EE320 - Electromagnetic Theory

Assignment in Plane Waves

W.M.B.S.K.Wijenayake (E/19/445)

09/01/2024

Question 1

Given that,

$$\underline{E}(y) = 5e^{-j10\pi y}\hat{z}$$

a. For the direction of propagation, considering,

$$\underline{E}(y) = 5e^{-j\pi y}\hat{z}$$

The direction $= +y$

b. For the wave number k,

$$\begin{aligned} k &= 10\pi \\ &= 31.41\ rads^{-1} \end{aligned}$$

c. Velocity of propagation v,

$$\begin{aligned} v &= \frac{\omega}{k} \\ &= \frac{2\pi f}{k} \\ &= \frac{2 \times \pi \times 10^9}{10\pi} \\ &= 2 \times 10^8 ms^{-1} \end{aligned} \tag{1}$$

d. We also know that the velocity of propagation v,

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_r \epsilon_0}} \tag{2}$$

In the free space waves travel at c,

$$c = 3 \times 10^8 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Thus by 1 and 2,

$$
\begin{aligned}
2 \times 10^8 &= \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \\
\epsilon_r &= 1.5^2 \\
&= 2.25
\end{aligned}
$$

e. For intrinsic impedance η of the medium,

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} \tag{3}$$

For free space,

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \tag{4}$$

Using 3,4,

$$
\begin{aligned}
\eta &= \frac{120\pi}{\sqrt{\epsilon_r}} \\
&= \frac{120\pi}{1.5} \\
&= 80\pi \\
&= 251.32\ \Omega
\end{aligned}
$$

f. For the magnetic field intensity,

$$
\begin{aligned}
\underline{H} &= \frac{\hat{y} \times \underline{E}}{\eta} \\
&= \frac{1}{\eta} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1 & 0 \\ 0 & 0 & 5e^{-j10\pi y} \end{vmatrix} \\
&= \frac{5e^{-j10\pi y} \hat{x}}{80\pi} \\
&= \frac{e^{-j10\pi y}}{16\pi} \hat{x}
\end{aligned}
$$

This gives,

$$
\begin{aligned}
\underline{B} &= \mu H \\
&= \mu_0 \frac{e^{-j10\pi y}}{16\pi} \hat{x}
\end{aligned}
$$