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Discussion

Daley–Kendal models in fake-news scenario

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ABSTRACT

The exponential growth of communication facilities and social networks has been radically changing human habits and influencing the interaction processes, such as purchasing products, reading, knowing new friends, teaching and, more recently, voting in democratic elections or plebiscites. This scenario has led the society to concerns on increase in dishonest activities and spreading of lies and their consequent contribution to polarization of views, disharmony and prejudice. The first important case surfaced with the *Facebook-Cambridge Analytica* scandal and there have since been many speculations about the influence of fake-news, transmitted through social networks in several elections, such as those in USA, Brazil and Nigeria. Consequently, checking the veracity of news propagated through social networks has become indispensable, and this responsibility must be borne by the social networks firms; accordingly, they have begun developing and operating rigorous electronic verifying algorithms. This is a microscopic approach toward the fake-news problem, and can be complemented with a macroscopic approach using some compartmental models that are commonly employed in epidemiology and rumor propagation. Specifically, the Daley–Kendall (DK) model, which appears to be a realistic compartmental model in rumor spreading and viral marketing, is suitable to describe fake-news propagation and can be equipped with a veracity checking compartment of population. In this study, the DK model is adopted to address the fake-news propagation problem, and the influence of the newly included veracity checking compartment on attenuating fake news is studied.

1. Introduction

The high-speed diffusion of information available in modern data networks and the technological advances have led to some important industrial and social progress, and have simultaneously created an environment conducive for dishonest activities and hate, that endanger either collective and individual lives or democracies. In concurrence with the perception of this phenomenon by people, regular and academic press have started publishing and researching this subject [1–3].

The 2016 presidential election of USA created large discussions on the influence of Russia on social networks that contributed to the electoral results [4]. Since then, evaluating the impact of fake-news on election results has come an object of study either using classical statistical tools [5] or by modeling connections between several actors, such as marketeers, voters, politicians and journalists [6,7].

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This way of promoting politic campaigns has been proliferating across the world. Social networks were apparently decisive in the Brazilian elections [8] and false information disturbed the Nigerian elections [9]. Therefore, identifying fake news is an important computational problem [10,11]; further, because it involves studying computer virus propagation, two possible approaches, namely microscopic and macroscopic studies, can be adopted from biology [12–14].

Following the microscopic approach, artificial intelligence algorithms are being successfully used [15] by news agencies and financial market actors [16,17] with pattern recognition and classical statistics tools to detect and combat lies and prejudice propagation [18]. However, this approach cannot predict the complete behavior of networks (physical or social) when a deleterious action is performed.

Therefore, preventive measures against malignant actions must be developed based on network events using the macroscopic approach [19]. Thus, fake-news propagation models must be derived based on their epidemiological counterparts considering the network connection graph to predict long-term behaviors.

Because both human behavior and spreading of diseases are caused by individual interaction, the dynamics of social events can be studied using epidemiological models.

For instance, the Susceptible–Infected–Removed (SIR) epidemiological model proposed by Kermack and McKendrick [20] inspired Daley and Kendall (DK) to propose a rumor spreading model, known as the Ignorant–Spreader–Stifler (ISS), which shares some common properties with SIR [21]; the ISS marks a major leap in rumor propagation studies.

Further, the ISS equations for ignorant and spreader populations are analogous to the susceptible and infected populations of the SIR model, respectively. The main difference between these models is the stiflers, which is not present in the SIR model. Stiflers prevent the propagation of rumors and remain in a steady state; differently from the removed from SIR, that can be transformed into susceptible.

In the original work of DK, populations are categorized into three: the first group of people never heard the rumor (ignorant), the second group know and spread rumors (spreader), and the third group knows the rumor but never spreads it (stifler). Then, the rumor spreading model was refined considering the forgetting and remembering mechanisms in complex networks [22,23].

Various mathematical models for rumor propagation have been developed with (a) denial and skepticism in social networks [24], (b) viral *meme* propagation [25], (c) transmission age-independent population [26], (d) limited information exchange [27], and (e) two simultaneously conflicting ideas in a crowd [28].

In this research, considering the ISS as a generalization of the DK model, rumor spreading is studied. A new category is included in this study, which contains true verification individuals (checkers), and is known as the Ignorant–Spreader–Stifler–Checker (ISSC) model; the ISSC can model the effects of fake news to avoid or attenuate them.

Although different dynamical propagation behaviors are possible based on the connection of nodes, the ISSC model is developed considering homogeneous mixing for social networks, thus providing plausible qualitative results [29]. Considering that the checkers become either stiflers or spreaders, a simple model is analyzed, which provides a bifurcation diagram that allows composition of new strategies to address this important and malignant new phenomenon-fake-news.

This paper is organized as follows. The model equations are studied and the stability conditions are derived in the following section. Then, numerical results are provided in the section that follows, which clarifies how true verifiers alter the effect of fake-news propagation in a population. Finally, conclusions and discussions are provided in the last section.

2. ISSC model: equations and equilibrium states

The DK model [21] was developed based on the compartmental SIR model described in [30] and analyzed in [31]. The asymptotic behaviors and bifurcation were developed in [32] considering the ISS model as a generalization of the DK model, and the total population T was categorized into three groups, namely, ignorant (I), spreaders (S), and stiflers (R).

By introducing the fake-news checker compartment, the social network population can be divided into, ignorant (I), which contains individuals who never heard rumors but are susceptible to receive them; spreaders (S), which contains individuals that spread rumors or fake news; checkers (C), which comprises individuals who check the veracity of the rumors or news; stiflers (R), who are individuals that know the rumor or news but do not spread them. In this study, it is considered that C can be converted into either R or S, depending on the accuracy of the information.

2.1. Model equations

Dynamic propagation depends on the way the individuals encounter each other. When a spreader meets an ignorant, the latter becomes a new spreader with the rate coefficient β. Similar to the SIR model, the decay of the spreading process could be either because of forgetting or because spreaders become aware that the rumor has lost its value.

If the checkers verify the veracity of one rumor, it is considered that spreaders become checkers with the rate coefficient δ; in addition, checkers transform into stiflers with the rate coefficient σ, and into spreaders with the rate coefficient ω (Fig. 1). The parameter α represents the transformation rate of spreaders directly into stiflers.

The dynamical behavior of the model shown in Fig. 1 can be described by:

$$\begin{cases} \dot{I} = -\beta IS; \\ \dot{S} = \beta IS - \alpha S(S+R) - \delta SC + \omega SC; \\ \dot{C} = \delta SC - \sigma CR - \omega SC; \\ \dot{R} = \alpha S(S+R) + \sigma CR. \end{cases} \tag{1}$$

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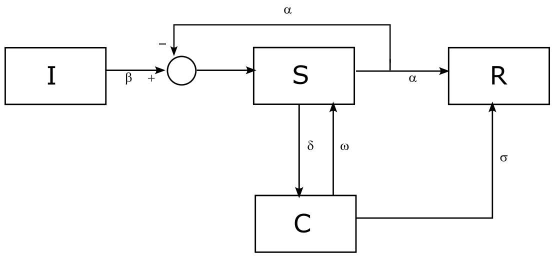


Fig. 1. ISSC model. I: ignorant population; S: spreader population; C: checker population; R: stifler population. The round block indicates an output combining the positive effect of the transformation of I into $S(+)$ and the negative effect of the transformation of S into $R(-)$.

It can be seen that, for the model represented by (1), the total population $T = I + S + C + R$ remains constant. Consequently, in spite of the description using four state variables, the model description has only three dimensions.

2.2. Equilibrium points

To investigate the way by which the checkers influence the propagation of rumors, the equilibrium points related to the model described by (1) must be determined and their stability discussed.

It is considered that $\alpha \neq 0$, i.e., spreaders can be converted into stiflers. Despite being an assumption, it is realistic for social networks, and implies that the fourth equation from (1) reaches an equilibrium state, if $S = 0$ and $C = 0$ or $R = 0$.

In the case of $S = C = 0$, the other three equations present possible equilibrium states for $I \neq 0$ and $R \neq 0$. If $S = R = 0$, the other three equations present possible equilibrium states for $I \neq 0$ and $C \neq 0$. Besides, examining (1), if $S = R = C = 0$, the state corresponding to $I \neq 0$ is an equilibrium point.

Therefore, in this case, it can be concluded that the possible equilibria are free of spreaders and are expressed as follows.

- $P_1 = (I, S, C, R) = (I^*, 0, 0, R^*)$;
- $P_2 = (I, S, C, R) = (I^*, 0, C^*, 0)$;
- $P_3 = (I, S, C, R) = (I^*, 0, 0, 0)$.

indicating that the introduction of checker combats fake-news propagation effectively.

Thus, it can be reasonably considered that checkers are converted only into stiflers ($\omega = 0$). This case is included in the former analysis with the same expressions for the equilibrium points.

The local stability of these points can be analyzed using the Hartman–Grobman theorem [33] and the Jacobian eigenvalues calculated for each equilibrium point can be examined.

To analyze the stability of these points, the general Jacobian (J) for the model (1) can be constructed as follows:

$$J = \begin{bmatrix} -\beta S & -\beta I & 0 & 0 \\ \beta S & \beta I - 2\alpha S - \alpha R - \delta C + \omega C & -\delta S + \omega S & -\alpha S \\ 0 & \delta C - \omega C & \delta S - \omega S - \sigma R & -\sigma C \\ 0 & 2\alpha S + \alpha R & \sigma R & \alpha S + \sigma C \end{bmatrix}.$$

The Jacobian calculated in the equilibrium point P_1 is given by:

$$J_{P_1} = \begin{bmatrix} 0 & -\beta I^* & 0 & 0 \\ 0 & \beta I^* - \alpha R^* & 0 & 0 \\ 0 & 0 & -\sigma R^* & 0 \\ 0 & \alpha R^* & \sigma R^* & 0 \end{bmatrix},$$

has two zero eigenvalues: λ_1, because the system is three-dimensional, and $\lambda_4 = 0$ indicating the existence of a central manifold [33]. The direction related to $\lambda_3 = -\sigma R^*$ indicates asymptotic stability.

Because $\lambda_2 = \beta I^* - \alpha R^*$, considering $T = I^* + R^*$, the equation can be rewritten as $\lambda_2 = I^*(\beta + \alpha)$. Consequently, it is responsible for an asymptotically stable direction if $I^* < \alpha T/(\beta + \alpha)$. However, if $I^* > \alpha T/(\beta + \alpha)$, the equilibrium point is unstable.

Using the same reasoning for P_2, the Jacobian is given by:

$$J_{P_2} = \begin{bmatrix} 0 & -\beta I^* & 0 & 0 \\ 0 & \beta I^* - \delta C^* + \omega C^* & 0 & 0 \\ 0 & \delta C^* - \omega C^* & 0 & -\sigma C^* \\ 0 & 0 & 0 & \sigma C^* \end{bmatrix}.$$

MATLAB R2013a [34] provides the following eigenvalues: $\lambda_1 = 0$, $\lambda_2 = \beta I^* - \delta C^* + \omega C^*$, $\lambda_3 = 0$, and $\lambda_4 = \sigma C^*$. Consequently, one real and positive eigenvalue exists for all possible parameter values, implying that P_2 is unstable.

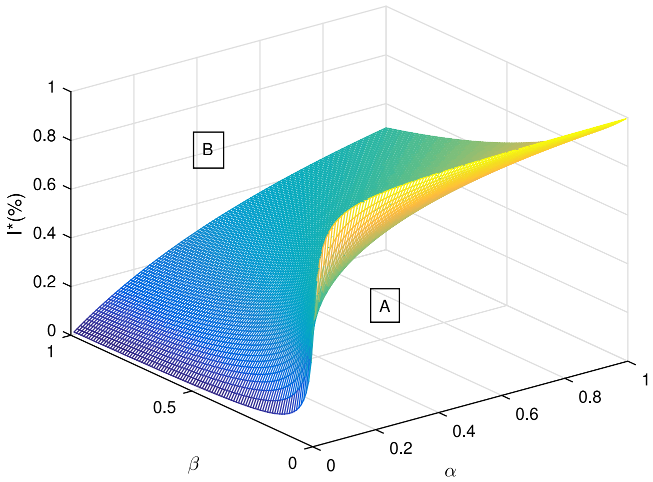


Fig. 2. ISSC models: equilibrium bifurcation.

Considering the equilibrium point P_3, the Jacobian is given by:

$$J_{P_3} = \begin{bmatrix} 0 & -\beta I^* & 0 & 0 \\ 0 & \beta I^* & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

MATLAB R2013a [34] provides the following eigenvalues: $\lambda_1 = 0$, 0, $\lambda_3 = 0$, and $\lambda_4 = \beta I^*$. Consequently, one real and positive eigenvalue exists for all possible parameter values, implying that P_3 is unstable.

If $\alpha = 0$, other equilibrium points are possible. However, only the case where $\alpha \neq 0$ is considered, corresponding to the situation that there is always a finite probability of undergoing a transformation from a spreader to a stifler.

2.3. Equilibrium bifurcation

As discussed already, the ISSC model considered in this study presents three equilibrium points: two that are unstable for any parameter combination (P_2; P_3), and the other (P_1), containing only ignorant and stifler individuals, whose stability depends on the ratio of the total ignorant population.

Because the stability condition depends on the combination $\frac{\alpha}{\alpha+\beta}$, a bifurcation diagram is provided in Fig. 2, which considers the percentage of I^*, related to the total population. The points in region A of the figure correspond to asymptotic stability of the equilibrium P_1. If the dynamics correspond to region B of the figure, P_1 is unstable.

3. Numerical simulations

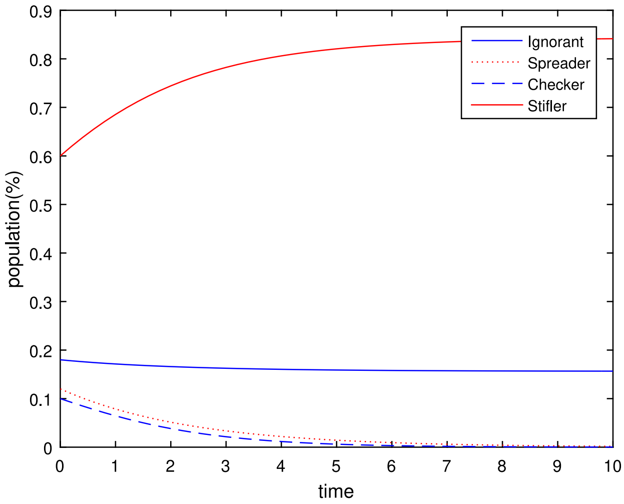
To quantitative analyze the influence of verification nodes on the rumor network, numerical simulations of the model are performed considering the unitary total population ($T = 1$). Thus, the instantaneous values of the populations I, S, C, and R are expressed in percentage.

First , the model is simulated to verify the stability of P_1, with the parameters belonging to region A of the bifurcation diagram shown in Fig. 2 being $\alpha = .6$, $\beta = .6$, and population I with an initial value near 20%.

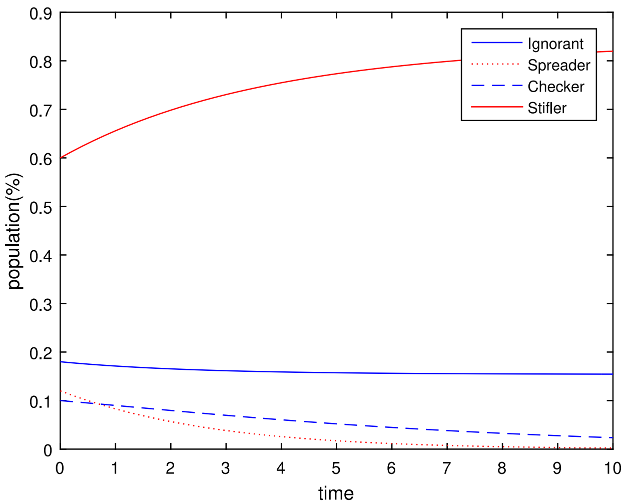
By adjusting the initial populations as, $I = 0.18$, $R = 0.60$, $C = 0.10$, and $S = 0.12$, and the parameters as $\sigma = 0.5$ and $\delta = 0.5$, the temporal evolution of the population for the model with checkers transform only into stiflers ($\omega = 0$), as shown in Fig. 3a. The parameters σ and δ do not change the behavior qualitatively, as shown in Fig. 3b; with their values set to 0.8, they change only the time relaxation.

By adjusting the initial populations as $I = 0.18$, $R = 0.60$, $C = 0.10$, $S = 0.12$, and the parameters as $\sigma = 0.8$, $\delta = 0.8$, and $\omega = 0.5$, the temporal evolution of the population of the model with checkers transform into stiflers and spreaders, as shown in Fig. 4a. The parameters ω, δ, and ω do not change the behavior qualitatively, as shown Fig. 4b; further, for $\omega = 0.8$, only the time relaxation is changed.

Following the simulations, two cases ($\omega = 0$ and $\omega \neq 0$) are studied with the parameters belonging to region B of the bifurcation diagram shown in Fig. 2, and related to point P_1, set as $\alpha = 1.0$, $\beta = 0.4$, and population I with an initial value near 80%. By setting the initial population as $I = 0.80$, $R = 0.01$, $C = 0.18$, and $S = 0.01$, and the parameters as $\sigma = 0.5$ and $\delta = 0.5$, the temporal evolution of the population of the model with checkers transforms only into stiflers



(a) Simulating near stable equilibrium ($\alpha = 0.6$; $\beta = 0.6$ $\sigma = 0.5$; $\delta = 0.5$; $\omega = 0$).



(b) Changing σ and δ ($\alpha = 0.6$; $\beta = 0.6$ $\sigma = 0.8$; $\delta = 0.8$; $\omega = 0$).

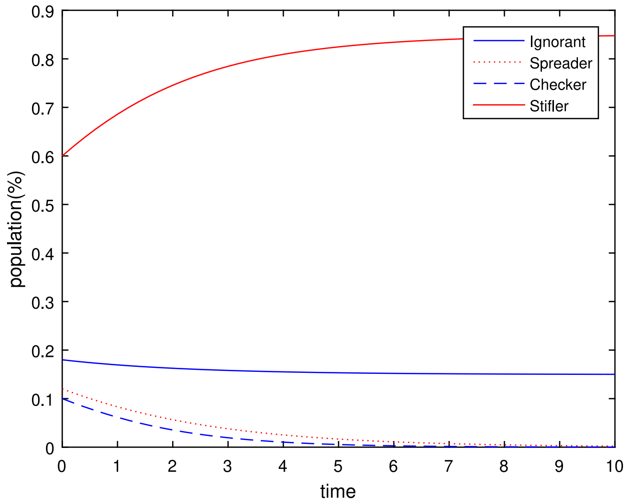
Fig. 3. Time evolution for populations starting near P_1 with checkers transformed only into stiflers.

($\omega = 0$), as shown in Fig. 5a. For the model whose checkers transform into stiflers and spreaders ($\omega \neq 0$), the temporal evolution for $\omega = 0.8$ is shown in Fig. 5b.

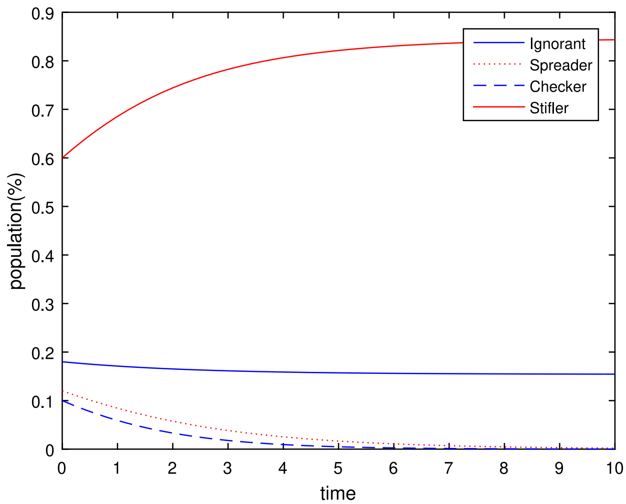
To complete several dynamical evolutions of the proposed model, which comprises a verifying category of population, simulations starting near the unstable point P_2 must be performed. Because the models with $\omega = 0$ and $\omega \neq 0$ present very similar behaviors, only the model, whose checkers transformed only into spreaders ($\omega = 0$), is simulated. By setting $\alpha = 0.6$, $\beta = 0.6$, $\delta = 0.5$, and $\sigma = 0.5$, with the initial populations $I = 0.70$, $C = 0.12$, $S = 0.08$, and $R = 0.10$, the populations are shown in Fig. 6a as functions of time. The results obtained for exchanging the initial populations I and C are shown in Fig. 6b.

Simulations near P_3 were not conduct because it is a simple case of instability. Any small perturbation of P_3 implies a dynamical behavior similar to the formerly simulated.

Because the DK rumor model was studied without true verification in former works [32], simulations of the model with only the ignorant, spreader, and stifler population can be used for comparison. Fig. 7a shows the time evolution of the model without checkers, starting with $I = 0.80$, $R = 0.01$, and $S = 0.19$, with the parameters α and β adjusted to



(a) Simulating near stable equilibrium ($\alpha = 0.6$; $\beta = 0.6$ $\sigma = 0.8$; $\delta = 0.8$; $\omega = 0.5$).



(b) Changing ω ($\alpha = 0.6$; $\beta = 0.6$ $\sigma = 0.8$; $\delta = 0.8$; $\omega = 0.8$).

Fig. 4. Time evolution for populations starting near P_1 with checkers transformed into stiflers and spreaders.

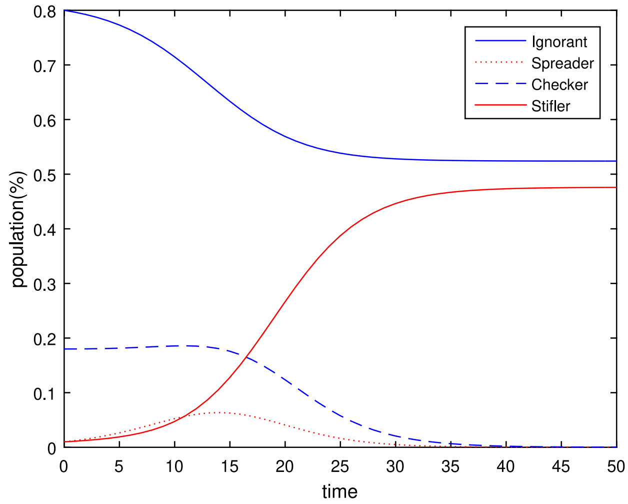
0.6., The resulting time evolution obtained by changing the initial populations to $I = 0.20$, $R = 0.30$, and $S = 0.50$ is shown in Fig. 7b.

4. Discussion of results

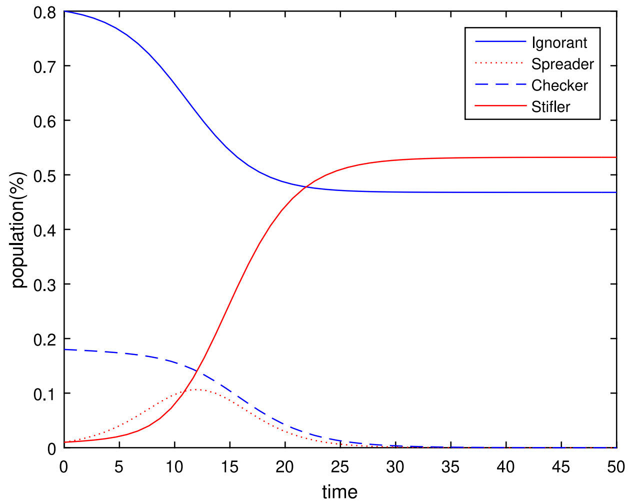
An important question of whether the introduction of true verification individuals (checkers) to the whole population avoids propagation of fake news needs to be answered.

A comparison of Fig. 5a and b (corresponding to the models containing checkers) with Fig. 7a (corresponding to the model without checkers) under the same initial condition (i.e., simulations are performed starting with high ignorant populations) indicates that the final ignorant population with checkers is considerably higher than that without checkers, indicating that the presence of checkers prevents the propagation of fake news. Moreover, a part of the final stiflers were originally checkers, and were consequently not influenced by fake news.

Considering the simulations starting with low ignorant populations, the results of the models containing checkers (Fig. 4a and b), present a slightly higher performance than the model without checkers (Fig. 7b). The final ignorant



(a) Time evolution for populations with checkers transformed only into stiflers ($\alpha = 1.0$; $\beta = 0.4$ $\sigma = 0.8$; $\delta = 0.8$; $\omega = 0$).



(b) Time evolution for populations with checkers transformed into stiflers and spreaders ($\alpha = 1.0$; $\beta = 0.4$ $\sigma = 0.8$; $\delta = 0.8$; $\omega = 0.8$)

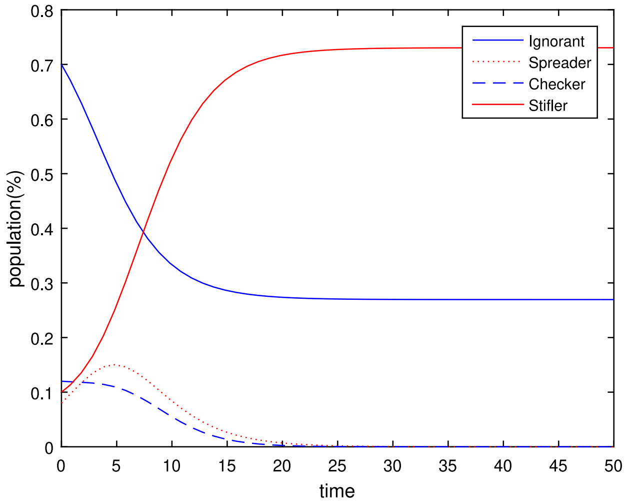
Fig. 5. Simulating with parameters related to unstable equilibrium for P_1.

population with checkers is higher than that without checkers, and the final stifler population of the model with checkers contains a large number of individuals aware of fake news.

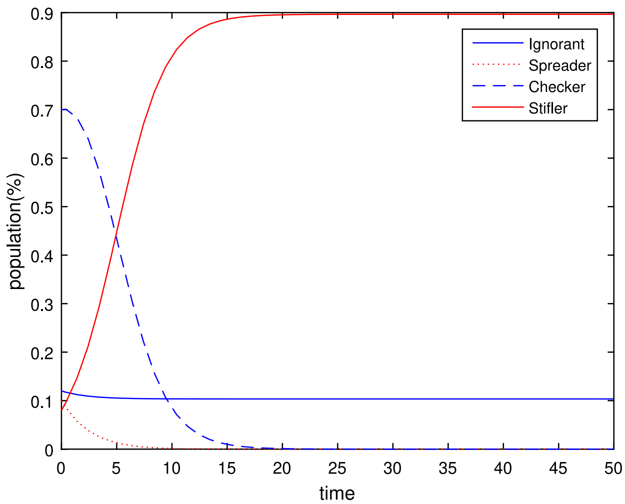
Therefore, including true verification individuals considerably attenuates the effect of fake news propagation across the entire population. Further, by observing Figs. 3 and 4, which are related to low ignorant initial populations, it can be concluded that the final results of the models, whose checkers transformed either into stiflers or into stiflers and spreaders, are equivalent.

However, an analysis of the instability situations (Figs. 5 and 6) indicates that fake news is more efficiently avoided if checkers are transformed only into stiflers. Further, the transition coefficients of different populations (α, β, δ, σ, and ω) do not affect the final results in each case, and change only the transition times.

Further, the transition coefficients of different populations (α, β, δ, σ, and ω) do not affect the final results in each case, and change only the transition times.



(a) Time evolution for populations starting with small number of checkers ($C_0 = 0.12$) ($\alpha = 0.6$; $\beta = 0.6$ $\sigma = 0.5$; $\delta = 0.5$; $\omega = 0$).



(b) Time evolution for populations starting with high number of checkers ($C_0 = 0.70$) ($\alpha = 0.6$; $\beta = 0.6$ $\sigma = 0.5$; $\delta = 0.5$; $\omega = 0$).

Fig. 6. Simulating with parameters related to unstable equilibrium for P_2.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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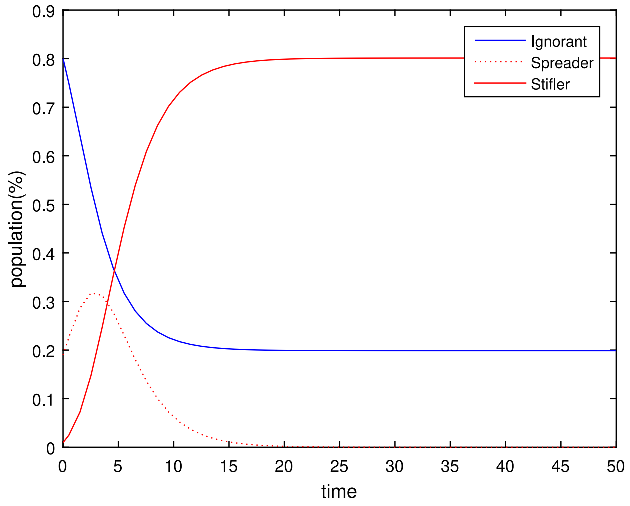
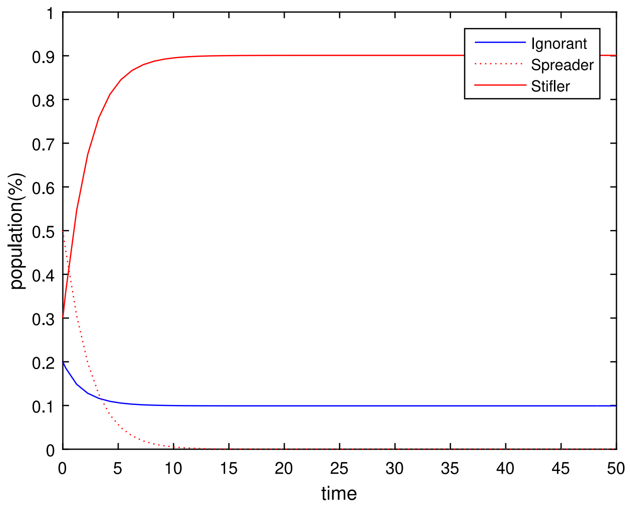


(a) Time evolution for populations starting with high ignorant population (Model without checkers, $\alpha = \beta = 0.6$).



(b) Time evolution for populations starting with low ignorant population (Model without checkers, $\alpha = \beta = 0.6$).

Fig. 7. Simulating model without checkers.

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