An Analysis of Mergesort - Computation Time and Number of Comparisons

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Brief

The goal of this report is to test the performance of Mergesort in terms of computation time and number of comparisons. We use a Java implementation, and timing is done internally using Java's built-in system time.

Test Cases

We ran 6 different kinds of tests:

- Random
- Already sorted
- Almost sorted
- Sorted in reverse
- Almost sorted in reverse
- Random with 10 copies of each entry

with sizes ranging from 100,000 to 1,000,000 (incrementing by 100,000) and three instances of each combination of test kind and size.

Critical Code Analysis

Method 0: `void sort(int[])`

```
void sort(int[] array) {
    comp = 0;
    comp++;
    if(array.length > 1) {
        int halfSize = array.length / 2;
        int[] leftArray = new int[halfSize];
        int[] rightArray = new int[array.length - halfSize];
        System.arraycopy(array, 0, leftArray, 0, halfSize);
        System.arraycopy(array, halfSize, rightArray, 0, array.length - halfSize)

        sort(leftArray);
```

```
        sort(rightArray);

        merge(array, 0, leftArray, rightArray);
    }
}
```

Code	Cost	Frequency
`void sort(int[] array) {`	0	1
`comp = 0;`	c_1	1
`comp++;`	c_2	1
`if(array.length > 1) {`	c_3	1
`int halfSize = array.length / 2;`	c_4	1
`int[] leftArray = new int[halfSize];`	c_5	1
`int[] rightArray = new int[array.length - halfSize];`	c_6	1
`System.arraycopy(array, 0, leftArray, 0, halfSize);`	n	1
`System.arraycopy(array, halfSize, rightArray, 0, array.length - halfSize);`	n	1
`sort(leftArray);`	$T_s(n/2)$	1
`sort(rightArray);`	$T_s(n/2)$	1
`merge(array, 0, leftArray, rightArray);`	$T_m(n)$	1
`}`	0	1
`}`	0	1

Method 1: `void merge(int[], int, int[], int[])`

```
private void merge(int[] outputSequence, int dest, int[] leftSequence, int[] righ
    int i = dest;
    int j = 0;
    int k = 0;

    while(i < leftSequence.length && j < rightSequence.length) {
        comp++;
        if(leftSequence[i] < rightSequence[j])
            outputSequence[k++] = leftSequence[i++];
```

```java
            else
                outputSequence[k++] = rightSequence[j++];
        }

        while(i < leftSequence.length) {
            comp++;
            outputSequence[k++] = leftSequence[i++];
        }
        comp++;

        while(j < rightSequence.length) {
            comp++;
            outputSequence[k++] = rightSequence[j++];
        }
        comp++;
    }
```

Code	Cost	Frequency
`private void merge(int[] outputSequence, int dest, int[] leftSequence, int[] rightSequence) {`	0	1
`int i = dest;`	c_1	1
`int j = 0;`	c_2	1
`int k = 0;`	c_3	1
`while(i < leftSequence.length && j < rightSequence.length) {`	c_4	n
`comp++;`	c_5	n
`if(leftSequence[i] < rightSequence[j])`	c_6	n
`outputSequence[k++] = leftSequence[i++];`	c_7	n
`else`	0	n
`outputSequence[k++] = rightSequence[j++];`	c_8	n
`}`	0	1
`while(i < leftSequence.length) {`	c_9	n
`comp++;`	c_{10}	n
`outputSequence[k++] = leftSequence[i++];`	c_{11}	n
`}`	0	1

Code	Cost	Frequency
`comp++;`	c_{12}	1
`while(j < rightSequence.length) {`	c_{13}	n
`comp++;`	c_{14}	n
`outputSequence[k++] = rightSequence[j++];`	c_{15}	n
`}`	0	1
`comp++;`	c_{16}	1
`}`	0	1

Adding all these up gives us $c_{17}n + c_{18}$. Therefore, $T_m(n) = O(n)$

Total

Define,

$$2^i = n$$

$$i = log_2(n)$$

$$i = lg(n)$$

Then, we can find the time complexity of Mergesort.

$$T(1) = 1$$

$$T(n) = c + 2T_{MS}(\frac{n}{2}) + n$$

$$= 2T_{MS}(\frac{n}{2}) + n$$

$$= 2(2T_{MS}(\frac{n}{2^2}) + \frac{n}{2}) + n$$

$$= 2^2T_{MS}(\frac{n}{2^2}) + 2n$$

$$= 2^2(2T_{MS}(\frac{n}{2^3} + \frac{n}{2^2}) + 2n$$

$$= 2^iT_{MS}(\frac{n}{2^i}) + in$$

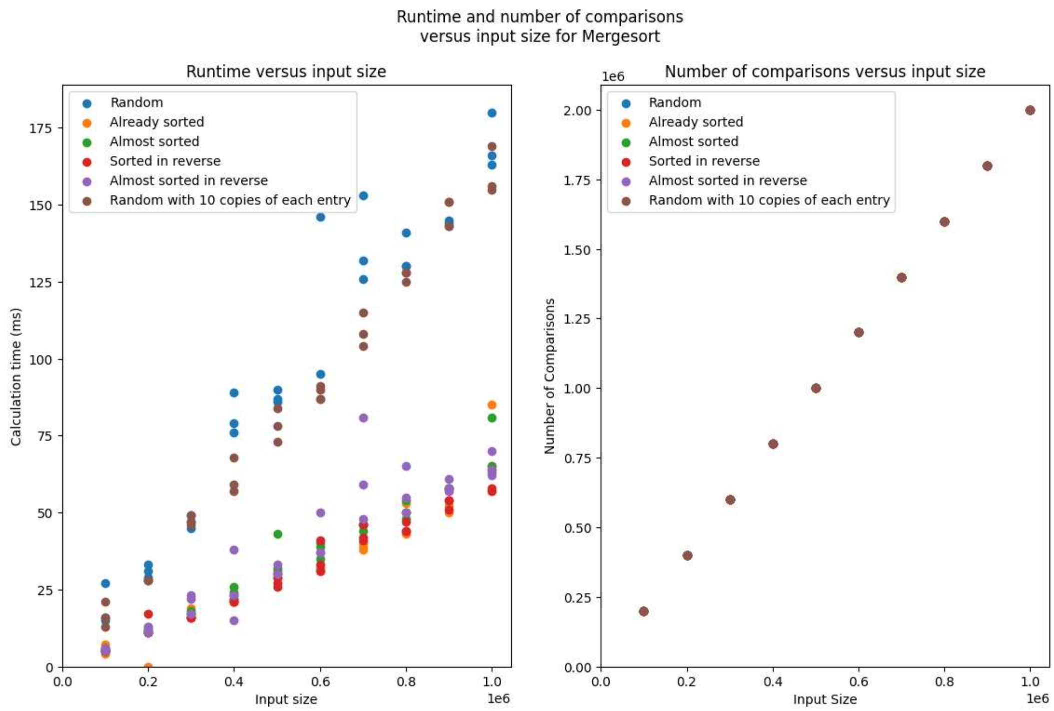
$$= 2^{lg(n)}T_{MS}(\frac{n}{lg(n)}) + nlg(n)$$

$$= n1 * T_{MS}(1) + nlg(n)$$

$$= nlg(n)$$

$$\Theta(nlg(n))$$

Impirical Analysis



The number of comparisons increased linearly with the size of input (independent of the organization), but the performance depended on how organized the input was. Inputs that were already Sorted or Sorted in Reverse resulted in the lowest times with Almost Sorted and Almost Sorted in Reverse were very close in many cases, and both random cases performed much worse.

Conclusion

One interesting outcome from this analysis of the merge sort is that the initial state of the data had an impact on the performance of the sort. This is interesting because the merge sort always breaks down the problem into an equal number of sub-problems, as seen by the constant number of comparisons for each iteration of a given input size. It could be that the merge sort works best with large input sizes, and as we "zoom out" on the graph by increasing the input size the difference between the sorted variations and the random variations decreases. After all, the benefits of log-based algorithms are best seen with large input times. Alternatively, it may be that there is some optimization going on which is hidden from us that allows working with sorted lists or near sorted lists to run faster on my particular machine. Either way, we should keep in mind that the difference in sorting time was minimal - only a handful of milliseconds between input sizes of up to 1,000,000. The best conclusion to take from this analysis is that the merge sort is consistant in terms of cost while still providing a very fast sort, particularly for large input sizes.

Appendix: Raw Data

Case 0: Random

Input Size	Processing Time (ms)	Number of Comparisons
100000	15	200040
100000	27	200040
100000	16	200040
200000	29	400042
200000	31	400042
200000	33	400042
300000	47	600044
300000	45	600044
300000	49	600044
400000	79	800044
400000	76	800044
400000	89	800044
500000	87	1000045
500000	90	1000045
500000	86	1000045
600000	146	1200046
600000	95	1200046
600000	87	1200046
700000	153	1400046
700000	126	1400046
700000	132	1400046
800000	141	1600046
800000	130	1600046
800000	130	1600046
900000	145	1800044

Input Size	Processing Time (ms)	Number of Comparisons
900000	143	1800044
900000	144	1800044
1000000	163	2000047
1000000	180	2000047
1000000	166	2000047

Case 1: Already sorted

Input Size	Processing Time (ms)	Number of Comparisons
100000	7	200040
100000	5	200040
100000	4	200040
200000	11	400042
200000	0	400042
200000	11	400042
300000	16	600044
300000	16	600044
300000	19	600044
400000	21	800044
400000	21	800044
400000	22	800044
500000	29	1000045
500000	30	1000045
500000	29	1000045
600000	32	1200046
600000	32	1200046
600000	31	1200046
700000	40	1400046
700000	38	1400046

Input Size	Processing Time (ms)	Number of Comparisons
700000	39	1400046
800000	50	1600046
800000	53	1600046
800000	43	1600046
900000	57	1800044
900000	52	1800044
900000	50	1800044
1000000	85	2000047
1000000	65	2000047
1000000	57	2000047

Case 2: Almost sorted

Input Size	Processing Time (ms)	Number of Comparisons
100000	5	200040
100000	5	200040
100000	6	200040
200000	11	400042
200000	11	400042
200000	13	400042
300000	17	600044
300000	18	600044
300000	17	600044
400000	23	800044
400000	24	800044
400000	26	800044
500000	31	1000045
500000	32	1000045
500000	43	1000045

Input Size	Processing Time (ms)	Number of Comparisons
600000	39	1200046
600000	35	1200046
600000	40	1200046
700000	46	1400046
700000	44	1400046
700000	46	1400046
800000	50	1600046
800000	54	1600046
800000	48	1600046
900000	58	1800044
900000	58	1800044
900000	58	1800044
1000000	81	2000047
1000000	63	2000047
1000000	65	2000047

Case 3: Sorted in reverse

Input Size	Processing Time (ms)	Number of Comparisons
100000	5	200040
100000	5	200040
100000	5	200040
200000	17	400042
200000	12	400042
200000	11	400042
300000	16	600044
300000	16	600044
300000	16	600044
400000	22	800044

Input Size	Processing Time (ms)	Number of Comparisons
400000	21	800044
400000	21	800044
500000	27	1000045
500000	26	1000045
500000	29	1000045
600000	41	1200046
600000	33	1200046
600000	31	1200046
700000	41	1400046
700000	46	1400046
700000	42	1400046
800000	44	1600046
800000	47	1600046
800000	44	1600046
900000	54	1800044
900000	51	1800044
900000	54	1800044
1000000	64	2000047
1000000	58	2000047
1000000	57	2000047

Case 4: Almost sorted in reverse

Input Size	Processing Time (ms)	Number of Comparisons
100000	5	200040
100000	6	200040
100000	6	200040
200000	12	400042
200000	11	400042

Input Size	Processing Time (ms)	Number of Comparisons
200000	13	400042
300000	17	600044
300000	22	600044
300000	23	600044
400000	23	800044
400000	15	800044
400000	38	800044
500000	33	1000045
500000	30	1000045
500000	30	1000045
600000	37	1200046
600000	37	1200046
600000	50	1200046
700000	59	1400046
700000	48	1400046
700000	81	1400046
800000	65	1600046
800000	50	1600046
800000	55	1600046
900000	57	1800044
900000	58	1800044
900000	61	1800044
1000000	62	2000047
1000000	64	2000047
1000000	70	2000047

Case 5: Random with 10 copies of each entry

Input Size	Processing Time (ms)	Number of Comparisons
100000	16	200040
100000	21	200040
100000	13	200040
200000	28	400042
200000	28	400042
200000	28	400042
300000	46	600044
300000	49	600044
300000	47	600044
400000	68	800044
400000	57	800044
400000	59	800044
500000	73	1000045
500000	78	1000045
500000	84	1000045
600000	87	1200046
600000	90	1200046
600000	91	1200046
700000	108	1400046
700000	115	1400046
700000	104	1400046
800000	125	1600046
800000	128	1600046
800000	128	1600046
900000	151	1800044
900000	151	1800044
900000	143	1800044
1000000	169	2000047

Input Size	Processing Time (ms)	Number of Comparisons
1000000	155	2000047
1000000	156	2000047