# Assignment 1

## Signals and Systems


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### E/19/445


## *Libraries*

In [121…

```
import numpy as np
import matplotlib.pyplot as plt
from sympy import *
from scipy import signal
from matplotlib import style
```

In [122…

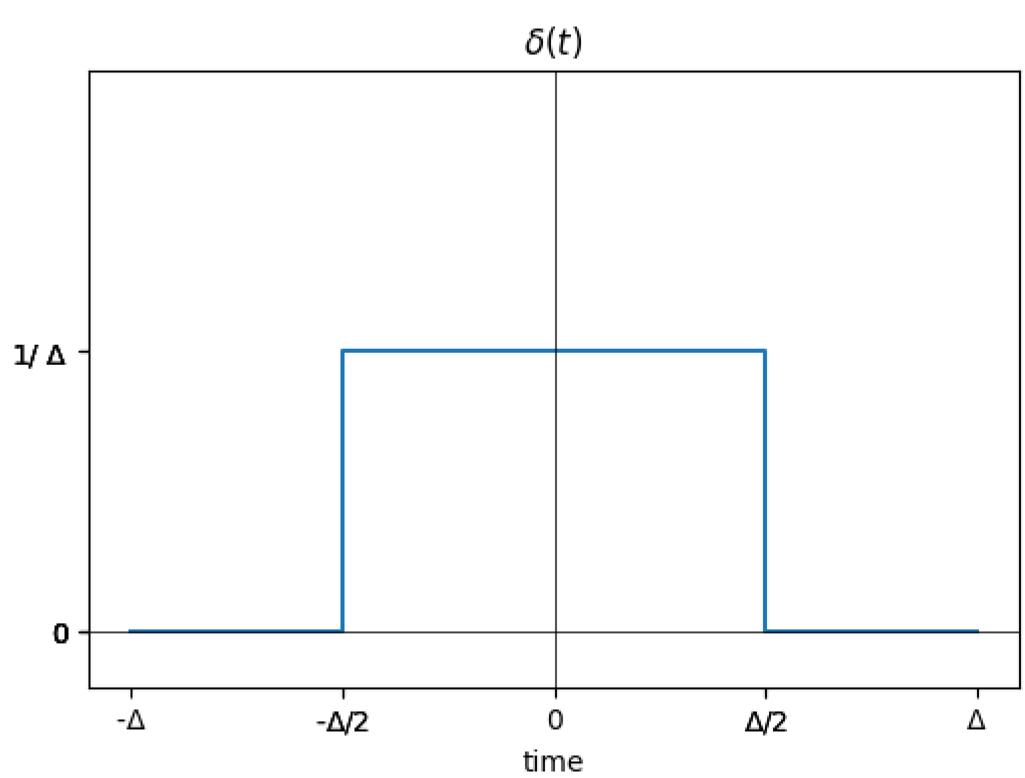
```
class buddhi_waves:
  def enlarged_impulse_1(self):
    plt.figure(figsize=(6, 4))
    y_values=[0,0,1,1,1,0,0]
    x_values=[-2,-1,-1,0,1,1,2]
    plt.plot(x_values,y_values)
    plt.ylim(-0.2,2)
    plt.axhline(y=0, color='black',linewidth = 0.5)
    plt.axvline(x=0, color='black',linewidth = 0.5)
    plt.xlabel("time")
    plt.title("$\delta(t)$")
    y_labels=[0,0,"1/ $\Delta$","1/ $\Delta}$","1/ $\Delta$",0,0]
    labels=["-$\Delta$","-$\Delta$/2","-$\Delta$/2","0","$\Delta$/2","$\Delta$/2","$\Delta$"]
    plt.xticks(x_values,labels,rotation="horizontal")
    plt.yticks(y_values,y_labels,rotation="horizontal")
    plt.show()

  def enlarged_impulse_2(self):
    plt.figure(figsize=(6, 4))
    y_values=[0,0,1,1,1,0,0]
    x_values=[-2,-1/2,-1/2,0,1/2,1/2,2]
    plt.plot(x_values,y_values)
    plt.ylim(-0.2,2)
    plt.axhline(y=0, color='black',linewidth = 0.5)
    plt.axvline(x=0, color='black',linewidth = 0.5)
    plt.xlabel("time")
    plt.title("$\delta(t)$")
    y_labels=[0,0,"1/ $\Delta$","1/ $\Delta}$","1/ $\Delta$",0,0]
    labels=["-$\Delta$/2","-$\Delta$/4","-$\Delta$/4","0","$\Delta$/4","$\Delta$/4","$\Delta$/2"]
    plt.xticks(x_values,labels,rotation="horizontal")
    plt.yticks(y_values,y_labels,rotation="horizontal")
    plt.show()

  def question_3_xt(self,shift):
    plt.figure(figsize=(6, 4))
    y_values=[0,0,1,1,1,0,0]
    arr=[-2,-1,-1,0,1,1,2]
    x_values=[(t-shift) for t in arr]
    plt.plot(x_values,y_values)
    plt.ylim(-0.2,2)
    plt.axhline(y=0, color='black',linewidth = 0.5)
    plt.axvline(x=0, color='black',linewidth = 0.5)
    plt.xlabel("time")
    plt.title("$x(t)$")
    labels=["-2T","-T","-T","0","T","T","2T"]
    y_labels=[0,0,1,1,1,0,0]
    plt.yticks(y_values,y_labels,rotation="horizontal")
    plt.xticks(arr,labels,rotation="horizontal")
    plt.show()


  def question_3_ht(self,shift):
    plt.figure(figsize=(6, 4))
    y_values=[0,0,1,1,1,1,1,0,0]
    arr=[-3,-2,-2,-1,0,1,2,2,3]
    x_values=[(t-shift) for t in arr]
    plt.plot(x_values,y_values)
    plt.ylim(-0.2,2)
    plt.axhline(y=0, color='black',linewidth = 0.5)
    plt.axvline(x=0, color='black',linewidth = 0.5)
    plt.xlabel("time")
    plt.title("$h(t)$")
    labels=["-3T","-2T","-2T","-T","0","T","2T","2T","3T"]
    y_labels=[0,0,1,1,1,1,1,0,0]
    plt.yticks(y_values,y_labels,rotation="horizontal")
    plt.xticks(arr,labels,rotation="horizontal")
    plt.show()

  def question_3_convolution(self,shift):
    plt.figure(figsize=(6, 4))
    y_values=[0,0,0,0,1,1,1,1,1,0,0,0,0]
```

```
    arr=[-5,-4,-3,-2,-2,-1,0,1,2,2,3,4,5]

    y_values_for_x=[0,0,0,0,0,1,1,1,0,0,0,0,0]
    arr_x=[-5,-4,-3,-2,-1,-1,0,1,1,2,3,4,5]

    x_x_values=[(t-shift) for t in arr_x]

    plt.plot(arr,y_values)
    plt.plot(x_x_values,y_values_for_x)
    plt.ylim(-0.2,2)
    plt.axhline(y=0, color='black',linewidth = 0.5)
    plt.axvline(x=0, color='black',linewidth = 0.5)
    plt.xlabel("time")
    plt.title("convolution")
    labels=["-5T","-4T","-3T","-2T","-2T","-T","0","T","2T","2T","3T","4T","5T"]
    y_labels=[0,0,0,0,1,1,1,1,1,0,0,0,0]
    plt.yticks(y_values,y_labels,rotation="horizontal")
    plt.xticks(arr,labels,rotation="horizontal")
    plt.show()

  def impulse(self):
    plt.figure(figsize=(6, 4))
    y_values=[0,0,0,0,10,0,0,0,0]
    arr=[-3,-2,-2,0,0,0,2,2,3]
    plt.plot(arr,y_values)
    plt.ylim(-0.2,2)
    plt.axhline(y=0, color='black',linewidth = 0.5)
    plt.axvline(x=0, color='black',linewidth = 0.5)
    plt.xlabel("time")
    plt.title("$\delta(t)$")
    plt.show()

buddhi=buddhi_waves()
```

## 1. Question 1.a

In [123…

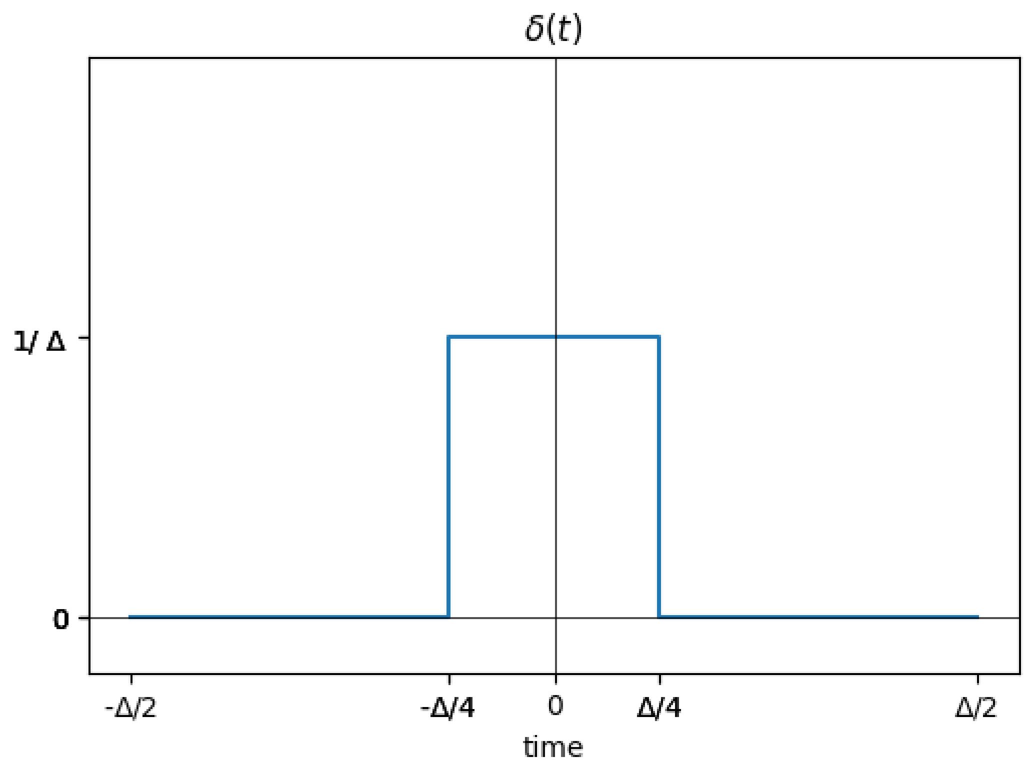
```
# Generating enlarge impulse
buddhi.enlarged_impulse_1()
```




$$
\begin{aligned}
\int_{\infty}^{\infty} \delta(t) &= \int_{\infty}^{\infty} [\lim_{\Delta \to 0} \delta_{\Delta}(t)] dt && (1)\\
&= \lim_{\Delta \to 0} \int_{\infty}^{\infty} \delta_{\Delta}(t) dt && (2)\\
&= \lim_{\Delta \to 0} [\frac{1}{\Delta} \times \Delta] && (3)\\
&= \lim_{\Delta \to 0} 1 && (4)\\
&= 1 && (5)
\end{aligned}
$$

In [124…

```
#Genrating same impulse with scaling
buddhi.enlarged_impulse_2()
```




$$\begin{aligned}
\int_{\infty}^{\infty} \delta(2t) &= \int_{\infty}^{\infty} [\lim_{\Delta \to 0} \delta_{\Delta}(2t)]dt && (6) \\
&= \lim_{\Delta \to 0} \int_{\infty}^{\infty} \delta_{\Delta}(2t)dt && (7) \\
&= \lim_{\Delta \to 0} [\frac{1}{\Delta} \times \frac{\Delta}{2}] && (8) \\
&= \lim_{\Delta \to 0} \frac{1}{2} && (9) \\
&= \frac{1}{2} && (10)
\end{aligned}$$

Thus,

$$\begin{aligned}
\int_{\infty}^{\infty} \delta(t) &= 2 \times \int_{\infty}^{\infty} \delta(2t) && (11) \\
\int_{\infty}^{\infty} \delta(t) &= \int_{\infty}^{\infty} 2\delta(2t) && (12) \\
\delta(t) &= 2\delta(2t) && (13)
\end{aligned}$$

### Question 1.b

$$x(t) = \sum_{n \ -\infty}^{\infty} e^{-(2t-n)} \tag{14}$$

Consider for $T \in \mathbb{R}$,

$$\begin{aligned}
x(t) &= x(t+T) && (15) \\
\sum_{n \ -\infty}^{\infty} e^{-(2t-n)} &= \sum_{n \ -\infty}^{\infty} e^{-(2(t+T)-n)} && (16) \\
e^{-(2t-n)} &= e^{-(2(t+T)-n)} && (17) \\
e^{2T} &= \frac{e^{-2t}}{e^{-2t}} && (18) \\
e^{2T} &= 1 && (19)
\end{aligned}$$

This only has solutions when $T = 0$. Thus the signal $x(t)$ is not periodic.

## 2. Question 3

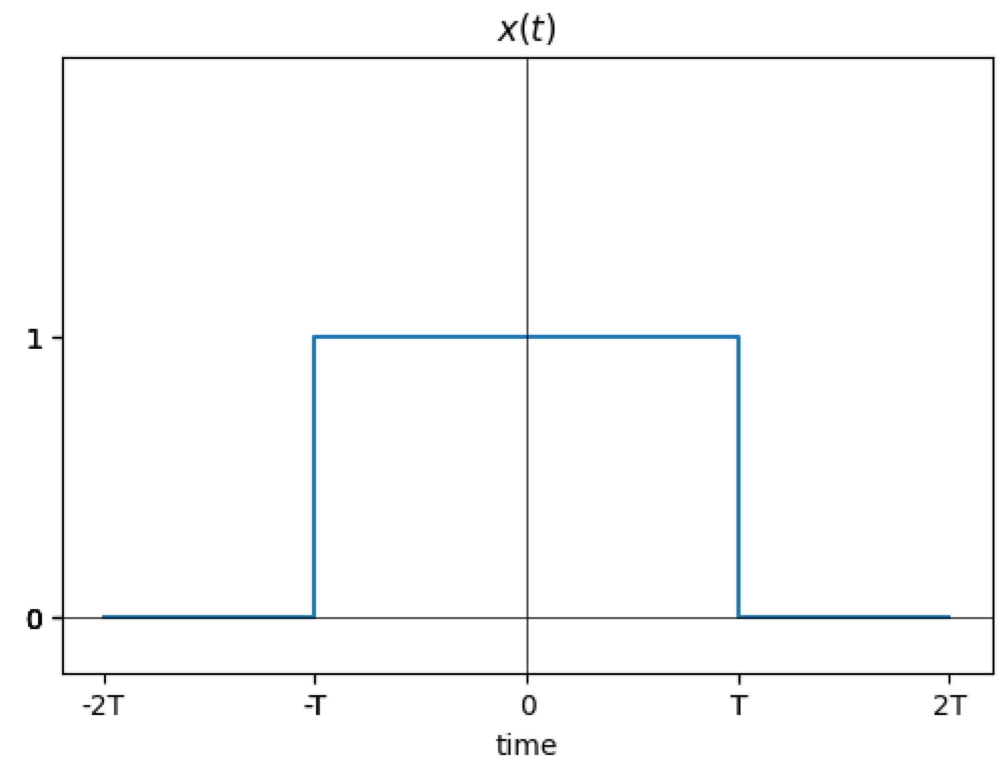
$$y(t) = x(t) * h(t) \tag{20}$$

Using the property.

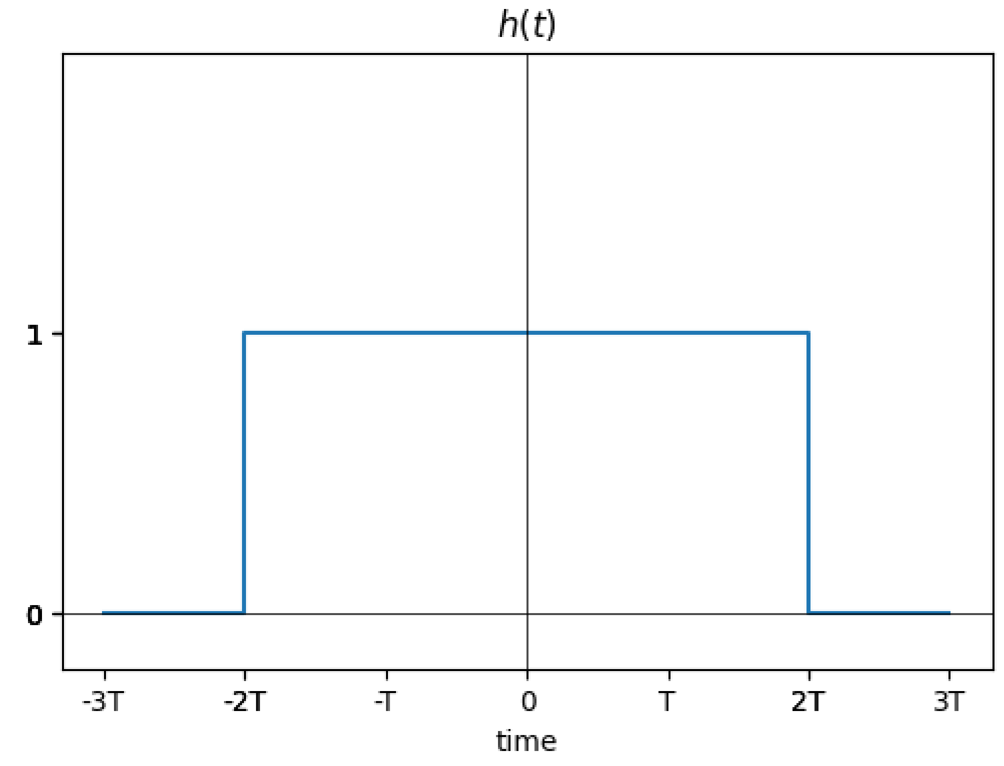
$$\begin{aligned}
y(t) &= h(t) * x(t) && (21) \\
&= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau && (22)
\end{aligned}$$

In [125…

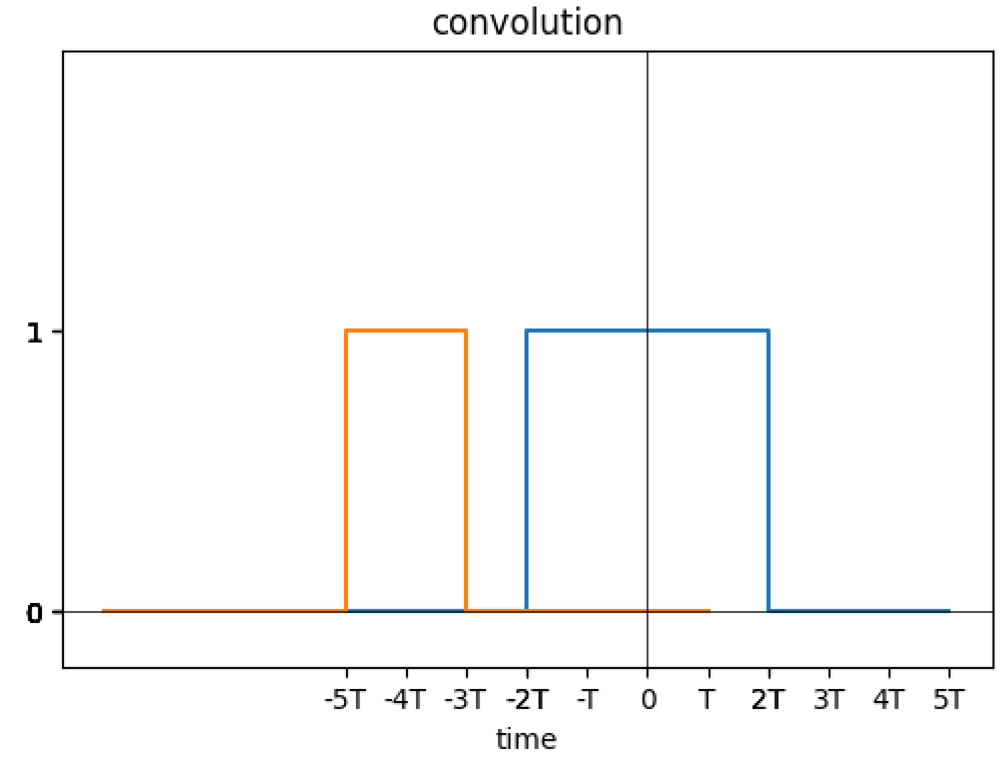
```
#vishualizing x(t)
buddhi.question_3_xt(0)
```




In [126…
```
#vishualizing h(t)
buddhi.question_3_ht(0)
```



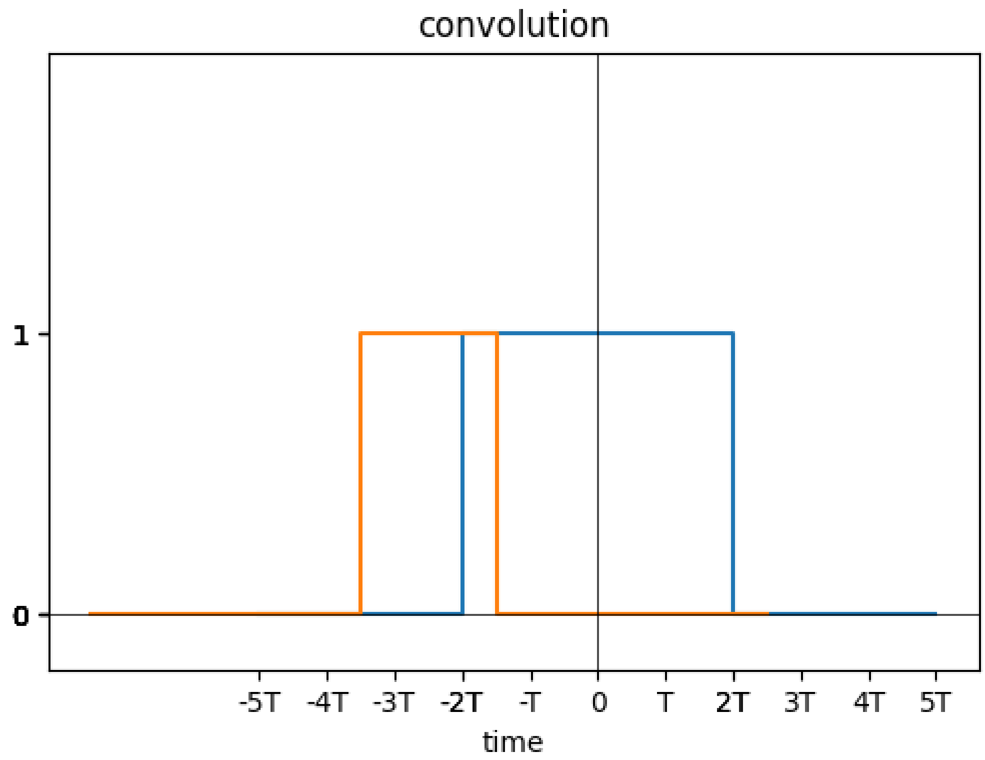

In [127…
```
buddhi.question_3_convolution(4)
```




When $t < -3T$,

$$y(t) = 0 \tag{23}$$

In [128…
```
buddhi.question_3_convolution(2.5)
```

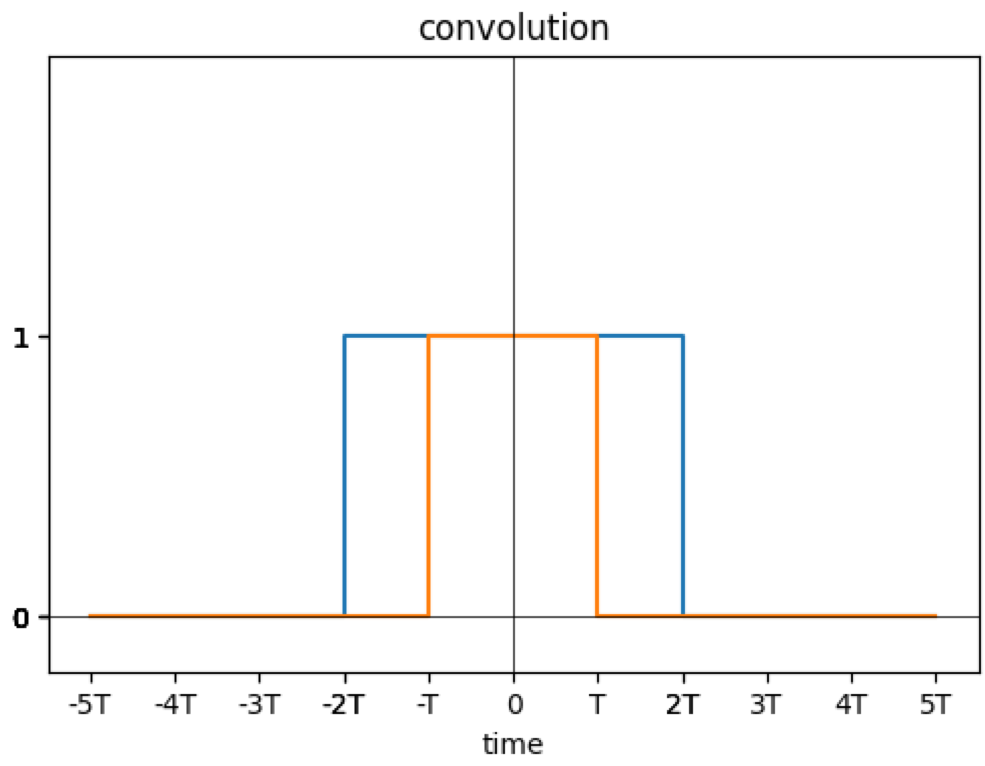



When $-3T \leq t < -2T$,

$$
\begin{aligned}
y(t) &= \int_{-2T}^{t+T} 1 d\tau && (24) \\
&= t + T - (-2T) && (25) \\
&= t + 3T && (26)
\end{aligned}
$$

In [129…

```
buddhi.question_3_convolution(0)
```

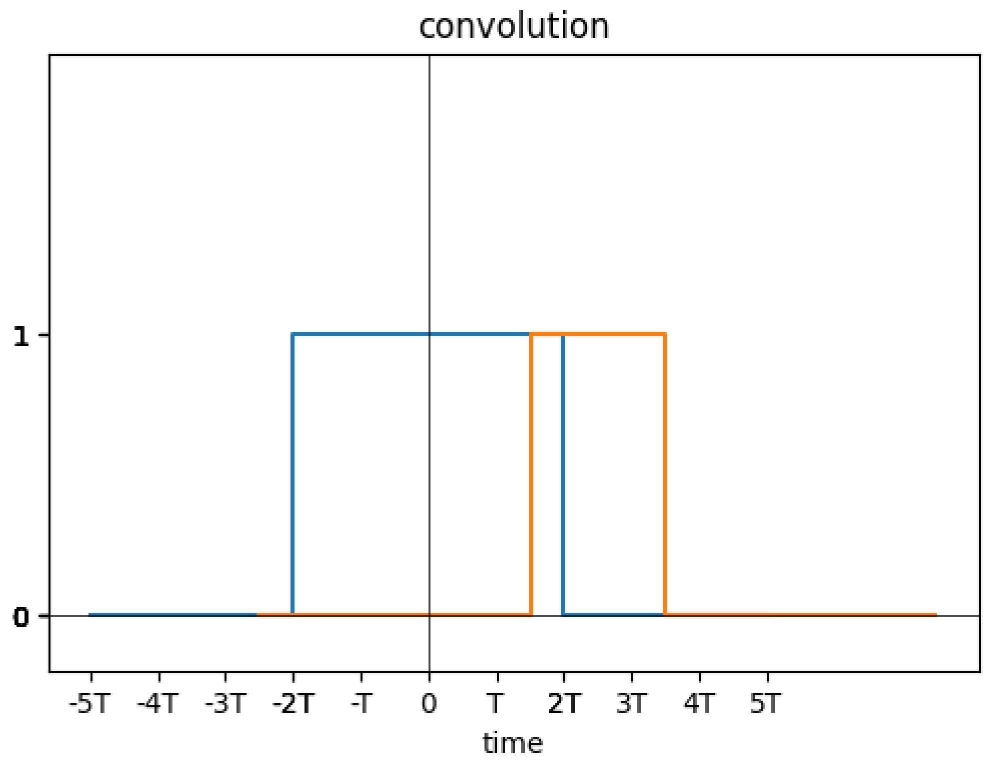



When $-2T \leq t < 2T$,

$$y(t) = 1 \qquad (27)$$

In [130…

```
buddhi.question_3_convolution(-2.5)
```

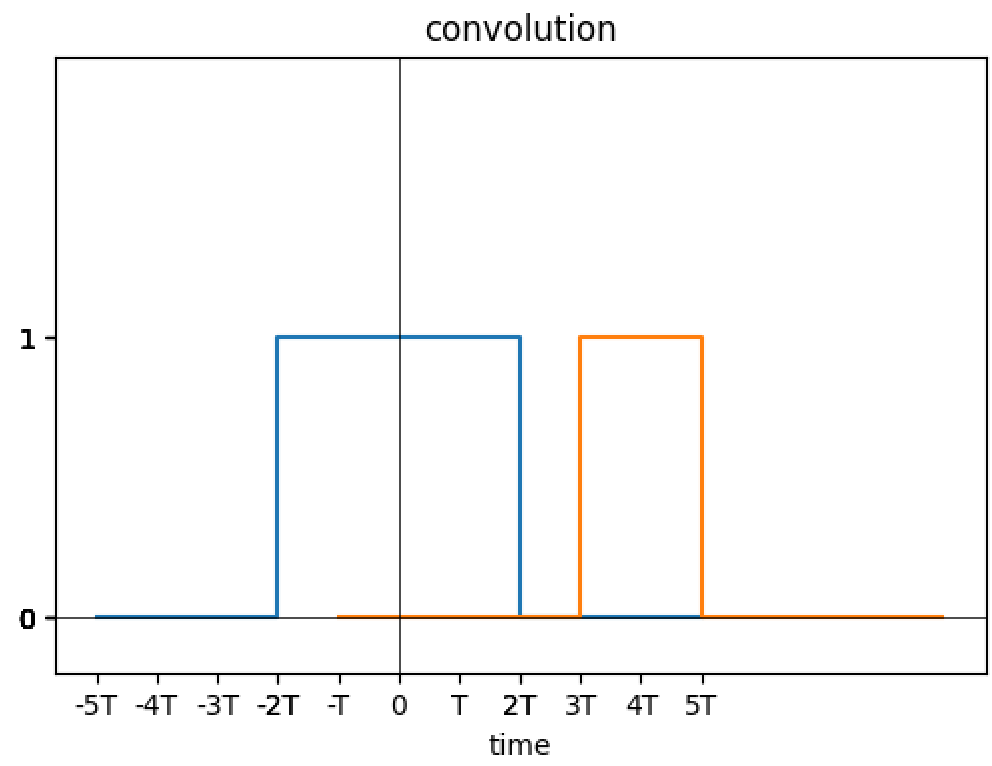



When $2T \leq t < 3T$,

$$
\begin{aligned}
y(t) &= \int_{t-T}^{2T} 1 d\tau && (28)\\
&= 2T - (t - T) && (29)\\
&= 3T - t && (30)
\end{aligned}
$$

In [131…

```
buddhi.question_3_convolution(-4)
```




When $3T \leq t$,

$$
y(t) = 0 \tag{31}
$$

Thus,

$$
y(t) = \begin{cases} 0 & t \leq -3T \\ t + 3T & -3T \leq t \leq -2T \\ 1 & -2T \leq t \leq 2T \\ 3T - t & 2T \leq t \leq 3T \\ 0 & t \geq 3T \end{cases} \tag{32}
$$

## 3. Question 4

### Part 4.a

$$
\begin{aligned}
x[n] &= \alpha^n U[n] && (33)\\
& && (34)\\
g[n] &= x[n] - \alpha x[n-1] && (35)\\
&= \alpha^n U[n] - \alpha \times \alpha^{n-1} U[n-1] && (36)
\end{aligned}
$$

for $n < 0$,

$$
g[n] = 0 \tag{37}
$$

for $n = 0$,

$$
\begin{aligned}
g[0] &= \alpha^0 U[0] - \alpha \times \alpha^{-1} U[-1] && (38)\\
&= 1 && (39)
\end{aligned}
$$

for $n > 0$,

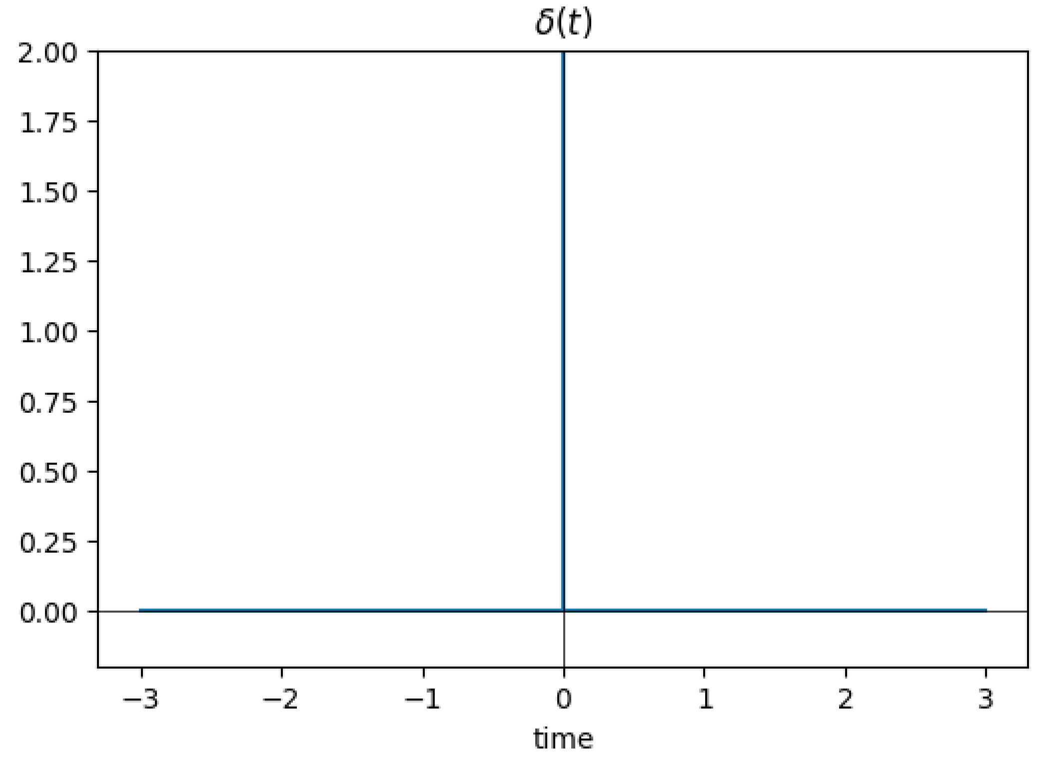
$$
\begin{aligned}
g[n] &= \alpha^n - \alpha \times \alpha^{n-1} && (40)\\
&= 0 && (41)
\end{aligned}
$$

Thus,

$$
g[n] = \delta[n] \tag{42}
$$

In [132…

```
#Impulse
buddhi.impulse()
```




## Part 4.b

Consider,

$$U[n+2]-U[n-2]=\delta[n+2]+\delta[n+1]+\delta[n]+\delta[n-1] \tag{43}$$

Using Property,

$$x[n]\delta(n-n_0)=x[n_0]\delta(n-n_0) \tag{44}$$

Thus,

$$(\frac{1}{2})^n\delta(n+2)=4\delta(n+2) \tag{45}$$
$$(\frac{1}{2})^n\delta(n+1)=2\delta(n+1) \tag{46}$$
$$(\frac{1}{2})^n\delta(n)=\delta(n) \tag{47}$$
$$(\frac{1}{2})^n\delta(n-1)=\frac{1}{2}\delta(n-1) \tag{48}$$

This yeilds,

$$x[n]*h[n]=4\delta(n+2)+2\delta(n+1)+\delta(n)+\frac{1}{2}\delta(n-1) \tag{49}$$

Using answer in part a,

$$\delta(n)=x[n]-\alpha x[n-1] \tag{50}$$
$$\delta(n-n_0)=x[n-n_0]-\alpha x[n-n_0-1] \tag{51}$$
$$=x[n]*\delta[n-n_0]-\alpha x[n]*\delta[n-n_0-1] \tag{52}$$
$$=x[n]*[\delta(n-n_0)-\alpha\delta(n-n_0-1)] \tag{53}$$

Hence,

$$\delta(n+2)=x[n]*[\delta(n+2)-\alpha\delta(n+1)] \tag{54}$$
$$\delta(n+1)=x[n]*[\delta(n+1)-\alpha\delta(n)] \tag{55}$$
$$\delta(n)=x[n]*[\delta(n)-\alpha\delta(n-1)] \tag{56}$$
$$\delta(n-1)=x[n]*[\delta(n-1)-\alpha\delta(n-2)] \tag{57}$$

Thus, \begin{align} x[n] \ast h[n]&=4 \times x[n] \ast [\delta(n+2)-\alpha \delta (n+1)] +\ 2 \times x[n] &\ast [\delta(n+1)-\alpha \delta (n)]

- x[n] \ast [\delta(n)-\alpha \delta (n-1)] +\frac{1}{2}\times x[n] \ast [\delta(n-1)-\alpha \delta (n-2)] \end{align} This gives,

$$x[n]*h[n]=x[n]*[4\delta(n+2)-4\alpha\delta(n+1)+2\delta(n+1)-2\alpha\delta(n)+\delta(n)-\alpha\delta(n-1)+\frac{1}{2}\delta(n-1)-\frac{1}{2}\alpha\delta(n-2)]$$
$$=x[n]*[4\delta(n+2)+(2-4\alpha)\delta(n+1)+(1-2\alpha)\delta(n)+(\frac{1}{2}-\alpha)\delta(n-1)-\frac{1}{2}\delta(n-2)]$$

This shows that,

$$h[n]=4\delta(n+2)+(2-4\alpha)\delta(n+1)+(1-2\alpha)\delta(n)+(\frac{1}{2}-\alpha)\delta(n-1)-\frac{1}{2}\delta(n-2) \tag{60}$$

# 4. Question 5

$$A_v = \int_{-\infty}^{\infty} v(t)dt \tag{61}$$

Since,

$$y(t) = x(t) * h(t) \tag{62}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \tag{63}$$

By integrating both sides with respect to $t$,

$$\int_{-\infty}^{\infty} y(t)dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau dt \tag{64}$$

$$A_y = \int_{-\infty}^{\infty} x(\tau)\int_{-\infty}^{\infty} h(t-\tau)dtd\tau \tag{65}$$

Let $t - \tau = k$,

Thus,

$$A_y = \int_{-\infty}^{\infty} x(\tau)\int_{-\infty}^{\infty} h(k)dk\ \ d\tau \tag{66}$$

$$= \int_{-\infty}^{\infty} x(\tau)A_h d\tau \tag{67}$$

$$= A_h\int_{-\infty}^{\infty} x(\tau)d\tau \tag{68}$$

$$= A_h A_x \tag{69}$$

This gives us,

$$A_y = A_x A_h \tag{70}$$

## 5. Question 7

$$y[n] + 2y[n-1] = x[n] \tag{71}$$

\

| n | $y[n] = x[n] - 2y[n-1]$ | $y[n-1]$ | $x[n] = \delta[n]$ |
|---|---|---|---|
| 0 | $1$ | $0$ | $1$ |
| 1 | $-2$ | $1$ | $0$ |
| 2 | $4$ | $-2$ | $0$ |
| 3 | $-8$ | $4$ | $0$ |
| 4 | $16$ | $-8$ | $0$ |
| 5 | $32$ | $16$ | $0$ |

Thus,

$$h[n] = (-2)^n U[n] \tag{72}$$