Option Pricing

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1 Monte Carlo Simulation

The initial stock price is chosen to be S0=153.6900 and the daily variance is calculated to be 0.01268. The sigma of the simulation is set to be annualized variance by multiply square root of 252 and daily variance.
The European option has strike price equal to the initial stock price S0 under time to maturity equal to 1 year and no dividend.
The accuracy of approximation increases as n increases.

2 Binomial Tree

The European option has strike price equal to the initial stock price S0 under time to maturity equal to 1 year and no dividend.
As we increase the value of m the accuracy of the approximation also increases, and according to the result, American call should worth exactly the same as European call given there is no dividend, thus should not be exercise early. American put usually worth more than the European put and can be optimal to exercise early.
*Under m=100, the factorial of 500 is out of the limitation of R thus the number was recognized as NaN.

3 Multi-asset Option

A basket option is an option that provides a payoff dependent on the value of a portfolio of assets. Specifically, the value of a basket call option at its maturity T is

$$\max(\sum_{i=1}^{N}\omega_i S_i(T)-K,0) \tag{1}$$

where L is the number of assets in the basket and ω_i is the weight of the ith asset in the basket.

Suppose $\omega_i = 1/L$, then we can define a corresponding geometric basket option with

$$\max((\prod_{i=1}^{L} S_i(T))^{1/L}-K,0) \tag{2}$$

Let $G(t) = (\prod_{i=1}^{L} S_i(T))^{1/L}$, then we have:

$$\exp^{-rT}\hat{E}(G(T)-K)^{+} = \exp^{-rT}\hat{E}(G(T))\phi(d_1) - K\exp^{-rT}\phi(d_2)$$

where:

$$E(\prod_{i=1}^{L} S_i(T))^{1/L} = G(t)\exp((r-\frac{1}{2L}\sum_{i=1}^{L}\sigma_i^2)(T-t)+\frac{1}{2}\sigma^2(T-t))$$

$$\sigma^2 = \frac{1}{L^2}\sum_{i,j=1}^{L}\rho_{i,j}\sigma_i\sigma_j$$

$$d_1 = \frac{\log\frac{G(t)}{K} + (r+\sigma^2-\frac{1}{2L}\sum_{i=1}^{L}\sigma_i^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Consider the case where $L = 2$, $\omega_1 = \omega_2 = 1/2$, $K = S_0$, $\sigma_1 = \sigma_2 = 0.3$,$\rho = 0.5$,$T = 1$ and $r = 5\%$, I am going to compute the estimates of the value of a basket call on two assets given by the binomial and Monte Carlo(with and without antithetic variate) for various lattice steps(M) and number of simulations(N).