Inotroduction to Numerical Analysis
-- Numerical methods for ODE #3 --

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Today's contents

■ Observe the convergence of the numerical solution to the true solution.

In particular, focus on the relationship between the accuracy of each method and the speed of convergence

Theorem on convergence in the previous class:
"the k-th accuracy" leads " $\max_{\{0 \le n \le N\}} |e_n| = O(h^k)$"

Exercise 1 (test problem)

■ Test problem (IVP1) *1:

$$\frac{dx}{dt} = ax(t), \quad 0 < t < T,$$
$$x(0) = x_0.$$

- Check the source file “euler.c” (Euler’s method)
- Implement Heun’s method and the 4th order Runge-Kutta method in C/C++.

- Case 1: Let $a = 1, T = 1, x_0 = 1.$ (Growth case)
  Compute a numerical solutions by the above 3 methods and an exact solution for the case $h = \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$, where h is a mesh size.

- Case 2: Let $a = -1, T = 1, x_0 = 1.$ (Decay case)
  Compute a numerical solutions by the 3 methods and an exact solution for the case $h = \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$.

- Calculate $\max_{\{0 \le n \le N\}} |e_n|$ $(e_n = x(t_n) - X_n)$ for all cases in Growth case. Here $t_n = 0 + nh$, $t_N = T$ and X_n is an approximate value of $x(t_n)$.
  Consider the relation between the above errors and the mothods.
  keywords: aqquracy, convergence rate

＊1 [Malthus model] (IVP1) describes a very simple population dynamics proposed by Thomas Robert Malthus.

Exponential notation (E notation)

- 1.2345e6 = 1.2345$\times 10^{6}$ = 12345600
- -1.2345e-10 = -1.2345 $\times 10^{-10}$ = -0.000000000123

Exercise 2 (main)

- Target problem (IVP2)*1:

$$\frac{dx}{dt} = x(t) - x^2(t)\,, \quad 0 < t < \mathrm{T},$$
$$x(0) = x_0.$$

- Implement Euler's method, Heun's method and the 4th order Runge-Kutta method in C/C++.
- Case 1: Let $T = 5, x_0 = 0.1.$ (Growth case)

Compute a numerical solutions by the above 3 methods and an exact solution for the case $h = \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$, where h is a mesh size.

- Case 2: Let $T = 5, x_0 = 2.$ (Decay case)

Compute a numerical solutions by the 3 methods and an exact solution for the case $h = \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$.

- Calculate $\max_{\{0 \le n \le \mathrm{N}\}} |e_n|$ $(e_n = x(t_n) - X_n)$ for all methods and mesh sizes in Case 1.

＊1 [Logistic equation] (IVP2) is a very famous nonlinear model on population dynamics proposed by Pierre-François Verhulst. The solution curve in the growth case (More precisely, the case with $x_0 < 0.5$.) is called "logstic curve", "Sigmoid curve" or "S-curve".

Today’s Assignment

- Summarize the results of Exercise 2 and describe your observations on errors and convergence.

- Insert your C/C++ program using the 4^{th} o r der Runge-Kutta method for Exercise 2 in your report.

- File type: PDF only.

- Language: English only

- File name: YourName-ODE1.pdf
  For example: Ishiwata-ODE1.pdf

- Hand in your PDF file via “Ishiwata's part: Assignment 2 (computation part 1)” in ScombZ.

- Deadline ⇒ See Today’s content in ScombZ/LMS

How to save the output to a file.

- use the redirection “>” in command shell

    euler.exe > data.txt

    ⇒All output is saved to a file “data.txt”

Schedule

- TNA #13 (Ishiwata's part 4/4)　:

  Numerical computation 2

  System of ODEs

  ※preparation:　Please install "gnuplot" into your PC,

  if you use your own PC.

Tips in gnuplot

- plot a graph of a function

  ex) plot sin(x)

- plot a graph of the data

  ex) plot “data.txt” using 1:2 with linespoints

- Upper left button of the window for the graph
  - save the graph image in an image file.
  - copy to clipboard（⇒paste it on Word file)