Contents

1 DONE Done

Pertubation Methods:

- Multiple Scales Expansion.
- Anharmonic Oscillator example.

`https://www.math.arizona.edu/~ntna2007/Perturbation_Methods.pdf`

2 NEXT Reading

Title: **PHY-892 The Many-Body problem, from perturbation theory to dynamical-mean Öeld theory (lecture notes)**.
Author: André-Marie Tremblay
`https://pitp.phas.ubc.ca/confs/sherbrooke2018/archives/N-corps-2017.pdf`

3 TODO Todo

`http://galileoandeinstein.phys.virginia.edu/7010/CM_22_Resonant_Nonlinear_Oscillations.html` `https://chem.libretexts.org/Bookshelves/`

Physical_and_Theoretical_Chemistry_Textbook_Maps/Supplemental_Modules_(Physical_and_Theoretical_Chemistry)

4 Spring-mass

Following **FYS 3120: Classical Mechanics and Electrodynamics**, considering:

4.1 The equation of motion

$$\ddot{q} = f(q, \dot{q}) \tag{1}$$

4.1.1 Mass-spring

In our case, mass-pring:

$$\begin{aligned} &m.\ddot{q} = -k.q \\ &\Leftrightarrow \ddot{q} = -\frac{k}{m}q \wedge f(q, \dot{q}) = -\frac{k}{m}q \end{aligned} \tag{2}$$

4.2 Deviation from Equilibrium

Let the variation from equilibrium be: $\rho = q - q_0$.

$$\ddot{\rho} = f(q_0 + \rho, \dot{\rho}) \tag{3}$$

e.g.,

$$q = \rho + q_0 \implies \dot{q} = \dot{\rho} \wedge \ddot{q} = \ddot{\rho} \tag{4}$$

4.2.1 Mass-spring

In our spring-mass case,

$$\ddot{\rho} = -\frac{k}{m}(q_0 + \rho) \tag{5}$$

4.3 Power expansion - Expansion around $(q_0, 0)$

$$\ddot{\rho} = f(q_0, 0) + \rho\frac{\partial f}{\partial \rho}(q_0, 0) + \dot{\rho}\frac{\partial f}{\partial \dot{\rho}}(q_0, 0) \tag{6}$$

$f(q_0, 0) = 0 \because q_0$ is equilibrium point: The equation of motion would be zero in this point.

We also neglect second order or higher terms.

4.3.1 Mass-spring

In our case,

$$\ddot{\rho} = \rho\frac{\partial f}{\partial \rho}(q_0, 0) + \dot{\rho}\frac{\partial f}{\partial \dot{\rho}}(q_0, 0) \tag{7}$$

in which,

$$\left(\frac{\partial f}{\partial \rho}(\rho, \dot{\rho}) = -\frac{k}{m}\right) \wedge \left(\frac{\partial \dot{f}}{\partial \dot{\rho}}(\rho, \dot{\rho}) = 0\right) \tag{8}$$

Therefore, in our case, the pertubation equation is:

$$\ddot{\rho} = -\frac{k}{m}\rho \tag{9}$$

$$\implies \rho(t) = A\sin\left(\sqrt{\frac{k}{m}}t\right) + Bcos\left(\sqrt{\frac{k}{m}}t\right) \tag{10}$$

1. Particular solution Particular case if $t_0 = 0$,

$$\rho_0 = B \implies \rho(t) = \rho_0\cos\left(\sqrt{\frac{k}{m}}t\right) \tag{11}$$

2. General initical condition

   If $\rho(t_0) = \rho_0$,

$$
\begin{aligned}
\rho_0 &= A\sin\left(\sqrt{\frac{k}{m}}t\right) + Bcos\left(\sqrt{\frac{k}{m}}t\right) \\
\Leftrightarrow \rho_0 &= \left(\frac{A\sin\left(\sqrt{\frac{k}{m}}t_0\right) + Bcos\left(\sqrt{\frac{k}{m}}t_0\right)}{\sqrt{A^2+B^2}}\right).\sqrt{A^2+B^2}
\end{aligned}
\tag{12}
$$

Let the right-triangle with sides oposite side A and adjacent side B, with thus hippotenuse, $\sqrt{A^2+B^2}$. This triangle define a angle $\alpha = \arctan(\frac{A}{B})$. So, $\sin(\alpha) = \frac{A}{\sqrt{A^2+B^2}}$ and $\cos(\alpha) = \frac{B}{\sqrt{A^2+B^2}}$.

$$
\begin{aligned}
\implies \rho_0 &= (\sqrt{A^2+B^2})\left(\sin(\alpha)\sin\left(\sqrt{\frac{k}{m}}t_0\right) + \cos(\alpha)cos\left(\sqrt{\frac{k}{m}}t_0\right)\right) \\
\therefore \rho_0 &= (\sqrt{A^2+B^2})\cos\left(\sqrt{\frac{k}{m}}t_0 - \alpha\right)
\end{aligned}
\tag{13}
$$