Unimetry: A Quaternionic Gravito–Electromagnetic Formulation

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Abstract

Contents

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1 Introduction

2 Introduction

2.1 Context and motivation

Unimetry is a phase–space reformulation of special relativity (SR) in which the fundamental objects are not worldlines in a pre-given Minkowski space, but stationary flows in an underlying Euclidean proto-space $\mathcal{E}$. A dimensionless scalar phase potential $\Phi : \mathcal{E} \to \mathbb{R}$ and its gradient define a normalized flow direction, while the familiar Minkowski metric and relativistic kinematics emerge as effective descriptions of how this flow is seen by observers tied to local "rest frames" of massive objects. In this sense, unimetry treats space–time not as a primitive arena, but as a derived structure obtained from the orientation of a flow in phase space.

The present paper extends this viewpoint to the problem of unifying gravity and electromagnetism (EM). At the classical level, gravito–electromagnetic (GEM) analogies are well known: in the weak-field, slow-motion limit of general relativity, the Einstein equations can be written in a Maxwell-like form, and moving masses generate a "gravitomagnetic" field. Similarly, quaternions and related formalisms have long been used to compactly encode rotations and the Maxwell equations. What unimetry adds to this landscape is a concrete physical interpretation: a single quaternionic object encodes both the temporal and spatial parts of a flow, and bilinear forms of such objects naturally split into scalar, symmetric vector, and axial (vorticity-like) channels.

Our goal here is to exploit this structure to construct a quaternionic GEM formulation in which gravitational and electromagnetic interactions arise from the *same* bilinear machinery acting on appropriately dressed "body quaternions". In particular, we show that Newton and Coulomb potentials appear as two branches of a single scalar form, while the magnetic and gravitomagnetic sectors are associated with a vortical bilinear form whose physical calibration reveals a natural role for the constants ε_0, μ_0, G and c. The resulting picture is entirely Euclidean at the level of the proto-space, yet reproduces relativistic kinematics and GEM fields in the observable three-space.

2.2 Relation to the base unimetry paper

This work is a direct sequel to the base unimetry paper, *"Unimetry: A Phase-Space Reformulation of Special Relativity"* (henceforth "Paper I"). In Paper I the phase potential Φ, the phase 1-form $\alpha = d\Phi$, the normalized flow vector $\widehat{\boldsymbol{\chi}}$, and the calibration $\boldsymbol{\chi} = c\,\widehat{\boldsymbol{\chi}}$ are introduced in detail, together with the derivation of the Minkowski interval and standard SR effects from a purely Euclidean proto-space. The unimetrical D-rotation, which encodes Lorentz boosts as Euclidean rotations in a suitable plane of the proto-space, is also defined there.

The present paper assumes familiarity with the conceptual framework of Paper I, but is written to be as self-contained as reasonably possible. We briefly recall the key definitions of the phase proto-space, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations. All proofs and derivations that are essential for the GEM construction are reproduced or adapted here; more detailed discussions of SR and cosmological applications remain in Paper I and are only referenced when needed.

2.3 Main results

The main technical contributions of this paper can be summarized as follows.

- We introduce *metrically dressed body quaternions* $\tilde{\mathsf{q}}_i = L_{E,i}\,\hat{h} + L_{G,i}\,\widehat{\boldsymbol{n}}_i$, whose components have the dimension of length. The "electric" and "gravitational" lengths

$$L_{E,i} = \sqrt{\frac{G}{4\pi\varepsilon_0 c^4}}\,Q_i, \qquad L_{G,i} = \frac{G}{c^2}\,m_i$$

  encode the charge Q_i and mass m_i of the body in a unified geometric fashion. The unit vector $\widehat{\boldsymbol{n}}_i$ represents the spatial flow direction associated with the body.

- We show that the scalar bilinear form

$$A(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2) = L_{E,1}L_{E,2} - \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

  (with $\boldsymbol{S}_i = L_{G,i}\widehat{\boldsymbol{n}}_i$) yields, after a single global calibration by c^4/G and a geometric $1/r$ factor, the combined Newton–Coulomb potential:

$$U(r) = \frac{c^4}{G}\,\frac{A}{r} = \frac{1}{4\pi\varepsilon_0}\,\frac{Q_1Q_2}{r} - G\,\frac{m_1m_2}{r}.$$

  Thus gravity and electrostatics arise as two channels of a single invariant scalar form.

- We identify two vector-valued bilinear forms, $\boldsymbol{B}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$ and $\boldsymbol{C}(\tilde{\mathsf{q}}_1, \tilde{\mathsf{q}}_2)$, corresponding to the symmetric and axial parts of the quaternion product. In the dressed setting these naturally describe current-like and vortical channels. In particular, the vortical form $\boldsymbol{C}$ reproduces the geometry of magnetic and gravitomagnetic fields generated by moving charges and masses.

- We construct a quaternionic GEM field $\mathcal{F}_{\mathrm{GEM}}(\boldsymbol{x})$ over the observable three-space by combining dressed source quaternions with purely imaginary distance quaternions. Its scalar channel reproduces the gravitational and electrostatic potentials, while its vortical channel yields a physically natural "phase-vortical" field C_{phys} with the same dimension as $\boldsymbol{E}$. The standard magnetic field $\boldsymbol{B}$ in SI units then appears as

$$\boldsymbol{B} = \frac{1}{c} C_{\mathrm{phys}},$$

  so that the familiar μ_0 and ε_0 can be interpreted in terms of linear and areal stiffness of the vacuum, combined into an effective volumetric stiffness proportional to $1/(\varepsilon_0 c^3)$.

- We analyze the action of unimetrical D-rotations and ordinary spatial rotors on dressed quaternions. Pure spatial rotations act in the usual way on the vector channels and leave the scalar form A invariant, while D-rotations mix the scalar channel and the longitudinal component of $\boldsymbol{B}$ in a two-dimensional "energy–current" plane. This provides a quaternionic encoding of relativistic kinematics in the GEM setting, with Lorentz-consistent transformation properties of the fields.

- Finally, we outline a Hamiltonian and Lagrangian formulation of the quaternionic GEM theory in terms of the self-form A and the norm-squares of $\boldsymbol{B}$ and $\boldsymbol{C}$, and discuss how the standard Maxwell Lagrangian and linearized GEM equations arise in appropriate limits.

2.4 Structure of the paper

The paper is organized as follows. In **??** we recall the basic quaternion algebra and introduce the bilinear forms A, $\boldsymbol{B}$, and $\boldsymbol{C}$ that arise from the quaternion product, together with their matrix representation and geometric interpretation. **??** provides a brief overview of the unimetrical phase proto-space, the phase potential, the flow vector, and the two calibrations of the flow that lead to kinematic and energetic interpretations.

In **??** we introduce metrically dressed body quaternions and define the electric and gravitational lengths L_E and L_G. **??** shows how the scalar form A for dressed quaternions reproduces the static Newton and Coulomb potentials. In **??** we construct a quaternionic GEM field over the observable three-space and identify the scalar and vortical channels with gravitational, electric, and magnetic sectors.

?? analyzes the action of spatial rotors and D-rotors on dressed quaternions and on the GEM field, clarifying the relativistic transformation properties of the scalar, current-like, and vortical channels. **??** is devoted to the calibration of $\boldsymbol{E}$ and $\boldsymbol{B}$, to the definition of the phase-vortical field C_{phys}, and to the interpretation of ε_0, μ_0, and c in terms of vacuum stiffness.

In **??** we outline Hamiltonian and Lagrangian formulations of quaternionic GEM, and in **??** we compare the resulting equations with the standard Maxwell and linearized GEM formalisms. Finally, **??** discusses limitations and open questions, and sketches possible extensions towards non-Abelian interactions and cosmological applications.