EE352 - AUTOMATIC CONTROL

Week 5 Activity 2

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Given that,

$$\frac{d^2}{dt^2}r(t) = r(t)\omega^2 - \frac{\beta^2}{r^2(t)} + u(t) \tag{1}$$

Introducing the state vector as,

$$x(t) = \begin{bmatrix} r(t) \\ \dot{r}(t) \end{bmatrix}$$

We can select state equations as follows,

$$\ddot{r}(t) = f_1(r, \dot{r}, u) = r(t)\omega^2 - \frac{\beta^2}{r^2(t)} + u(t) \tag{2}$$

$$\dot{r}(t) = f_2(r, \dot{r}, u) = \frac{d}{dt}r(t) \tag{3}$$

Given that the stationary point conditions as,

$$\begin{bmatrix} r(0) \\ \dot{r}(0) \\ u(0) \end{bmatrix} = \begin{bmatrix} r^0 \\ 0 \\ 0 \end{bmatrix}$$

Using equation 1 at stationary point,

$$0 = r^0\omega^2 - \frac{\beta^2}{(r^0)^2}$$

$$(r^0)^3 = \frac{\beta^2}{\omega^2}$$

$$r^0 = \left(\frac{\beta^2}{\omega^2}\right)^{\frac{1}{3}}$$

For equation 2,

$$\frac{\partial f_1}{\partial r} = \omega^2 + 2\frac{\beta^2}{r^3} \qquad \frac{\partial f_1}{\partial u} = 1 \qquad \frac{\partial f_1}{\partial \dot{r}} = 0$$

$$\frac{\partial f_1}{\partial r}\Big|_{r=r^0} = 3\omega^2 \qquad \frac{\partial f_1}{\partial u}\Big|_{u=0} = 1 \qquad \frac{\partial f_1}{\partial \dot{r}}\Big|_{\dot{r}=0} = 0 \tag{4}$$

For equation 3, at stationary point conditions

$$\frac{\partial f_2}{\partial r} = \frac{\partial f_2}{\partial u} = 0 \qquad \frac{\partial f_2}{\partial \dot{r}} = 1 \tag{5}$$

Linearizing the space equation around the stationary point conditions this gives,

$$\begin{bmatrix} \frac{d}{dt}\Delta r \\ \frac{d^2}{dt^2}\Delta r \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial r} & \frac{\partial f_1}{\partial \dot{r}} \\ \frac{\partial f_2}{\partial r} & \frac{\partial f_2}{\partial \dot{r}} \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta \dot{r} \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix} \Delta u$$

Thus using 4,5 this gives,

$$\frac{d}{dt}\Delta x(t) = \begin{bmatrix} 3\omega^2 & 0 \\ 0 & 1 \end{bmatrix} \Delta x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta u(t)$$

For the output $r(t)$,

$$r(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \Delta x(t)$$