EE352 - AUTOMATIC CONTROL

Week 3 Activity 3

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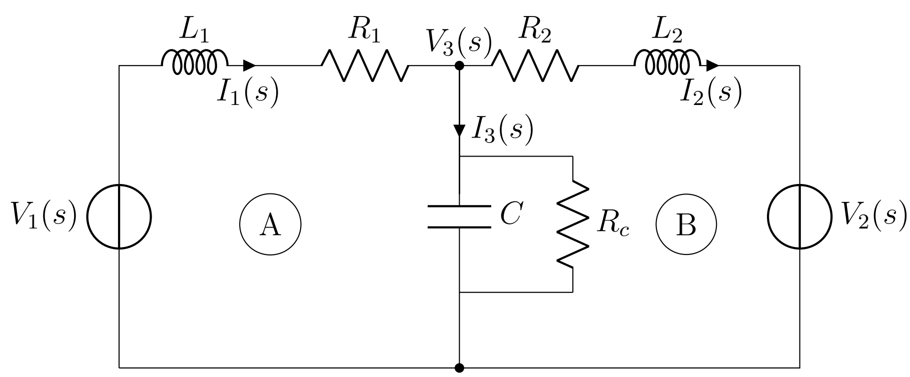

0.1

Consider loop A,

$$v_1(t) = L_1\frac{di_1(t)}{dt} + i_1(t)R_1 + v_3(t)$$
$$\frac{di_1(t)}{dt} = -\frac{R_1}{L_1}i_1(t) + \frac{1}{L_1}v_1(t) - \frac{1}{L_1}v_3(t)$$

Consider loop B,

$$v_3(t) = i_2(t)R_2 + L_2\frac{di_2(t)}{dt} + v_2(t)$$
$$\frac{di_2(t)}{dt} = -\frac{R_2}{L_2}i_2(t) + \frac{1}{L_2}v_3(t) - \frac{1}{L_2}v_2(t)$$

For $i_3(t)$,

$$i_3(t) = C\frac{dv_3(t)}{dt} + \frac{v_3(t)}{R_c}$$
$$\frac{dv_3(t)}{dt} = \frac{1}{C}i_3(t) - \frac{1}{CR_c}v_3(t)$$

$$\frac{dv_3(t)}{dt} = \frac{1}{C}i_1(t) - \frac{1}{C}i_2(t) - \frac{1}{CR_c}v_3(t)$$

Thus, selecting the state vector as,

$$x(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \\ v_3(t) \end{bmatrix}$$

Gives us,

$$\begin{bmatrix} \frac{di_1(t)}{dt} \\ \frac{di_2(t)}{dt} \\ \frac{dv_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & -\frac{1}{CR_c} \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ v_3(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} v_1(t) + \begin{bmatrix} 0 \\ -\frac{1}{L_2} \\ 0 \end{bmatrix} v_2(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ v_3(t) \end{bmatrix}$$

This gives us,

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bv_1(t) + Cv_2(t) \\ y(t) &= Dx(t) \end{aligned}$$

Converting into Laplace Domain, gives,

$$sX(s) = AX(s) + BV_1(s) + CV_1(s) \tag{1}$$

$$Y(s) = DX(s) \tag{2}$$

Using 1,

$$\begin{aligned} (sI - A)X(s) &= BV_1(s) + CV_2(s) \\ X(s) &= (sI - A)^{-1}\left[BV_1(s) + CV_2(s)\right] \end{aligned}$$

Substituting to 2,

$$Y(s) = D(sI - A)^{-1}\left[BV_1(s) + CV_2(s)\right]$$

Consider $H = sI - A$,

$$H = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & -\frac{1}{CR_c} \end{bmatrix}$$

$$= \begin{bmatrix} s + \frac{R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & s + \frac{R_2}{L_2} & -\frac{1}{L_2} \\ -\frac{1}{C} & \frac{1}{C} & s + \frac{1}{CR_c} \end{bmatrix}$$

Determinant of the matrix H,

$$det(H) =$$

$$s^3 + s^2\left(\frac{R_2}{L_2} + \frac{R_1}{L_1} + \frac{1}{cR_c}\right) + \frac{R_1 R_2}{L_1 L_2}s + \frac{1}{CL_1L_2R_c}\left[s(L_1(R_2 + R_c) + L_2(R_1 + R_c)) + R_1(R_2 + R_c) + R_2R_c\right]$$

Hence, the inverse of matrix $H = P =$,

$$\frac{1}{det(H)}\begin{bmatrix} s^2 + \frac{R_2 s}{L_2} + \frac{s}{CR_c} + \frac{R_2}{CL_2R_c} + \frac{1}{CL_2} & \frac{1}{CL_1} & \frac{-L_2s - R_2}{L_1L_2} \\ \frac{1}{CL_2} & s^2 + \frac{R_1 s}{L_1} + \frac{s}{CR_c} + \frac{R_1}{CL_1R_c} + \frac{1}{CL_1} & \frac{L_1 s + R_1}{L_1 L_2} \\ \frac{L_2 s + R_2}{CL_2} & \frac{-L_1 s - R_1}{CL_1} & s^2 + \frac{R_2 s}{L_2} + \frac{R_1 s}{L_1} + \frac{R_1R_2}{L_1L_2} \end{bmatrix}$$

Thus, $V_3(s)$,

$$V_3(s) = DP\left[BV_1(s) + CV_2(s)\right]$$

We know that,

$$BV_1(s) + CV_2(s) = \begin{bmatrix} \frac{V_1(s)}{L_1} \\ \frac{V_2(s)}{L_2} \\ 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

By substituting,

$$V_3(s) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} P \begin{bmatrix} \frac{V_1(s)}{L_1} \\ \frac{V_2(s)}{L_2} \\ 0 \end{bmatrix}$$

This gives,

$$V_3(s) = \frac{1}{det(H)} \left[\frac{V_{2(s)}(-L_1L_2R_cs - L_2R_1R_c)}{CL_1L_2^2R_c} + \frac{V_{1(s)}(L_1L_2R_cs + L_1R_2R_c)}{CL_1^2L_2R_c} \right]$$

0.2

When $v_2(t)$ is short-circuited,

$$\begin{aligned} V_2(s) &= 0 \\ V_3(s) &= \frac{1}{det(H)} \left[\frac{V_{1(s)}(L_1L_2R_cs + L_1R_2R_c)}{CL_1^2L_2R_c} \right] \\ \frac{V_3(s)}{V_1(s)} &= \frac{1}{det(H)} \left[\frac{(L_1L_2R_cs + L_1R_2R_c)}{CL_1^2L_2R_c} \right] \\ &= \frac{1}{det(H)} \left(\frac{s}{CL_1} + \frac{R_2}{CL_1L_2} \right) \end{aligned}$$

Thus,

$$G(s) = \frac{1}{det(H)} \left(\frac{s}{CL_1} + \frac{R_2}{CL_1L_2} \right)$$