G$ is the gravito–electric field (spatial channel, sourced by $\rho_G, \mathbf{j}_G$);
- $\mathcal{C}$ is the vortical part associated with the vector bilinear form C and will give rise, after calibration, to magnetic and gravito–magnetic fields.

Inserting (132) and (140) into (137) and collecting the scalar and vector parts yields, after projection on the temporal (electric) and spatial (gravitational) channels, the following schematic system:

$$\nabla \cdot \mathcal{E}_E = \frac{\rho_E}{\epsilon_0}, \qquad \nabla \cdot \mathcal{E}_G = -\,4\pi G\,\rho_G, \tag{141}$$

$$\nabla \times \mathcal{C}_E - \frac{1}{c^2}\frac{\partial \mathcal{E}_E}{\partial t} = \mu_0\,\mathbf{j}_E, \qquad \nabla \times \mathcal{C}_G - \frac{1}{c^2}\frac{\partial \mathcal{E}_G}{\partial t} = -\,\frac{4\pi G}{c^2}\,\mathbf{j}_G, \tag{142}$$

where $\mathcal{C}_E$ and $\mathcal{C}_G$ are the electric and gravito–magnetic parts of $\mathcal{C}$ selected by projecting on the temporal and spatial dressing channels.

Similarly, the homogeneous constraint arising from (138) gives Bianchi-type identities

$$\nabla \cdot \mathcal{C}_E = 0, \qquad \nabla \cdot \mathcal{C}_G = 0, \tag{143}$$

$$\nabla \times \mathcal{E}_E + \frac{\partial \mathcal{C}_E}{\partial t} = 0, \qquad \nabla \times \mathcal{E}_G + \frac{\partial \mathcal{C}_G}{\partial t} = 0. \tag{144}$$

In this way the standard Maxwell equations and their gravito–electromagnetic analogue appear simply as different projections of the quaternionic field equation (137).

The calibration constants ϵ_0, μ_0 and G are related to the dressing constants λ_E, λ_G and the global unimetrical scale by the same relations as in Sections 7.2 and 8; structurally they are conversion factors between the dimensionless quaternionic forms and laboratory units.

11.4 Covariance under D-rotors and propagation

Because both $\mathcal{D}$ and $\mathbf{F}$ transform by conjugation under D-rotors, the quaternionic field equation (137) is covariant under unimetrical boosts:

$$\mathbf{D}\,(\mathcal{D} \circ \mathbf{F})\,\mathbf{D} = \mathcal{D}' \circ \mathbf{F}' = \mathbf{J}', \tag{145}$$

where primed quantities are measured in the boosted frame. This ensures that the projected 3+1 system (141)–(144) reproduces the familiar Lorentz transformation properties of electric, magnetic, gravito–electric and gravito–magnetic fields.

Furthermore, in charge- and mass-free regions ($\mathbf{J} = 0$) the field equation reduces to

$$\mathcal{D} \circ \mathbf{F} = 0, \tag{146}$$

which, upon applying $\mathcal{D}$ again and using the unimetrical calibration of the flow speed, yields a wave equation for each component of $\mathbf{F}$ with propagation speed c. Thus the quaternionic GEM field naturally propagates as waves in the observable three–space with the same universal speed that sets the scale in the D-rotor kinematics.

11.5 Summary and outlook

In this section we have:

- introduced a quaternionic differential operator $\mathcal{D}$ and a source current $\mathbf{J}$ combining mass and charge densities and currents;
- postulated a single quaternionic field equation $\mathcal{D} \circ \mathbf{F} = \mathbf{J}$ together with a potential representation $\mathbf{F} = \mathcal{D} \circ \mathbf{A}$;
- shown that the 3+1 decomposition of this equation reproduces, after appropriate calibration, the standard Maxwell equations for electromagnetism and their Newton–Heaviside analogue for gravity;
- observed that the whole system is covariant under D-rotors and that GEM disturbances propagate at the same universal speed as the unimetrical flow.

From the unimetrical perspective, these equations are not fundamental but emerge as an effective description of the averaged dynamics of streamlets in proto-space. A more microscopic treatment would start from the 1-form of phase, $\alpha = d\Phi$, and derive the quaternionic GEM equations as effective constraints on coarse-grained flow variables. We leave this refinement for future work and focus here on the structural unification already captured by the quaternionic GEM formulation.

12 Hamiltonians and Lagrangians of quaternionic GEM

The bilinear forms A, B, C, E provide a natural language for constructing Hamiltonians and Lagrangians for both particles and fields. Structurally, the picture is:

- the scalar form A plays the role of a Lorentzian (Minkowski-like) quadratic form and is suited for Lagrangians;
- the Euclidean self-form E plays the role of a Hamiltonian (energy) functional;
- for multi-body systems, the cross term $A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$ in $E(\tilde{\mathsf{q}}_1 + \tilde{\mathsf{q}}_2, \tilde{\mathsf{q}}_1 + \tilde{\mathsf{q}}_2)$ encodes the interaction energy;
- for fields, quadratic combinations of A, B, C over the GEM field quaternion $\mathbf{F}(\boldsymbol{x}, t)$ yield a Lagrangian density whose 3+1 decomposition reproduces the standard EM and linearized GEM Lagrangians.

12.1 Free-body Hamiltonian

For a single body B_i we use the metrically dressed quaternion introduced in Section 7:

$$\tilde{\mathsf{q}}_i = T_i \, \hat{h} + S_i \, \hat{\mathbf{u}}_i, \qquad T_i = \lambda_E q_i, \quad S_i = \lambda_G m_i, \tag{147}$$

with T_i, S_i having units of length and $\hat{\mathbf{u}}_i$ the free unit imaginary aligned with the effective spatial channel of the body in its rest frame.

The Euclidean self-form of $\tilde{\mathsf{q}}_i$ is

$$E(\tilde{\mathsf{q}}_i, \tilde{\mathsf{q}}_i) = T_i^2 + S_i^2, \tag{148}$$

with the natural Euclidean norm

$$\|\tilde{\mathsf{q}}_i\|_E := \sqrt{E(\tilde{\mathsf{q}}_i, \tilde{\mathsf{q}}_i)}. \tag{149}$$

Since $\widetilde{\mathsf{q}}_i$ is metrically dressed, this norm has dimension of length. To obtain an energy we multiply by a universal stiffness scale with dimension energy per length. A convenient choice is to express this scale in terms of the Planck force c^4/G:

$$H_{\text{free}}^{(i)} := \kappa_{\text{H}} \, \|\widetilde{\mathsf{q}}_i\|_E, \qquad \kappa_{\text{H}} \sim \frac{c^4}{G} \frac{1}{L_*}, \tag{150}$$

where L_* is a reference length scale (for example a Compton-like length associated with the object).

In the unimetrical calibration used in Section 4 and **??**, the internal flow rate $\dot{H}$ and the volumetric coefficient κ are chosen such that in the rest frame

$$H_{\text{free}}^{(i)} \equiv E_{\text{rest}}^{(i)} = m_{0,i} \, c^2, \tag{151}$$

with $m_{0,i}$ defined by the streamlet structure of the body. Thus, up to the universal conversion factor κ_{H}, the free-body Hamiltonian is simply the Euclidean self-form of the metrically dressed quaternion.

Formally, this rest energy defines a proper frequency of the global phase χ:

$$\omega_{0,i} := \frac{H_{\text{free}}^{(i)}}{\hbar} \sim \frac{m_{0,i} \, c^2}{\hbar}, \tag{152}$$

which for an electron would correspond to the usual Compton frequency when the dressing parameters λ_E, λ_G are chosen appropriately. In this sense the unimetrical free-body Hamiltonian provides a bridge between the internal geometric flow (through $\widetilde{\mathsf{q}}_i$) and the proper-time oscillation familiar from relativistic quantum mechanics.

12.2 Interaction Hamiltonian

For a two-body system with dressed quaternions $\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2$, the Euclidean self-form of the total dressed quaternion $\widetilde{\mathsf{q}}_{\text{tot}} := \widetilde{\mathsf{q}}_1 + \widetilde{\mathsf{q}}_2$ splits as

$$E(\widetilde{\mathsf{q}}_{\text{tot}}, \widetilde{\mathsf{q}}_{\text{tot}}) = E(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_1) + E(\widetilde{\mathsf{q}}_2, \widetilde{\mathsf{q}}_2) + 2 \, A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2), \tag{153}$$

so the cross term is precisely the scalar bilinear form A.

In the static regime, where the bodies are separated by a distance r, the interaction Hamiltonian can be defined as

$$H_{\text{int}}(r) := \kappa_{\text{int}} \, \frac{A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2)}{r}, \tag{154}$$

with κ_{int} a universal constant. Using the two-channel dressing (147), we obtain

$$A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2) = T_1 T_2 - S_1 S_2 = \lambda_E^2 q_1 q_2 - \lambda_G^2 m_1 m_2. \tag{155}$$

The first term corresponds to the temporal (electromagnetic) channel, the second to the spatial (gravitational) channel.

It is convenient to express κ_{int} in terms of a fundamental force scale c^4/G:

$$\kappa_{\text{int}} = \frac{c^4}{G} \, \alpha_{\text{int}}, \tag{156}$$

with α_{int} a dimensionless structural factor built from λ_E, λ_G. Then

$$H_{\text{int}}(r) = \frac{c^4}{G} \frac{\alpha_{\text{int}}}{r} \, (\lambda_E^2 q_1 q_2 - \lambda_G^2 m_1 m_2). \tag{157}$$

By an appropriate choice of $\alpha_{\text{int}} \lambda_E^2$ and $\alpha_{\text{int}} \lambda_G^2$ one recovers the familiar Newton and Coulomb potentials:

$$H_{\text{int}}(r) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} - G \, \frac{m_1 m_2}{r}, \tag{158}$$

as already discussed in Section 7.2. The advantage of writing (157) is that both gravitational and electromagnetic couplings appear as projections of a single Planck-scale stiffness c^4/G dressed by dimensionless geometric factors.

From the Hamiltonian point of view, the total energy of the two-body system in the static limit is

$$H_{\text{tot}} = H_{\text{free}}^{(1)} + H_{\text{free}}^{(2)} + H_{\text{int}}(r), \tag{159}$$

with $H_{\text{free}}^{(i)}$ obtained from the self-form E as in Section 12.1. In the full unimetrical picture, boosts implemented by D-rotors add the kinetic (relativistic) contribution by changing the decomposition of the flow between temporal and spatial channels, while leaving the internal structure of $\widetilde{\mathsf{q}}_i$ intact.

12.3 Strong–field gravitational sector and Schwarzschild limit

In the weak–field limit the quaternionic GEM recovers the Newtonian potential and the standard linearized metric,

$$g_{tt} \simeq -c^2\left(1 + \frac{2\Phi_{\text{G}}}{c^2}\right), \qquad g_{ij} \simeq \delta_{ij}\left(1 - \frac{2\Phi_{\text{G}}}{c^2}\right), \qquad \Phi_{\text{G}}(\mathbf{x}) = -\frac{GM}{r},$$

where Φ_{G} is extracted from the gravitational part of the bilinear $A(\widetilde{\mathsf{q}}_1, \widetilde{\mathsf{q}}_2)$ in the interaction Hamiltonian.

To extend this picture to strong fields without leaving the Euclidean proto–space, we introduce a *gravitational angle* ζ_{G}, which measures the fraction of proto–flow locked into spatial directions by the gravitational field. In the static case we define it by

$$\sin^2 \zeta_{\text{G}}(\mathbf{x}) := -\frac{2\,\Phi_{\text{G}}(\mathbf{x})}{c^2}, \qquad \Phi_{\text{G}} < 0. \tag{160}$$

Thus the Newtonian potential is *exactly*

$$\Phi_{\text{G}}(\mathbf{x}) = -\frac{c^2}{2}\sin^2 \zeta_{\text{G}}(\mathbf{x}), \tag{161}$$

and the dimensionless quantity $1 - \cos 2\zeta_{\text{G}} = 2\sin^2 \zeta_{\text{G}}$ plays the role of a geometric "budget" of spatialized flow.

12.3.1 Spherically symmetric source and Schwarzschild sector

For a static, spherically symmetric source of mass M, the weak–field limit of the interaction Hamiltonian,

$$H_{\text{int}}^{\text{grav}}(r) = -\frac{GMm}{r} = \frac{c^4}{G}\,\frac{A_{\text{grav}}(\widetilde{\mathsf{q}}_M, \widetilde{\mathsf{q}}_m)}{r}, \tag{162}$$

fixes the gravitational part of the bilinear A_{grav} and gives the usual Newtonian potential $\Phi_{\text{G}}(r) = -GM/r$. Substituting this into the angle definition (160) we obtain

$$\sin^2 \zeta_{\text{G}}(r) = \frac{2GM}{rc^2} = \frac{r_{\text{s}}}{r}, \qquad r_{\text{s}} := \frac{2GM}{c^2}, \tag{163}$$

so that

$$\cos^2 \zeta_{\text{G}}(r) = 1 - \sin^2 \zeta_{\text{G}}(r) = 1 - \frac{r_{\text{s}}}{r}. \tag{164}$$

Equivalently,

$$1 - \cos 2\zeta_{\text{G}}(r) = 2\sin^2 \zeta_{\text{G}}(r) = \frac{2r_{\text{s}}}{r}, \tag{165}$$

which makes the structural similarity to the Schwarzschild factor explicit.

In Unimetry the observable metric components are Euclidean proto–components rescaled by the temporal share of the flow. For the static spherical case we can therefore write the effective line element as

$$ds^2 = -\, c^2 \cos^2 \zeta_{\mathrm{G}}(r)\, dt^2 + \cos^{-2} \zeta_{\mathrm{G}}(r)\, dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2). \tag{166}$$

With (163) this becomes

$$ds^2 = -c^2\left(1 - \frac{r_{\mathrm{s}}}{r}\right) dt^2 + \left(1 - \frac{r_{\mathrm{s}}}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{167}$$

i.e. exactly the Schwarzschild metric in standard coordinates.

Thus, in the spherical sector the *same* Newtonian potential $\Phi_{\mathrm{G}}(r) = -GM/r$, when encoded via the flow angle $\zeta_{\mathrm{G}}(r)$, reproduces both: (i) the weak–field limit of quaternionic GEM, and (ii) the full Schwarzschild strong–field geometry.

12.3.2 Strong–field GEM equations

In the linear GEM regime we work directly with Φ_{G} and the gravitational "electric" field $\mathbf{g} = -\nabla\Phi_{\mathrm{G}}$, with sources determined by the scalar part of the bilinear $A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$. The strong–field extension consists simply in *not* expanding ζ_{G} for small angles:

- the scalar potential is still defined by (161),

  $$\Phi_{\mathrm{G}}(\mathbf{x}) = -\frac{c^2}{2}\sin^2\zeta_{\mathrm{G}}(\mathbf{x}),$$

  with ζ_{G} sourced by the same quaternionic charge through the bilinear A;

- the GEM fields follow as $\mathbf{g} = -\nabla\Phi_{\mathrm{G}} = \frac{c^2}{2}\nabla(\sin^2\zeta_{\mathrm{G}})$, and the vector/torsional fields (the B and C forms of the quaternionic product) remain linear in the sources;

- the electromagnetic sector stays strictly linear (Maxwell–like): no trigonometric dressing is introduced there; nonlinearities from extreme EM fields appear only via their contribution to the same gravitational angle ζ_{G} through the total energy content.

In this way the quaternionic GEM keeps its linear Maxwell structure in both the gravitational and electromagnetic field equations at the level of sources, while the *geometry* seen by null and timelike probes is upgraded from the linearized metric to the exact strong–field metric (166) through the single flow angle ζ_{G}.

12.4 Field Lagrangian in terms of A, B, C

The quaternionic GEM field $\mathbf{F}(\boldsymbol{x}, t)$ introduced in Section 8 can be used to build a field Lagrangian in direct analogy with the standard EM Lagrangian. The basic idea is that:

- the scalar form $A(\mathbf{F}, \mathbf{F})$ plays the role of a Lorentzian invariant quadratic form (analogous to $F_{\mu\nu}F^{\mu\nu}$);

- the norms of the vector forms $B(\mathbf{F}, \mathbf{F})$ and $C(\mathbf{F}, \mathbf{F})$ distinguish radial and vortical channels and allow us to control separately the electric/gravito–electric and magnetic/gravito–magnetic sectors;

- the Euclidean self-form $E(\mathbf{F}, \mathbf{F})$ acts as an energy density (Hamiltonian density) after appropriate calibration.

We introduce a field Lagrangian density of the schematic form

$$\mathcal{L}_{\text{field}} := \alpha_A\, A(\mathbf{F},\mathbf{F}) + \alpha_B\, \|B(\mathbf{F},\mathbf{F})\|^2 + \alpha_C\, \|C(\mathbf{F},\mathbf{F})\|^2, \tag{168}$$

where $\alpha_A, \alpha_B, \alpha_C$ are constants with dimensions chosen so that $\mathcal{L}_{\text{field}}$ has units of energy density. The full action is

$$S_{\text{field}} = \int \mathcal{L}_{\text{field}}\, d^3x\, dt. \tag{169}$$

To relate (168) to the standard electromagnetic Lagrangian, we decompose $\mathbf{F}$ into its electric and magnetic components as in Section 11.3:

$$\mathbf{F} = \Phi\, \hat{h} + \mathcal{E}_E + \mathcal{E}_G + \mathcal{C}, \tag{170}$$

and then restrict to the purely electromagnetic sector by setting the gravitational dressing to zero ($\mathcal{E}_G = 0$) and projecting onto the temporal channel. With a suitable calibration one can choose the coefficients so that

$$\mathcal{L}^{\text{EM}}_{\text{field}} := \mathcal{L}_{\text{field}}\big|_{\text{EM sector}} = \frac{\epsilon_0}{2}\,\big(\|\mathbf{E}\|^2 - c^2\|\mathbf{B}\|^2\big), \tag{171}$$

where $\mathbf{E}$ and $\mathbf{B}$ are extracted from $\mathbf{F}$ via the appropriate projections of $B(\mathbf{F},\mathbf{F})$ and $c\,C(\mathbf{F},\mathbf{F})$.

Similarly, in the purely gravitational (linearized) sector one can project onto the spatial dressing channel and obtain a Lagrangian density of the form

$$\mathcal{L}^{\text{GEM}}_{\text{field}} \sim \frac{1}{8\pi G}\,\big(\|\mathcal{E}_G\|^2 - c^2\|\mathcal{B}_G\|^2\big), \tag{172}$$

where $\mathcal{E}_G$ and $\mathcal{B}_G$ are the gravito–electric and gravito–magnetic fields obtained from $\mathbf{F}$. The precise coefficients again depend on the chosen calibration of the dressing and the normalization of $\mathbf{F}$, but structurally the form is identical to the Maxwell Lagrangian, with ϵ_0 replaced by an effective gravitational stiffness $\sim 1/G$.

Finally, the total Lagrangian of a system of bodies interacting through the GEM field is naturally written as

$$L_{\text{total}} = \sum_i L^{(i)}_{\text{free}} + \sum_{i<j} H^{(ij)}_{\text{int}} + \int \mathcal{L}_{\text{field}}\, d^3x, \tag{173}$$

where:

- $L^{(i)}_{\text{free}}$ is the free-body Lagrangian of the i-th body, which in the unimetrical calibration is generated by the scalar self-form $A(\tilde{\mathsf{q}}_i, \tilde{\mathsf{q}}_i)$ and reproduces the usual relativistic free Lagrangian;
- $H^{(ij)}_{\text{int}}$ is the interaction Hamiltonian (154) derived from the cross form $A(\tilde{\mathsf{q}}_i, \tilde{\mathsf{q}}_j)$;
- $\mathcal{L}_{\text{field}}$ is the field Lagrangian density of the form (168).

In this way the same quaternionic bilinear forms A, B, C, E govern:

- the free dynamics of bodies (through A and E);
- their mutual interactions (through cross terms in A);
- the dynamics of the GEM field itself (through quadratic combinations of A, B, C).

This closes the algebraic circle: the Hamiltonian and Lagrangian structures of the GEM sector emerge directly from the quaternionic decomposition of the flow, without introducing separate ad hoc field tensors or potentials beyond the unified quaternionic objects already present in Unimetry.

13 Relation to Maxwell's equations and linearized GEM

13.1 Static equations and Poisson-type equations

13.2 Quasi-stationary regime and continuity equations

13.3 Comparison with the classical GEM formalism

14 Discussion and outlook

The quaternionic gravito–electromagnetic (GEM) formulation presented above grew out of the unimetrical viewpoint, in which all observable structure is traced back to a single underlying flow in a Euclidean proto–space. In this closing section we summarize what is actually achieved by the present construction, which parts remain explicitly provisional, and where it may be useful to push the framework further.

14.1 What is unified, and in what sense

At a purely algebraic level, the quaternionic GEM formulation brings together several ingredients that are usually treated separately:

- The quaternion product is decomposed into four bilinear forms A, B, C, E, which naturally separate into scalar, radial vector, vortical vector and Euclidean norm channels.
- Metrically dressed body quaternions encode mass and charge as effective lengths in temporal and spatial channels of the same quaternionic object, without introducing separate "kinds" of charge in the algebra.
- Static Newton and Coulomb interactions are recovered as different contributions of a single scalar form A, once mass and charge are encoded by the dressing and the familiar constants G, ϵ_0 are interpreted as calibration factors between quaternionic and laboratory units.
- The Euclidean self-form E of dressed quaternions plays the role of a Hamiltonian channel: for a two-body system it splits into two self-energy terms and an interaction term proportional to A, providing a compact energy bookkeeping scheme.
- The vector forms B and C supply, respectively, the radial and vortical channels that become, after 3+1 decomposition, the gravito–electric/electric and gravito–magnetic/magnetic fields.
- D-rotors act as fully geometric implementations of relativistic boosts, preserving E and reshuffling the contributions of A, B, C in a way that reproduces the Lorentz structure of forces and the mixing of electric and magnetic fields.
- A single quaternionic field equation $\mathcal{D} \circ \mathbf{F} = \mathbf{J}$ generates, after projection, both Maxwell's equations and their Newton–Heaviside analogue for gravity.

In this sense the unification achieved here is structural: gravity and electromagnetism are not merged into one new interaction, but they are realized as different channels of the same quaternionic objects and the same underlying bilinear forms. The distinction between the "gravitational" and "electromagnetic" sectors is attributed to the temporal versus spatial dressing of the body quaternions, and to how the free unit imaginary vector $\hat{\mathbf{u}}$ is deployed, not to different algebraic rules.

From the unimetrical viewpoint, this is a natural extension: special relativity and GEM are no longer separate layers (Minkowski metric plus Maxwell plus Newton), but different aspects of a single Euclidean phase–space kinematics with D-rotors.

14.2 Relation to existing frameworks

On the electromagnetic side, the quaternionic GEM construction is closely related to familiar biquaternion and Clifford-algebra formulations of Maxwell's theory, where electric and magnetic fields are combined into a single multicomponent object and the Maxwell equations take a compact first-order form. The difference here is that:

- we keep the underlying metric Euclidean and interpret the Minkowski structure as an emergent consequence of the unimetrical flow;
- we include mass and gravity on the same footing as charge and electromagnetism, via the spatial channel of the dressing and the gravitational constants in the calibration;
- D-rotors are treated as geometric rotations of the real and imaginary parts rather than as abstract Lorentz transformations on a separate spacetime manifold.

On the gravitational side, the connection to general relativity is more indirect. The present GEM system corresponds to a linearized or Newton–Heaviside regime on a fixed background: it captures gravito–electric and gravito–magnetic fields generated by slowly moving masses, but it does not yet reproduce the full non-linear Einstein–Hilbert dynamics of the metric. Instead, gravitational effects are encoded in:

- the spatial dressing of body quaternions (how much proto-flow is locked into spatial loops);
- the D-rotor field $\mathbf{D}(\boldsymbol{x}, t)$ that rotates local time and space directions in response to mass distributions;
- the gravito–electric and gravito–magnetic parts of the quaternionic field $\mathbf{F}$ in three-space.

A genuine replacement for Einstein's equations would require promoting the D-rotor field (or its derivative invariants) to the primary gravitational variable and deriving its dynamics from unimetrical flow conservation at the streamlet level. This step is not carried out in the present text.

14.3 Limitations and open conceptual questions

Several limitations of the present formulation are worth stating explicitly.

Linear regime and weak fields. The GEM equations derived from the quaternionic field equation are of Maxwell–Heaviside type and are therefore intrinsically linear in the fields. Non-linear gravitational phenomena — strong-field effects, black holes, cosmological solutions — lie outside this regime. The unimetrical picture suggests that such effects should be encoded in non-linear constraints on the D-rotor field and the streamlet ensemble statistics, but this has not been made explicit.

Backreaction and self-energy. The use of dressed body quaternions and bilinear forms provides a compact description of interaction energies, but it sidesteps the delicate question of self-energy and radiation reaction. In particular, the feedback of the GEM field $\mathbf{F}$ on the internal streamlet structure of a body (and hence on its mass functional) is not modelled. A consistent unimetrical treatment would have to specify how $\mathbf{C}_B$ and the structural angle ζ_B evolve under emission and absorption of GEM radiation.

Geometry of the vacuum and "rion" medium. At several points the construction tacitly assumes the existence of a background flow or condensate (the "rion" medium) whose effective stiffness sets the values of the gravitational and electromagnetic constants. While the unified dressing suggests that G and ϵ_0 should be related to different projections of the same vacuum quaternion, no explicit relation is derived here. Understanding this link would require a microscopic model of the vacuum as a streamlet ensemble and an analysis of how its second moments determine the effective GEM calibration constants.

Global hyperbolicity and causal structure. The unimetrical framework treats time as a geometric direction emerging from the phase flow, and the Minkowski-like interval arises from the Euclidean norm of the flow vector and the cyclic interpretation of the temporal axis. However, the global causal structure — horizons, singularities, and cosmological boundary conditions — is not addressed. It remains to be seen how far the emergent Minkowski picture can be pushed before explicit non-Euclidean geometry becomes unavoidable.

14.4 Possible observational and theoretical tests

Although the present work is largely structural, it hints at several directions where concrete predictions or consistency checks might be extracted.

Relations between coupling constants. The unified dressing and the appearance of a single vacuum scale in both the gravitational and electromagnetic sectors suggest that dimensionless combinations of G, ϵ_0, c and possibly $\hbar$ could be interpreted as ratios of geometric invariants of the underlying streamlet ensemble. If the same structural parameter k (or the same class of invariants built from $\mathbf{C}_B$) controls both mass and charge channels, this may lead to constraints on how these constants can vary in cosmological settings driven by changes in the background flow.

GEM in strong-field but slow-motion regimes. In settings where velocities are small but gravitational fields are strong (e.g. near compact objects, in certain binary systems), the Maxwell–Heaviside picture is often used as an effective approximation. The quaternionic GEM formulation provides an alternative parametrization of such regimes. Comparing its predictions for gravito–magnetic effects (precession, frame-dragging, waveforms) with those of standard post-Newtonian GR could reveal whether the unimetrical dressing introduces any subtle, testable differences.

Structure of massive bodies from streamlet ensembles. On the microscopic side, the identification of rest mass with functionals of the spatial second moment $\mathbf{C}_B$ suggests that different classes of objects (elementary particles, composite nuclei, macroscopic bodies) may correspond to distinct families of streamlet configurations. Even simple toy models of such ensembles could be used to check whether reasonable mass spectra and scaling relations can be reproduced without fine-tuning.

14.5 Future directions

Several natural extensions of the present work suggest themselves.

From GEM to full Unimetry. Here we treated GEM as a sector of Unimetry, focusing on how the existing Newton–Coulomb–Maxwell phenomenology fits into the unimetrical phase-space picture. A more ambitious project is to invert the logic: to start from the unimetrical postulates about phase 1-forms, streamlet ensembles and D-rotors and derive GEM, special relativity and parts of general relativity as emergent effective descriptions. This would require, in particular, a systematic treatment of the mass functional $m_0(\zeta_B, \mathbf{C}_B)$ and the dynamics of $\mathbf{C}_B$ under interactions.

Non-linear D-rotor dynamics and effective Einstein equations. If the gravitational field is fundamentally a field of D-rotors $\mathbf{D}(\boldsymbol{x}, t)$, then one can attempt to derive effective Einstein-like equations by constructing curvature-like invariants from spatial gradients and time derivatives of $\mathbf{D}$. Such invariants would replace the Riemann tensor as descriptors of how local time and space are rotated relative to the global proto-space flow. The challenge is to connect these invariants back to observable quantities in three-space and to the stress-energy content encoded in dressed quaternions and $\mathbf{J}$.

Quantization and relation to quantum theory. The quaternionic structure and the role of phase in Unimetry suggest uneasy but intriguing parallels with quantum mechanics. Whether the streamlet picture can provide a realist underpinning for quantum amplitudes, or whether the quaternionic GEM fields can be related to spinor or gauge-field descriptions at the quantum level, remains an open question. Any such attempt would have to respect the well-tested structure of quantum field theory while offering a genuinely new geometric interpretation.

In summary, the quaternionic GEM formulation proposed here is a step towards expressing gravity and electromagnetism in a language entirely adapted to the unimetrical phase-space picture: Euclidean at the fundamental level, but capable of reproducing relativistic kinematics and field dynamics in the observable three-space. It is neither a complete theory of gravity nor a replacement for quantum field theory, but a structural bridge between familiar phenomenology and a different way of thinking about flow, time and interaction. Further work will show whether this bridge can carry quantitative weight or remains primarily a conceptual tool for organizing known physics.

Particle	m [GeV/c^2]	$\lambda = (m/m_P)^2$	$I = (m_P/m)^2$	$\log_{10} I$	$\ln I$
e	5.10999×10^{-4}	1.75×10^{-45}	5.71×10^{44}	44.76	103.06
μ	1.05658×10^{-1}	7.49×10^{-41}	1.34×10^{40}	40.13	92.39
τ	1.77686	2.12×10^{-38}	4.72×10^{37}	37.67	86.75
u	2.16×10^{-3}	3.13×10^{-44}	3.19×10^{43}	43.50	100.17
d	4.70×10^{-3}	1.48×10^{-43}	6.75×10^{42}	42.83	98.62
s	9.3×10^{-2}	5.80×10^{-41}	1.72×10^{40}	40.24	92.65
c	1.273	1.09×10^{-38}	9.20×10^{37}	37.96	87.41
b	4.18	1.17×10^{-37}	8.53×10^{36}	36.93	85.04
t	1.7276×10^{2}	2.00×10^{-34}	4.99×10^{33}	33.70	77.59
$W^{\pm}$	8.03692×10^{1}	4.33×10^{-35}	2.31×10^{34}	34.36	79.12
Z^0	9.11876×10^{1}	5.58×10^{-35}	1.79×10^{34}	34.25	78.87
H	1.2510×10^{2}	1.05×10^{-34}	9.52×10^{33}	33.98	78.24
p	9.38272×10^{-1}	5.91×10^{-39}	1.69×10^{38}	38.23	88.02
n	9.39565×10^{-1}	5.92×10^{-39}	1.69×10^{38}	38.23	88.02

Table 2: $I = (m_P/m)^2$.

A Quaternion algebra and matrix representation (details)

B Extended notation table

C Dimensional analysis and numerical estimates

D D-rotors, Lorentz transformations and Wigner–Thomas rotation