Contents

1 Equation of continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \tag{1}$$

2 Navier-Stokes one-dimensional

Cited [?] abud [?]

$$\rho \left[\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right] = \frac{\partial \left(\mu \frac{\partial v}{x} \right)}{\partial x} - \frac{\partial p}{\partial x} + X \tag{2}$$

$$\begin{cases} \rho : \text{Car density} \\ p : \text{Local car pressure} v : \text{Car instant velocity} \\ \mu : \text{Viscosity} \\ X : \text{Sum of all inner particle interaction forces} \end{cases} \tag{3}$$

3 Mathematical meanings of X and p

3.1 Relaxation process meaning

- If the perception is that current velocity v is too slow compared to what can be safely achieved, then X is positive.

  – If the perception is that current velocity v is too fast and

  dangerous compared to the traffic condition, then X is negative.

- Oscillatory behavior can appear.

3.2 Consider the time independent homogeneous condition

$$
\begin{cases}
\langle\frac{\partial v}{\partial x}\rangle = 0 \quad \text{(Time independent)} \\
\langle\dfrac{\partial\left(\mu\frac{\partial v}{\partial x}\right)}{\partial x}\rangle = 0 \quad \text{(Time independent and Homogeneous)} \\
\because \langle\dfrac{\partial\left(\mu\frac{\partial v}{\partial x}\right)}{\partial x}\rangle = \langle\dfrac{\partial \mu}{\partial x}\left(\frac{\partial v}{\partial x}\right)\rangle + \langle\mu\left(\dfrac{\partial^2 v}{\partial^2 x}\right)\rangle \\
\left((\langle\dfrac{\partial \mu}{\partial x}\rangle = 0 \quad \text{Homogeneous}) \wedge (\langle\dfrac{\partial^2 v}{\partial^2 x}\rangle = 0 \quad \text{Time independent and Homogeneous})\right) \\
= 0 \\
\langle\frac{\partial p}{\partial x}\rangle = 0 \quad \text{(Time independent)}
\end{cases}
\tag{4}
$$

3.3 X and acceleration; considerations of instant velocity

According to 5 and X definition, under 4, we will have:

$$
\begin{aligned}
&(\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = \frac{\partial\left(\mu\frac{\partial v}{\partial x}\right)}{\partial x} - \frac{\partial p}{\partial x} + X) \wedge (X = \rho.\frac{(V(\rho) - v)}{\tau}) \\
&\implies \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{V(\rho) - v}{\tau}
\end{aligned}
\tag{5}
$$