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Geomath

exercise 128

1. a. $5x^5$ b. $2,4x^{3,5}$ c. $\frac{1}{x}$

Jawab

a. $y = 5x^5$

$\frac{dy}{dx} = 5 \cdot 5x^{5-1} = 25x^4$

b. $y = 2,4x^{3,5} =$

$\frac{dy}{dx} = 2,4 \times 3,5 \, x^{3,5-1} = 8,4x^{2,5}$

c. $y = \frac{1}{x} = x^{-1}$

$y = x^{-1} \Rightarrow \frac{dy}{dx} = -1 \cdot x^{-2} = -\frac{1}{x^2}$

5. a. $-4 \cos 2x$ b. $2e^{6x}$ c. $\frac{3}{e^{5x}}$

Jawab

a. Turunan dari $-4 \cos 2x$

$\frac{d}{dx} \cos kx = -k \sin kx$

$\frac{d}{dx}(-4 \cos 2x) = -4 \cdot (-2 \sin 2x)$

$= 8 \sin 2x$

b. $2e^{6x}$

Gunakan aturan turunan eksponensial

$\frac{d}{dx} e^{kx} = ke^{kx}$

$\frac{d}{dx}(2e^{6x}) = 2(6e^{6x})$

$= 12e^{6x}$

c. ~~$\frac{3}{dx}$~~ $\frac{3}{e^{5x}}$

$\frac{3}{e^{5x}} = 3e^{-5x} = \frac{d}{dx} e^{-5x} = (-5)e^{-5x}$

$\frac{d}{dx}(3e^{-5x}) = 3(-5e^{-5x}) = -\frac{15}{e^{5x}}$

Exercise 16.4

7. The Volume of a cone of height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$. Determine

$$\frac{\partial V}{\partial h} \text{ and } \frac{\partial V}{\partial r}$$

Jawab

- Turunan parsial $\frac{\partial V}{\partial h}$

$$\frac{\partial V}{\partial h} = \frac{\partial}{\partial h}(\pi r^2 h) = \pi r^2$$

$$\frac{\partial V}{\partial h} = \frac{1}{3}\pi r^2$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{\partial V}{\partial h} = \frac{1}{3}\pi r^2$$

- Turunan Parsial $\frac{\partial V}{\partial r}$

$$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r}\left(\frac{1}{3}\pi r^2 h\right)$$

gunakan aturan turunan

$$\frac{1}{3}\pi h \cdot \frac{d}{dr}(r^2)$$

$$\frac{1}{3}\pi h \cdot 2r = \frac{2}{3}\pi r h$$

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9. Diketahui fungsi

$$z = \sin\left(\frac{n\pi x}{L}\right) \times \left[k\cos\left(\frac{n\pi b}{L}\right)t + c\sin\left(\frac{n\pi b}{L}\right)t\right]$$

Mencari $\frac{\partial z}{\partial x}$

$$\frac{d}{dx}\sin u = \cos u \cdot \frac{du}{dx}$$

$$\frac{du}{dx} = \frac{n\pi}{L}$$

Sehingga: $\frac{d}{dx}\sin\left(\frac{n\pi x}{L}\right) = \cos\left(\frac{n\pi x}{L}\right) \cdot \frac{n\pi}{L}$

- Terapkan aturan perkalian

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

$$\cdot f(x) = \sin\left(\frac{n\pi x}{L}\right)$$

$$g(x) = k\cos\left(\frac{n\pi b}{L}\right)t + c\sin\left(\frac{n\pi b}{L}\right)t$$

$$f'(x) = \cos\left(\frac{n\pi x}{L}\right) \cdot \frac{n\pi}{L}$$

Jadi

$$\frac{\partial z}{\partial x} = \left(\frac{n\pi}{L}\right)\cos\left(\frac{n\pi x}{L}\right) \times \left[k\cos\left(\frac{n\pi b}{L}\right)t + c\sin\left(\frac{n\pi b}{L}\right)t\right]$$

2. Mencari $\frac{\partial z}{\partial y}$

$$\frac{d}{dy}\left[k\cos\left(\frac{n\pi b}{L}\right)t + c\sin\left(\frac{n\pi b}{L}\right)t\right]$$

$$\frac{d}{dy}\cos v = -\sin v \cdot \frac{dv}{dy}$$

dari turunan $\cos u$ adalah:

$$\frac{d}{dy}\cos u = -\sin u \cdot \frac{du}{dy}$$

$$\frac{d}{dy}\sin u = \cos u \cdot \frac{du}{dy}$$

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$$\frac{du}{dy} = \frac{n\pi}{L}$$

Sehingga

$$\frac{\partial z}{\partial y} = \left(\frac{n\pi}{L}\right) \cos\left(\frac{n\pi x}{L}\right) \times \left[k \cos\left(\frac{n\pi b}{L}\right) t + c \sin\left(\frac{n\pi b}{L}\right) t\right]$$

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Exercise 128

6) a) $4 \ln 9u$

b) $\frac{e^u - e^{-u}}{2}$

c) $\frac{1 - \sqrt{u}}{u}$

Jawab

a) $4 \ln 9u$

$= \frac{d}{du} \ln f(u) = \frac{1}{f(u)} \cdot f'(u)$

$\frac{d}{du}(4 \ln 9u) = 4 \times \frac{1}{9u} \times 9$

$= \frac{4}{u}$

b) $\frac{e^u - e^{-u}}{2}$

$= \frac{2}{u}$, aturan turunan u^n, diinvers

$\frac{2}{u} = 2u^{-1}$

$\frac{d}{du}(2u^{-1}) = 2(-1)u^{-2}$

$= -\frac{2}{u^2}$

c) $\frac{1 - \sqrt{u}}{u}$ ubah bentuk pecahan ke eksponen:

$\frac{1}{\sqrt{u}} = u^{-\frac{1}{2}}$

$\frac{d}{du}(u^{-\frac{1}{2}}) = -\frac{1}{2} u^{-\frac{3}{2}}$

$= -\frac{1}{2\sqrt{u^3}}$

11) Dik: energi potensial sistem

$$P = \frac{1}{2}kx^2 - mgx$$

$$\frac{dP}{dx} = \frac{d}{dx}\left(\frac{1}{2}kx^2 - mgx\right)$$

Gunakan aturan turunan dasar

$$\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = kx, \quad \frac{d}{dx}(-mgx) = -mg$$

Sehingga;

$$\frac{dP}{dx} = kx - mg$$

Sistem setimbang saat $\frac{dP}{dx} = 0$,

maka: $kx - mg = 0$

$$kx = mg$$

$$x = \frac{mg}{k}$$

Practice 12g

1. $x \sin x$

$$\frac{d}{dx} uv = u'v + uv'$$

$u = x, \quad u' = 1.$

$v = \sin x, \quad v' = \cos x.$

Maka,

$$\frac{d}{dx}(x \sin x) = (1 \cdot \sin x) + (x \cdot \cos x)$$

$$= \sin x + x \cos x$$

3. $x^2 \ln x$

$u = x^2 \qquad u' = 2x$

$v = \ln x \qquad v' = \frac{1}{x}$

$$\frac{d}{dx}(x^2 \ln x) = (2x \ln x) + \left(x^2 \cdot \frac{1}{x}\right)$$

$$= 2x \ln x + x$$

Practice 13i

1. $(2x-1)^6$

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Jika $u = 2x-1$, maka u^6.

$$\frac{d}{du}(u^6) = 6u^5 = 6(2x-1)^5$$

$$\frac{d}{dx}(2x-1)^6 = 6(2x-1)^5 \cdot 2 = 12(2x-1)^5$$

5. $\frac{1}{(x^3-2x+1)^{-5}}$

Jika $(x^3-2x+1)^{-5}$,

$u = x^3-2x+1 \Rightarrow u^{-5}$.

$$\frac{d}{du}(u^{-5}) = -5u^{-6} = -\frac{5}{(x^3-2x+1)^6}$$

$$\frac{du}{dx} = 3x^2-2.$$

$$\frac{d}{dx}(x^3-2x+1)^{-5} = -\frac{5}{(x^3-2x+1)^6}(3x^2-2)$$

$$= -\frac{5(3x^2-2)}{(x^3-2x+1)^6}$$

Practice 130

1. $\frac{\sin x}{x}$

$u = \sin x \qquad u' = \cos x$

$v = x \qquad v' = 1$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2}$$

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{(\cos x \cdot x) - (\sin x \cdot 1)}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

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11 $x = 6,54 \cos(0,3t - 0,15) + 3,2$

$v = \frac{dx}{dt} = -54 \cdot 0,3 \sin(0,3t - 0,15)$

$v = -0,162 \sin(0,3t - 0,15)$

Jika $t = 0$

$v = -0,162 \sin(0,3 \cdot 0 - 0,15)$

$v = -0,162 \sin(-0,15)$

$v = -0,162 (-0,1494)$

$v = 0,0242$ m/s //

Jika $t = 2$

$v = -0,162 \sin(0,3 \cdot 2 - 0,15)$

$v = -0,162 \sin(0,6 - 0,15)$

$v = -0,162 \sin(0,45$

$v = -0,162 (0,4348))$

$v = -0,0709$ m/s //

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evaluasi $t = 0,1$

$\left.\frac{di}{dt}\right|_{0,1} = -15\sin(3 \cdot 0,1) + 9,5 \cdot 0,1 \cos(3 \cdot 0,1)$

$= 15 \sin 0,3 + 9,5 \cos 0,3$

Exercise 13.1

3. $F(\theta) = 2\sin(3\theta - 2)$

$\frac{d}{d\theta}[\sin(v)] = \cos v \cdot \frac{dv}{d\theta}$

$\frac{dv}{d\theta} = \frac{d}{d\theta}(3\theta - 2) = 3$

$\frac{d}{d\theta}[2\sin v] = 2 \cdot \cos v \cdot \frac{dv}{d\theta}$

$\frac{d}{d\theta}[2\sin(3\theta - 2)] = 2\cos(3\theta - 2) \cdot 3$

$= 6\cos(3\theta - 2)$

Jadi turunan dari $2\sin(3\theta - 2)$ terhadap θ adalah $6\cos(3\theta - 2)$.

7. $F(t) = 2\cot(5t^2 + 3)$

$\frac{d}{dt}[\cot v] = -\operatorname{cosec}^2 v \cdot \frac{dv}{dt}$

$v = 5t^2 + 3$

$\frac{dv}{dt} = \frac{d}{dt}(5t^2 + 3) = 10t$

$\frac{d}{dt}[2\cot v] = 2 \cdot (-\operatorname{cosec}^2 v) \cdot \frac{dv}{dt}$

$\frac{d}{dt}[2\cot(5t^2 + 3)] = 2(-\operatorname{cosec}^2(5t^2 + 3)) \cdot 10t$

$\frac{d}{dt}[2\cot(5t^2 + 3)] = -20t \operatorname{cosec}^2(5t^2 + 3)$

Jadi turunan $2\cot(5t^2 + 3)$ terhadap t: $-20t \operatorname{cosec}^2(5t^2 + 3)$

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Exercise 12.9

5. $\sqrt{x^3}\ln 3x = x^{3/2}\ln 3x$

$\frac{d}{dx}[u.v] = u'.v + u.v'$

$u = x^{3/2}$ $\quad v = \ln 3x$

$u' = \frac{3}{2}x^{1/2}$ $\quad v' = \frac{1}{x}$

maka $F'(x) = \left(\frac{3}{2}x^{1/2}\right).\ln 3x + x^{3/2}.\left(\frac{1}{x}\right)$

$= \frac{1}{2}\sqrt{x}.\ln(3x) + \sqrt{x}$

$= \sqrt{x}\left(\frac{3}{2}\ln 3x + 1\right)$

7. $R(\theta) = e^{a\theta}.\ln 3\theta$

$\frac{d}{dx}u.v = u.v' + u'.v$

$u = e^{a\theta}$ $\quad v = \ln 3\theta$

$u' = ae^{a\theta}$ $\quad v' = \frac{1}{\theta}$

maka $F'(\theta) = ae^{a\theta}.\ln(3\theta + e^{a\theta}.\frac{1}{\theta}$

$= e^{a\theta}\left(a\ln 3\theta + \frac{1}{\theta}\right)$

9. $T = 15t.\sin 3t$

$\frac{dT}{dt} = \frac{d}{dt}u.v = u'.v + u.v'$

$u = 15t$ $\quad v = \sin 3t$

$u' = 15$ $\quad v' = 3\cos 3t$

maka $\frac{dT}{dt} = 15.\sin 3t + 15t.3\cos 3t$

$= 15\sin 3t + 45t\cos 3t$

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Kelas B

Geomatematika

Date.

Practice 130 (5, 7, 9)

5.) $\frac{3\sqrt{(-)^3}}{2 \sin 2(-)} = \frac{3(-)^{3/2}}{2 \sin 2(-)}$

$u = 3(-)^{3/2}$

$v = 2 \sin 2(-)$

- $u' = \frac{d}{d(-)}(3(-)^{3/2})$
  $= 3 \cdot 3/2 (-)^{3/2-1}$
  $= \frac{9}{2}(-)^{1/2}$
  $= \frac{9}{2}\sqrt{(-)}$

- $v' = \frac{d}{d(-)}(2 \sin 2(-))$
  $= 2 \cdot \cos(2(-)) \cdot 2$
  $= 4 \cos 2(-)$

- $f'(-) = \frac{u'v - uv'}{v^2} = \frac{(9/2\sqrt{(-)})(2 \sin 2(-)) - (3(-)^{3/2})(4 \cos 2(-))}{(2 \sin 2(-))^2}$

$= \frac{9\sqrt{(-)} \sin 2(-) - 12(-)^{3/2} \cos 2(-)}{4 \sin^2 2(-)}$

$= \frac{3\sqrt{(-)}(3 \sin 2(-)) - 4(-) \cos 2(-)}{4 \sin^2 2(-)}$

7.) $f(x) = 2xe^{4x} \sin x$

$u(x) = 2x \longrightarrow u'(x) = 2$

$v(x) = e^{4x} \longrightarrow v'(x) = 4e^{4x}$

$w(x) = \sin x \longrightarrow w'(x) = \cos x$

- $f'(x) = u'(x)v(x)w(x) + u(x)v'(x)w(x) + u(x)v(x)w'(x)$

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$= (2)(e^{4x})(\sin x) + (2x)(4e^{4x})(\sin x) + (2x)(e^{4x})(\cos x)$

$f'(x) = 2e^{4x}\sin x + 8xe^{4x}\sin x + 2xe^{4x}\cos x$

$f'(x) = 2e^{4x}(\sin x + 4x\sin x + 2x\cos x)$

9. $y = \dfrac{2x^2+3}{\ln 2x}$ $\quad u = 2x^2+3$ $\quad x = 2,5$

$v = \ln 2x$

- $\dfrac{du}{dx} = \dfrac{d}{dx}(2x^2+3) = 4x$
- $\dfrac{dv}{dx} = \dfrac{d}{dx}(\ln 2x) = \dfrac{1}{2x}\cdot 2 = \dfrac{1}{x}$
- $\dfrac{dy}{dx} = \dfrac{\ln 2x \cdot 4x - (2x^2+3)\cdot 1/x}{(\ln 2x)^2}$

$= \dfrac{4x\ln 2x - \frac{2x^2+3}{x}}{(\ln 2x)^2}$

$= \dfrac{4x\ln 2x - (2x + 3/x)}{(\ln 2x)^2}$

$= \dfrac{4x\ln(2x) - 2x - 3/x}{\ln(2x)^2}$

- $x = 2,5 \rightarrow \dfrac{4(2,5)\ln(2(2,5)) - 2(2,5)}{[\ln(2(2,5))]^2}$

$= \dfrac{10\ln(5) - 5 - 1,2}{[\ln(5)]^2} = \dfrac{10\ln(5) - 6,2}{[\ln(5)]^2}$

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• $\ln(5) \approx 1.609 4 \rightarrow \frac{16(1.609 4) - 6.2}{1.609 4^2}$

$= \frac{16.094 - 6.2}{2.5901} = \frac{9.894}{2.5901} \approx 3.819 9$ atm. 3.82

Exercise 16.4 (1, 3, 5)

1.) $z = 2xy$

• $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(2xy) \rightarrow \frac{\partial z}{\partial x} = 2y \frac{\partial}{\partial x}(x)$

$= \frac{\partial z}{\partial x} = 2y \cdot 1 = 2y$

• $\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(2xy) \rightarrow \frac{\partial z}{\partial y} = 2y \frac{\partial}{\partial y}(y)$

$= \frac{\partial z}{\partial y} = 2x \cdot 1 = 2x$

3.) $z = \frac{x}{y}$

• $z = \frac{1}{y} \cdot x \rightarrow \frac{\partial z}{\partial x} = \frac{1}{y} \cdot \frac{\partial x}{\partial x} = \frac{1}{y} \cdot 1 = \frac{1}{y}$

• $z = x \cdot \frac{1}{y} = x \cdot y^{-1} \rightarrow \frac{\partial z}{\partial y} = x \cdot \frac{2}{\partial y}(y^{-1}) = x \cdot (-1 \cdot y^{-2})$

$= -x \cdot \frac{1}{y^2} = -\frac{x}{y^2}$

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5. $z = x^3 y^2 - \frac{y}{x^2} + \frac{1}{y}$

• Turunan terhadap x

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^3 y^2) - \frac{\partial}{\partial x}\left(\frac{y}{x^2}\right) + \frac{\partial}{\partial x}\left(\frac{1}{y}\right)$$

$= \frac{\partial}{\partial x}(x^3 y^2) = 3x^2 y^2$ $\quad = \frac{\partial}{\partial x}\left(\frac{y}{x^2}\right) = y\frac{\partial}{\partial x}(x^{-2})$

$= \frac{\partial}{\partial x}\left(\frac{1}{y}\right) = 0$ $\quad = y(-2x^{-3})$

maka $\frac{\partial z}{\partial x} = 3x^2 y^2 + \frac{2y}{x^3}$

• Turunan terhadap y

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(x^3 y^2) - \frac{\partial}{\partial y}\left(\frac{y}{x^2}\right) + \frac{\partial}{\partial y}\left(\frac{1}{y}\right)$$

$= \frac{\partial}{\partial y}(x^3 y^2) = x^3(2y) = 2x^3 y$

$= \frac{\partial}{\partial y}\left(\frac{y}{x^2}\right) = \frac{1}{x^2}\frac{\partial}{\partial y}(y) = \frac{1}{x^2}$

$= \frac{\partial}{\partial y}\left(\frac{1}{y}\right) = \frac{\partial}{\partial y}(y^{-1}) = -y^{-2} = -\frac{1}{y^2}$

maka $\frac{\partial z}{\partial y} = 2x^3 y - \frac{1}{x^2} - \frac{1}{y^2}$

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2- menentukan titik stasioner dan jenisnya:

(a) $f(x,y) = x^2 + y^2 - 2x + 4y + 8$

* $\frac{\partial f}{\partial x} = 2x - 2, \quad \frac{\partial f}{\partial y} = 2y + 4$

: $2x - 2 = 0 \Rightarrow x = 1, \quad 2y + 4 = 0 \Rightarrow y = -2$

: $(1, -2)$

$\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = 2, \quad \frac{\partial^2 f}{\partial x \partial y} = 0$

Diskriminan:

$D = (2)(2) - (0)^2 = 4 > 0$

Karena $D > 0$ dan $\frac{\partial^2 f}{\partial x^2} > 0$, maka $(1, -2)$ adalah minimum lokal.

(b) $f(x,y) = x^2 - y^2 - 2x + 4y + 8$

$\frac{\partial f}{\partial x} = 2x - 2, \quad \frac{\partial f}{\partial y} = -2y + 4$

: $2x - 2 = 0 \Rightarrow x = 1, \quad -2y + 4 = 0 \Rightarrow y = 2$

: $(1, 2)$

$\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = -2, \quad \frac{\partial^2 f}{\partial x \partial y} = 0$

Diskriminan: $D = (2)(-2) - (0)^2 = -4 < 0$

Karena $D < 0$, maka $(1, 2)$ adalah titik pelana (saddle point).

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(c) $f(x,y) = 2x + 2y - 2xy - x^2 - y^2 + 4$

$\frac{\partial f}{\partial x} = 2 - 2y - 2x$, $\frac{\partial f}{\partial y} = 2 - 2x - 2y$

$= 2 - 2y - 2x = 0 \Rightarrow x + y = 1$

$= 2 - 2x - 2y = 0 \Rightarrow x + y = 1$

$x + y = 1$

3. menentuuan nilai stusioner

$f(x,y) = x^3 - 6x^2 - 8y^2$

$\frac{\partial f}{\partial x} = 3x^2 - 12x$, $\frac{\partial f}{\partial y} = -16y$

$3x^2 - 12x = 0 \Rightarrow 3x(x-4) = 0 \Rightarrow x = 0$ atau $x = 4$

$-16y = 0 \Rightarrow y = 0$

$= (0,0)$ $(4,0)$

$\frac{\partial^2 f}{\partial x^2} = 6x - 12$, $\frac{\partial^2 f}{\partial y^2} = -16$, $\frac{\partial^2 f}{\partial x \partial y} = 0$

$(0,0) = \frac{\partial^2 f}{\partial x^2} = 6(0) - 12 = -12$

$D = (-12)(-16) - (0)^2 = 192 > 0$: maksimum lokal

$(4,0) = \frac{\partial^2 f}{\partial x^2} = 6(4) - 12 = 12$

$D = (12)(-16) - (0)^2 = -192 < 0$: titik pelana (saddle point)