EE352 - AUTOMATIC CONTROL

Week 4 Activity 4

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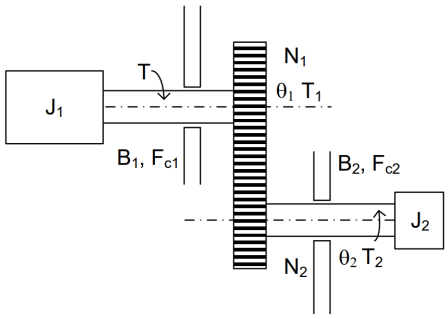

Figure 1: Reduction gearbox connected between a motor shaft and a load

For J_2,

$$T = J\alpha$$

$$T_2(t) = B_2\frac{d\theta_2}{dt} + F_{c2}\frac{\frac{d\theta_2}{dt}}{|\frac{d\theta_2}{dt}|} + J_2\frac{d^2\theta_2}{dt^2} \tag{1}$$

For J_1,

$$T = J\alpha$$

$$T(t) = T_1(t) + B_1\frac{d\theta_1}{dt} + F_{c1}\frac{\frac{d\theta_1}{dt}}{|\frac{d\theta_1}{dt}|} + J_1\frac{d^2\theta_1}{dt} \tag{2}$$

Considering the gear train, assuming power is conserved,

$$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{T_1}{T_2}$$

This gives,

$$T_1(t) = T_2(t)\frac{N_1}{N_2} \tag{3}$$

$$\theta_2 = \theta_1\frac{N_1}{N_2} \tag{4}$$

Using equation 1,

$$T_1(t) = \frac{B_2 N_1^2}{N_2^2}\frac{d\theta_1}{dt} + \frac{F_{C2} N_1}{N_2}\frac{\frac{d\theta_1}{dt}}{|\frac{d\theta_1}{dt}|} + \frac{J_1 N_1^2}{N_2^2}\frac{d^2\theta_1}{dt^2}$$

Substituting to equation 2,

$$\begin{aligned} T(t) &= \left(B_1 + B_2\frac{N_1^2}{N_2^2}\right)\frac{d\theta_1}{dt} + \left(F_{c1} + F_{c2}\frac{N_1}{N_2}\right)\frac{\frac{d\theta_1}{dt}}{|\frac{d\theta_1}{dt}|} + \left(J_1 + J_2\frac{N_1^2}{N_2^2}\right)\frac{d^2\theta_1}{dt^2} \\ &= B_{eq}\frac{d\theta_1}{dt} + F_{eq}\frac{\frac{d\theta_1}{dt}}{|\frac{d\theta_1}{dt}|} + J_{eq}\frac{d^2\theta_1}{dt^2} \end{aligned}$$